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ACCELERATOR BEAM DOSIMETRY*

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Introduction

The phrase radiation dosimetry implies, from an etymological point of view, the measurement of radiation dose. In the broader sense of the phrase, radiation dosimetry also concerns itself with the estimation of energy deposited in an irradiated medium by the interaction of ionizing radiation with matter. Both theoretical and applied radiation dosimetry have been treated adequately in the texts by Fitzgerald, Brownell, and Mahoney[1] and by Attix, Roesch, and Tochilin.[2-5] In the present lecture we will concern ourselves with understanding what it is that we are measuring when we place an ionization chamber, for example, in a beam of high energy charged particles. In particular, we will take the point of view that a comprehensive knowledge of the physical interactions leading to excitation, ionization, and the production of delta-rays greatly aids in this understanding. A large part of what will be presented here is detailed in Chapter 3 of the text by Kase and Nelson.[6]

Secondary beams of charged particles are very common around high energy accelerators and each beam is generally well understood. An example might be a 10 GeV/c (± 4%) beam of pions with a flux density of 10 particles/cm^2-sec and with a beam spot size of about 1 cm^2 that falls off radially in a Gaussian fashion. The contamination of the pion beam by electrons, muons, etc., might also be known. With all this information it is rather straightforward to evaluate the dose rate in the beam. Dose rate measurements using ion chambers, for example, might yield results in disagreement with these calculations, and many times it is due to the fact that the energy escaping the detector is not correctly accounted for.

In this lecture, we will only consider the problem of charged particles losing energy by collision (i.e., excitation and ionization). We will also refer to the text by Kase and Nelson[6] for the physics and related formulae concerned with the collision process itself.

Collision with Free Electrons

The differential collision probability $\phi_{col}(T,T')dT'dx$ is defined as the probability for a charged particle of kinetic energy T, traversing a thickness dx (g-cm^{-2}), to transfer an energy dT' about T' to an atomic electron (assumed free). For high energy charged particles ($T \gg m$), several formulas have been derived for these hard collisions,[6] depending on the type of incident particle.[7] For an electron

$$\phi_{col}(T,T')dT' = 2Cm\frac{T^2 dT'}{(T-T')^2(T')^2}\left[1-\frac{T'}{T}+\left(\frac{T'}{T}\right)^2\right]^2 \qquad (1)$$

= probability that either electron is in dT' about T' (the Møller cross section),

where

$$C = \pi N_o (Z/A) r_o^2 = 0.150\ Z/A \qquad (\text{cm}^2\text{-g}^{-1}),$$

Z = atomic number,

A = atomic weight.

The Unrestricted Mass Stopping Power

The average energy loss per unit path length (also known as the average stopping power) from ionization (and excitation) is determined from

$$\frac{1}{\rho}\frac{dT}{dx} = \int_{T'_{min}}^{T'_{max}} T'\phi_{col}(T,T')dT'$$

$$= \int_{T'_{min}}^{H} T'\phi_{col}^{S}\,dT' + \int_{H}^{T'_{max}} T'\phi_{col}^{H}\,dT' \ (\text{MeV-cm}^2\text{-g}^{-1})$$

$$= \left.\frac{1}{\rho}\frac{dT}{dx}\right)_S + \left.\frac{1}{\rho}\frac{dT}{dx}\right)_H \qquad (2)$$

where we have broken up the integration into a "soft" (distant) contribution and a "hard" (close) contribution,[6] and H is an arbitrarily chosen energy. We can calculate the hard-collision term by using Eq. (1). The soft-collision term is more difficult due to the binding effects of the electrons to the atoms. As derived by Bethe[8,9]

$$\left.\frac{1}{\rho}\frac{dT}{dx}\right)_S = \frac{2Cm}{\beta^2}\left\{\ln\left[\frac{2m\beta^2 H}{I^2(1-\beta^2)}\right] - \beta^2\right\} . \qquad (3)$$

If we now combine the soft and hard contributions, we get

$$\frac{1}{\rho}\frac{dT}{dx} = \frac{2Cm}{\beta^2}\left\{\ln\left[\frac{\tau^2(\tau+2)}{2(I/m)^2}\right] + 2\ln 2 - \beta^2 + [\tau^2/8 - (2\tau+1)\ln 2]/(\tau+1)^2 - \delta\right\}$$

$$= L_\infty \qquad (4)$$

where $\tau = T/m$, and where we have also included a density effect[10] correction term, δ. This is the equation used in the tables of Berger and Seltzer.[11] The unrestricted stopping power is numerically equivalent to LET_∞ (or L_∞).

The Restricted Mass Stopping Power

The difference between the restricted and the unrestricted stopping power involves the upper limit of integration in the previous derivation of the hard-collision contribution. Mathematically, the restricted stopping power is defined by

$$\left.\frac{1}{\rho}\frac{dT}{dx}\right)_\Delta = \left.\frac{1}{\rho}\frac{dT}{dx}\right)_S + \int_H^{T'_c} T'\phi_{col}\,dT' \ (\text{MeV-cm}^2\text{-g}^{-1}) \qquad (5)$$

where T'_c is the kinetic energy of the delta-ray that just escapes the region of interest (such as the sensitive volume of an ion chamber). Using the nomenclature of Berger and Seltzer,[11] the restricted stopping power for electrons is

$$L^-(\tau,\Delta) = \frac{2Cm}{\beta^2}\left\{\ln\left[\frac{2(\tau+2)}{(I/m)^2}\right] + F^-(\tau,\Delta) - \delta\right\} \qquad (6)$$

$$(\text{MeV-cm}^2\text{-g}^{-1})$$

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where

$$F^-(\tau,\Delta) = -1-\beta^2+\ln\left[(\tau-\Delta)\Delta\right]+\tau/(\tau-\Delta)$$
$$+\left[\Delta^2/2+(2\tau+1)\ln(1-\Delta/\tau)\right]/(\tau+2)^2 \qquad (7)$$

and where $\tau = T/m$ and $\Delta = T'_c/m$. The restricted stopping power is numerically equivalent to LET_Δ, although the latter is usually expressed in units of energy per linear path length.

Application

A recent paper by Kase and Domen[12] illustrates very nicely the use of the restricted stopping power and the related equations. In this experiment, a portable carbon calorimeter built at the National Bureau of Standards was used in a 19.5-GeV electron beam at SLAC to measure absorbed dose. The dose measurements were normalized to a given number of incident electrons by monitoring the beam with a transmission ion chamber as indicated in Fig. 1. The ion

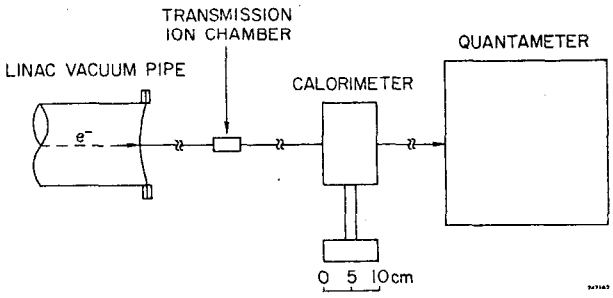

Fig. 1. Setup for Kase and Domen experiment.[12]

chamber was calibrated against a quantameter during which time the calorimeter was removed from the beam. The calorimeter measurements gave an energy deposition per incident electron of

$$\epsilon_{meas} = 0.815\ (\pm 8\%)\ \text{MeV/electron}$$

for a carbon thickness of 452 mg-cm^{-2}. Using the unrestricted stopping power formula, we get (for carbon)

$$L_\infty = 2.36\ \text{MeV-cm}^2\text{-g}^{-1},$$

and correspondingly, for a thickness of 452 g-cm^{-2},

$$\epsilon_\infty = 1.07\ \text{MeV/electron},$$

which is 24% higher than the measured value.

A better estimate can be made by using the restricted stopping power, $L^-(\tau,\Delta)$, which represents the energy per unit path length transferred from a primary electron of energy T to secondary electrons with energies less than $T'_c(=m\Delta)$. It is assumed that any secondary that receives an energy greater than T'_c escapes the core and deposits no energy, and that all secondaries with energies less than or equal to T'_c deposit all of their energy in the core. An illustration of the distance travelled, $t(T',x)$, by a delta-ray of energy T' generated at position x in the core of thickness X, is given in Fig. 2. If we equate $t(T',x)$ with the range, $R(T'_c(x))$, of an electron with energy T'_c, we can write

$$\cos\theta = \frac{X-x}{R\left(T'_c(x)\right)} \quad . \qquad (8)$$

Furthermore, it is easy to show from kinematics that

$$T' \approx \frac{2m\cos^2\theta}{1-\cos^2\theta} \quad . \qquad (9)$$

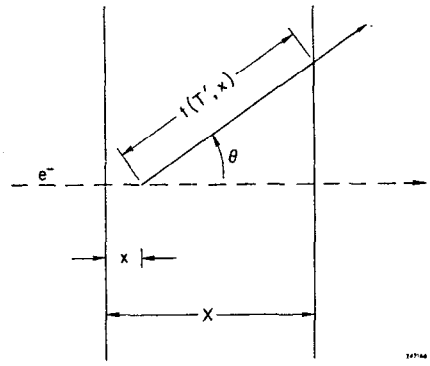

Fig. 2. Illustration of the distance travelled by a secondary electron generated in the core of thickness X.

By setting $T' = T'_c$, we can obtain the transcendental equation

$$T'_c(x) = \frac{2m(X-x)^2}{R^2\left[T'_c(x)\right]-(X-x)^2} \qquad (10)$$

which can be used to calculate T'_c, as a function of x, with the help of range-energy tables.[11] To obtain the energy deposited in the core by secondaries with energies T'_c or less, one simply integrates $L^-(\tau,\Delta)$ over x

$$\epsilon_\Delta = \int_0^X L^-(\tau,\Delta)\,dx \qquad (11)$$

Using X = 452 mg-cm^{-2}, the integral was evaluated numerically with the result

$$\epsilon_\Delta = 0.718\ \text{MeV/electron},$$

which is 12% lower than the measurement.

The next step in the calculation is to correct for the fact that we have ignored those secondaries having energies greater than T'_c, even though they do lose some energy in the core. This energy can be estimated from the equation

$$\epsilon_e = \int_0^X \int_{T'_c(x)}^{T'_{max}} \phi_{col}(T,T')S(T')t(T',x)\,dt'\,dx \qquad (12)$$

where

$\phi_{col}(T,T')dT'$ = Møller cross section (Eq. (1)) for electron-electron collisions,

$S(T')$ = unrestricted mass stopping power for electrons having energy T',

and

$$T'_{max} = T/2 = 9.75\ \text{GeV}.$$

This integral was also evaluated numerically with the result

$$\epsilon_e = 0.021\ \text{MeV/electron},$$

which is a 3% addition to the estimation, in the direction of the measured value.

A third calculation is made by Kase and Domen[12] in order to account for the energy deposited in the core by secondaries that are produced in the 635 mg-cm^{-2} thick

carbon jacket-assembly that is positioned immediately upstream. The details of the calculation are given in their paper and the method is very similar to that above. They obtain

$$\epsilon_w = 0.029 \text{ MeV/electron}$$

which is another 3-4% addition to the total. Energy deposition by secondaries generated in the air path upstream of the calorimeter, as well as the backscatter contribution, was found to be negligible in this particular situation. This may not always be true in general.

The total mean energy deposited in the core by a 19.5-GeV electron is therefore estimated to be

$$\epsilon = \epsilon_\Delta + \epsilon_e + \epsilon_w = 0.768 \text{ MeV/electron} ,$$

which is only 5.5% smaller than the measured value of 0.815 MeV/electron, and certainly within the maximum uncertainty in the measurements of 8%.

In the above calculational technique, the change in the stopping power, S, along the track of the secondaries has been ignored. Furthermore, it has been assumed that the secondaries travel in straight lines, and we know that multiple scattering will add to the total path length of each delta-ray. Both of these effects will lead to an underestimation of the energy deposition, ϵ , and the 5.5% discrepancy can in theory be lowered even further.

Summary and Concluding Remarks

By means of a recent investigation by Kase and Domen,[12] we have demonstrated that high energy charged particle beam dosimetry can be understood using rather simple models and basic concepts of physics. This does not mean that all beam dosimetry situations are this simple. We have not, for example, discussed recombination problems that might be associated with ion chambers. In a recent paper by Kase, Nelson, and Keller,[13] the Boag theory of recombination loss in a pulsed beam[2] is extended and used for electron beams whose dimensions are smaller than the ion chamber. Another paper by Dinter and Tesch[14] is also of interest for measurements that are made in pulsed fields of electromagnetic radiation.

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