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A SIMPLE DESCRIPTION OF THE $J^P = 1^+ K^\pm \pi^+ \pi^-$ SYSTEM IN THE REACTIONS $K^\pm p \to K^\pm \pi^+ \pi^- p$ *

R. K. Carnegie,† R. J. Cashmore,†† M. Davier,‡ W. M. Dunwoodie, T. A. Lasinski, D. W. G. S. Leith and S. H. Williams

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A model in which Q_1 and Q_2 resonance contributions add coherently to a gaussian background is shown to reproduce the mass dependence of the $J^P = 1^+ K^*\pi$ and ρK partial waves in $K^\pm p \to K^\pm \pi^+ \pi^- p$ at 13 GeV/c. Through a fit to the data, the mass and total width for Q_1 are found to be $m = 1289 \pm 3 \pm (25)$ MeV, $\Gamma = 150 \pm 9 (\pm 70)$ MeV and for Q_2, $m = 1404 \pm 3 (\pm 10)$ MeV, $\Gamma = 142 \pm 4 (\pm 15)$ MeV, where estimated systematic errors are given in parentheses. While a significant background is required for the $1^+ K^*\pi$ system, none is needed for the $1^+ \rho K$ system.

*Work supported by the Energy Research and Development Administration.
†Now at the Physics Department, Carleton University, Ottawa, Ontario, Canada K15 5B6.
††Now at the Physics Department, Oxford University, Oxford, England.
‡Now at Laboratoire de l'Accélérateur Linéaire, Orsay, France.

In previous letters [1, 2] we have presented results for the $J^P = 1^+ K^*\pi$ and ρK partial waves obtained from an analysis of the $K\pi\pi$ system observed in the reactions

$$K^{\pm}p \rightarrow K^{\pm}\pi^{+}\pi^{-}p \tag{1}$$

at 13 GeV/c. The structure observed in the mass dependence of the cross section and relative phase of these 1^+ partial waves was qualitatively interpreted as due to the presence of two axial vector Q mesons and a low mass "Deck" background. In this paper we demonstrate that a straightforward model which coherently adds background and two resonance contributions does indeed quantitatively reproduce all the mass dependent features of these partial waves. From fits of this model to the data, we obtain values for the mass, width, and decay couplings of the Q_1 and Q_2 mesons. A good knowledge of these parameter values for the Q mesons provides an important clue in predicting the properties of the missing axial vector mesons such as the A_1.

We first review those features of the data motivating a description in terms of two resonances and a background contribution. The $1^+ K^*\pi$ and ρK partial waves are shown as a function of $K\pi\pi$ mass in figs. 1 and 2. The wave notation $J^P M^\eta$ denotes the $K\pi\pi$ spin, parity, magnetic substate, and the exchange naturality. The Q_1 meson is associated with the peak ~200 MeV wide in the $1^+\rho K$ partial waves centered at ~1300 MeV (fig. 2). If Q_1 has a small $K^*\pi$ coupling, the large forward phase variation (~70° relative to $1^+0^+K^*\pi$) which these waves exhibit in this region would substantiate a resonance interpretation. The Q_2 meson is associated with the structure observed in the cross section for the $1^+0^+K^*\pi$ waves at ~1400 MeV with a width of ~160 MeV (fig. 1a, b). Assuming Q_2 has a small coupling to ρK, the backward relative phase motion of the $1^+\rho K$ waves (fig. 2) in the 1400 MeV region is further evidence for the second resonance. In addition, the 2^+1^+ $K^*\pi$ wave describing the K*(1420) meson shows little phase variation relative to the 1^+0^+ $K^*\pi$ wave (fig. 1e,f). Finally the large peak in the 1^+ $K^*\pi$ waves at ~1200 MeV is attributed to a "Deck" background contribution.

We now introduce a simple model which describes the experimental features of the data† summarized above. We write each partial wave amplitude, N_{ij}, as the sum of a background or "Deck" component, D_{ij}, and two resonance contributions, R_{ij},

$$N_{ij} = D_{ij} + R_{ij}^{(1)} + R_{ij}^{(2)} \quad . \tag{2}$$

The subscript i denotes $J^P M^\eta$ and j, the isobar channel ($K^*\pi$ or ρK). We assume that each resonance contribution factorizes into a "production" part depending only on p-p momentum transfer (t') and a Breit-Wigner part depending only on $K\pi\pi$ mass m:

$$R_{ij}^{(n)} = a_i^{(n)} e^{i\phi_i^{(n)}} \frac{C_j \gamma_j^{(n)} \langle q_j^{2L_j+1} \rangle^{1/2}}{m_n^2 - m^2 - i m_n \Gamma_n} \quad , \qquad n=1,2 \; . \tag{3}$$

The total width, Γ_n, is related to the reduced partial width couplings, $\gamma_j^{(n)}$, by

$$m_n \Gamma_n = \sum_j \left(\gamma_j^{(n)}\right)^2 \langle q_j^{2L_j+1} \rangle / m \quad . \tag{4}$$

In eqs. (3) and (4), $\langle q_j \rangle$ denotes the momentum of the Q meson decay products averaged over a Breit-Wigner intensity for the isobar, and L_j is the orbital angular momentum between the decay products. The C_j are isospin coefficients for the $K^*\pi$ (-2/3) and ρK ($1/\sqrt{3}$) channels. The parameters $a_i^{(n)}$, $\phi_i^{(n)}$ correspond

†Similar features have also recently been observed in an analysis combining K-bubble chamber data at 10, 14, and 16 GeV/c (ref. [3]).

to the production part of the amplitude averaged over the momentum transfer interval $-t' < 0.3$ GeV^2. We emphasize that the features of the $K\pi\pi$ partial waves when studied as a function of t' differ significantly with beam particle [2] and isobar channel [2, 3]. Consequently the only constraint we place on the $a_i^{(n)}$, $\phi_i^{(n)}$ is that resonance production be independent of decay channel. The parametrization of eq. (3) was chosen so that the different production properties [2, 3] of the $1^+K^*\pi$ and $1^+\rho K$ systems may be accommodated by further parametrizing $a_i^{(n)}$, $\phi_i^{(n)}$ as functions of t'. For the background contribution we write

$$D_{ij} = A_{ij}\, e^{i\Phi_{ij}} \langle q_j \rangle^{1/2} \exp\left[\frac{-(m - m_{0ij})^2}{2\sigma_{ij}^2}\right] . \tag{5}$$

Here we assume a gaussian mass dependence for the "Deck" contribution and specify the averaged t' dependence by means of A_{ij} and Φ_{ij}.

For each partial wave the cross section is given by

$$\left.\frac{d^2\sigma}{dmdt'}\right|_{ij} = |N_{ij}|^2 / C^2_{ij,k\ell} \tag{6}$$

and the relative phase between partial waves by

$$\phi_{rel} = \arg\,(N_{ij} N^*_{k\ell}) \quad . \tag{7}$$

In eq. (6) $C_{ij,k\ell}$ is the coherence parameter between two waves defined by

$$C_{ij,k\ell} = \frac{|\rho_{ij,k\ell}|}{|\rho_{ij,ij}|^{1/2}\, |\rho_{k\ell,k\ell}|^{1/2}} \quad , \tag{8}$$

where $\rho_{ij,k\ell}$ is the density matrix element between waves ij and $k\ell$. The observation [5] that the 1^+0^+, 1^+1^+ and $2^+1^+K^*\pi$ waves are to a good approximation coherently produced relative to one another leads us to set the coherence parameter to the value one for these waves. For the 1^+0^+ and $1^+1^+ \rho K$ waves, we use values of the coherence parameter consistent with our measurements.†

The values of the parameters are determined by least squares fits to the K^- and K^+ data separately, with the resonance parameters $\gamma_j^{(n)}$ and m_n constrained to be the same in both cases. The fits were performed using the data over the full mass range for the $1^+K^*\pi$ partial waves, the region $1.21 \leq m \leq 1.46$ GeV for the $1^+\rho K$ waves, and $1.33 \leq m \leq 1.50$ GeV for the $2^+1^+ K^*\pi$ waves. Phase data for $1^+1^+K^*\pi$ were not used for $m \leq 1.10$ GeV. To use eqs. (2)-(5) in fits to the data, one phase parameter for both K^+ and K^- must be fixed. We chose to make D_{ij} purely real for the $1^+0^+K^*\pi$ amplitude ($\Phi(1^+0^+K^*\pi) = 0$). Thus all phases are measured relative to this background component. The K*(1420) mass and width were fixed at the nominal values [4].

The curves resulting from our fit are shown in figs. 1 and 2. The overall χ^2 is 772 for 252 data points. The errors are statistical only; systematic uncertainties due to the neglect of small partial waves and isospin 3/2 contributions, for example, have not been included. The overall agreement of the curves with the data is satisfactory, the most noticeable systematic discrepancy being in the 1.2 GeV region for the $1^+0^+K^*\pi$ waves. A somewhat more flexible background than eq. (5) does do better in describing the low mass peaks. In addition we recall that there are ambiguous solutions [1] in the K^- case for $1.14 \leq m \leq 1.25$ GeV; these are indicated by crosses in figs. 1 and 2. The contributions of

†The $1^+\rho K$ system is found [1, 3] not to be coherently produced relative to $1^+0^+K^*\pi$. The coherence parameters (with $k\ell = 1^+0^+K^*\pi$) used for the $1^+\rho K$ waves are for K^+, 0.75 ($1^+0^+\rho K$), 0.85 ($1^+1^+\rho K$) and for K^-, 0.76 ($1^+0^+\rho K$), 0.87 ($1^+1^+\rho K$). The measured t-channel coherence parameters [5] are essentially independent of $K\pi\pi$ mass. We nevertheless identify the $1^+\rho K$ relative phase data with our model through eq. (7). This is justified provided the magnitude of the true [5, 6] $1^+0^+K^*\pi$ nucleon helicity nonflip amplitude is large compared to that of the helicity flip amplitude.

background and Q_2 (Q_1 is negligible) to $1^+0^+K^*\pi$ are shown in figs. 1a and b by dotted and dashed curves, respectively. Our background is considerably narrower than that expected for "Deck" mechanisms. However, it has been noted by several authors [7, 8] that a "Deck" amplitude should in principle be unitarized in the presence of $K^*\pi$ or ρK resonance effects; such a procedure could modify a broad input background to agree with that found here. The model successfully describes the different K^+ and K^- mass dependence of both 1^+0^+ and 1^+1^+ $K^*\pi$ waves near 1400 MeV. We note that the incoherently combined background and resonance contributions account for no more than ~60% of the observed $1^+0^+K^*\pi$ cross sections, indicating sizeable interference contributions.

The $1^+\rho K$ partial cross sections in fig. 2 are reproduced essentially by a single Breit-Wigner corresponding to Q_1. Any attempt to include a background of the form eq. (5) is rejected by the fit, essentially because of the large relative phase motion between the $1^+0^+K^*\pi$ and $1^+\rho K$ waves. The <u>absolute</u> phase of the model amplitude for the $1^+0^+K^*\pi$ wave does not start to move until about 1.3 GeV, where it then increases according to the Breit-Wigner description of Q_2 to about 110^o at 1.5 GeV. Thus the <u>relative</u> phase motion shown in fig. 2 corresponds essentially to the difference of two Breit-Wigner phase curves for resonances of similar widths but separated by ~100 MeV in mass.

The parameters of our fit are presented in tables 1-2. The partial widths of Q_1 and Q_2 (table 2) were calculated as indicated in eq. (4) using $\langle q_{K^*}\rangle = 306$ MeV, $\langle q_\rho\rangle = 141$ MeV for Q_1 and $\langle q_{K^*}\rangle = 395$ MeV, $\langle q_\rho\rangle = 279$ MeV for Q_2. The errors in parentheses for the resonance parameters (table 2) are estimates of systematic uncertainties in our parametrization. They are based on the spread of values we have observed from fits with different width parametrizations, different background shapes, and different choices of K^- solutions in the ambiguous region [1]. The large systematic uncertainty in $\Gamma_{\rho K}(Q_1)$ is due principally to the fact that removing the factor of m in eq. (4) results in a larger value for this width; this sensitivity to width parametrization for Q_1 is not surprising in view of its proximity to the nominal ρK threshold. In addition we note that there is some coupling [5] of Q_1 to $\kappa\pi$ and, possibly ϵK; the inclusion of these smaller modes in our fit would decrease $\Gamma_{\rho K}(Q_1)$ by ~25%, while leaving the total width unchanged. Similarly, $\Gamma_{K^*\pi}(Q_2)$ would decrease by ~20% were the ϵK channel [5] included in the fit. The most striking feature of the present results is the nearly complete decoupling of Q_1 from the $K^*\pi$ mode and of Q_2 from the ρK mode. In a conventional mixing scheme for Q_A and Q_B, this decoupling pattern of Q_1 and Q_2 suggests a mixing angle near 45^o (modulo π) and is not unexpected from quark model estimates [9].

In conclusion we have presented a simple model which quantitatively reproduces the features in the mass dependence of the $1^+ K^*\pi$ and ρK partial waves. We emphasize that the structure of the model allows for the different production features observed in the data for the $1^+ K^*\pi$ and ρK systems [2, 3] as well as the difference between the K^+ and K^- reactions [2]. We find that the $1^+ K^*\pi$ system is described by a coherent sum of contributions from a low mass (~1.2 GeV) background and, essentially, a single resonance (Q_2) with a mass of 1404 MeV and a width of 142 MeV. In contrast, the $1^+\rho$K system is almost entirely described in terms of a single resonance (Q_1) of mass 1289 MeV and width 150 MeV. While the model we have presented is admittedly not unique [7,8], it nevertheless provides a simple description of the data. In addition it demonstrates that the inclusion of interference effects between the resonance production and "Deck" background amplitudes is probably essential to a quantitative understanding of the experimental measurements.

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FIGURE CAPTIONS

1. Comparison of the 1^+0^+, 1^+1^+, and $2^+1^+ K^*\pi$ cross sections and relative phases with the results (solid curves) of the fit described in the text. The reference wave is $1^+0^+K^*\pi$. The crosses denote the K^- ambiguous solutions (lower likelihood). The dotted and dashed curves in (a) and (b) correspond to the background and Q_2 contributions, respectively.

2. Comparison of the 1^+0^+ and $1^+1^+ \rho K$ cross sections and relative phases with the results of our fit. The reference wave is $1^+0^+K^*\pi$. The crosses denote the K^- ambiguous solutions (lower likelihood).

Table 1

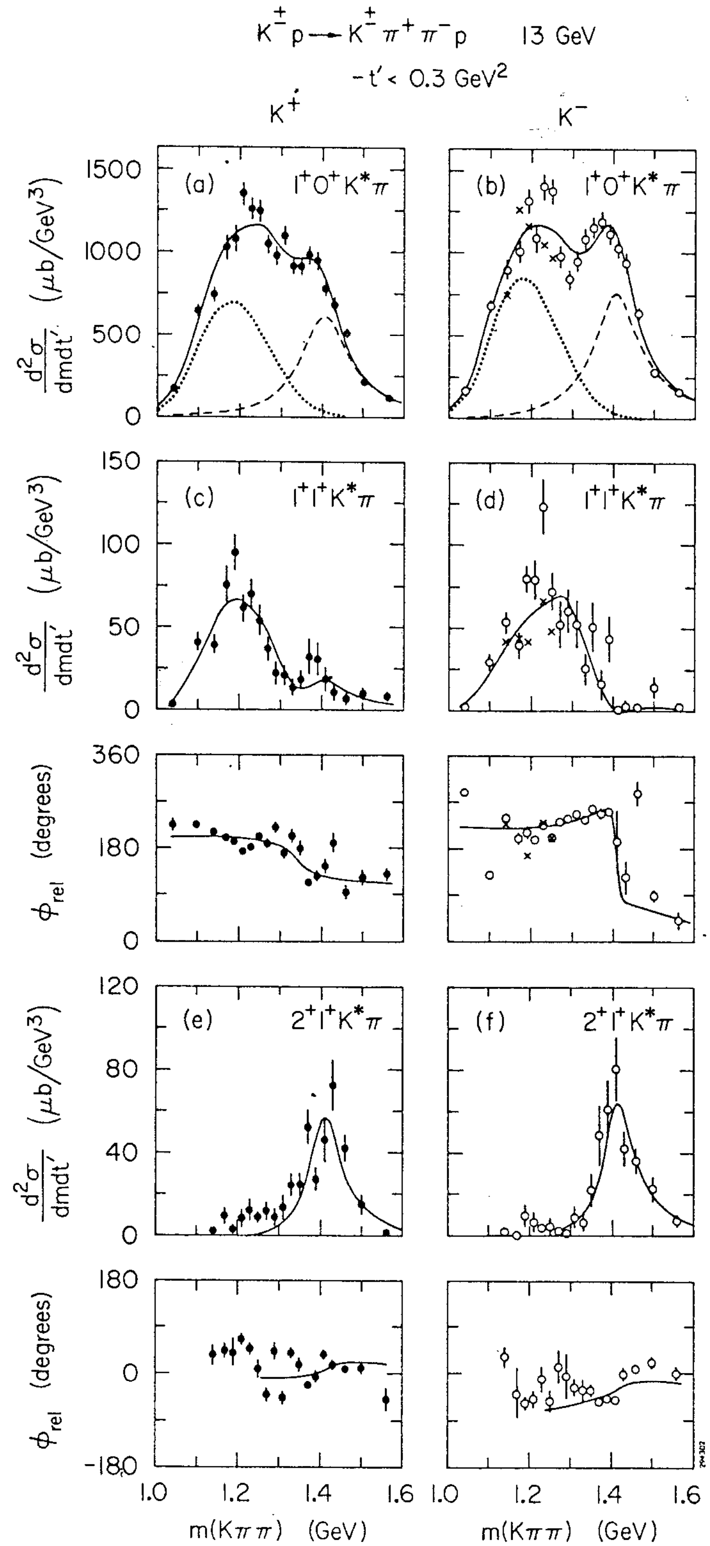
Background parameters.

$J^P M$	Beam	$A(\mu b^{\frac{1}{2}}/GeV^2)$	Φ(deg.)	m_0(MeV)	σ(MeV)
1^+0^+	K^+	60.0±0.9	0	1135±4	134± 4
	K^-	68.5±1.3	0	1136±3	131± 4
1^+1^+	K^+	18.7±0.5	-154±2	1154±5	98± 4
	K^-	17.5±0.5	-151±3	1190±9	147±11

Table 2

Resonance parameters. The total width for Q_1 is 150 ± 9 (±70) MeV and for Q_2, 142 ± 4 (±15) MeV. Estimated systematic errors for masses and widths are given in parentheses.

Meson	$J^P M^\eta$	Beam	$a (\mu b^{\frac{1}{2}}/GeV)$	ϕ (deg.)	m (MeV)	$\Gamma_{K^*\pi}$ (MeV)	$\Gamma_{\rho K}$ (MeV)
Q_1	1^+0^+	K^+	11.2 ± 0.5	-79 ± 5	1289 ± 3 (±25)	5 ± 3 (±5)	145 ± 9 ± (70)
		K^-	8.7 ± 0.5	-88 ± 4		(γ = -167 ± 44)	(γ = 1309 ± 42)
	1^+1^+	K^+	6.7 ± 0.3	108 ± 4			
		K^-	6.3 ± 0.3	71 ± 4			
Q_2	1^+0^+	K^+	13.8 ± 0.3	139 ± 9	1404 ± 3 (±10)	140 ± 4 (±15)	2 ± 1 (±2)
		K^-	15.6 ± 0.3	100 ± 6		(γ = 836 ± 11)	(γ = 126 ± 25)
	1^+1^+	K^+	2.1 ± 0.2	-118 ± 12			
		K^-	2.5 ± 0.3	112 ± 4			
K*(1420)	2^+1^+	K^+	3.6 ± 0.1	158 ± 6	1420	Γ_{tot} = 100 MeV (γ = 1.03 GeV^{-1})	
		K^-	3.7 ± 0.2	85 ± 4			



K± p → K± π+ π− p 13 GeV
−t' < 0.3 GeV²
K+
K−
(a) 1+0+K*π
(b) 1+0+K*π
(c) 1+1+K*π
(d) 1+1+K*π
(e) 2+1+K*π
(f) 2+1+K*π
d²σ/dmdt' (μb/GeV³)
φrel (degrees)
m(Kππ) (GeV)

Fig. 1

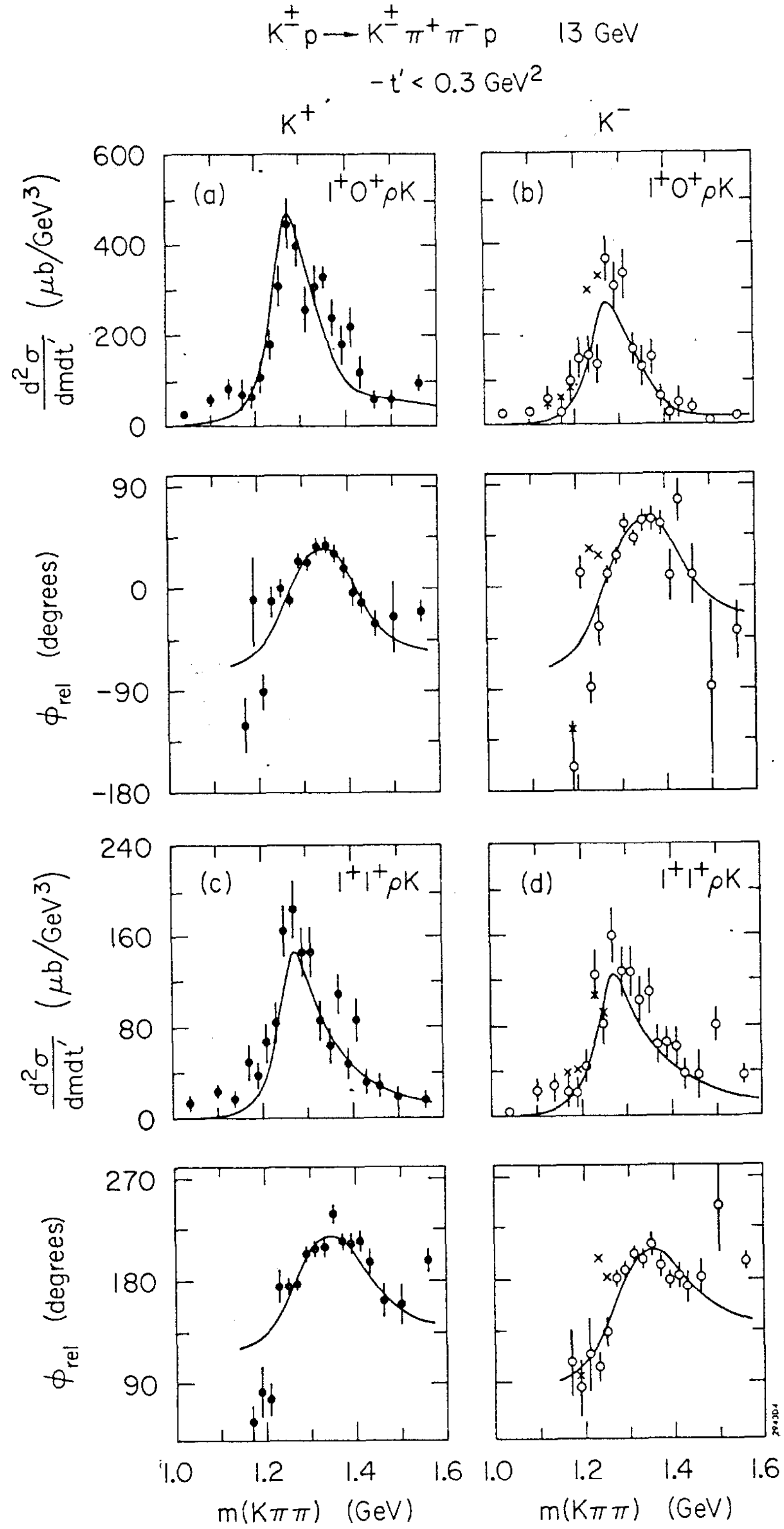

$K^{\pm}p \rightarrow K^{\pm}\pi^+\pi^-p$ 13 GeV
$-t' < 0.3$ GeV^2
K^+
K^-
(a) $1^+0^+\rho K$
(b) $1^+0^+\rho K$
(c) $1^+1^+\rho K$
(d) $1^+1^+\rho K$
$\frac{d^2\sigma}{dmdt'}$ (μb/GeV^3)
ϕ_{rel} (degrees)
600
400
200
0
90
0
-90
-180
240
160
80
0
270
180
90
1.0
1.2
1.4
1.6
$m(K\pi\pi)$ (GeV)

Fig. 2