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ON THE RADIATION OF SOFT GLUONS*

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ABSTRACT

We demonstrate that the cross section for the radiation of soft gluons in Quantum Chromodynamics does not obey a Poisson distribution (even when the colours of the gluons are summed over). This implies that any cancellation of infrared divergences between radiative corrections from real and virtual gluons in QCD must be more complicated than in QED. We present an example of such a cancellation.

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Recently there has been considerable effort to produce a theory of strong interactions in which the constituents of hadrons are permanently confined. The leading candidates for such a theory are the non-Abelian gauge theories [1]. These theories, because they are asymptotically free [2], are able to explain the phenomenon of Bjorken scaling, and it has been suggested that they may have severe infrared divergences which would somehow provide the dynamics of confinement. However, low order perturbation theory studies of the infrared divergences in these theories [3] have not provided any clues as to how confinement is to be possible.* Although it has recently been shown [5] that the cross section for the near forward scattering of coloured particles with the radiation of soft gluons is more singular in QCD than for the analogous process in QED, this is not the case when the initial state consists only of particles which are colour singlets, as is the case in experimental situations. The simplest such process is $\mathcal{J} \to \gamma \to q\bar{q}$ + gluons, where $\mathcal{J}$ is some external electromagnetic source, and it has been demonstrated [3] that the cross section for this process, to fourth order in the strong coupling constant (g), is finite, as required by the Kinoshita-Lee-Nauenberg theorem [6]. This may be readily seen by studying the photon vacuum polarization diagrams in terms of Feynman parameters, and using the standard techniques for evaluating the asymptotic behaviour of Feynman diagrams [7,8]. In this case the asymptotic variables are s/λ^2 and m^2/λ^2, where s is the square of the invariant mass of the virtual photon, m is the mass of the quarks, and λ is the fictitious mass of the gluons. Contributions to the cross section from particular final states will be infrared divergent, but these divergences cancel in the sum over final states. In QED this cancellation occurs in a simple way (not only in fourth order, but to all orders in α), but as we

*See however the projection to higher orders and interpretation of Cornwall and Tiktopoulos [4].

demonstrate below, the cancellation is more complicated in QCD.

We start by considering the process $\mathscr{J} \rightarrow \gamma \rightarrow e^+e^-$ + soft photons in QED. We regulate the infrared divergences by working in $4 + \epsilon$ dimensions. All the results below can be readily generalized to other processes. The cross section for the emission of n soft photons, each with energy less than ΔE, is given by

$$\sigma_n = \sigma_0 \, e^{2\alpha B} \frac{(2\alpha \widetilde{B})^n}{n!} \tag{1}$$

where (for small ϵ)

$$B = \frac{i}{(2\pi)^3} \int \frac{d^{4+\epsilon}k}{k^2} \left[\frac{(2p'-k)_\mu}{2p'\cdot k - k^2} - \frac{(2p-k)_\mu}{2p\cdot k - k^2} \right]^2$$

and

$$\widetilde{B} = -\frac{1}{(8\pi)^2} \int \frac{d^{3+\epsilon}k}{\omega} \left[\frac{p'_\mu}{p'\cdot k} - \frac{p_\mu}{p\cdot k} \right]^2 \tag{2}$$

and σ_0 is the elastic Born cross section. p and p' are the momenta of the electron and positron respectively. Both B and $\widetilde{B}$ behave like $1/\epsilon$ for small ϵ. Soft photons are thus radiated according to a Poisson distribution, i.e., independently. When the sum over n is performed, the cross section is $\sigma_0 e^{2\alpha(\widetilde{B}+B)}$, and all the infrared divergences have cancelled, since the coefficients of $1/\epsilon$ in B and $\widetilde{B}$ are equal in magnitude but opposite in sign [9,10].

It has been shown to sixth order in g that in Quantum Chromodynamics the elastic form factor, at high energy, exponentiates like $e^{2\alpha_s C_F B}$, where C_F is the eigenvalue of the quadratic Casimir operator for the quark representation, and $\alpha_s = \frac{g^2}{e^2}\alpha$ [4,11]. In these calculations only the most divergent terms are kept in each order of perturbation theory. It is therefore natural to ask whether the entire cancellation mechanism is identical to that in QED, αB and $\alpha\widetilde{B}$ being replaced now by $\alpha_s C_F B$ and $\alpha_s C_F \tilde{B}$. Indeed this has been assumed recently to be the case, with successful experimental predictions [12].

We now study the cross section for the process $\mathcal{J} \to \gamma \to q\bar{q}$ + soft gluons. In second order in the strong coupling constant, the diagrams which contribute are similar to those in QED, the only difference being the factor of C_F coming from the summation over gluons. Thus in this order the cancellation of infrared divergences proceeds exactly as in QED. In fourth order we distinguish 3 types of contributions, those with no gluons in the final state (type 0), those with one gluon in the final state (type 1), and finally those with two gluons in the final state (type 2). In order to obtain a Poisson distribution for the emission of soft gluons the sum of the type 2 contributions must be equal to $(2\alpha_s C_F \tilde{B})^2/2$, which for small ϵ behaves as $1/\epsilon^2$. We now demonstrate the existence of $1/\epsilon^3$ singularities for type 2 contributions, which do not cancel among themselves, thereby proving that a Poisson distribution is not obeyed. We start by calculating the type 2 contribution of Fig. 2(a). This contribution to the cross section is

$$\sigma_a = \frac{g^4\sigma_0}{(2\pi)^6} \frac{C_A C_F}{2} \int \frac{d^{3+\epsilon}k_1}{2\omega_1} \int \frac{d^{3+\epsilon}k_2}{2\omega_2} \frac{[p_1\cdot p_2\, p_1\cdot(k_2-k_1)+m^2\, p_2\cdot(k_1-k_2)]}{(k_1+k_2)^2[p_1\cdot(k_1+k_2)][p_2\cdot k_1][p_1\cdot k_2]} \tag{3}$$

where C_A is gluon Casimir operator. These integrations can be performed and one finds

$$\sigma_a = \frac{g^4\sigma_0}{64\pi^4\epsilon^3} \frac{C_F C_A}{2} \left(\frac{p_1\cdot p_2}{2Ep} \ln\frac{E+p}{E-p} - 1 \right) . \tag{4}$$

We work in the Breit frame and E and p are defined by $p_1 = (E,0,0,p)$, $p_2 = (E,0,0,-p)$. Thus σ_a is more divergent than any of the type 2 contributions in QED, which are never more divergent than $1/\epsilon^2$. The third factor of $1/\epsilon$

arises from the angular divergences from the region where $\underline{k}_1$ and $\underline{k}_2$ are parallel, since in that case if θ is the angle between $\underline{k}_1$ and $\underline{k}_2$, $(k_1 + k_2)^2 = 2\omega_1\omega_2(1-\cos\theta)$, and the integration in (3) is divergent as $\cos\theta \to 1$. The type 2 contribution of Fig. 2(b) is found to be $3\sigma_a$. These are found to be the only independent type 2 cuts which have $1/\epsilon^3$ divergences and they do not cancel among themselves. There are other type 2 contributions which have an angular divergence leading to a factor $1/\epsilon$, but in these cases the energy integrals are found to give at most one more factor of $1/\epsilon$. Thus we have demonstrated that the emission of soft gluons does not obey a Poisson distribution.

Since we know that the fourth order cross section must be finite, these $1/\epsilon^3$ divergences must be cancelled by diagrams with virtual gluons, and we have checked that this is indeed the case. The cancellation takes place between the type 2 cuts of Fig. 2 and the type 1 cuts of Fig. 3. The contribution from Fig. 3(a) is

$$\tilde{\sigma}_a = -\frac{g^4\sigma_0}{16\pi^5}\frac{C_A C_F}{2}\int\frac{d^{3+\epsilon}k_1}{2\omega_1}\int_0^1 d\alpha\, d\beta\, d\gamma \times$$

$$\times \frac{[1-2\beta][p_1\cdot p_2\, p_1\cdot k_1 - m^2 p_2\cdot k_1]\,\delta(1-\alpha-\beta-\gamma)}{(2p_1\cdot k_1)(2p_2\cdot k_1)[2\beta\gamma p_1\cdot k_1 - \gamma^2 m^2]^{1-\epsilon/2}} \tag{5}$$

and when the integrations are performed one finds $\tilde{\sigma}_a = -\sigma_a$. Similarly the type 1 contribution of Fig. 3(b) cancels that of Fig. 2(b).

We have demonstrated in this letter that although the cancellation of infrared divergences between real and virtual gluons does take place in the process $\mathcal{J} \to \gamma \to q\bar{q}$ + soft gluons, the mechanism of cancellation is more complicated than that in QED. Angular divergences, which arise due to the coupling of massless particles, contribute to these complications.

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References

[1] E. S. Abers and B. W. Lee, Phys. Rep. 9 (1973) 1.

[2] D. J. Gross and F. Wilczek, Phys. Rev. Lett. 26 (1973) 1343; H. D. Politzer, Phys. Rev. Lett. 26 (1973) 1346.

[3] T. Appelquist, J. Carrazone, H. Kluberg Stern, and M. Roth, Phys. Rev. Lett. 36 (1976) 768.

[4] J. M. Cornwall and G. Tiktopoulos, Phys. Rev. D 13 (1976) 3370.

[5] C. T. Sachrajda, Stanford Linear Accelerator Center Report No. SLAC-PUB-1743 (1976), to be published in Phys. Rev. D.

[6] T. Kinoshita, J. Math. Phys. 3 (1962) 650; T. D. Lee and M. Nauenberg, Phys. Rev. 133B (1964) 1549.

[7] R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, The Analytic S-Matrix (Cambridge University Press, New York, 1966).

[8] N. Nakanishi, Graph Theory and Feynman Integrals (Gordon and Breach, New York, 1971).

[9] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. 13 (1961) 397.

[10] D. R. Yennie, in Lectures on Strong and Electromagnetic Interactions, ed. K. Ford (Gordon and Breach, New York, 1964).

[11] J. J. Carazzone, E. R. Poggio, and H. R. Quinn, Phys. Rev. D 11 (1975) 2286; Phys. Rev. D 12 (1975) 3368(E).

[12] S. J. Brodsky and J. F. Gunion, Stanford Linear Accelerator Report No. SLAC-PUB-1749 (1976).

Figure Captions

1. The two 4th order diagrams whose imaginary parts have contributions from the unitarity sum $\sim 1/\epsilon^3$. (The dashed line represents a virtual photon.)
2. The two independent type 2 cuts which $\sim 1/\epsilon^3$.
3. The two independent type 1 cuts which $\sim 1/\epsilon^3$.

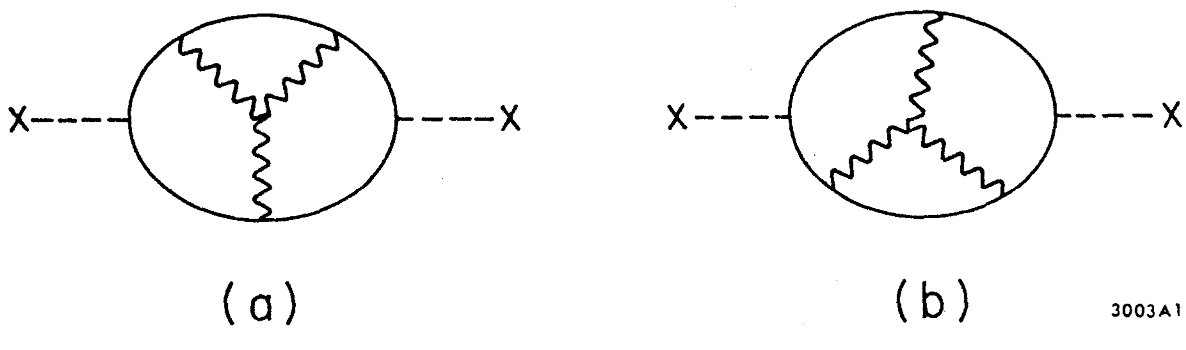
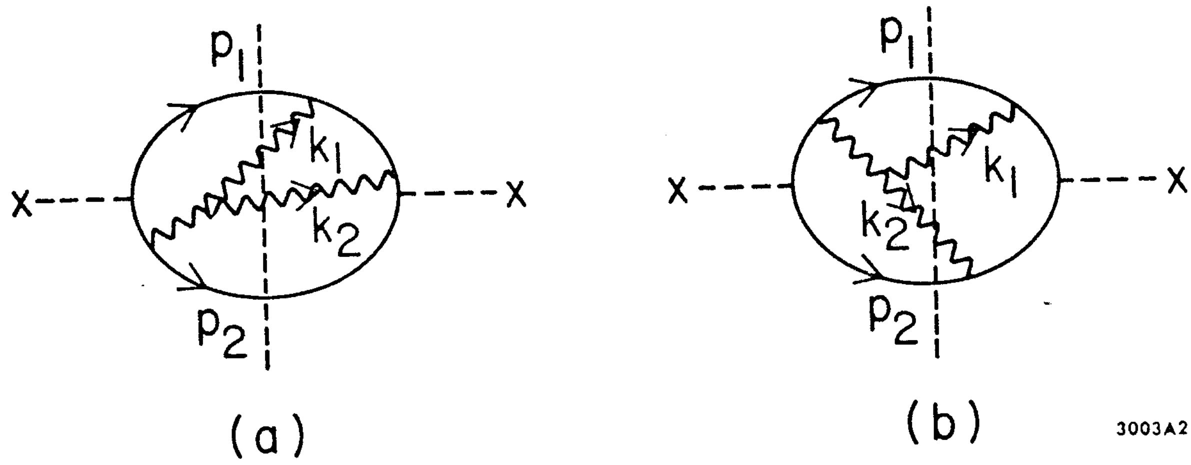
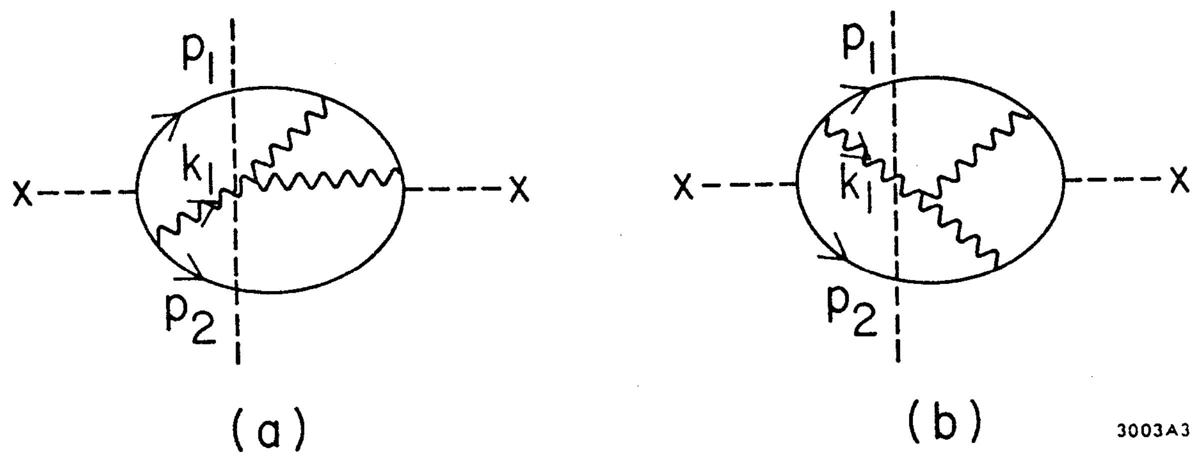


Fig. 1



p_1
k_1
k_2
x
x
p_2
(a)
p_1
k_1
k_2
x
x
p_2
(b)
3003A2

Fig. 2



p_1
k_1
x
x
p_2
(a)
p_1
k_1
x
x
p_2
(b)
3003A3

Fig. 3