# KINEMATICAL AMBIGUITIES IN SCATTERING BY DEUTERON* 

Erasmo M. Ferreira<br>Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 and<br>Pontifícia Universidade Católica, Rio de Janeiro, Brazil ${ }^{\dagger}$<br>L. Pinguelli Rosa and Zieli D. Thome ${ }^{\prime}$<br>Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 and<br>COPPE and Instituto de Fisica, Universidade Federal de Rio de Janeiro, Brazil ${ }^{\dagger}$


#### Abstract

We discuss the kinematical ambiguities occurring in the definition of the twobody amplitudes in particle-deuteron scattering. We show how the value of the energy parameter is uniquely determined through the proper reduction from the three-body to the two-body matrix elements. This value is compared to previously used prescriptions, and shown to give superior fitting when compared with experimental data on elastic pion deuteron scattering at low and medium energies.


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The multiple scattering calculations are affected by technical details which may have strong influence on the results obtaincd. Here we include the off-shell behavior of the amplitudes, and the specification of the kinematics for the twobody interactions. In the present work we are mainly concerned with these points. For definiteness we refer from now on to the $\pi \mathrm{d}$ system.

Let the three particles be called $1,2,3$. Channel $l$ is defined forming a pair $(2,3)$ and treating particle 1 separately. Define the momentum variables $\overrightarrow{\mathrm{K}}, \vec{k}_{1}$, $\vec{q}_{1}$ where $\vec{K}$ is the total momentum of the system, $\vec{k}_{1}$ is the internal momentum in the $(2,3)$ pair, and $\vec{q}_{1}$ is the momentum of particle 1 in the center of mass frame. In the center of mass system ( $\overrightarrow{\mathrm{K}}=0$ ) the kinetic energy can be written

$$
\begin{equation*}
\mathrm{H}_{0}=\mathrm{k}_{\mathrm{l}}^{2} / 2 \mu_{1}+\mathrm{q}_{\mathrm{l}}^{2} / 2 \mathrm{M}_{\mathrm{l}} \tag{1}
\end{equation*}
$$

where
and

$$
\begin{gather*}
\mu_{1}=m_{2} m_{3} ;\left(m_{2}+m_{3}\right)  \tag{2}\\
M_{1}=m_{1}\left(m_{2}+m_{3}\right) /\left(m_{1}+m_{2}+m_{3}\right) \tag{3}
\end{gather*}
$$

Channels 2 and 3 are similarly defined. The channel hamiltonian $h_{\alpha}$ contains $\mathrm{H}_{0}$ plus the interaction between the two particles forming the pair in channel $\alpha$. The channel resolvent is $\mathrm{g}_{\alpha}(\mathrm{z})=(\mathrm{z}-\mathrm{h})^{-1}$. The matrix elements of channel operators between three particle states can be expressed in terms of matrix elements of reduced two-body operators. Calling $\hat{\mathrm{h}}_{\alpha}$ the two body hamiltonian for the particles forming the pair in channel $\alpha$, and $\hat{\mathrm{g}}_{\alpha}=\left(\mathrm{z}-\hat{\mathrm{h}}_{\alpha}\right)^{-1}$ the two body resolvent, we can write

$$
\begin{equation*}
<\overrightarrow{\mathrm{k}}_{\alpha} \overrightarrow{\mathrm{q}}_{\alpha}\left|\mathrm{g}_{\alpha}(\mathrm{z})\right| \overrightarrow{\mathrm{k}}_{\alpha}^{\prime} \overrightarrow{\mathrm{q}}_{\alpha}^{\prime}>=\delta\left(\overrightarrow{\mathrm{q}}_{\alpha}-\overrightarrow{\mathrm{q}}_{\alpha}\right)<\overrightarrow{\mathrm{k}}_{\alpha}\left|\hat{\mathrm{g}}_{\alpha}\left(\mathrm{z}-\frac{\mathrm{q}_{\alpha}^{2}}{2 \mathrm{M}_{\alpha}}\right)\right| \overrightarrow{\mathrm{k}}_{\alpha}^{\prime}> \tag{4}
\end{equation*}
$$

The shift in the value of the argument must be noted. A similar shift appears when re relate the matrix element of the collision operator $t_{\alpha}(z)$ to the matrix element of the two body operator $\hat{\mathrm{t}}_{\alpha}(\mathrm{z})$.

A multiple scattering series for the amplitude $T(z)$ representing particledeuteron scattering can be obtained by iteration of the coupled Faddeev equations. For the elastic scattering of particle 1 by the $(2,3)$ bound pair we have

$$
\begin{equation*}
\mathrm{T}(\mathrm{z})=\mathrm{t}_{2}(\mathrm{z})+\mathrm{t}_{3}(\mathrm{z})+\mathrm{t}_{2}(\mathrm{z}) \mathrm{g}_{0}(\mathrm{z}) \mathrm{t}_{3}(\mathrm{z})+\mathrm{t}_{3}(\mathrm{z}) \mathrm{g}_{0}(\mathrm{z}) \mathrm{t}_{2}(\mathrm{z})+\ldots \tag{5}
\end{equation*}
$$

where all operators are in the three particle Hilbert space. The quantity

$$
\begin{equation*}
\mathrm{g}_{0}(\mathrm{z})=\left(\mathrm{z}-\mathrm{H}_{0}\right)^{-1} \tag{6}
\end{equation*}
$$

is the resolvent for three free particles. To evaluate the terms of the series, we introduce the shift corresponding to the energy of the particle which, in each term, does not participate in the collision. To show explicitly how this reduction affects the calculation, let E be the value of the total kinetic energy for the particle-deuteron scattering in the center of mass system. Let $\vec{P}\left(\vec{P}^{\prime}\right)$ be the nucleon, called particle 3, initial (final) momentum in the deuteron rest frame, and $\vec{p}\left(\overrightarrow{p^{\prime}}\right)$ be the initial (final) meson, called particle 1 , momentum in the laboratory system, in which the deuteron is initially at rest. Then, for the term in which the other nucleon (particle 2) is a spectator, we obtain

$$
\begin{equation*}
<\overrightarrow{\mathrm{P}^{\prime}},-\overrightarrow{\mathrm{P}^{\prime}}, \overrightarrow{\mathrm{p}^{\prime}}\left|\mathrm{t}_{2}(\mathrm{E})\right| \overrightarrow{\mathrm{P}},-\overrightarrow{\mathrm{P}}, \overrightarrow{\mathrm{p}}>=\delta\left(\overrightarrow{\mathrm{q}_{2}^{\prime}}-\overrightarrow{\mathrm{q}_{2}}\right) \delta\left(\overrightarrow{\mathrm{K}^{\prime}}-\overrightarrow{\mathrm{K}}\right)<\overrightarrow{\mathrm{k}}_{1}\left|\hat{\mathrm{t}}_{2}\left(\mathrm{E}-\frac{\mathrm{q}_{2}^{2}}{2 \mathrm{M}_{2}}\right)\right| \overrightarrow{\mathrm{k}}_{1}>. \tag{7}
\end{equation*}
$$

Here $\overrightarrow{\mathrm{K}}\left(\overrightarrow{\mathrm{K}}^{\prime}\right)$ is the total initial (final) momentum of the three particles, $\overrightarrow{\mathrm{q}}_{2}\left(\overrightarrow{\mathrm{q}_{2}^{\prime}}\right)$ is the initial (final) momentum of the spectator with respect to the center of mass of the whole system, $\overrightarrow{\mathrm{k}}_{1}\left(\overrightarrow{\mathrm{k}}_{1}^{\prime}\right)$ is the initial (final) momentum of the meson with respect to the center of mass of the interacting meson-nucleon system, and $\mathrm{t}_{2}$ is the usual two-body collision operator. The reduced mass is in this case

$$
\begin{equation*}
M_{2}=m_{N}\left(m_{N}+m_{\pi}\right) /\left(2 m_{N}+m_{\pi}\right) \tag{8}
\end{equation*}
$$

In terms of the quantities defined in the laboratory system, the argument of the
two-body collision operator in Eq. (7) is given by

$$
\begin{equation*}
E-\frac{q_{2}^{2}}{2 \mathrm{M}_{2}}=E-\left[\left(2 m_{N}+m_{\pi}\right) \stackrel{\rightharpoonup}{\mathrm{P}}+\mathrm{m}_{\mathrm{N}} \overrightarrow{\mathrm{p}}\right]^{2} /\left[2 \mathrm{~m}_{\mathrm{N}}\left(\mathrm{~m}_{\mathrm{N}}+\mathrm{m}_{\pi}\right)\left(2 \mathrm{~m}_{\mathrm{N}}+\mathrm{m}_{\mathrm{N}}\right)\right] \tag{9}
\end{equation*}
$$

Similarly evaluated shifts must be introduced in the evaluation of double scattering terms in the series.

We must remark that pions are relativistic even at rather low energies, while the formalism developed above is completely non-relativistic. However, the approximation involved in the use of Eq. (7) is expected to be very reasonable, as the spectator particle, whose energy is subtracted from the total energy available, is always a non-relativistic nucleon.

The prescription thus established by Eq. (7) for the determination of the value of the energy parameter has safe theoretical foundation in non-relativistic particle-deuteron scattering, and it is interesting to compare its predictions with calculations made with other usual approaches.

The most commonly used prescription for the treatment of the two-body collision amplitudes appearing in the multiple scattering series consists in considering that the incident particle collides with an on-shell physical nucleon. The relative energy for the collision is evaluated through a Lorentz transformation applied to the system of two moving physical particles. In this evaluation no account is made for the energy carried by the spectator nucleon.

A third prescription which has been used to solve the kinematical ambiguity is suggested by the experiments in which there is a breakup of the deuteron, and where an identification can be made between the spectator and the struck nucleons. These experiments show that the spectator nucleon recoils with a momentum distribution which is approximately the same as determined by the deuteron
wavefunction. The consequent assumption is then that the spectator nucleon behaves like an on-shell particle. Energy conservation is imposed stating that the struck nucleon carries an energy amount given by the deuteron mass minus the energy $m_{N}+P^{2} / 2 m_{N}$ carried by the spectator nucleon.

In our calculation the off-shell behavior of the partial wave amplitude is fixed writing $f_{\ell J}\left(k, k^{\prime}, E\right)$ in the form $1 /\left(k^{\prime}\right)^{\frac{1}{2}} \sin \delta_{\ell J} \exp \left(\mathrm{i} \delta_{\ell J}(E)\right)$, where $k, k^{\prime}$ are respectively the initial and final relative momenta evaluated with physical masses for the two colliding particles.

## Results of Calculations and Experimental Data in $\pi d$ Elastic Scattering

We now compare the results obtained from these three kinematical prescriptions with experimental results obtained in $\pi d$ elastic scattering. Our calculations account for fermi motion dependence of the amplitudes in the single scattering terms. We evaluate both single and double scattering contributions, allowing for nucleon recoil, and including both the delta function and the principal value parts originated from the pole in the propagator. As the contributions from double scattering terms to the differential cross section are here never larger than 10 percent, the fermi motion dependence can be safely neglected in these terms. We use the Moravesik deuteron wavefunction with about 7 percent d wave component. Except for Eq. (7) all our treatment is relativistic.

The $\pi d$ elastic differential cross section falls rapidly as a function of the scattering angle. The product $\psi(\overrightarrow{\mathrm{P}}) \psi(\overrightarrow{\mathrm{P}}+\Delta / 2)$, where $\overrightarrow{\mathrm{P}}$ is the nucleon fermi momentum, $\vec{\Delta}$ is the pion momentum transfer, and $\psi$ is the deuteron wavefunction in momentum space, appears in the integrand of the expression for the differential cross section. Due to the short range of the momentum distribution,
the value of the differential cross section for large values of $|\vec{\Delta}|$ is very sensitive to the $\vec{P}$ dependence of other terms occurring in the integrand. As a consequence, different kinematical prescriptions give very different results in large angle scattering. In Fig. 1 we plot the experimental data for $\mathrm{d} \sigma / \mathrm{d} \Delta$ at $160^{\circ}$ laboratory scattering angle ${ }^{1-12}$ together with the results of our calculations. We see from the figure that the prescription using the multiple scattering series based on Faddeev's equation gives a good description of the data in the region from 140 to 250 MeV . The other two prescriptions fail in this energy region, and seem to be more reasonable at the lower energies (see the experimental points at 47.5 and 61 MeV ). We think that this good fitting at these lowest energies is purely accidental, as a multiple scattering calculation, without a considerable extra care for binding corrections, is not expected to give good results at such low energies.

Above 250 MeV all calculations made give values for the backward cross section which are too low if compared to the experimental data. It is not difficult to mention possible reasons for this disagreement, as the calculated values are highly sensitive to details of the deuteron structure, to changes in the kinematics due to relativistic corrections, and to dynamical effects, such as can be introduced assuming a different off-the-energy shell behavior of the twobody amplitudes.

In Fig. 2 we plot the angular distributions at 142 and 182 MeV , together with the experimental data at these energies. ${ }^{6,7}$ We see from these figures that the kinematic prescription derived from a proper treatment of the threebody system gives clearly better results than the other two prescriptions. Due to the strong variation in the value of the differential cross sections as we go from forward to backward angles, we have used two separate scales for the
vertical axis. We have thus avoided the use of a logarithmic scale, trying to exhibit more clearly the existing discrepancies. It seems reasonable to us to conclude that the theoretical calculations in the framework of the multiple scattering series are able to give a good fitting to the experimental data at these medium energies.

## FIGURE CAPTIONS

Fig. 1 Energy dependence of backward differential cross section for elastic $\pi$ d scattering. The experimental points are from References 1-12. The solid curve represents results obtained from the kinematical prescription derived from the proper reduction from three-body to twobody matrix elements. For the dotted curve the struck nucleon is treated as an on-shell physical nucleon. The dashed curve shows the results obtained with on-shell spectators.

Fig. 2 Curves for the differential cross section for $\pi d$ elastic scattering at 142 and 182 MeV , obtained in a multiple scattering calculation involving single and double scattering terms, and accounting for fermi motion and nucleon recoil effects. The solid curves are calculated using the value of the energy parameter obtained from a proper treatment of the three body kinematics, as described in the text. The dotted curve is obtained with the prescription which puts the struck nucleon on the mass shell, while for the dashed curve the spectator nucleon is on shell. The experimental results are from references 6 and 7 .

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    $\dagger$ Permanent addresses.

