SLAC-PUB-1788
August 1976
(T/E)

RELATIVISTIC FORM FACTORS FOR CLUSTERS WITH NON-RELATIVISTIC WAVE FUNCTIONS*

A. N. Mitra**

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Indra Kumari

Department of Physics
University of Delhi, Delhi-7, India

(Submitted to Phys. Rev.)

* Work supported in part by the Energy Research and Development Administration.

** University Grants Commission (India) National Fellow.
Permanent address: Department of Physics, University of Delhi, Delhi-7, India.

ABSTRACT

Using a simple variant of an argument employed by Licht and Pagnamenta (L-P) on the effect of Lorentz contraction on the elastic form factors of clusters with non-relativistic wave functions, it is shown how their result can be generalized to inelastic form factors so as to produce (i) a symmetrical appearance of Lorentz contraction effects in the initial and final states, and (ii) asymptotic behavior in accord with dimensional scaling theories. A comparison of this result with a closely analogous parametric form obtained by Brodsky and Chertok from a propagator chain model leads, with plausible arguments, to the conclusion of an effective mass M for the cluster, with M^2 varying as the number n of the quark constituents, instead of as n^2. A further generalization of the L-P formula obtained for an arbitrary duality diagram vertex, again with asymptotic behavior in conformity with dimensional scaling. The practical usefulness of this approach is emphasized as a complementary tool to those of high energy physics for phenomenological fits to data up to moderate values of q^2.

1. Introduction and Summary

Form factors represent a theoretically powerful and experimentally convenient index for judging the degree of compositeness of hadronic and multi-hadronic systems. The concept of an extended hadronic structure started getting serious attention after Hofstadter's classic experiments but most of the investigations on hadronic structure with field-theoretic techniques had been centered on the concept of nucleon and meson fields as the basic entities in a bootstrap spirit.[1] Form factors for few-nucleon systems had also been developed in the language of effective fields for their external kinematics and that of orthodox nuclear physics for their internal structures through appropriate overlap integrals of their (non-relativistic) wave functions.[2] It is only in more recent times that the quark constitution of hadrons and their composites has come under more serious attention for the determination of their form factors with considerable support from experiment.[3,4] Powerful techniques of field theory have recently been employed to determine the asymptotic behaviour of hadronic form factors in terms of quark constituents.[3]

While such results are of great value for providing general guidance on the functional behaviour to be expected from physical form factors, the actual ranges (even down to moderate energies) of experimental interest are such as to require enormous details for which (at the present state of development of field theory) a good deal of parametrization is probably unavoidable in order to obtain numerical fits to the data. For the sources of such parametrization one should probably look at both ends - the techniques of high energy physics to provide fits mainly at large q^2, and the traditional tools of nuclear physics for the complementary region.

One should also expect a substantial overlap region for moderate q^2, which should hopefully serve as a proving ground for consistency in description from these two complementary points of view, and thus as a valuable check on plausible ideas of a smooth transition from low to high energy physics. Attempting to interpolate from high q^2 to medium q^2 regions, Brodsky and Chertok[5] have just given a phenomenological analysis of hadronic and few-nucleonic form factors in terms of quark constituents, with a striking degree of success. This still leaves a substantial transition region that should presumably be more amenable to a complementary description, the philosophy of which represents the motivation for this paper.

For a completely satisfactory description of the data up to moderate q^2 one would probably require a full-fledged relativistic theory of multi-particle systems, <u>without</u> the simplifications that accrue from a high energy treatment. While there is evidence of serious relativistic treatments for the deuteron with nucleonic and mesonic constituents[6], such treatments do not appear to be feasible at the quark level, and their fare with experiment in the transition region of q^2 is as yet less than encouraging on the whole, despite some recent results.[7] For a quark level description in a feasible manner up to the transition region there is probably little viable alternative to a basically three-dimensional approach with N.R. wave functions. Nevertheless one must give a relativistic meaning, howsoever limited, to such a wave function before it can be used for the data. While the external kinematics of the system can still be treated in a relativistically invariant manner[2], it is on the <u>internal</u> wave function that one requires a more tractable scheme of relativization.

Of particular interest in this context are certain semi-intuitive ideas on Lorentz contraction of clusters in motion, suggested by some authors to fit the proton form factors[8,9]. Of these the L-P method is somewhat more general and yields a simple recipe for the matrix element for an arbitrary cluster undergoing transition in an external field. In particular for elastic scattering of a cluster of n particles the relativistiv form factor $F_R(q^2)$ is related to the corresponding N.R. form $F(\vec{q}^{\,2})$ by

$$F_R(q^2) = (1+\eta)^{\frac{1+\eta}{2}} F(q^2/(1+\eta)), \quad \eta = \tfrac{1}{4}q^2M^{-2}. \tag{1}$$

This formula was applied with a fair amount of success to fit the proton form factor up to moderately high q^2. However, before indulging in an indiscriminate application of the formula to high q^2 one should pause to check its asymptotic behaviour. Unfortunately, this goes like q^{1-n}, as against the 'expected' behaviour as q^{2-2n} on deeper theoretical grounds.[4] In view of the basically non-relativistic premises of the formula this should hardly cause any surprise and should merely help set limits on its applicability. On the other hand, it would be far more interesting for phenomenological applications on an extended scale if the considerations leading to the formula (1) could be appropriately modified so as to lead exactly to the asymptotic behaviour ($\sim q^{2-2n}$) expected from dimensional scaling. Indeed one would almost instinctively feel the practical utility of such a 'relativistic' form factor conceived within the three-dimensional framework of quantum mechanics, with all the built-in advantages of nuclear wisdom hopefully incorporated within a judiciously chosen wave function.

The purpose of this paper is to present a suitably modified version of the L-P argument designed to achieve the following results:

(i) Generalization of the L-P result for the elastic form factor to an arbitrary inelastic ($A \neq B$) vertex with a symmetrical appearance of the Lorentz contraction factors in more explicit correspondence with the individual states.

(ii) Exhibition of exact asymptotic behaviour ($\sim q^{2-2n}$) expected from dimensional scaling, as a consequence of (i).

(iii) Derivation of a corresponding formula for an arbitrary duality diagram ($A \rightarrow B + C$), again with asymptotic behaviour expected from dimensional scaling.

No attempt is made in this paper to give numerical results which will be reported separately. However, the algebraic formulae appear to be sufficiently simple and transparent to merit separate recording as a complementary tool to high energy techniques, more appropriate to medium q^2 regions. The original L-P arguments and their modifications to obtain results (i) and (ii) are summarized in Secs. 2 and 3 respectively. Result (iii) is given in Sec. 4. A comparison of result (i) with a similar parametric formula obtained by Brodsky and Chertok (referred to as BC) on the basis of a propagator chain model leads to a plausible conclusion of an <u>effective</u> mass $\underline{M}$ for the cluster, varying as $\sqrt{n}$ (where $\underline{n}$ is the number of quark constituents) instead of as $\underline{n}$. This is discussed in Sec. 5.

2. The Licht-Pagnamenta Argument

Let us first quickly recall the essential <u>assumptions</u> of L-P in making a transition from a non-relativistic to a relativistic matrix

element, as distinct from a strict 'derivation'. They argue that there is a preferred frame (the Breit frame) in which the interaction of the individual members of a cluster with an external potential (or radiation quantum) may be regarded as instantaneous to a good approximation. In the N.R. theory, one considers the emission or absorption of radiation by a cluster at rest, with its individual members in small relative motion with respect to the center of mass. According to the Licht-Pagnamenta argument, such a picture can with a good approximation be taken over to the Breit frame for the cluster motion as a whole, involving concomitant Lorentz-transformation effects on the cluster coordinates. Now since there is only one time-coordinate for the cluster, formal relativistic invariance can be maintained only up to its external kinematics (cluster motion as a whole), but the frame-dependence would nevertheless show up in the structure of the matrix element due to internal motion.

To see more clearly their proposed modification of the internal matrix element, we first write down its non-relativistic form for a transition $A \to B$, with suitably normalized internal coordinates $\vec{x}_i$ as ($q = 2p$ in the Breit frame):

$$F_{AB}(\vec{q}^{\,2}) = \int \prod_{i}^{n-1} d\vec{x}_i \; \psi_B^*(\vec{x}_i)\, e^{i\vec{q}\cdot\vec{x}_1}\, \psi_A(\vec{x}_i) \tag{2}$$

Since the states A and B are now in different Lorentz frames, their proposal consists in the effective replacements

$$\vec{x}_i \to \vec{x}_{Ai}' \text{ (in } \psi_A); \; \vec{x}_i \to \vec{x}_{Bi}' \text{ (in } \psi_B) \tag{3}$$

appropriate to the Lorentz frames the cluster finds itself in, before and after the interaction. Taking the common direction of motion as the z-direction, the Lorentz transformations (at $t = 0$) are expressed by

$$z_{Ai}' = z_i E_A/M_A; \; z_{Bi}' = z_i E_B/M_B \tag{4}$$

while the transverse coordinates (x_i, y_i) are unaffected. Note that <u>after</u> the replacements (3) the matrix element is now <u>physically</u> different from the original N.R. form (2), and it is precisely at this point that the L-P proposal amounts to a departure from the physical premises of N.R. theory. This leads to Eq. (1) for the elastic case, noting that the contraction factor, which is just $(M_A/E_A)^{n-1}$, arises from the $z_i \to z_i'$ substitutions in the integration variables.

3. <u>Modified L-P Argument</u>

Apart from the difficulty of an inadequate asymptotic behaviour $(\sim q^{1-n})$ noted earlier, the L-P prescription does not quite offer a symmetrical treatment between the initial and final states, a feature which remains effectively hidden for the elastic case considered by them, but would nevertheless show up for an inelastic transition $(A \neq B)$ through an apparent lack of decision as to which one of the two sets in (4) should be used for the $dz_i \to dz_i'$ transformation. However, a simple variant of the L-P prescription helps restore both the desired asymptotic behaviour as well as an explicit symmetry in the treatment of the coordinates in the A and B states as implied by their transformation (4). The trick consists in rewriting the N.R. matrix element (2) in the alternative form

$$F_{AB}(\vec{q}^{\,2}) = \int \prod_{1}^{n-1} d\vec{x}_{Ai} \; d\vec{x}_{Bi} \; \delta(\vec{x}_{Ai} - \vec{x}_{Bi}) \; \psi_B^*(\vec{x}_{Bi}) \; e^{ip(z_{A1}+z_{B1})} \psi_A(\vec{x}_{Ai}) \tag{5}$$

which still has the same physical content as (2) but helps bring out a more symmetrical appearance of the A- and B- coordinates. However, we now have the extra option of regarding the δ-function as a function of

the mixed coordinates $(\vec{x}_A, \vec{x}_B)$ which is also subject to the L-P transformation (4), just like the wave functions.

If we choose to exercise this option, the L-P transformation on (5), instead of on (2), yields the following relativistic prescription for an arbitrary $(A \neq B)$ transition as an alternative to (1):

$$F_{AB}(\vec{q}^{\,2}) \rightarrow \left(\frac{M_A M_B}{E_A E_B}\right)^{n-1} F_{AB}(4p^2\gamma_p^2) \tag{6}$$

where

$$2\gamma_p = M_A E_A^{-1} + M_B E_B^{-1}; \; E_{A,B}^2 = p^2 + M_{A,B}^2 \tag{7}$$

and p is the Breit frame momentum which equals $\frac{1}{2}q$ in the equal mass case.

Not only is the result (6) explicitly symmetrical between the parameters of the A and B states, and hence more transparent than (1), but it also exhibits a more 'acceptable' asymptotic behaviour $\sim q^{2-2n}$ as $q^2 \rightarrow \infty$. Indeed, the result is so simple as to encourage the illusion of a kinematical significance, but for the recognition that the δ-function transformation has probably imitated some effective features of a "good" field theory, which a more fundamental derivation of the BF type brings out more explicitly. Nevertheless the phenomenological usefulness of a formula like (6) in the intermediate energy region, with a shape determined from essentially N.R. premises, should require little defence with the added confidence of a hopefully correct asymptotic behaviour.

4. Form Factor Under Quark Pair Creation Hypothesis

It is tempting to look for a similar formula for an arbitrary quark diagram.[10] As with the radiation quantum (RQ) hypothesis such as above, QPC (quark pair creation) matrix elements with harmonic oscillator wave

functions have also been employed in recent literature with a fair amount of success at both the N.R.[11,12] and relativistic levels[13], and we assume the physical validity of an N.R. matrix element analogous to (2) in the QPC case as well. On the other hand, the question of physical validity L-P type modification on the individual wave functions is probably more serious in this case. A possible view that can be taken is that the QPC analogue of the Breit frame, which ensures good instantaneous overlap of the 'pancakes' in the RQ case, is a collinear frame which should roughly achieve a similar objective. (Only there are more varieties now!) For a specified collinear frame, the Lorentz transformations on the internal coordinates of the different clusters in the process $A \to B + C$ are defined by their respective velocities (or energies) as in Eq. (4) and provide the necessary ingredients for the L-P modifications on the corresponding wave functions.

To be more specific, consider an arbitrary quark diagram, Fig. I, for a transition $A \to B + C$, with the number of quark lines in the different clusters given by

$$n_A = n_{A_1} + n_{A_2}, \; n_B = n_{A_1} + n_Q$$
$$n_C = n_{A_2} + n_{\bar{Q}}, \; n_Q = n_{\bar{Q}} \tag{8}$$

We use the following collective notation for the N.R. form of the internal matrix element, as the QPC analogue[14] of Eq. (2):

$$M_{A\to BC} = \int dx_A dx_Q \; \psi_C^*(x_{A2}, x_{\bar{Q}}) \; \psi_B^*(x_{A1}, x_Q) \; \psi_A(x_A) \quad \exp(ix\cdot q) \tag{9}$$

where the x's denote the (independent) internal 3-coordinates, and the notations

$$x_A = (x_{A1}, x_{A2}), \; x_Q = -x_{\bar{Q}}, \text{ etc.}, \tag{10}$$

indicate the manner of their appearance in the respected wave functions in accordance with Fig. I. Now an L-P type modification on (9) would correspond to one on (2), with presumably the same symmetry and asymptotic problems as before. To give a 'natural' representation to all the coordinates of the different clusters, one must rewrite Eq. (9) in a form analogous to Eq. (5), viz.,

$$\int dx_A dx_Q \cdot dx_{\bar{Q}}\ \delta(x_Q + x_{\bar{Q}})\ dx_{B1} dx_{C2}\ \delta(x_{A1} - x_{B1})\ \delta(x_{A2} - x_{C2})$$
$$\times\ \psi_C^*(x_{C2}, x_{\bar{Q}})\ \psi_B^*(x_{B1}, x_Q)\ \psi_A(x_A)\ \exp(ix\cdot q) \tag{11}$$

and subject the coordinates appearing in the δ-functions <u>and</u> the wave functions to the L-P transformations like (4), viz.,

$$z_{A,B,C} \rightarrow z'_{A,B,C} = \frac{E_{A,B,C}}{M_{A,B,C}}\ z_{A,B,C} \tag{12}$$

where z represents the (common) direction of the Lorentz transformation, while the transverse components remain unaffected. The overall contraction factor now comes from a counting of the internal z-integration variables in each cluster in the form

$$F_{ABC} = (M_A/E_A)^{n_A - 1}\ (M_B/E_B)^{n_B - 1}\ (M_C/E_C)^{n_C - 1} \tag{13}$$

an expression which is not only symmetric in the different clusters but also has the correct asymptotic behaviour predicted by BF from dimensional scaling arguments. The scalar function multiplying (13), which is now the QPC analogue of the N.R. form factor $F_{AB}(\vec{q}^{\,2})$ of Eq. (6), tends to a finite limit as $q^2 \rightarrow \infty$, so that the aymptotic form is still determined by Eq. (13). Its detailed expressions for $n_A = 2,3$ and $n_q = 1$ are given elsewhere.[14]

Because of its simple and transparent form, Eq. (13) instinctively appeals to intuition despite the heuristic (almost naive) nature of the

derivation; its "correct" asymptotic form, independently of the details of dynamics, seems to be an extra bonus. However, unlike the RQ case of Eq. (6), the QPC result suffered from some ambiguity in the choice of the "best" (collinear) frame, one which cannot be resolved on theoretical grounds alone. In a phenomenological application, considerable caution is needed: e.g., in an $N^* \to N\pi$ decay process, a literal application of the contraction factor M_C/E_C for the pion would cause the rate to be suppressed, probably requiring remedies more in conformity with an L-P transformation on Eq. (9) than one on Eq. (11). It is also possible in principle to consider choices intermediate between (9) and (11), but no clearer guidance than asymptotic behaviour seems to be available from general theory which favours Eq. (11).

Recently, in a different kind of application[14], viz. the asymptotic behaviour of total widths of high mass resonances as a function of the $(\text{mass})^2$ or excitation quantum number $\underline{N}$ as a possible means of distinguishing between the RQ and QPC hypotheses, we had occasion to use the original L-P recipe on RQ and QPC matrix elements. The former predicts an exponential increase of Γ_{tot} with N, while the latter keeps it below the exponential level. The modification of the L-P recipe suggested here will not of course affect this qualitative result.

5. Comparison with the Brodsky-Chertok Model

While a quantitative comparison with data is not the subject of this paper, some qualitative features of the formula (6) under the radiation quantum hypothesis are of immediate interest. Especially interesting is its striking similarity with an effective parametrization given by Brodsky and Chertok[5] on the basis of their high-energy model of propagator chains. Before discussing this similarity and the possible inferences

therefrom, we first recall the fits given by Licht and Pagnamenta to the nucleon form factor (n=3), with their formula (2), which differs from (6) only to the extent of an extra power $(1+\eta)^{-1}$. Since the main source of agreement of the L-P formula with the data is the propagator $(1+q^2 m_\rho^{-2})^{-1}$ arising from vector meson dominance (VMD) and the power $(1+\eta)^{-1}$ varies rather slowly with q^2, Eq. (6) should give about the same qualitative fit to the proton form factor as Eq. (2), noting a parametric flexibility in the Gaussian damping constant a in their wave function. The more important issue in the L-P fit, on the other hand, is the crucial role of the VMD propagator.

A more interesting possibility arises from a comparison of the formula (6) with the proposal of BC, viz., $(1+q^2 m^{-2})^{1-n}$ for large q^2, obtained from entirely different considerations. Indeed, the algebraic similarity is so close that one would be tempted to identify the cluster mass M^2 parametrically with BC's m_n^2, thus providing an interesting form of "bridge", howsoever empirical, between the high and low q^2 points of view. However, such a correspondence would not be literally acceptable without heavy qualification. In the BC picture, m_n^2 goes like n, while in the L-P picture it effectively goes like n^2. The latter feature causes a rather slow decrease due to this factor alone, thus necessitating the VMD propator for the proton form factor in the L-P description. For the deuteron form factor, the damping due to this factor would be even slower, and the observed precipitous fall[4] would therefore require — over and above the VMD effect — a heavy strain on the wave function manifesting through strong core effects, as inferred from certain types of fits to the e-D data[4]. On the other hand, the simpler fits to the

data for large q^2 with the BC parametrization would suggest the possibility of a gentler wave function for a more quantitative description at moderate q^2 values. We hasten to add that in the BC description there is no place for the VMD propagator.

We now rephrase the question raised in the preceding paragraph: Is it possible to reconcile the L-P and the BC points of view in some physical sense so as to retain the parametric advantages of both the low and high q^2 descriptions? An intuitive answer that suggests itself requires the concept of an effective mass. Such a concept is not new, having been used extensively in the theory of nuclear matter[15], and possibly in other areas of physics as well. At any event, it seems to provide a rather simple way out of the present situation, if one insists that the cluster mass M of the L-P description should be regarded—from the lessons of the BC analysis—as an effective mass varying as $\sqrt{n}$ instead of as n, which a naive counting based on N.R. ideas would otherwise seem to suggest.

We would like to offer at least one argument in support of such an interpretation, based on the analogy with the hadron mass spectrum in a relativistic harmonic oscillator quark model[16]: the variation of M^2 with the total excitation quantum number N (not N^2) in good agreement with the data. Here the number n has a different origin - the total number of quark constituents in the system. Nevertheless the lesson is quite valuable and at the very least should serve as a caution against a hasty conclusion, such as $M \sim n$, based on entirely N.R. premises. Let us also recall that the L-P result is not entirely kinematical: The validity of the approximations and assumptions made depends on the

fundamental dynamics of the quark. These are involved in the replacements (3) of the internal coordinates of the wave function, a prescription carried a step further in this paper through the introduction of additional δ-functions for purposes of some more replacements. It is interesting to ask if such derivation and the more concrete results of BC (on the basis of a propagator chain model) have a common dynamical origin characteristic of some 'good' field theoretic models. We don't know the answer, but the strong algebraic correspondence of the BC and L-P results is certainly suggestive.

On the basis of these considerations we would be strongly inclined to suggest a variation of M^2 like n, the number of quark constituents in the cluster. As to the constant of proportionality, one possibility is to identify it with BC's parameter, $\beta^2 = 0.24$ Gev^2, without any further role of the VMD parameter. With the inclusion of a VMD propagator (and in view of so many successes of VMD, it would be premature to dismiss this effect too quickly without close scrutiny), one would presumably need a different constant. These alternative possibilities—form factors with or without VMD effects—are probably linked with the details of the wave function employed for specific calculations. In this respect we believe that while the order-of-magnitude consistency of the high q^2 results with the wave function at zero distance, as found by BC, is a most useful check, it still leaves a lot of scope for a more dynamical role of the wave function at non-zero distances. The philosophy of the present approach, on the other hand, is based on the latter alternative for the dynamics of the form factor for moderate q^2.

6. Concluding Remarks

We have tried to suggest a simple extension of the L-P argument for the construction of relativistic, albeit frame-dependent, form factors in a form which is not only symmetrical between the initial and final states but exhibits the correct asymptotic behaviour expected from theories of scale invariance. Having its base in a non-relativistic quantum mechanical wave function, the approach is closer to the spirit of nuclear physics and may be regarded as a complementary form of bridge between high and low energy physics to a recent proposal by Brodsky and Chertok from the high energy end. However, a closer comparison of the two formulae leads to the plausible concept of an effective cluster mass varying as $\sqrt{n}$, and not as n, where n is the number of quarks in the cluster. For the case of QPC matrix elements, our derivation has been limited to more general considerations bearing only on the Lorentz contraction factors without purporting to go into the detailed structure of the matrix elements. This would prevent us from attempting off-hand interpretations of apparent paradoxes such as different asymptotic behaviour of $\gamma\pi\pi$ and $\gamma\pi\rho$ matrix elements, inferred from exclusive-inclusive connections between structure functions[17], but we suspect that such results are of more dynamical origin. Currently separate efforts are under way with the radiation quantum hypothesis, to calculate the predictions of formula (6) on the form factors of very light nuclei (D, He^3, He^4) in a multi-quark model with necessary symmetries under all degrees of freedom (space, SU(6), and color) taken into account, to test the validity of this alternative approach.[18]

One of us (AM) is indebted to R. Blankenbecler and G. Farrar for some valuable comments and suggestions, and also for reading the manuscript. He is also grateful to Professors S. D. Drell and W. Panofsky for the warm hospitality of SLAC.

REFERENCES

1. Some sample references are:
A. Jaffe, Phys. Rev. Lett. 17, 661 (1966);
J. Harte, Phys. Rev. 165, 1557 (1968);
J. Ball and F. Zachariasen, Phys. Rev. 170, 1541 (1968).
2. See, e.g., R. Blankenbecler et al., Nucl. Phys. 12, 629 (1969);
M. Gourdin et al., Nuovo Cimento 37, 524 (1965
3. S. Brodsky and G. Farrar, Phys. Rec. Lett. 31, 1153 (1973);
Phys. Rev. D11, 1309 (1975), hereafter referred to as BF; see also
V. Matveev et al., Nuovo Cimento Lett. 7, 719 (1973).
4. R. Arnold et al., Phys. Rev. Lett. 35, 776 (1975).
5. S. Brodsky and B. Chertok, SLAC-PUB-1759 (1976) - to be published;
hereafter referred to as BC.
6. Some recent references are:
R. Blankenbecler and J. Gunion, Phys. Rev. D4, 718 (1971); F. Gross,
Phys. Rev. D10, 223 (1974); also R. Arnold - private communication.
7. See, e.g., R. Woloshyn, Phys. Rev. Lett. 36, 220 (1976).
8. K. Fujimura et al., Prog. Theo. Phys. 44, 193 (1970).
9. A. Licht and A. Pagnamenta, Phys. Rev. D2, 1150 (1970); hereafter
referred to as L-P.
10. H. Harari, Phys. Rev. Lett. 29, 562 (1969); J. Rosner, ibid.,
689 (1969).

11. A. Le Yaonanc et al., Phys. Rev. D8, 2223 (1973).

12. A. N. Mitra, Phys. Rev. D14 (1976) - in press.

13. Y. S. Kim and M. Noz, Phys. Rev. D12, 129 (1975)

14. Indra Kumari and A. N. Mitra - submitted to Phys. Rev. D.

15. See, e.g., H. A. Bethe, Phys. Rev. 103, 1353 (1956).

16. See, e.g., R. P. Feynman et al., Phys. Rev. D3, 2706 (1971).

17. E.g., G. Farrar and D. Jackson, Phys. Rev. Letters 35, 1416 (1975).

18. B. F. Bayman and A. N. Mitra, in preparation.

FIGURE CAPTION

1. Quark diagram for the process $A \rightarrow B + C$.

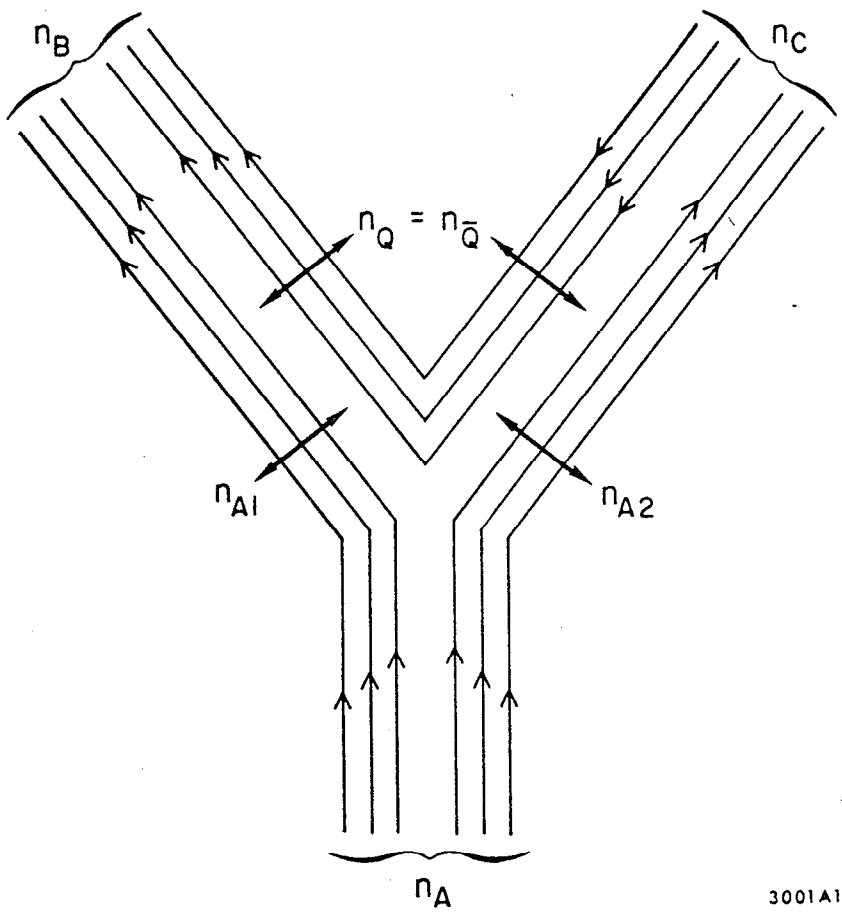

Fig. 1