SLAC-PUB-1793
August 1976
(T/E)

MASS DIFFERENCES OF CHARMED HADRONS

Kenneth Lane*

Laboratory of Nuclear Studies
Cornell University, Ithaca, New York 14853

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Steven Weinberg†

Lyman Laboratory of Physics
Harvard University, Cambridge, Massachusetts 02138

(Revised Version.)

ABSTRACT

The mass difference $m(D^+) - m(D^0)$ of the charmed pseudoscalar meson doublet is estimated as about 6.7 MeV, as compared with the value 15 MeV given by De Rújula, Georgi, and Glashow. Mass differences are also estimated for charmed baryons.

(Submitted to Phys. Rev. Letters.)

---

*Work supported in part by the National Science Foundation and by the Energy Research and Development Administration under Contract No. E(04-3)-515.

†Work supported in part by the National Science Foundation under Grant No. MPS75-20427.

The new particle at 1865 MeV discovered[1] at SPEAR may well be the long-sought bound state of a charmed c and an anti-up-quark $\bar{u}$, known to theorists[2] as the D^0, or its antiparticle, the $\bar{D}^0$. If this is the case, then the other member $D^+ \equiv c\bar{d}$ or $D^- \equiv \bar{c}d$ of the same isodoublet should also soon be found, and we can look forward to a measurement of the $D^+ - D^0$ mass splitting. This splitting will provide an interesting test of our ideas about the origin of isospin nonconservation. De Rújula, Georgi, and Glashow[3] have estimated the mass difference between the D^+ and D^0 as about 15 MeV, and have pointed out important consequences of such a large isospin splitting for the production rates of various charmed particles. In this comment, we wish to suggest a slightly different method of calculating the $D^+ - D^0$ splitting, which leads to a rather different numerical result, about 6.7 MeV.

In any renormalization theory of strong interactions based on quarks and flavorless gauge bosons, the only sources of isospin breaking are the quark mass differences and ordinary one-photon exchange.[4] However, it is not in general obvious how to evaluate these two contributions in actual hadron states. De Rújula et al.[3] employ a nonrelativistic atomic model: the mass difference within any isospin multiplet consists of a mass term, equal to the difference in the masses of the constituent quarks, plus a Coulomb term, equal to the difference of the products of the quark charges times a constant[5] $\langle 1/r \rangle$. They determine the u-d quark mass difference and $\langle 1/r \rangle$ from the $\pi^+ - \pi^0$ and $K^+ - K^0$ mass differences, and then use the same parameters to calculate the $D^+ - D^0$ mass difference, obtaining 13 MeV. Taking into account the fact that $\langle 1/r \rangle$ is likely to be a little larger for D-mesons, their estimate is then increased to 15 MeV.

This nonrelativistic atomic model may be reasonable for the heavy pseudoscalar D-mesons, and possibly even for the K-mesons, but it seems to us unlikely that it could be applicable to the π-mesons. On the other hand, there is a way that we can use the $\pi^+ - \pi^0$ mass difference to separate the mass and Coulomb parts of the $K^+ - K^0$ splitting. Dashen's theorem[6] tells us that the photon part of the mass-squared splittings are the same for the K and π. Furthermore, the u-d quark mass difference does not contribute to the $\pi^+ - \pi^0$ mass splittings. It follows that the "photon exchange" part of the K-mass splitting is $[m^2(\pi^+) - m^2(\pi^0)]/2m(K)$, or 1.27 MeV. (De Rújula et al.[3] get 3 MeV.) If we interpret this as due to the Coulomb interaction in a nonrelativistic quark model, then $\langle 1/r \rangle$ would be 520 MeV, and the observed $K^+ - K^0$ mass difference would require a quark mass difference $m(u) - m(d)$ of -5.27 MeV. However, photon exchange contributes not only to the Coulomb force between quarks but also to the quark masses themselves. Using a phenomenological Lagrangian with zero quark masses to carry out calculations in the chiral $SU(3) \otimes SU(3)$ limit, we find that the diagrams which contribute to the squared-mass of the K's consist of a $K^0 \to d\bar{s} \to K^0$ or $K^+ \to u\bar{s} \to K^+$ quark loop, with the photon exchanged either (a) between opposite sides of the quark loop, or (b) on the same side of the quark loop, or (c) from one side of the quark loop to a separate charmed quark bubble. (Gluons and quark bubbles are inserted in diagrams of each type in all possible ways.) We will neglect diagrams of type (c) because the charmed quark bubble must be connected to the s and u or d quark lines by at least three gluon lines, and the gluon-quark coupling is relatively weak at energies of the order of the charmed quark mass. Denoting the sum of the other two classes of diagrams by A and B respectively, we easily see that $A(K^+) = -2A(K^0)$ and $B(K^+) = 5B(K^0)/2$. But $A(K^0) + B(K^0)$ must vanish,[6] so the "Coulomb" part $A(K^+) - A(K^0)$ of the $K^+ - K^0$ mass-squared difference is 2/3 of the total photon exchange contribution $A(K^+) + B(K^+) - A(K^0) - B(K^0)$, and $[m(K^+) - m(K^0)]_{\text{Coul.}} = 0.85$ MeV. Equating this to $\frac{1}{3}\alpha\langle\frac{1}{r}\rangle$ gives

$$\langle\frac{1}{r}\rangle \simeq 350 \text{ MeV} \tag{1}$$

The u-d quark mass difference can also be estimated from the observed K^+-K^o mass difference as

$$m(u) - m(d) \simeq \left[m(K^+) - m(K^o)\right]_{obs} - \left[m(K^+) - m(K^o)\right]_{Coul} \simeq -4.84 \text{ MeV} \tag{2}$$

We can now calculate the D^+-D^o mass splitting, using the formula

$$m(D^+) - m(D^o) = m(d) - m(u) + \frac{2}{3}\alpha\langle\frac{1}{r}\rangle \tag{3}$$

Taking the same values (1) and (2) for the parameters in Eq. (3), we find a mass splitting of 6.5 MeV.

It is interesting to compare the value (2) for $<1/r>$ with the value expected in a nonrelativistic potential model. Schnitzer[7] has estimated that for a linear potential $V(r) = ar$, the ground-state expectation value of $1/r$ is

$$\langle\frac{1}{r}\rangle \simeq \left[\frac{32\mu a}{3\pi^2}\right]^{1/3} \tag{4}$$

where μ is the reduced mass. For the force constant a, we will use the estimate of Kang and Schnitzer,[8] $a \simeq 0.3 \text{ GeV}^2$. If we adopt masses of 340 MeV and 540 MeV for the masses of the u and s quarks, the reduced mass for a K-meson is $\mu \simeq 210$ MeV, and Eq. (4) gives a value of 410 MeV for $<1/r>$. Our value (1) is 15% less. On the other hand, the use by De Rújula et al.[3] of the π^+-π^o mass difference to estimate $<1/r>$ gives a value $<1/r> \simeq 1230$ MeV, three times greater than the value given by Eq. (4).

For a charmed quark mass of 1500 MeV, the reduced mass for a D-meson is 30% larger than for a K-meson, so (4) suggests that $<1/r>$ should be 10% larger for D-mesons than for K-mesons. Taking this into account in the Coulomb term of Eq. (3) gives a D^+-D^o mass difference of 6.7 MeV. The D* mass splitting should be comparable.

The same simple nonrelativistic approach can be applied to the baryons. The Σ and Ξ mass differences can be moderately well fit with $m(d) - m(u) \simeq 4.5$ MeV and with a common value $\langle 1/r \rangle \simeq 240$ MeV for all quark pairs. Using these parameters for the charm-one baryons described in Ref. 2 gives the mass splittings very roughly as

$$m\left(C_1^{++}\right) - m\left(C_1^{+}\right) = m(u) - m(d) + \frac{4}{3}\alpha\left\langle\frac{1}{r}\right\rangle \approx -2 \text{ MeV}$$

$$m\left(C_1^{+}\right) - m\left(C_1^{o}\right) = m(S^+) - m(S^o) = m(A^+) - m(A^o) = m(u) - m(d) + \frac{1}{3}\alpha\left\langle\frac{1}{r}\right\rangle \approx -4 \text{ MeV}$$

These are only very rough estimates, because for baryons there is a partial cancellation between the Coulomb and quark mass contributions.

We are grateful for helpful conversations with A. De Rújula, H. Georgi, S. L. Glashow, and H. J. Schnitzer. One of us (K. L.) thanks the Theoretical Physics Group of the Stanford Linear Accelerator Center for their hospitality this summer.

Note added in proof: Since completion of this paper, I. Peruzzi et al. (SLAC-PUB-1776, LBL-5340) have reported strong evidence for D^+ at 1876 ± 15 MeV. As expected, this is heavier than D^o (1865), but the mass difference is too imprecisely known to decide the issue we have addressed.

REFERENCES

1. G. Goldhaber *et al.*, to be published in Phys. Rev. Letters.
2. M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).
3. A. De Rújula, H. Georgi, and S. L. Glashow, Harvard preprint (1976). Also see E. Eichten and K. Lane, to be published.
4. S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); Phys. Rev. D 6, 4482 (1973). Note that the quark mass difference is not an effect of electromagnetism alone; in unified gauge theories it receives contributions of order αm also from the weak interactions.
5. Relativistic corrections to hadron mass splittings in quark models are discussed by A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975); W. Celmaster, Harvard preprint (1976). Part of these corrections may be incorporated in a redefinition of the constant $\langle 1/r \rangle$.
6. R. Dashen, Phys. Rev. 183, 1245 (1969). Corrections to this theorem are discussed by P. Langacker and H. Pagels, Phys. Rev. D 8, 4620 (1973).
7. H. J. Schnitzer, Phys. Rev. D 13, 74 (1976). Schnitzer's analytic approximation to $\langle 1/r \rangle$ has been found to be accurate for a linear potential to within 2% in a computer calculation by J. Borenstein, Harvard University Ph.D. thesis, June 1975 (unpublished). In order to apply Schnitzer's result to bound states of unequal mass quarks, replace the quark mass in his Eqs. (3.6) and (3.20) with twice the reduced mass.
8. J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 461 (1975). A value $a \simeq 0.2$ GeV is given by E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975), while a value 0.211 GeV is given by B. J. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 168 (1975). Using 0.2 GeV for a instead of 0.3 GeV would lead to even better agreement with our estimate of $\langle 1/r \rangle$, and even worse agreement with the estimate of De Rújula *et al.*