SLAC-PUB-1794
August 1976
(T/E)

THE ψ SPECTROSCOPY OF A CHARM STRING*

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ABSTRACT

We report the results of the application of the quark-confining string model to the ψ spectrum. The model is defined by a relativistic invariant action of quarks and color SU(3) gauge fields. In this work, we consider only the charm quark, so that there are only two parameters, the charm quark mass m and the quark-gluon coupling e. In the Schrödinger limit, this string model reduces to the charmonium model(with a linear potential) in the absence of string vibrations. String vibrations provide additional levels. The first of these come in at about 4 GeV, and the density of states increases rapidly as a function of energy. Relativistic splittings of the low lying states are calculated to be of order 0.1 GeV. We predict two levels at around 4.4 GeV in the e^+e^- channel.

(Submitted to Phys. Rev. Letters.)

*Work supported by the Energy Research and Development Administration.

This letter outlines and summarizes the results of calculations of the ψ spectrum[1] in the quark-confining string (QCS) model recently proposed by one of us (SHT)[2]. This model attempts to synthesize two-dimensional quantum chromodynamics (QCD) and the string model. It is hoped that this synthesis will provide a model of hadron dynamics which maintains the desirable features of two-dimensional QCD and of the string, and in which meaningful calculations can be performed.

In two-dimensional QCD, quark confinement follows from the presence of the linearly rising Coulomb potential; there are no massless gluons; in the $\frac{1}{N}$ approximation[3] at least (for color SU(N)), the theory is asymptotically free, exhibits scaling, Okubo-Zweig-Iisuka rule and power-like form factors. The world, on the other hand, is four dimensional. Unfortunately, to demonstrate quark confinement in four-dimensional QCD and then calculate in a believable way its properties is a very formidable challenge.[4] QCS provides a different approach to the problem of four-dimensional chromodynamics.

The geometric formalism of the Nambu-Goto (N-G) string[5] describes the dynamics of a one space-one time dimensional world sheet of constant energy density embedded in higher (e.g., four) space-time dimensions. We use such a geometric formalism to lift two-dimensional QCD to four-dimensional Minkowski space. The resulting QCS describes two-dimensional QCD on such a world sheet embedded in Minkowski space. Relativistic invariance requires the quarks to be four-component Dirac fields.

Classical QCS is defined by the action:[2]

$$S = \int d^2u \sqrt{-g} \left\{ \bar{\psi} \rho^{\alpha} \left(\frac{i}{2} \overleftrightarrow{\partial}_{\alpha} - eB^a_{\alpha} T^a \right) \psi - \bar{\psi} m \psi - \frac{1}{4} F^a_{\alpha\beta} F^{a\alpha\beta} \right\} \tag{1}$$

where[6] coordinates u^0, u^1 parametrize the embedding $R_\mu(u)$, μ=0, 1, 2, 3, of the string in four dimensions. The local geometry of this embedding is described by the tangent vectors $\tau^\mu_\alpha = (\partial R^\mu/\partial u^\alpha)\left(\not{\tau}_\alpha \equiv \gamma_\mu \tau^\mu_\alpha\right)$, the induced metric $g_{\alpha\beta}=\tau_\alpha \cdot \tau_\beta$, $g=\det(g_{\alpha\beta})$, and its inverse $g^{\alpha\beta}=(g_{\alpha\beta})^{-1}$ with $\tau^\alpha_\mu = g^{\alpha\beta}\tau_{\beta\mu}$. The quark fields ψ are color triplets of four-component fermions. They come in different flavors. $\left\{B^a_\alpha(u):\ \alpha=0,1;\ a=1,2,\ldots 8\right\}$ are two-dimensional color SU(3) gauge fields. The parameters are the quark masses m_j (j is the flavor index) and the quark-gluon coupling constant e.

The action (1) is invariant under reparametrization, Lorentz and gauge transformations. The string coordinates are unbounded and the embedding is taken to be topologically equivalent to two-dimensional Minkowski space. If the embedding were in two-dimensional Minkowski space, the action (1) is identical to that of two-dimensional QCD in the coordinate system $R^0=u^0$, $R^1=u^1$ (where $-g=1$). Indeed, in the absence of string dynamics (i.e., no curvature), QCS is equivalent to two-dimensional QCD with four component quarks.

The motions of the string are determined by the energy-momentum distribution on its surface. In the case of the N-G string,[5] $S_{N-G} = \int d^2u\sqrt{-g}(-C)$, the constant energy density C is introduced as the fundamental parameter which characterizes the spectrum. It is related to the Regge slope, $\alpha' = 1/2\pi C$. In QCS, there is <u>no</u> independent string constant. The field energy-momentum density plays a role analogous to the string constant of the N-G string. In a classical picture with quarks and antiquarks represented by wave-packets along the string, physical color singlet solutions appear as in Fig. 1. In particular, a q-$\bar{q}$ pair generates a constant energy density between them due to the color electric flux.

A key difference between the standard QCD and QCS is that the latter has no independent gluonic degrees of freedom; this implies, in particular, that there are no glueballs. Hence, if QCS is to be considered as a phenomenological model derivable from QCD, the derivation must be highly nontrivial. For practical purposes, we take QCS as a working model where various properties can be calculated.

Electromagnetic and weak interactions can be introduced easily via minimal coupling to the quarks. Intuitively, many of the parton model properties are expected (e.g., the jet structure); in particular, the (neutral) string degrees of freedom carry a finite fraction of the momentum in the infinite momentum frame. The quark-gluon interaction leads to nontrivial quark-quark scatterings within a string which may characterize hadron-hadron scattering. Since the action (1) is reparametrization invariant, duality in scattering amplitudes might be expected in a consistent quantum theory. Regge trajectories are asymptotically straight.

In this letter we report the results[7] of a preliminary investigation of QCS, namely its nonrelativistic (i.e., Schrödinger) limit applied to the ψ spectrum recently discovered at SPEAR and elsewhere.[1] Since we shall limit ourselves to the study of the ψ spectrum only, we can neglect all flavors except charm.[8] We refer to the resulting string as the charm string. The nonrelativistic limit is that in which the quark mass is large compared to e. We consider the charm string in two steps.

First, we consider the charm string in the absence of string vibrations. In this restricted case, the nonrelativistic string is straight and its motions consist only of translations and rotations. The resulting Schrödinger equation for the ψ meson bound state wave function along the string, f(r) (where r is the distance between the charm quark-antiquark), is

$$\left\{-\frac{1}{m}\frac{\partial^2}{\partial r^2} + 2m + kr + \frac{\ell(\ell+1)}{mr^2}\right\} f(r) = Ef(r) \tag{2}$$

where the state has mass E. This is equivalent to the charmonium model[9] that has been studied extensively in relation to the ψ spectroscopy. $\ell = 0, 1, 2, \ldots$ is the orbital angular momentum and $k \equiv \frac{1}{2}\left(\frac{e}{2}\right)^2 \frac{N^2-1}{N} = \frac{e^2}{3}$. The simple Schrödinger equation (2) arises from the action (1) only after rather tedious algebra. The essential elements of the derivation are:[7]

(a) Assume that the string is straight in the rest frame of the meson and is therefore described only by its position and orientation.

(b) Transform the Dirac field ψ by a local boost, $\psi(u^\alpha) = S(u^\alpha)\chi(u^\alpha)$. Qualitatively, χ is the wave function of the quark in the local rest frame of the string.

(c) Perform a Foldy-Wouthuysen transformation on χ to separate the nonrelativistic quark and antiquark wave functions. Drop all relativistic correction terms and spin effects. Approximate the $q\bar{q}$ interaction by the instantaneous (linear) Coulomb interaction. We choose the gauge $A_1^a = 0$.

(d) Introduce the $q\bar{q}$ bound state wave function and quantize the string position and orientation. In the zero momentum frame, the orbital angular momentum is quantized with integer values ℓ.

(e) Demonstrate that the one-dimensional relative wave function f(r) is related to the usual three-dimensional relative wave function by: $\phi_{3D}(r, \theta, \varphi) = \frac{f(r)}{r} Y_\ell^m(\theta, \varphi)$.

Next we consider the vibrational modes. This is more difficult to tackle; instead of solving the nonlinear coupled string and Dirac equations, we content ourselves with a crude WKB estimate of the vibrational energy as a function of $q\bar{q}$ separation, r, and insert it as an effective potential into the Schrödinger equation

$$\left\{-\frac{1}{m}\frac{\partial^2}{\partial r^2} + 2m + V_n(r) + \frac{\ell(\ell+1)}{mr^2}\right\} f(r) = Ef(r) \tag{3a}$$

where n is the vibrational mode quantum number.

$$V_n(r) = kr/\sqrt{2-\alpha_n^2} \tag{3b}$$

$$\alpha_n^2 = 1 + \frac{2n\pi}{2n\pi + k\left[(r-2d)^2 + 4d^2\right]} \tag{3c}$$

d is the correction due to the finite quark mass and is given by ($1 \leq \alpha_n^2 \leq 2$)

$$d(r) = \frac{kr^2\alpha_n}{4} \; \frac{1}{2m + kr\,\alpha_n} \tag{3d}$$

For n=0, $\alpha_n^2=1$ and $V_0(r)=kr$ so that we get back the charmonium equation (2). We have two parameters m and k. They are fitted by the masses of $\psi(3.095)$ and $\psi'(3.684)$ so that m=1.154 GeV and k=0.21 GeV^2 (e=0.8 GeV). The levels of the charm string equation (3) are shown in Fig. 2. All states are further split by spin effects. The $n\neq 0$, $\ell \neq 0$ levels actually have couplings between the vibrational and the rotational modes, which have been neglected.

The vibrational levels (that are absent in charmonium) start coming in at around 4 GeV. Comparing the wave functions at the origin, we expect the vibrational states to have smaller leptonic widths. For higher energy states, the Schrödinger approximation breaks down. The density of states also increases rapidly as we go to higher energies. A simple estimate gives the asymptotic Regge slope $\alpha' \simeq \frac{1}{2\pi k} \simeq 0.8$ GeV^{-2}. In QCS, this is the universal Regge slope. (A better estimate of α' requires the inclusion of the relativistic corrections in fitting ψ and ψ' and the spreading of quark wave function instead of treating it as point-like in the string equation.)

To check the validity of the Schrödinger approximation, we have calculated some of the relativistic corrections and find they are small. In particular, the S-L (spin-orbit) splitting of the 1 P state is of the order

$$E(\ell=1,\ J=2) - E(\ell=1,\ J=0) \sim 0.14 \text{ GeV} \quad .$$

Comparing this with the binding energy ($E(\ell=1) - 2m \sim 1.1$ GeV), the nonrelativistic approximation is justified aposteriori. To test the validity of QCS, it is important to complete the leading order relativistic correction (e.g., spin-spin splitting) calculation for the charm string and compare with the data.

To summarize, we note that the charmonium model with a linearly rising potential can be obtained from a relativistic invariant field theoretic (albeit unconventional) model. Furthermore, relativity requires the introduction of string variables (via $g_{\alpha\beta}$) which give additional physical states even in the Schrödinger limit.

Triplet-singlet and spin-orbit splittings are expected to occur in the levels shown in Fig. 2 due to relativistic corrections, which are, in principle, determined by the action (1).

Even in the absence of the evaluation of such terms, we see that the spectrum of the charm string has some attractive features vis a vis the data. Namely, we expect resonance structures around and above 4 GeV due to vibrational excitations of the ψ and ψ' .

In this region, one also expects structures due to charm thresholds (and possibly S-D mixing). Whether the observed structure can be entirely accounted for in this way remains to be seen. We note one prediction at this stage: there are two states around 4.4 GeV in the e^+e^- channel.

Acknowledgments

We thank our colleagues, especially J. D. Bjorken, S. D. Drell, F. J. Gilman, A. Hanson, and H. C. Tze, for useful discussions.

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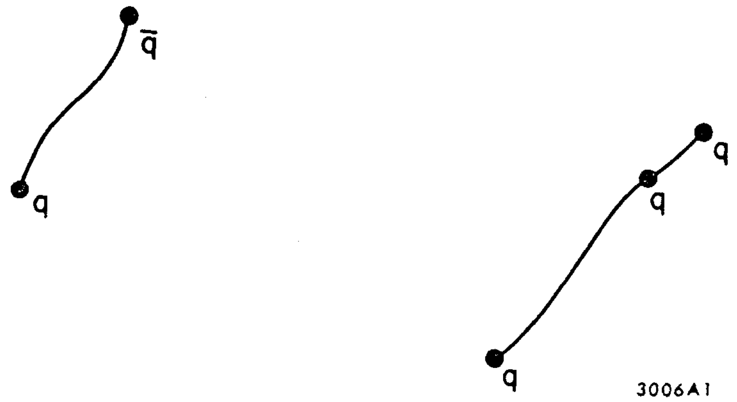
FIGURE CAPTIONS

1. The physical picture of the color singlet states, namely, mesons and baryons, in a classical picture with the dots representing quark wave packets.

2. Level solutions of Eq. (3), $\psi(3.1)$ and $\psi(3.7)$ are fitted to obtain m=1.154 GeV and k=0.21 GeV^2; spin-spin and spin-orbit interactions will split the levels. The dotted lines are the vibrational states. The solid lines are those also present in the charmonium model. States with energies $E > 4.5$ GeV or $\ell > 2$ are not shown.



q̄
q
q
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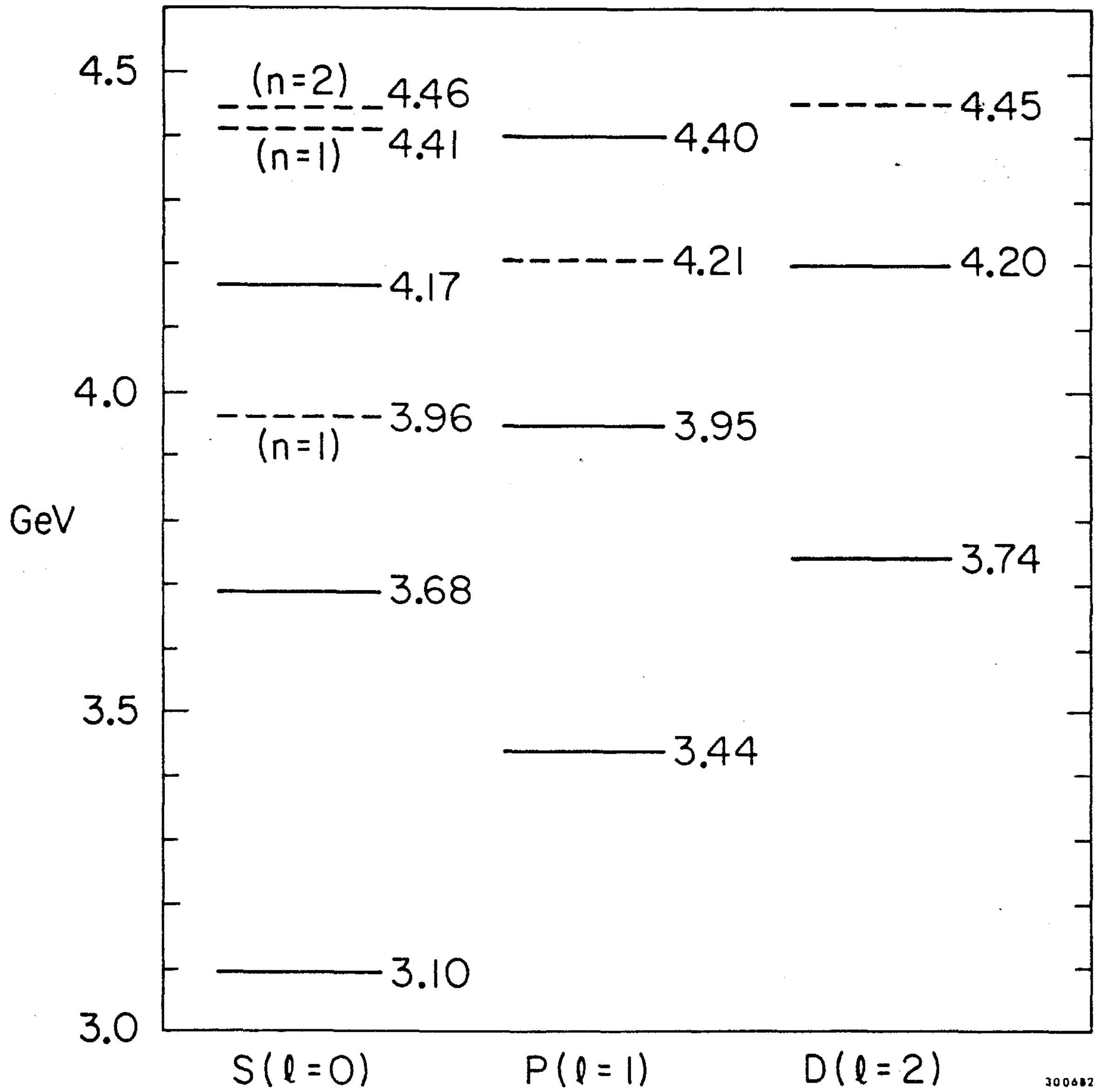
Fig. 1



4.5
4.0
3.5
3.0
GeV
(n=2)
4.46
4.41
(n=1)
4.17
3.96
(n=1)
3.68
3.10
4.40
4.21
3.95
3.44
4.45
4.20
3.74
S(ℓ=0)
P(ℓ=1)
D(ℓ=2)

Fig. 2