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BARYON EXCITATIONS IN THE BAG MODEL†

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I wish to discuss two recent spectroscopic applications of the bag model. The first is a study of the place of multiquark (≥ 4) states (exotics and crypto-exotics) in meson and baryon spectroscopy. The second is an attempt by Claudio Rebbi and (from a more phenomenological standpoint) by Tom DeGrand and myself to sort out the P-wave baryon excitations in a bag model. Before describing this work I would like to take some pains to compare the bag-quark model with more conventional quark models. Otherwise it will not be apparent why we go to great effort to confront problems which quark enthusiasts have generally dismissed out of hand.

The bag model began with little reference to spectroscopy.[1] Our concern was to construct a relativistic theory consistent with the parton model and with the apparent permanent confinement of quarks. Only later did we learn that very light, weakly interacting but permanently confined quarks might be consistant with at least the low end of the hadron spectrum.[2,3] The quark model, on the other hand, began as a spectroscopic mnemonic but has been taken progressively more seriously over the years. Many of the features which were puzzling but forgivable when it was a mnemonic have become embarassments for a theory. Most notably the generally accepted quark model[4] (GAQM) is rooted in nonrelativistic potential theory rather than relativistic field theory. Its remarkable phenomenological success is intimately connected with some of these theoretically embarassing features. A model like the bag, which is more closely related to relativistic field theory, will either fail phenomenologically or mimic ("explain") the GAQM often through some subtle effect not apparent at first glance. So far we have examples of both possibilities.

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Let me review the motivation for the bag theory in order to understand the origin of its spectroscopic predictions. Bjorken scaling requires that the mass scale which characterizes quarks and their interactions be less than (generously) a GeV. Quarks cannot be very heavy—in the sense of the old quark model—and their interactions must be negligible at distances less than about .3 fm. The forces which confine quarks must be long range. They must confine not only quarks but also all states with triality not equal to zero. All this can be accomplished by adding a single term to the hadron stress tensor[5]

$$T_{\mu\nu} = \left(T^{o}_{\mu\nu} - g_{\mu\nu} B\right) \theta_s$$

Here $T^{o}_{\mu\nu}$ is the conventional stress tensor of any relativistic quantum field theory. $g_{\mu\nu} B$ has the form of a pressure ($g_{00} = -1$, $B > 0$). B is a universal constant. Such a field theory cannot be defined over all space, since the total energy, $\int d^3x\, T_{00}$, would diverge. Hence the function θ_s, which is 1 inside some region (the bag) defined by a spatial surface S and zero outside. Equations of motion and boundary conditions follow from demanding conservation of four momentum: $\partial^{\mu} T_{\mu\nu} = 0$.[1] The classical theory is Lorentz invariant. The fields which enter $T^{o}_{\mu\nu}$ are defined only over the interior of the bag, much like the phonon field is defined only over a crystal lattice.

If we let $T^{o}_{\mu\nu}$ be the stress tensor describing light, colored quarks weakly coupled ala Yang-Mills to massless, colored, vector mesons, then all the desederata of the previous paragraph are achieved: Only triality zero (color singlet) solutions exist.[1] Hadrons are extended structures characterized by a length $B^{-1/4} \sim 1$ fm.[1,2] The quarks inside are weakly coupled at distances of the order 1 fm. Bjorken scaling is obtained.[6] The asymptotically free character of the Yang-Mills theory[7] guarantees that the coupling only gets weaker at shorter distances.

Most importantly, the hadron states still exist in the limit $g \rightarrow 0$ (g being the gauge coupling constant). The long-range confining forces are provided by B, not g. It therefore makes sense to calculate hadron properties order by order in g, where zeroeth order is not a free field theory but instead an extended system with rich structure. Of course the value of g determined by phenomena must turn out small enough to be consistent with a perturbative expansion.

The problem with this theory, and with every nontrivial nonlinear field theory, is that it cannot be solved classically, much less quantized. Most of the applications of the bag have relied on a semiclassical quantization of a set of spherical solutions in three dimensions. Recent work by Rebbi,[8] though, includes small excursions from the spherical configuration and has important phenomenological consequences discussed below.

Once this approximation is accepted a quark model follows directly.[3] It is not, however, the GAQM. It differs in the following essential ingredients.

First, it is impossible to formulate a quark model from the bag theory unless the quarks are very light.[9] Colored hadrons simply do not exist in the bag theory and in color singlet states there are no super strong (confinement related) interactions between the quarks[10] to cancel the quark masses. The mass of a hadron is the sum of the quark masses, their kinetic energies, the bag energy BV and small (~25%) corrections from gluon exchange etc. Given the momentum required by the uncertainty principle $\left(\frac{2.04}{R} < P < \frac{\pi}{R}\right.$ for a massless (2.04) or very heavy (π) quark confined to a sphere of radius R$\left.\right)$ there is no room in a hadron of mass 1 GeV and radius 1 fm for much quark mass.

Support for light quarks has been accumulating for many years. Perhaps the most convincing evidence is the success of chiral $SU(2)\times SU(2)$.[11] While no one has successfully incorporated chiral symmetry in a quark model it seems clear that massless quarks are a precondition. Closer to home, support comes from the relativistic corrections to g_A/g_V ($g_A/g_V = 5/3$ in the nonrelativistic quark model[4]). It is sensitive to the lower components of the quark spinor. As the quark mass drops so does g_A/g_V.[12] This is illustrated in Figure 1 with the bag-quark model calculation. The relativistic correction to the kinetic energy is also shown. Reasonable values of g_A/g_V select relativistic regimes. Parenthetically

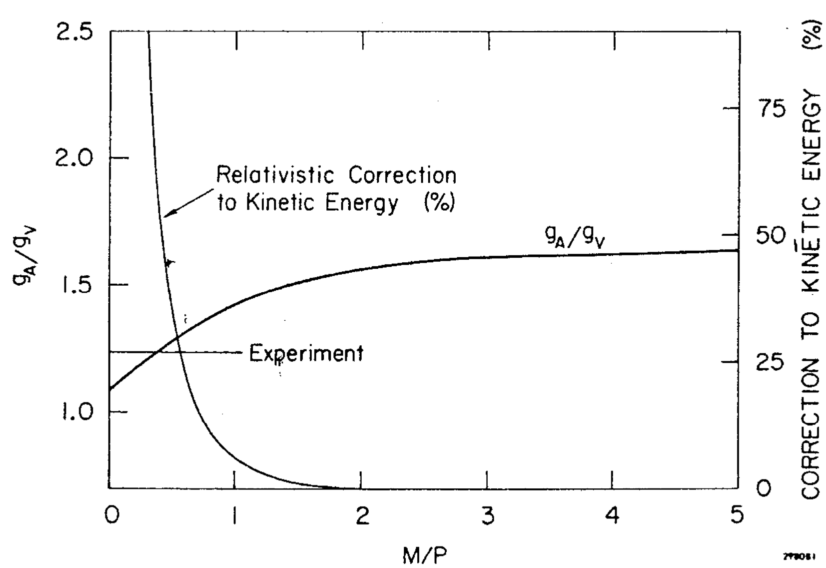

Figure 1. g_A/g_V and the relativistic correction to a quark's kinetic energy, both plotted as functions of the ratio of the quark's mass to its momentum.

it should be noted that g_A is not pion pole dominated so that this is a separate issue from the failure of chiral symmetry in quark models with heavy quarks.

The phenomenological price of massless (or light) quarks is high. Apparently the $SU(6) \times O(3)$ static classification scheme is lost—$\vec{L}$ and $\vec{S}$ are not separately good quantum numbers. Furthermore the small $L \cdot S$ splittings of baryon multiplets as conventionally assigned[13] to the L=1 [70] argue against light quarks. A major task for light quark enthusiasts is to resurrect the spectroscopic classification scheme of the GAQM in a relativistic context.

A second unconventional feature of the bag-quark model is the possibility of excitations of quarks relative to the bag. Dynamically the bag acts like a very heavy scalar object to which the quarks are bound. Being very heavy we do not observe dynamical excitations of the object itself but detect it by the presence of quark excitations relative to it. The archtype for such states is a group of P-wave baryons with the quantum numbers of an L=1 [56].

This problem will arise in any field theoretic description of hadrons.[14] Field theories inherently contain many (infinite) numbers of degrees of freedom. The idea that a baryon consists of 3 quarks emerges only in a static, nonrelativistic limit. In a more conventional field theory the bag would be replaced by some self consistent field generated by the quarks and gluons. Clearly an excitation of the quarks relative to the collective coordinates defining the self-consistent field is possible. The presence of low lying states with funny quantum numbers will be a headache for anyone attempting a relativistic field-theoretic description of hadrons.

A third departure from the GAQM is a boon to calculation. There need be no very strong forces between quarks to obtain confinement. It has always been a puzzle why quark model estimates of static moments (g_A/g_V, μ_P/μ_N, etc.) and transition amplitudes[13] remain unrenormalized in the presence of strong confining forces. The bag-quark model provides a possible explanation for this. We can go one step further and calculate processes first order in gluon exchange. Surprisingly we find much of the spin structure within the lowest multiplets is successfully described by lowest order perturbation theory.

Keeping in mind these distinctions I will review the quark-bag model and then turn to its applications. Since the model has been described extensively elsewhere[3, 15, 16] I will be exceedingly brief. The up and down quarks are massless.[17] The strange quark has a small mass, m_s. The mass of a hadron receives contributions from four terms. First the quark energy, determined by its rest mass and bag boundary conditions. The quark energy (in units of the bag radius) is given in Figure 2 for the three lowest eigenmodes $S_{1/2}$, $P_{1/2}$ and $P_{3/2}$.[18] Second is the bag energy, BV, representing, if you wish, the energy stored in the self-consistent confining fields. Third is a phenomenological estimate of the zero point energy generated by fields fluctuating in a finite region of space.[19]

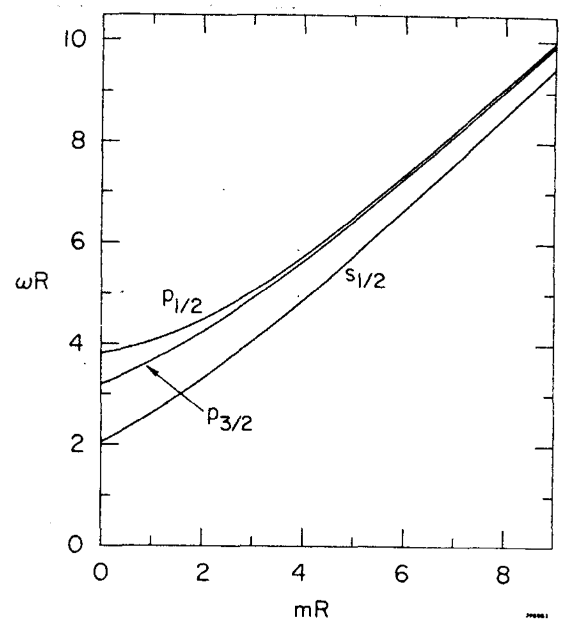

Figure 2. Energy as a function of mass (both scaled by the bag radius) for the three lowest quark modes.

So far the spectrum of even the lowest states looks only vaguely like reality. Eigenstates are diagonal in the number of strange quarks (n_s). Hadron masses go approximately like the 3/4 power of the number of quarks. This situation is shown in Figure 3. Notice not only the poor description of the S-wave baryons (Q^3) and mesons ($Q\bar{Q}$) but also the large numbers of light, perhaps stable, $Q^2\bar{Q}^2$ states only a fraction of which are shown in Figure 3.

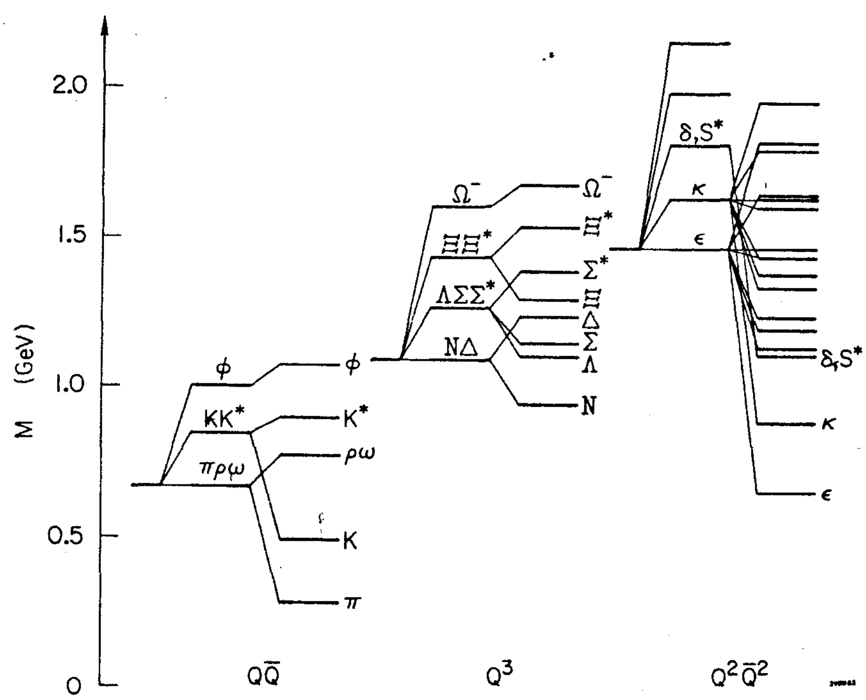

Figure 3. Hadron splittings: The first are those induced by giving the S-quark a mass 280 MeV; the second are produced by gluon interactions. Not all $Q^2\bar{Q}^2$ states are shown.

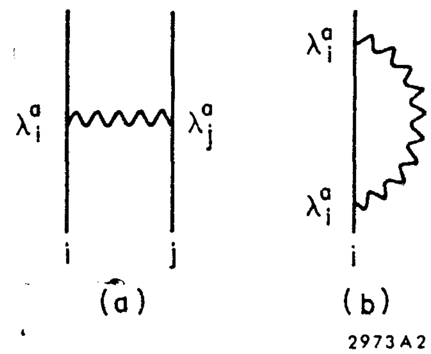

Figure 4. Lowest order gluon exchange.

The inclusion of the first order corrections due to the exchange of colored gluons improves the situation immensely. To lowest order only the Born graph of Figure 4a and the self energy of Figure 4b contribute. Only the static magnetic contribution is important:[20]

$$E_g = \frac{-g^2}{2} \sum_{i \neq j} \sum_a \int d^3x \, \vec{B}_i^a(\vec{x}) \cdot \vec{B}_j^a(\vec{x}) \tag{1}$$

$\vec{B}_i^a(x)$ is the color ($a=1,2\ldots 8$) magnetic field of the ith quark. In Reference 3 this integral was evaluated[21]

$$E_g = -\frac{\alpha_c}{R} \sum_{i>j} \sum_a \vec{\sigma}_i \cdot \vec{\sigma}_j \, \lambda_i^a \lambda_j^a \, M(m_i R, m_j R) \tag{2}$$

$M(m_i R, m_j R)$ is the result of an integral over Dirac wave functions given in Figure 3 of Reference 3. $\vec{\sigma}_i$ and λ_i^a are the spin and color SU(3) matrices of the ith quark.

Equation (2) has a form reminiscent of the exchange interactions of nuclear physics. The effect on the spectrum (S-waves only) is also shown in Figure 3. The color matrices which appear in Equation (2) are essential in ordering the spectrum consistent with reality.[22] Without them the nucleon would be heavier than the Δ and the Λ heavier than the Σ.

The model has four parameters—B, α_c ($g^2/4\pi$), m_s and z_0[23]—which were fit to the masses of the N, Δ, Ω and ω. The rest of the S-wave states (the $1/2^+$ octet, $3/2^+$ decuplet, 0^- and 1^- nonets) are well fitted, except for the η and η' which no one is certain how to treat in a quark model and the π which is probably too light to be treated semiclassically. Wherever relativistic effects enter they improve matters[3] (g_A/g_V for the entire octet; μ_Λ/μ_N, etc.). So far SU(6) structure is preserved—the lightest baryons are a [56]; the lightest mesons, a [36]—though it's not the usual SU(6). This SU(6) is generated by flavor SU(3) and the SU(2) of relativistic j=1/2 S-wave quarks (which is not spin). Having completed this brief review let me turn to the recent work on spectroscopy.

I. Multiquark Hadrons

The bag model may be used to shed some light on the old problem of exotics. Most of the distinctions between it and the GAQM are irrelevant for this discussion. The advantage of the bag is that it provides more dynamical information. Effects which would be lost in a maze of reduced matrix elements in a more general approach stand out clearly in the bag.

In the bag model the mass of a hadron increases roughly in proportion to the total number of quarks it contains. It was realized long ago that Coulomb-like (electric) color forces saturate.[24] That is, there are no strong, confinement related forces between color singlet mesons and baryons. There are however color-magnetic forces. Two color-singlet hadrons sitting side by side are not an eigenstate of the magnetic gluon exchange operator of Equation (2). They can exchange a gluon becoming in the process color octets (still coupled to an overall color singlet). This force mixes and splits multiquark states. Since the spin splittings among Q^3 baryons and $Q\overline{Q}$-mesons are a substantial fraction of their masses, the splittings induced among $Q^2\overline{Q}^2$ mesons, $Q^4\overline{Q}$ baryons, etc. should be spectroscopically important. Occasionally one might expect a multiquark state to lose so much energy as to be bound relative to the decay into normal (Q^3 or $Q\overline{Q}$) hadrons. The problem of exotics becomes one of finding the eigenstates of Equation (2).

We proceed exactly as we did in the $Q\overline{Q}$ and Q^3 sectors.[25] There are no new parameters or approximations. The results are surprising and encouraging. There are a set of Hund's rules which may be distilled from Equation (2). These work to suppress the spectroscopic importance of multiquark states even though they are not elevated to very great masses.

For the sake of discussion we ignore the dependence of M in Equation (2) on quark masses. Then the color-spin[26] operator may be rewritten in terms of Casimir operators of color ($SU(3)_c$), spin ($SU(2)$) and colorspin ($SU(6)_{cs}$), which is the SU(6) generated by $SU(3)_c \times SU(2)$:

$$\begin{aligned} E_g = \frac{\alpha_c}{R} M \Big[& 8N + \frac{1}{2} C_6(\mathrm{tot}) - \frac{4}{3} S_{tot}(S_{tot}+1) \\ & + \frac{8}{3} S_Q (S_Q + 1) + C_3(Q) - C_6(Q) \\ & + \frac{8}{3} S_{\overline{Q}} (S_{\overline{Q}} + 1) + C_3(\overline{Q}) - C_6(\overline{Q}) \Big] \end{aligned} \tag{3}$$

C_6 and C_3 are the quadratic Casimirs of SU(3) and SU(6) respectively.[27] The Casimirs of colorspin dominate Equation (3). They are generally much larger than those of spin or color. For a given number of quarks and antiquarks ($Q^m\overline{Q}^n$) the lowest lying multiplet obeys the following rules:

1. The quarks and antiquarks are separately coupled to the largest possible representation (hence Casimir) of colorspin.
2. The colorspin Casimir of the total system (quarks plus antiquarks) is minimized.

These rules make multiquark states relatively elusive. The quarks must be antisymmetrized in colorspin and flavor. Large colorspin representations (with large Casimirs) are as symmetric as possible.[28] Antisymmetry requires a conjugate representation for flavor, as antisymmetric as possible. With the noted exception[28] the state of lowest energy places the quarks in a $\underline{3}$ or $\underline{\bar{3}}$ of flavor. The same applies to the antiquarks. Consequently the entire system is a flavor nonet. Therefore the ground states of $Q^2\bar{Q}^2$ and $Q^4\bar{Q}$ are not exotic, they are nonets. They may be misclassified as conventional $Q\bar{Q}$ or Q^3 states. Exotics are heavier, further above threshold for decay to $(Q\bar{Q})(Q\bar{Q})$ or $Q^3(Q\bar{Q})$, and therefore very broad if indeed they are resonant at all.

Furthermore the ground state nonets of $Q^2\bar{Q}^2$ and $Q^4\bar{Q}$ contain more than their share of strange quarks. The weight diagrams and quark content for these multiplets is shown in Figure 5. Notice the $s\bar{s}$ pairs, which appear when many quarks are forced into small $SU(3)_f$ representations. Because of the $s\bar{s}$ pairs the nonstrange members of ground state multiplets of $Q^2\bar{Q}^2$ and $Q^4\bar{Q}$ are heavier than one would expect from their quantum numbers above (with the exception of $u\bar{u}d\bar{d}$ which lies just where we want it). They couple to channels with strange quarks which are harder to see experimentally.

I have discussed the lowest $Q^2\bar{Q}^2$ meson multiplet extensively elsewhere.[25] A case can be made on the basis of calculated masses and decay couplings that the lightest 0^+-mesons ($\epsilon(650)$, $S^*(993)$, $\delta(976)$, κ) are $Q^2\bar{Q}^2$ states. The P-wave $Q\bar{Q}$ 0^+-mesons

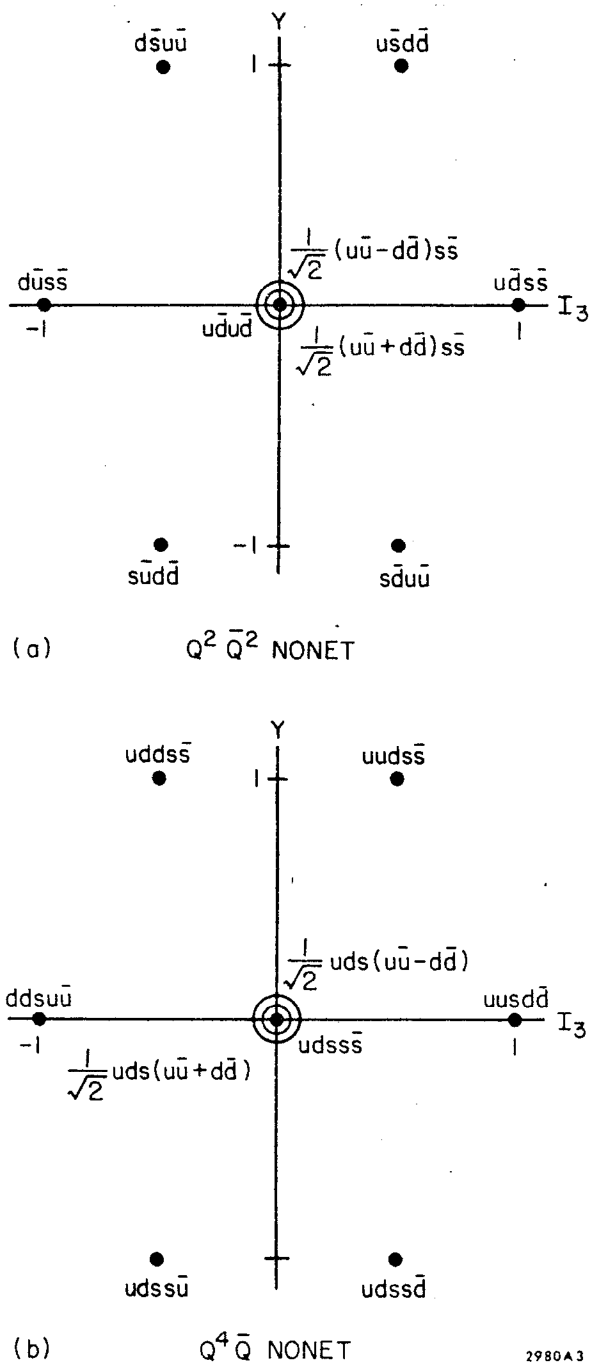

Figure 5. The quark content of $Q^2\bar{Q}^2$ and $Q^4\bar{Q}$ nonets.

would have to lie elsewhere. Perhaps the recent observation[29] of a 0^+-isovector at 1255 MeV and the reported $\epsilon'(1240)$[30] signal the appearance of this multiplet. Since this is a baryon conference let me focus on the lightest $Q^4\bar{Q}$ cryptoexotic nonet and its heavier exotic siblings.

The masses of the states in the $Q^4\bar{Q}$ $1/2^-$-nonet are estimated to be (in MeV):

$$1350 < M(\Lambda_5) \approx M(\Sigma_5) < 1450$$

$$1600 < M(N_5) \approx M(\Xi_5) < 1700$$

$$1800 < M(\Lambda_5^s) < 1900$$

The superscript s denotes the octet singlet mixture with udss$\bar{s}$ quark content. The uncertainty is theoretical: the vagaries of the spin and color couplings of the four quarks have yet to be worked out exactly. These will split the Λ_5 from the Σ_5.

The decays of these states should be dominated by S-wave channels into which the baryon merely falls apart. The complete recoupling calculation ($Q^4\bar{Q} \to Q^3(Q\bar{Q})$) has yet to be performed. The flavor recoupling has been done and the color-spin recoupling should be dominated by pseudoscalar color singlet—$1/2^+$-color singlet channels.[31]

$$\Lambda_5 = \sqrt{\frac{3}{8}}\, N\bar{K} + \frac{3}{4}\,\Sigma\pi - \frac{1}{4}\,\Lambda\eta_0$$

$$\Sigma_5 = \sqrt{\frac{3}{8}}\,(N\bar{K} + \Sigma\pi) - \frac{1}{4}\,\Lambda\pi - \frac{\sqrt{3}}{4}\,\Sigma\eta_0$$

$$N_5 = \frac{3}{4}\,\Sigma K - \frac{1}{4}\,\Lambda K - \sqrt{\frac{3}{8}}\, N\eta_s$$

$$\Xi_5 = \frac{3}{4}\,\Xi\pi + \frac{1}{2}\,\Lambda\bar{K} - \frac{\sqrt{3}}{4}\,\Xi\eta_0$$

$$\Lambda_5^s = -\frac{\sqrt{3}}{2}\,\Xi K + \frac{1}{2}\,\Lambda\eta_s$$

η_0 and η_s are short hand for $1/\sqrt{2}\ (u\bar{u}+d\bar{d})$ and $s\bar{s}$ respectively. The physical η and η' are linear combinations of η_0 and η_s.

A thorough phenomenological analysis of these states has not yet been attempted. Only the Λ_5 and Σ_5 should be prominant. The N_5 does not couple to πN but is broad into ΣK and $N\eta$ (unless it is lighter than we think). The Λ_5^s doesn't couple to $\bar{K}N$ and is broad into obscure channels. The Ξ_5 is broad. Where are the Λ_5 and Σ_5? States with the right quantum numbers lie nearby ($\Lambda(1405)$, $\Lambda(1670)$, $\Sigma(1620)$, etc.). These states are currently classified as Q^3 P-waves. If we classify known Y*'s as $Q^4\bar{Q}$, additional Y*'s are needed to fill out SU(3) multiplets in the L=1 [70].

The lightest exotics are heavier than the $1/2^-$-nonet. The lighter Z*'s predicted by the bag model are:

$$Z^*(I=0)\ \frac{1}{2}^- \quad M \approx 1700\ \text{MeV} \quad \text{couples to KN}$$

$$Z^*(I=1)\ \frac{1}{2}^- \quad 1850 \lesssim M \lesssim 1950 \quad \text{couples to KN}$$

$$Z^*(I=1)\ \frac{3}{2}^- \quad 1700 \lesssim M \lesssim 1850 \quad \text{couples to K}\Delta$$

Better estimates of the masses and fall-apart decay couplings of these states awaits more powerful group theory techniques for the color×spin×flavor couplings of $Q^4\bar{Q}$.

So far I have argued that S-wave multiquark hadrons are spectroscopically elusive because

1. For most[28] $Q^m\bar{Q}^n$ configurations, the lightest multiplet is not exotic (and may therefore be misclassified as $Q\bar{Q}$ or Q^3).
2. Many states of the lightest $Q^m\bar{Q}^n$ multiplet are heavier and coupled to more obscure channels than expected on the basis of quantum numbers above, because of their s-quark enrichment.

It would help greatly if we knew more about the dynamics of production and decay of multiquark states. Two questions immediately come to mind. First, if resonance formation is peripheral[32] then high mass S-wave $Q^4\bar{Q}$ states will be suppressed. As an estimate assume $\ell \cong PR$. P is the center of mass momentum, R is about 1 fm.[32] For S-waves we require $P \lesssim 200$ MeV or $M(Q^4\bar{Q}) \lesssim 1500$ MeV for KN scattering. The Z*'s listed above would be somewhat suppressed.

The second question is more basic. Are these states resonant at all? Consider the $Q^2\bar{Q}^2$ mesons. The lowest $Q^2\bar{Q}^2$ bag state available to two pions (in an I=0 S-wave) is at 650 MeV, about 350 MeV above the combined rest masses of the two pions. One might interpret this semiclassical calculation as indicating a repulsive interaction in this channel, leading one to expect a negative phase shift. However the 650 MeV state also has substantial coupling to $\eta\eta$ where it would appear to represent an attractive interaction. The effects of unitarity cannot be neglected. Perhaps the bag calculations should be interpreted as input to some multichannel unitarization scheme.

Much work remains to be done to understand better the resonant parameters and dynamics of multiquark states, to look for elusive states in experiment and to distinguish between alternative classifications of known states.

II. P-Wave Baryons

The distinctions between the bag and the GAQM become clear when the first baryon excitation is studied. Suppose we ignore the bag's desire to deform when P-wave modes are occupied. Then the lowest negative parity baryons are either $S^2_{1/2}P_{1/2}$ or $S^2_{1/2}P_{3/2}$, where the labels refer to modes in a spherical cavity. The recipe for hadron masses may be applied[33] to these configurations in the same manner as to the ground state $S^3_{1/2}$. All of the difficulties prefigured in the first part of the talk appear on schedule:

1. The resultant states are members of both an L=1 [70] and an L=1 [56]. $SU(6)\times O(3)$ is broken by relativity and gluon exchange so it is not possible to identify bag states with GAQM states. In any case there is no sign that one set (hopefully the [70]) lies generally lower than another set (hopefully the [56]).
2. The L-S splittings are too large and the entire multiplet ($S^2_{1/2}P_{1/2}$ and $S^2_{1/2}P_{3/2}$) is too light. Figure 2 anticipates this: for massless quarks the $P_{1/2}$-$P_{3/2}$ splitting is 40% of the S-P splitting, while the S-P splitting is only 70% of the energy of a single S-wave quark.

The first difficulty has recently been (partially) resolved by Rebbi.[8] He has shown that a more sophisticated treatment of the bag model mimics the nonrelativistic classification scheme of the GAQM. When small bag deformations are included he finds that the L=1 [56] of Reference 33 is promoted to higher energy while the states of the L=1 [70] remain more or less where they were. His work is not quantitatively successful—the L=1 [56] is not elevated enough to eliminate it from experimental detection, nor do the L=1 [70] states all lie where they should—but it strongly suggests that it is possible for a relativistic, field-theoretic quark model to mimic the nonrelativistic classification scheme of the GAQM.

Rebbi linearizes the bag equations about the static spherical configuration. This approximation becomes exact in the limit that the number of $S_{1/2}$-quarks approaches infinity with a fixed number of P-quarks. The case of interest (2 S-quarks, 1 P-quark) comes close enough to the limit for theorists if not for experimentalists. The soliton fervor of the past few years has taught us to expect zero-frequency modes among the small oscillations about static solutions of nonlinear field theories.[34] These represent translations: one of the infinitesimal P-wave (dipole) deformations of a sphere is not a deformation at all but instead a uniform infinitesimal translation of the ground state. Such a "deformation" gives rise to no restoring force—it has the same energy as the ground state.

Rebbi's spectrum therefore contains an L=1 [56] degenerate with the L=0 [56] ground state. This L=1 [56] is not a new state of the system but merely the infinitesimally translated ground state. Physical L=1 [56] excitations are heavier. This may be understood by the following simple minded (but essentially correct) argument.[8] The cavity wave functions ($1S_{1/2}$, $1P_{3/2}$, $1P_{1/2}$, $2S_{1/2}$, etc.) form a complete set. The translation mode may therefore be written as a linear combination of L=1 [56] cavity eigenstates. As expected the lowest cavity L=1 [56] dominates. The first "real" L=1 [56] in Rebbi's calculation overlaps principally with higher cavity L=1 56 multiplets and is therefore heavier. The situation is illustrated schematically in Figure 6. Rebbi's L=1 [70] is unaffected by this phenomenon. He finds states of the first L=1 [70] in the vicinity of 1400 MeV, while states of the first L=1 [56] are at about 1700 MeV.[35]

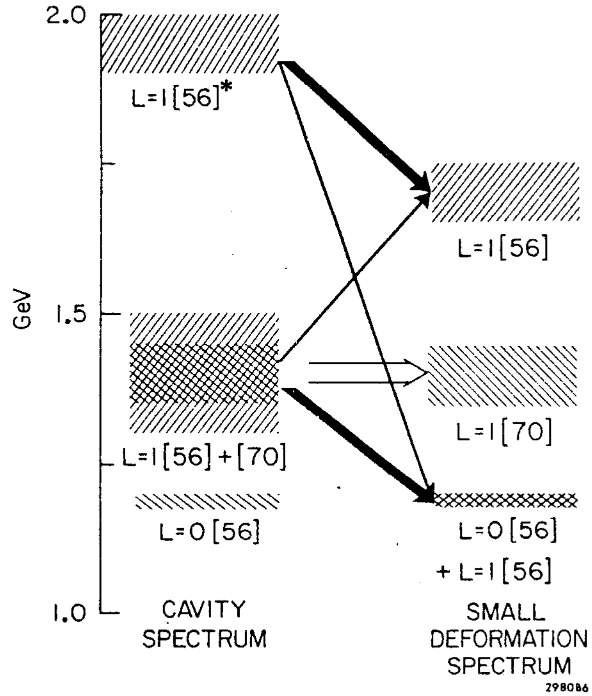

Figure 6. Schematic comparison of Rebbi's spectrum with that of a fixed cavity. The arrows denote overlap of Rebbi's states with cavity eigenstates. Multiplets are denoted by hatching rather than sharp lines to emphasize internal splittings.

DeGrand[36] has revised the calculations of Reference 33 in light of Rebbi's observation. He writes the states of the L=1 [70] in the $S^2_{1/2}P_{3/2}$, $S^2_{1/2}P_{1/2}$ basis. According to Rebbi the [70] may be reliably estimated using the spherical cavity wave functions and Hamiltonian. DeGrand then diagonalizes the cavity Hamiltonian in the L=1 [70] basis. The lightest physical L=1 [56] lies higher but cannot be reliably treated with cavity techniques. DeGrand's spectrum looks more like the known light negative parity baryons than that of Reference 33 but the L-S splittings remain too large and the multiplet as a whole is too light.

This is the second difficulty I mentioned. A glance back at Figure 2 puts the problem in perspective. For m=0 the $P_{3/2}$-$P_{1/2}$ splitting is nearly as large as eithers' splitting from the S-wave. To reduce this L-S splitting one would like to increase the quark mass. On the other hand as the quark mass is increased the distinction between S and P waves disappears and the L=1 [70] descends even further relative to the L=0 [56]. Something else must be happening. Either the L-S splittings of very light quarks are for some reason suppressed or the excitation spacing of heavy quarks is for some reason enhanced.

It is possible that a careful treatment of gluon interactions and zero point fluctuations in the context of Rebbi's small deformation approximation will mimic this behavior. But that is a very hard calculation. At the moment this piece of the phenomenology remains inexplicable from the viewpoint of the bag model.

To conclude: the bag-quark model is well-defined if approximate. It can be used as a tool to study some problems (e.g., multiquark states) which less well-defined schemes cannot handle. Many multiquark states may be obliterated by dynamical effects—e.g., peripherality and unitarity—presently outside the scope of the bag model. Those that are not should be seen. If so it will be necessary to revise conventional resonance classification schemes. If not, the model will be in trouble.

So far the bag model has only begun to shed some light on the puzzles of the GAQM. At least we can see an ordering: L=0 [56], L=1 [70], emerging from a dynamics which is neither nonrelativistic nor based on instantaneous two body interactions. Other quark model regularities—like the absence of large L-S splittings—remain, as yet, a mystery to us.

References

1. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).
2. A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).
3. T. DeGrand, R. J. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D 12, 2060 (1975).
4. See J. J. J. Kokkedee, The Quark Model (W. A. Benjamin, New York, 1969); H. J. Lipkin, Physics Reports C 8, 173 (1973); we include in this category $SU(6)_W$ and its various extensions discussed, for example, by J. L. Rosner, Physics Reports C 11, 189 (1974).
5. We could proceed from the Lagrangian formulation of field theory. The stress tensor approach is physically more transparent.
6. R. L. Jaffe, Phys. Rev. D 11, 1953 (1975); R .L. Jaffe and A. Patrascioiu, Phys. Rev. D 12, 1314 (1975).
7. H. D. Politzer, Phys. Rev. Letters 26, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Letters 26, 1343 (1973).

8. C. Rebbi, Phys. Rev. D 12, 2407 (1975); MIT preprint "The Spectrum of P-Wave Baryonic Excitations in a Model with Field Confinement," MIT-CTP-551.

9. One can ask what quark masses mean in a world where free quarks do not exist. For our purposes it suffices that they are a measure of the need for relativistic kinematics in our calculations.

10. Should a color singlet hadron attempt to fission into two color nonsinglets gluon "electric" forces would arise as color charge separated and prevent the fissioning. See Reference 1 for a graphic description of this.

11. H. Pagels, Physics Reports C 16, 219 (1975).

12. This seems to have been noticed first by P. N. Bogoliubov, Ann. Inst. H. Poincaré 8, 163 (1967); and independently rediscovered in Reference 2.

13. See, for example, J. L. Rosner, Reference 4.

14. See, for example, S. D. Drell and K. Johnson, Phys. Rev. D 6, 3248 (1972).

15. K. Johnson, Lectures at the XV Cracow School of Theoretical Physics, Zakopane, 1975; MIT preprint MIT-CTP-494.

16. V. F. Weisskopf, Lectures at the 1975 Summer School in Subnuclear Physics, "Ettore Majorana," Erice, 1975; CERN preprint Ref. TH. 2068-CERN.

17. One may give them a small mass to get g_A/g_V just right. The spectroscopy is insensitive to this modification, see E. Golowich, Phys. Rev. D 12, 2108 (1975); J. Donoghue and E. Golowich, Phys. Rev. D 12, 2875 (1975); D. J. Broadhurst, "Static Properties and Structure Functions of a Bag of Finite Mass Quarks," Open University preprint (August 1975).

18. The notation is borrowed from atomic spectroscopy. Relativistically only j and the parity are good quantum numbers. $S_{1/2}$ denotes the even parity $j=1/2$ mode which reduces to $\ell=0$ nonrelativistically. The $P_{1/2}$ and $P_{3/2}$ modes reduce to $\ell=1$ in the nonrelativistic limit.

19. H. Casimir, K. Ned. Akad. Wet. Versl. Gewone Vergod. Afd. Natuurkd. Ser. B., 793 (1948). See also Reference 15.

20. Other contributions—the electric energy and the self energy graphs—are either small (because the states are color singlets) or included in the definitions of the observable (renormalized) parameters of the model.

21. The quoted result refers to S-waves only. The analogous expressions for P-wave quarks were worked out in References 33 and 36.

22. A deRujula, H. Georgi and S. L. Glashow, Phys. Rev. D 12, 147 (1975) were the first to point this out (in the context of a nonrelativistic quark model).

23. z_0 parametrizes the energy of zero point fluctuations. The radius, R, of the cavity to which the fields are confined is not a free parameter. It is fixed for each hadron by a boundary condition.

24. Y. Nambu in Preludes in Theoretical Physics, ed. by A. de Shalit, H. Feshbach and L. van Hove (North Holland, Amsterdam, 1966).

25. R. L. Jaffe and K. Johnson, Phys. Letters B 60, 201 (1976). R. L. Jaffe, Stanford Linear Accelerator Center preprints, "Multiquark Hadrons I and II", SLAC-PUB-1772 and SLAC-PUB-1773 (1976).

26. We refer to the SU(2) of S-wave j=1/2 quarks loosely as spin.

27. The 35 generators of colorspin are

$$\frac{2}{3}\sigma^k;\ \lambda^a;\ \sigma^k\lambda^a;\ k=1,2,3;\ a=1,2\ldots 8 \quad .$$

The Casimirs are defined by

$$C_3(Q) = \sum_{a=1}^{8}\left(\sum_{i=1}^{N_Q}\lambda_i^a\right)^2 ; \quad C_6(Q) = \sum_{\mu=1}^{35}\left(\sum_{i=1}^{N_Q}\alpha_i^\mu\right)^2 \quad .$$

28. N_Q=3n is an exception. $Q^3\bar{Q}^3$ is the lowest configuration affected. The ground state is $\underline{8}\otimes\underline{8}$ in flavor. These states are heavy, broad and strongly coupled to 3-pseudoscalars, so they are most likely spectroscopically irrelevant.

29. N. M. Cason et al., "Observation of a New Scalar Meson," University of Notre Dame preprint (1976).

30. J. T. Carroll et al., Phys. Rev. Letters 28, 318 (1972); P. Estabrooks et al., in Proc. of the International Conference on $\pi\pi$ Scattering and Associated Topics at Tallahassee, ed. by P. K. Williams and V. Hagopian (AIP, New York, 1973).

31. The attractive magnetic energy in the lightest $Q^4\bar{Q}$ multiplet may be attributed to the $Q\bar{Q}$ and Q^3 components of the state primarily being in magnetically attractive (pseudoscalar rather than vector, octet rather than decuplet) configurations themselves. This is the case among $Q^2\bar{Q}^2$ mesons.

32. M. Davier and H. Harari, Phys. Letters B 35, 239 (1971).

33. T. A. DeGrand and R. L. Jaffe, MIT preprint MIT-CTP-529.

34. See, for example, R. F. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D 10, 4114 (1975); J. Goldstone and R. Jackiw, Phys. Rev. D 11, 1486 (1975).

35. It is amusing that the Particle Data Tables persistently show a $\Delta(5/2^-)$ at 1960 MeV. This is the one L=1 [56] state not also found in the L=1 [70].

36. T. A. DeGrand, MIT preprint MIT-CTP-543.