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# EEFECTS OF NEUTRAL WEAK CURRENTS IN $e^+$-$e^-$ ANNIHILATION*

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ABSTRACT

The effects of neutral weak currents are discussed in $e^+e^-$ annihilation into final states such as $\mu^+\mu^-$, $e^+e^-$, $\pi^+\pi^-$, many hadrons and also in the one particle inclusive production. According to current gauge models these weak effects which are manifested as various asymmetries, turn out to be around 10% in the energy range of PEP and PETRA. In identifying weak effects polarized incident beams are advantageous, in particular, the longitudinally polarized beams select parity violating asymmetries.

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In the past few years several proposals [2, 47, 48] have been made for detecting asymmetries coming from neutral weak currents in $e^+e^-$ collisions. Since important experimental evidence has been found in favor of the neutral weak current [1, 2], the experimental investigation of these weak asymmetries will provide further statistics as well as essential details on the structure of neutral weak currents allowing comparisons of the results with predictions of several theoretical models. In general, the photon-neutral weak boson Z interference influences the $e^+e^-$ annihilation in such a way that characteristic asymmetries should grow with the center-of-mass energy squared, therefore the high energy region is most promising for finding effects caused by Z. In gauge models, like the Weinberg-Salam, Lee-Prentki-Zumino ones, etc., the weak asymmetries are about 10% in the total center-of-mass energy range of 20-40 GeV.

From the theoretical point of view the simplest treatments concern final states of $\mu^+\mu^-$, $e^+e^-$ which avoid all the hadronic complications. Here, weak effects cause forward-backward and charge asymmetries, as well as outgoing particles that appear with nonvanishing average helicities due to $\gamma$-Z interference. Transversely or longitudinally polarized incident beams make the separation of weak terms easier and sometimes they enhance the effects. In particular, longitudinally polarized beams identify parity violating terms in the cross section.

The kinematics of $\gamma$-Z interference has been calculated also for two hadrons, many hadron final states and one particle inclusive production. The measurability depends crucially upon the luminosity and time-like electromagnetic form factors at high energies. Furthermore, at present the theoretical estimates for weak effects are rather uncertain for one particle inclusive production through $e^+e^-$

annihilation because of the nature of the scaling violation at higher energies and of the form factors of the Z-current.

By now, the existence of neutral weak currents is well established [1, 2] although many of their properties are still unknown. This has led to many experimental proposals (e.g., [2]); among others, colliding beam physics gives very clean possibilities for providing further statistics and establishing essential details about the underlying theoretical models. The expectation is that in the energy range of the next generation of $e^+e^-$ storage rings (PEP, PETRA), $q = \sqrt{s} \lesssim 40$ GeV, asymmetries due to neutral weak currents, will be about 10%. It is, however, not excluded that weak effects also manifest themselves at present energies.

The leptonic neutral weak currents can be traced in $e^+e^- \to \mu^+\mu^-$ [3 - 15], $e^+e^- \to e^+e^-$ [10, 11, 16-18] as well as in $e^\pm e^\pm \to e^\pm e^\pm$ [18-20], while the effects of hadronic neutral weak currents appear in exclusive hadronic final states [21-26] and also in inclusive hadron production [12, 27-37].

We use the Bjorken-Drell conventions [38].

A. Charge Asymmetry in $e^+e^- \to \mu^+\mu^-$

The relevant graphs are drawn in Fig. 1 corresponding to the reduced amplitudes

$$M_\gamma = -i \frac{1}{q^2} \langle \mu^+\mu^- | j_\nu^{elm} | 0 \rangle \langle 0 | j^{\nu elm} | e^+e^- \rangle \ ,$$

$$M_{Z_0} = -i \frac{g^{\alpha\beta} - q^\alpha q^\beta / m_Z^2}{q^2 - m_Z^2} \langle \mu^+\mu^- | j_\alpha^{Z'} | 0 \rangle \langle 0 | j_\beta^Z | e^+e^- \rangle \qquad (1)$$

where the hermitian electron (muon)-type neutral weak current $j^Z_\beta$ ($j^{Z'}_\alpha$) interacts with a neutral weak meson $Z_0$ of mass $m_Z$. Assuming pointlike leptons, the total amplitude is

$$M_\gamma + m_Z = -\frac{ie^2}{q^2}\bar{u}_3\gamma_\nu v_4 \quad \bar{v}_2\gamma^\nu u_1 - i\frac{g^{\alpha\beta}-q^\alpha q^\beta/m_Z^2}{q^2-m_Z^2}\bar{u}_3\gamma_\alpha(g'_V+g'_A\gamma_5)v_4 \cdot$$

$$\cdot\bar{v}_2\gamma_\beta(g_V+g_A\gamma_5)\,u_1 \quad . \tag{2}$$

In (2) only vector and axial vector currents are introduced [2] with coupling constants $g_V, g'_V$ and $g_A, g'_A$, respectively, $\mu$-e universality would require $g_V=g'_V$, $g_A=g'_A$.

If the $e^-$-beam is incident in the positive z-direction, $p_1^\nu = (E,0,0,E)$, and in the natural center-of-mass system of the storage ring $p_2^\nu = (E,0,0,-E)$, $\underline{p}_4 = -\underline{p}_3$. We assume, the $e^-$-$e^+$ beams are transversely polarized in the direction of the x-axis, $s_1^\nu = (0,\xi_1,0,0)$, $s_2^\nu = (0,\xi_2,0,0)$, $\xi_1\xi_2 < 0$ and $\underline{s}_1$ ($\underline{s}_2$) is antiparallel (parallel) to the guide magnetic field. The production angles $\theta,\phi$ of $\mu^-$ are defined by $p_3^\nu = (E, E\sin\theta\cos\phi,\ E\sin\theta\sin\phi,\ E\cos\theta)$; $q^2 = 4E^2$. We denote the helicities for the $\mu^-$ and $\mu^+$ by $h_3$ and $h_4$ respectively. Neglecting lepton masses from (2) the differential cross section is [7,21]

$$\frac{d\sigma}{d\Omega}(h_3,h_4) = \frac{\alpha^2}{16q^2}\Big\{(1-h_3h_4)\Big[F_1(1+\cos^2\theta) + F_2\xi_1\xi_2\sin^2\theta\cos 2\phi + F_3\cos\theta\Big] +$$

$$+ (h_3-h_4)\Big[F_4(1+\cos^2\theta) + F_5\xi_1\xi_2\sin^2\theta\cos 2\phi + F_6\cos\theta\Big]\Big\} \tag{3}$$

$\xi_1$ ($\xi_2$) is the polarization of the $e^-$ ($e^+$) beam. $F_{1-6}$ are defined by

$$F_1 = 1 + 2g_Vg'_VR + (g_V^2+g_A^2)\,(g'^2_V+g'^2_A)R^2$$

$$F_2 = 1 + 2g_Vg'_VR + (g_V^2-g_A^2)\,(g'^2_V+g'^2_A)R^2$$

$$F_3 = 4g_A g'_A R \left[1 + 2g_V g'_V R\right]$$

$$F_4 = 2g_V g'_A R + 2g'_V g'_A \left(g_V^2 + g_A^2\right) R^2$$

$$F_5 = 2g_V g'_A R + 2g'_V g'_A \left(g_V^2 - g_A^2\right) R^2$$

$$F_6 = 4g_A g'_V R + 4g_V g_A \left(g'^2_V + g'^2_A\right) R^2 \tag{4}$$

$$R = \frac{q^2}{e^2\left(q^2 - m_Z^2\right)}$$

Equation (3) yields the pure QED cross section if $F_1 = F_2 \to 1$ and $F_i \to 0$, and the other F terms describe the electromagnetic and weak interference as well as the pure weak effects. In the $q^2 \ll m_Z^2$ region the interference effects grow with $q^2$, therefore, the effects of $Z_0$ must be searched for at high energies.

A convenient way of identifying the weak interference is to measure the charge asymmetry

$$A_W^c = \frac{\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta)}{\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi + \theta)} \quad . \tag{5}$$

Summing over $\mu^{\pm}$ helicities, (3) gives

$$A_W^c = \frac{2F_3 \cos\theta}{F_1(1 + \cos^2\theta) - F_2 \,|\, \xi_1 \xi_2 \,|\, \sin^2\theta \cos 2\phi} \tag{6}$$

where $F_1 \approx 1$, $F_2 \approx 1$. $A_W^c$ measures the axial coupling $g_A g'_A$.

The maximum value of $A_W^c$,

$$A_W^c = \frac{F_3}{F_1} \frac{1}{\sqrt{1 - \left(\frac{F_2}{F_1} \xi_1 \xi_2 \cos 2\phi\right)^2}} \tag{7}$$

occurs at

$$\cos^2\theta = \frac{1 - \frac{F_2}{F_1}\,|\xi_1\xi_2|\cos 2\phi}{1 + \frac{F_2}{F_1}\,|\xi_1\xi_2|\cos 2\phi} \quad . \tag{8}$$

At $\phi=0$ or $\phi=\pi$, $A_W^c \approx 1.9\,F_3$ for maximum beam polarizations of $|\xi_1| = |\xi_2| = 0.924$.

To see the significance of the weak charge asymmetry let us remark that in the above case the forward-backward asymmetry is

$$\frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\frac{F_3}{F_1} \quad ,$$

where (9)

$$N_F = \int_0^{2\pi} d\phi \int_0^1 d(\cos\theta)\,\frac{d\sigma}{d\Omega} \quad ,$$

$$N_B = \int_0^{2\pi} d\phi \int_{-1}^0 d(\cos\theta)\,\frac{d\sigma}{d\Omega} \quad .$$

In order to have some insight into the numerical details of $A_W^c$, we invoke the Weinberg-Salam model [39, 40]. In this model $g_V = g'_V$, $g_A = g'_A$ and they are parametrized by the Weinberg angle $\Theta_W$

$$g_V = \frac{e\left(4\sin^2\Theta_W - 1\right)}{2\sin 2\Theta_W} \quad , \qquad g_A = \frac{e}{2\sin 2\Theta_W} \tag{10}$$

thus, to order $g_{V,A}^2/e^2$ and $q^2 \ll M_Z^2$

$$F_1 \doteq F_2 = 1 + 2\epsilon\left(4\sin^2\Theta_W - 1\right)^2 \quad ,$$

$$F_3 = 2\epsilon \quad , \tag{11}$$

$$\epsilon = \frac{q^2}{q^2 - M_Z^2}\,\frac{1}{4\sin^2 2\Theta_W} \approx \frac{-q^2 G_F}{8\sqrt{2}\,\pi\alpha} = -0.40\cdot 10^{-4}\,\frac{q^2}{M_P^2} \quad .$$

Consequently, in the present energy range, at $E = 3.5$ GeV, $A^c_{W\,max} \approx 3.8\epsilon \approx 1\%$, however, in PEP-region at $E = 15$ GeV, $A^c_{W\,max} \approx 15\%$, in the small angular region (see (8) ). In practice, it will be necessary to use detectors covering the entire angular region [41]. Because the decisive contribution to the total cross section comes from the photon piece, at 15 GeV we will still have about 200 events per day assuming an average luminosity of $2.5 \times 10^{31}$ cm$^{-2}$ sec$^{-1}$. The $\cos\theta$ asymmetry and relative change in rate $(\Delta F_1)$ are plotted in Fig. 2, using the Weinberg-Salam model at $E = 14$ GeV [7]. (From experiments $35^o \lesssim \Theta_W \lesssim 45^o$.)

The radiative corrections to the charge asymmetry arise from the interference of the one photon and two photon exchange diagrams as well as from the bremsstrahlung contribution [9,42]. In a medium-photon approximation [9] the electromagnetic charge asymmetry $A^c_{ELM}$ turns out to be dependent on $\ln(\Delta E/E)$ where $\Delta E$ is the energy resolution of the muon detector, on the production angles and $\xi_1\xi_2$. Typical curves of $A^c_{ELM}$ and $A^c_W + A^c_{ELM}$ are drawn in Fig. 3 showing a large electromagnetic background which dominates $A^c_W$. One can exploit, however, the strong energy dependence of $A^c_W$ at high energies. On the other hand, the large $\Delta E$-term of $A^c_{ELM}$ is independent of $\phi$, therefore, $A^c_{ELM}(\theta,\phi) - A^c_{ELM}(\theta,\phi')$ gives rise to only a small electromagnetic background when one measures the same difference for $A^c_{ELM+W}$.

B. Average Helicity of $\mu^-$

An alternative way of studying weak neutral currents is to measure the average helicity of the $\mu^-$ [3-6]

$$\langle h_3 \rangle = \frac{\sum_{h_4} \frac{d\sigma}{d\Omega}(+1,h_4) - \sum_{h_4} \frac{d\sigma}{d\Omega}(-1,h_4)}{\sum_{h_4} \frac{d\sigma}{d\Omega}(+1,h_4) + \sum_{h_4} \frac{d\sigma}{d\Omega}(-1,h_4)} \quad . \tag{12}$$

From (3) we have

$$\langle h_3 \rangle = \frac{F_4(1+\cos^2\theta) + F_5\xi_1\xi_2 \sin^2\theta \cos 2\phi + F_6 \cos\theta}{F_1(1+\cos^2\theta) + F_2\xi_1\xi_2 \sin^2\theta \cos 2\phi + F_3 \cos\theta} \quad . \tag{13}$$

In the Weinberg-Salam model

$$F_4 = F_5 = F_6/2 = 2\epsilon\left(4 \sin^2 \Theta_W - 1\right) \tag{14}$$

(neglecting $R^2$-terms). The asymmetry $\langle h_3 \rangle \rightarrow F_4$ can be seen in Fig. 2, the effect is about 3 - 8% at 15 GeV, $\sin^2 \Theta_W = 0.35 - 0.5$. The maximum value of $\langle h_3 \rangle$ is $2F_4$ for unpolarized beams and $2.9F_4$ for maximal incident polarizations. For experimental feasibility see [41].

There is an electromagnetic background to $\langle h_3 \rangle$ originating from $\gamma$-$2\gamma$ interference of the order $\alpha^3$ [43]. At $E \approx 10$-$15$ GeV this can be neglected relative to the weak term (13), furthermore, in certain angular regions the background vanishes (e.g., $\theta = \pi/2$, $\phi = 0$).

The effect of the Higgs particle turns out to be very small since the shift of the cross section (3) is proportional to $(m_e m_\mu)^2$.

If $2E \approx m_Z$, the width of the Z must be taken into account [6] by writing $(q^2 - m_Z^2)^{-1} \rightarrow (q^2 - m_Z^2 - i m_Z \Gamma)^{-1}$. So, the weak term in the cross section is enhanced by $m_Z^2/\Gamma^2$. This factor might be quite essential at resonance energy producing an increase of about four orders of magnitude in the weak cross section.

C. Longitudinally Polarized Beams

Parity violating effects can be projected out easily by making use of longitudinally polarized $e^+$-$e^-$ beams. If the spin vectors of $e^-, e^+$ are $s_1^\nu = p_-(\beta\gamma, 0, 0, \gamma)$, $s_2^\nu = p_+(-\beta\gamma, 0, 0, \gamma)$ with $\beta = |\underline{p}_1|/E$, $1/\gamma = \sqrt{1-\beta^2}$ then the

differential cross section of $e^+e^- \to \mu^+\mu^-$ has the form [12]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\Big[F_1(1+\cos^2\theta)(1+p_+p_-) + $$

$$+ F_3 \cos\theta\,(1+p_+p_-) + \Big(2\cos\theta\, F_4 + \frac{1}{2}(1+\cos^2\theta)\,F_6\Big)(p_+ + p_-)\Big] \tag{15}$$

($h_3, h_4$ are summed). $p_-$ ($p_+$) means the magnitude of the longitudinal polarization for $e^-$ ($e^+$).

The parity asymmetry would be indicated by a nonvanishing value of the longitudinal asymmetry $A^P$

$$A^P = \frac{\frac{d\sigma}{d\Omega}(\to\to) - \frac{d\sigma}{d\Omega}(\leftarrow\leftarrow)}{\frac{d\sigma}{d\Omega}(\to\to) + \frac{d\sigma}{d\Omega}(\leftarrow\leftarrow)} =$$

$$= \frac{p_+ + p_-}{1+p_+p_-}\;\frac{2\cos\theta F_4 + \frac{1}{2}(1+\cos^2\theta)F_6}{(1+\cos^2\theta)F_1 + \cos\theta\, F_3} \tag{16}$$

where the arrows indicate the polarization of the beams.

In the Weinberg-Salam model this has an order of magnitude given by For instance, at $E = 14$ GeV, $\sin^2\Theta_W = 0.35$ $2F_4 = 6\%$, a significant effect. However, in certain angular regions (16) is small, e.g., in the Weinberg-Salam model $A^P = O(g^4/e^4)$ for $\cos\theta = -1$.

For measuring the longitudinal asymmetry it is enough to have only one longitudinally polarized beam.

$A^P$ is a parity violating asymmetry, therefore the electromagnetic background will not disturb it.

Other interesting quantities which can be easily calculated from the above picture are $\langle h_3 \rangle$, $A^c$ and [13]

$$D(p_-, \theta) = \frac{\frac{d\sigma}{d\Omega}(0,0) - \frac{d\sigma}{d\Omega}(p_-, 0)}{\frac{d\sigma}{d\Omega}(0,0) + \frac{d\sigma}{d\Omega}(p_-, 0)} \tag{17}$$

where only the $e^-$-beam is polarized and (0, 0) denotes an unpolarized cross section. Here, we only remark the following. $\langle h_3 \rangle$ measures $g_A g_V'$ and $g_V g_A'$, the electromagnetic background is absent for $p_- = p_+$ (positive helicities) or $\theta = \pi/2$. At $\theta = \pi$ and $p_+ = p_-$, a measurement of $\langle h_3 \rangle$ would test the electron-muon universality [41]. $A_W^c$ depends on $\theta$, $p_\pm$, $q^2$; the higher order QED contribution does not contain $p_\pm$, hence, it is worthwhile to compare charge asymmetries with different beam polarizations (e.g., one unpolarized beam).

These quantities are plotted in Fig. 4 for $E = 15$ GeV, $\sin^2 \Theta_W = 0.4$. The largest effect is given by $A_W^c$, or by $\langle h_3 \rangle$.

In general, charged weak currents contribute to the asymmetries in higher order. It has been shown [14] that such effects are smaller by at least a factor $\alpha$ than the first order terms in the neutral weak current.

D. Bhabha Scattering

$e^+e^- \to e^+e^-$ is an advantageous process having a relatively large cross section. Unpolarized or transversely polarized incident beams, however, yield only very small effects due to neutral weak currents [11 ,16]. Namely, at $E = 14$ GeV, $\sin^2 \Theta_W = 0.38$ (about the expected $\Theta_W$)

$$\frac{d\sigma}{d\Omega} \left(\frac{d\sigma}{d\Omega}\right)^{-1}_{QED} - 1 \lesssim 1\% \ . \tag{18}$$

The longitudinal asymmetry is [16]

$$A^P = \frac{16\, E^2 g_V g_A (1+\cos\theta)^3 (1-\cos\theta)(p_+ + p_-)}{e^2 m_Z^2 \left\{(3+\cos^2\theta)^2 + \left[(3+\cos^2\theta)^2 - 16\right] p_+ p_-\right\}} \quad . \tag{19}$$

$A^P$ is drawn in Fig. 5 at $E = 14$ GeV for various $\sin^2\Theta_W$ and $p_+ = p_- = 0.924$. One can see that $A^P_{max}$ is about 5% for $\sin^2\Theta_W = 0.38$ ($\cos\theta \approx 1/5$). Thus, the realization of longitudinally polarized beams would be effective. A further discussion of the Bhabha scattering can be found in Ref. [17].

E. Møller Scattering

The process $e^-e^- \to e^-e^-$ is also sensitive to the neutral weak current. A general analysis of the weak corrections is provided by Ref. [20] where $\gamma$ and $Z_0$ exchanges are included with transverse and longitudinal beam polarizations.

The lowest order QED cross section for unpolarized beams is typically 399 nb at $E = 15$ GeV, $\theta = 10^o$, and 0.21 nb at $E = 15$ GeV, $\theta = 90^o$ [19]. The influence of the initial transverse polarization on this cross section and the weak electromagnetic interference term is very small, while for the radiative corrections it is a little larger at small scattering angles. For a soft photon cutoff of E/10, at $E = 15$ GeV, $\sin^2\Theta_W = 0.34$ the following results are found [19]: the relative shift in the cross section, due to $\alpha^3$-order QED corrections (weak electromagnetic interference) is estimated to be -21.1% (0) at $10^o$, -25.2% (1.3%) at $50^o$ and -26.1% (3.4%) at $90^o$. At present energies the weak shift is less than 1%. The longitudinal polarization of one of the final electrons is about 1% at 15 GeV, but there is an electromagnetic background of 1-2%, too.

Summarizing, in particular the $\mu^+\mu^-$ final state is promising for finding effects of neutral weak leptonic currents. At 15 GeV/beam measurable effects are expected, especially if longitudinally polarized beams can be developed.

F. Exclusive Hadronic Final States

The measurability of a two-hadron final state at 10 - 15 GeV depends on the energy dependence of the electromagnetic form factor which is not quite known. Therefore, it is not known a priori that weak effects will not manifest themselves at such high energies. For instance, assuming an $E^{-2}$ decrease for the electromagnetic form factor of the pion, 6 events/hour of $\pi^+\pi^-$ pairs at $E = 3$ GeV would be transformed into a few events per day assuming a one order of magnitude higher luminosity.

In what follows we consider effects of the hadronic neutral weak current $J_Z^\mu$ in the process $e^+e^- \to \pi^+\pi^-$ [24].

Three form factors enter into the relevant matrix elements

$$< \pi^+(p_4)\ \pi^-(p_3)\ |J_\gamma^\mu|0> = (p_3 - p_4)_\mu\ F \quad , \tag{20}$$

$$< \pi^+(p_4)\ \pi^-(p_3)\ |J_Z^\mu|0> = \left[(p_3 - p_4)_\mu\ F_V + (p_3 + p_4)_\mu \cdot H_V\right] \ , \tag{21}$$

$H_V$ measures the divergence of $J_Z^\mu$.

Let us assume that the incident $e^-$-beam runs in the positive z-direction, the scattering angles of $\pi^-$ are $\theta, \phi$ with respect to $e^-$. Denote the product of the polarization and the rest-frame spin vector of $e^-$ ($e^+$) by $\underline{n}$ ($\underline{n}'$), and also $q_\mu = (p_3 + p_4)_\mu$. The $\gamma + Z_0$ exchange gives for the reduced matrix elements

$$M_\gamma + M_Z = -ie^2 \frac{1}{q^2} (p_3 - p_4)_\nu\ \bar{v}\gamma^\nu \mu\ F\ -$$

$$-ig_Z \frac{g_{\alpha\beta} - \frac{q^\alpha q^\beta}{m_Z^2}}{q^2 - m_Z^2}\ \bar{v}\gamma_\alpha (g_V + g_A\gamma_5)\, u\left(F_V (p_3 - p_4)_\beta + H_V q_\beta\right) , \tag{22}$$

here $g_Z$ is the coupling constant of $J_Z^\mu$ to $Z_{0\mu}$ (one $g_Z$ is absorbed in $g_V$ and $g_A$).

The differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4e^2q^2}\left(1 - \frac{4m_\pi^2}{q^2}\right)^{1/2} T \quad ,$$

$$T = \left(\frac{q^2}{2} - 2m_\pi^2\right) q^2 \sin^2\theta \Bigg\{ \Big[ 1 + n_z n'_z + (n_y n'_y - n_x n'_x) \cos 2\phi - \\ - (n_x n'_y + n_y n'_x) \sin 2\phi \Big] A + 2g_A^2 \left(\frac{g_Z^2}{q^2 - m_Z^2}\right)^2 |F_V|^2 (1 + n_z n'_z) + \\ + \mathrm{Re}\, B\, (n_z + n'_z) + \Big[ \sin 2\phi\, (n_x n'_x - n_y n'_y) - \cos 2\phi\, (n_x n'_y + n_y n'_x) \Big] \mathrm{Im}\, B \Bigg\} \tag{23}$$

with

$$A = \left| \frac{e^2}{q^2} F + \frac{g_V g_Z}{q^2 - m_Z^2} F_V \right|^2 - g_A^2 \left(\frac{g_Z}{q^2 - m_Z^2}\right)^2 |F_V|^2 \quad ,$$

$$B = 2 \frac{e^2 g_Z g_A}{q^2 (q^2 - m_Z^2)} F^* F_V + 2 \left(\frac{g_Z}{q^2 - m_Z^2}\right)^2 g_A g_V |F_V|^2 \quad . \tag{24}$$

We have neglected $0(m_e/q)$ terms, $H_V$ drops out. Assuming TCP-invariance, $\mathrm{Im}\, B = 0$.

Special cases: 1. Transverse polarizations, $\underline{n} = (\xi_1, 0, 0)$, $\underline{n'} = (-\xi_2, 0, 0)$, $\xi_{1,2} > 0$ [21]. In this case the cross section depends on A and Im B which can be separated by identifying the $\phi$-dependence (azimuthal asymmetry). There is a large electromagnetic background to (23) [23].

2. Longitudinal polarizations, $\underline{n} = (0, 0, p_-)$, $\underline{n'} = (0, 0, p_+)$. Charge asymmetry is again vanishing. No $\phi$-dependence remains, but the weak term Re B can be separated by reversing the polarizations. The parity violating term in

the Hamiltonian is reflected by the presence of $A^P$ ((16)),

$$A^P = \frac{p_+ + p_-}{1 + p_+ p_-} \frac{\mathrm{Re}\, B}{A + 2g_A^2 \left(\frac{g_Z}{q^2 - m_Z^2}\right)^2 |F_V|^2} = A^P(\sigma_{tot}) \quad . \tag{25}$$

$A^P$ is determined by the ratio $F_V/F$. In many theories $F_V = cF$, c is a real constant. For instance, in the Weinberg-Salam model $c = \cos 2\Theta_W$ following from

$$J_Z^\mu = -2 \sin^2 \Theta_W J_\gamma^\mu + \frac{1}{2} \bar{q} R \gamma^\mu (1 + \gamma_5) q \tag{26}$$

where the matrix R and quark operator q are defined by

$$q = \begin{pmatrix} p' \\ p \\ n \\ 1 \end{pmatrix} , \qquad R = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} . \tag{27}$$

Putting $F_V/F = c$ into (25) yields

$$-A^P = -2\, c g_A g_Z \frac{q^2}{e^2 (q^2 - m_Z^2)} \left(1 - g_V c g_Z \frac{q^2}{e^2 (q^2 - m_Z^2)}\right) \frac{p_+ + p_-}{1 + p_+ p_-} + \\ + O\left(\left[g_Z q^2 / e^2 (q^2 - m_Z^2)\right]^3\right) \quad . \tag{28}$$

This form is plotted in Fig. 6 in the Weinberg-Salam model ($g_Z \sin 2\Theta_W = e$) at maximum polarizations, $q^2 \ll m_Z^2$, and it exhibits significant effects. Figure 7 shows $A^P$ in the Lee-Prentki-Zumino model [44,45].

The rate correction corresponding to (23) is shown in Fig. 8 [26] . The final state $p\bar{p}$ is analyzed in Refs. [21,26]. It is also shown that weak contributions to $e^+e^- \to e^+e^- \pi^+\pi^-$ can be completely neglected at high energies [26]. Furthermore, the polarization dependence of the QED cross section is inessential for the transverse case.

G. $e^+e^- \to \sum$ Hadrons in $\gamma + Z_0$ Exchange

The total cross section is determined by the spectral functions of currents, $\langle 0 | J^i_\mu(x)\, J^j_\nu(0) | 0 \rangle \to \rho^{ij}_{T,L}(s)$; $i, j = \gamma, Z$; T(L) is responsible for the transversal (longitudinal) part. Finally, we get [31]

$$\sigma(q^2) = \frac{\pi}{\sqrt{q^2(q^2-4m^2)}} \Bigg\{ \rho_T^{\gamma\gamma} \frac{e^4}{q^4} \left[q^2 - 2m^2(s's-1)\right] +$$

$$+ \left(\rho_T^{\gamma Z} + \rho_T^{Z\gamma}\right) \frac{e^2}{q^2} \frac{g_Z}{q^2-m_Z^2} \left[g_V(q^2 - 2m^2(s's-1)) + 2m\, g_A(p_2 s - p_1 s')\right] +$$

$$+ \left(\frac{g_Z}{q^2-m_Z^2}\right)^2 \Bigg\{ \rho_T^{ZZ} \Bigg[ g_V^2(q^2 - 2m^2(s's-1)) +$$

$$+ g_A^2 \left(q^2 - 4m^2 - \frac{4m^2}{q^2}(p_1 s')(p_2 s)\right) + 4 g_V g_A m\, (p_2 s - p_1 s') \Bigg] -$$

$$- \rho_L^{ZZ}\, 2m^2 \frac{\left(q^2-m_Z^2\right)^2}{q^2 m_Z^4} g_A^2 \left[q^2(1+s's) - 2(p_2 s)(p_1 s')\right] \Bigg\} \Bigg\} \qquad (29)$$

We can neglect the terms of order m/E. Equation (29) cannot be used around the mass of the Z-meson where finite width effects become important. $s, s'$ are the polarization four vectors for $e^-, e^+$ with the above convention for $\underline{n}, \underline{n}'$.

$$s^o = \frac{\underline{p}_1 \underline{n}}{m}, \qquad \underline{s} = \underline{n} + \frac{\underline{p}_1 \underline{n}}{m(E+m)} \underline{p}_1$$

$$s'^o = \frac{\underline{p}_2 \underline{n}'}{m}, \qquad \underline{s}' = \underline{n}' + \frac{\underline{p}_2 \underline{n}'}{m(E+m)} \underline{p}_2 \quad . \qquad (30)$$

For transverse polarizations ($\underline{p}_1\underline{n} = \underline{p}_2\underline{n}' = 0$, $\underline{n}\underline{n}' = -\xi_1\xi_2$, $\xi_1 > 0$, $\xi_2 > 0$) the $\xi$-dependence survives only in the last term of (29) which can be dropped out because $m^2 \rho_L^{ZZ}$ is negligible. Therefore, $\sigma(q^2)$ is independent of $\xi_{1,2}$.

However, for longitudinal polarizations ($\underline{p}_1\underline{n} = |\underline{p}_1|p_-$, $\underline{p}_2\underline{n}' = |\underline{p}_1|p_+$, $\underline{n}\underline{n}' = -p_+p_-$, $p_+ > 0$, $p_- > 0$) there remains a nontrivial $p_\pm$-dependence, whence

$$\sigma(\rightarrow\leftarrow) - \sigma(\leftarrow\rightarrow) = \frac{2\pi g_Z g_A (p_- - p_+)}{q^2 - m_Z^2}\left\{\frac{e^2}{q^2}\left(\rho_T^{\gamma Z} + \rho_T^{Z\gamma}\right) + \frac{2 g_Z g_V}{q^2 - m_Z^2}\,\rho_T^{ZZ}\right\} \tag{31}$$

measures the $\gamma$-Z interference and ZZ-terms.

In order to have a guess about the weak terms, let us assume one may calculate the spectral functions from free quark currents above $E \approx 10 - 15$ GeV. In general the Z-current can be decomposed as

$$J_Z^\mu = a\,J_\gamma^\mu + b\,V_R^\mu + c\,A_S^\mu \quad , \tag{32}$$

a, b, c are constants and

$$\begin{aligned} J_\gamma^\mu &= :\bar{q}\gamma^\mu Q q: \\ V_R^\mu &= :\bar{q}\gamma^\mu R q: \\ A_S^\mu &= :\bar{q}\gamma^\mu \gamma_5 S q: \end{aligned} \tag{33}$$

So that

$$\rho_T^{\gamma\gamma} = \frac{q^2}{12\pi^2}\,\mathrm{Tr}\,Q^2 \quad , \qquad \rho_T^{\gamma Z} = \rho_T^{Z\gamma} = \frac{q^2}{12\pi^2}\,(a\,\mathrm{Tr}Q^2 + b\,\mathrm{Tr}QR) \quad ,$$

$$\rho_T^{ZZ} = \frac{q^2}{12\pi^2}\,(a^2\,\mathrm{Tr}Q^2 + b^2\,\mathrm{Tr}R^2 + c^2\,\mathrm{Tr}S^2 + 2ab\,\mathrm{Tr}QR) \quad ,$$

$$\rho_L^{ZZ} = 0 \quad . \tag{34}$$

Substituting (34) into $\sigma(q^2)$, it follows that the cross section and the $\gamma$-Z interference exhibit a strong dependence on the details of the quarks and the

gauge model chosen. For example, in the Weinberg-Salam model

$$Q = \begin{pmatrix} \alpha & & & \\ & \alpha & & \\ & & \alpha-1 & \\ & & & \alpha-1 \end{pmatrix}, \qquad R = S = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix},$$

$$a = -2\sin^2\Theta_W \quad , \qquad b = c = \frac{1}{2} \quad . \tag{35}$$

The cross section gets large contributions from the $\gamma Z + ZZ$ terms in the region $\sin^2\Theta_W \approx 0.3 - 0.45$, they can reach even 50% (e.g., $\leftarrow\leftarrow$, $\alpha = \frac{2}{3}$ or $\frac{1}{2}$). Furthermore, we get large longitudinal asymmetries too (at maximum polarization). Similar results have also been found in the Lee-Prentki-Zumino model [31]. The role of Higgs scalars turns out to be negligible.

At energies high enough, $q^2 \ll m_Z^2$, the asymmetry parameter $A^P$ plays an important role. Numerical data are indicated in Fig. 9 for the case of Weinberg-Salam model with maximum polarizations and $m_Z \approx 70$ GeV. The longitudinal asymmetry possesses a strong dependence on the charge assignments of quarks. At $\sin^2\Theta_W \approx 0.4$, $A^P_{max} \approx -8\%$ for $E = 14$ GeV.

H. <u>One-Particle Inclusive $e^+e^-$-Annihilation</u>

According to Richter's analysis [41; p. 24], at PEP (15 GeV/beam) a large hadron yield is to be expected in the inclusive annihilation $e^+e^- \to hX$, h means a detected hadron with momentum $\underline{p}$. For a 30-day data run the yield is in turn $10^5$, $1.1\times10^4$, $5.3\times10^2$ at $|\underline{p}| = 1.5$, 6, 12 GeV.

The differential cross section of the one particle inclusive production has been calculated in Ref. [12] for arbitrary initial polarizations. It is not difficult to identify certain combinations of the structure functions as the charge asymmetry, forward-backward asymmetry, or the longitudinal asymmetry $A^P$. In this case, however, the numerical predictions depend on not only the nature of the neutral weak current but also the assumptions concerning the production of

the hadron h. As usual, assume that the $e^+e^-$ are coupled to a parton-antiparton pair by $\gamma$ and $Z_0$, and this pair produces the final hadron state. Furthermore, let us consider $h=\pi^+$ and assume the Dakin-Feldman model for parton distribution functions [46]. As a result, the ratio of the structure functions becomes independent of the parton distribution functions [28, 12].

Under these assumptions, in the Weinberg-Salam model the forward-backward asymmetry has a value of 0.014 for fractionally charged quarks ($\alpha=2/3$) and 0.026 for integrally charged quarks ($\alpha=1$) [27, 28]. At E = 14 GeV, $A^c_{max}=3.5\%$ and 6.8% for $\alpha=2/3$ and $\alpha=1$, respectively [28, 12], while the parity violating asymmetry $A^P$ lies near the forward direction between 14% and -6%, depending on the Weinberg angle [12].

As shown by Gatto and Preparata [27], there are two-photon effects contributing to the forward-backward asymmetry ($<2\%$ at E = 5 GeV), however, these may be separated since they are concentrated at small production angles and increase as $(\ln q^2)^{1/2}$ instead of $q^2$.

An expedient choice is $h=\Lambda$ which eliminates most of the disturbing interactions in the storage ring. Analogous to the case of $\mu^+\mu^-$, $\Lambda$ gets a nonvanishing average helicity which might be measurable [41, p. 356] through the decay distribution of $\Lambda$. Although it is difficult to predict this helicity, it is perhaps not unreasonable to expect an effect similar to the case of $\mu^-$ (Section B).

In conclusion, production of hadronic final states offers valuable possibilities for studying effects of hadronic weak neutral currents.

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## FIGURE CAPTIONS

1. $\gamma$ and $Z_0$ exchange diagrams for $e^+e^- \to \mu^+\mu^-$.
2. Relative changes in the rate, $\cos\theta$ asymmetry and helicity of $\mu^-$.
3. Charge asymmetry as a function of $\theta$ at $\phi=0$ (or $\pi$), maximum polarizations and $\Delta E/E=1\%$.
4. Weak asymmetries in the Weinberg-Salam model for E=15 GeV, $\sin^2\Theta_W=0.4$. (a) $A_W^c$ for $\frac{p_- - p_+}{1-p_-p_+} = 1/2$, (b) $A_W^c$ for $\frac{p_- - p_+}{1-p_-p_+} = -1/2$, (c) $\frac{1}{p_-} D(p_-, 0)$, (d) $\langle h_3 \rangle_{p_- = p_+}$.
5. Longitudinal asymmetry $A^P$ in the Weinberg-Salam model for various values of $\sin^2\Theta_W$.
6. Longitudinal asymmetry $(-A^P)$ in the Weinberg-Salam model for maximum polarizations.
7. Longitudinal asymmetry $A^P$ in the Lee-Prentki-Zumino model for maximum polarizations.
8. Correction of $\sigma_{tot}$ in the Weinberg-Salam model.
9. Longitudinal asymmetry $(-A^P)$ in the Weinberg-Salam model at maximum polarizations, $m_Z \approx 70$ GeV.

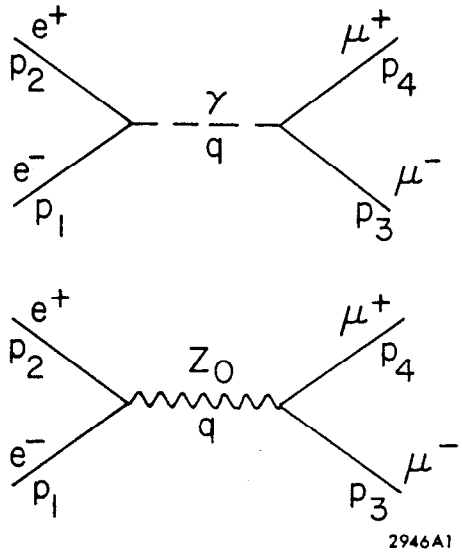


Fig. 1

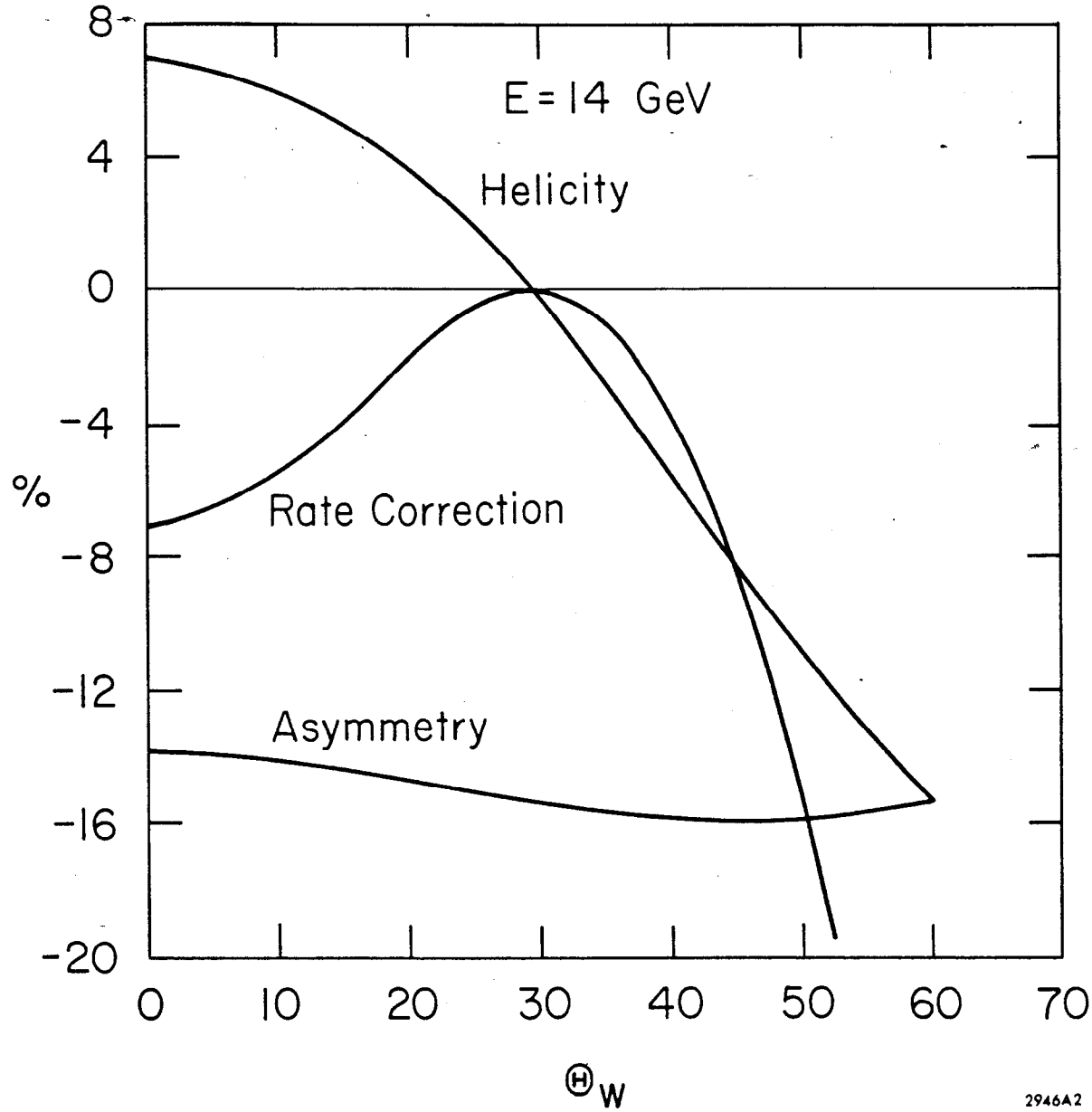
E = 14 GeV
Helicity
Rate Correction
Asymmetry
%
8
4
0
-4
-8
-12
-16
-20
0
10
20
30
40
50
60
70
Θ W
2946A2

Fig. 2

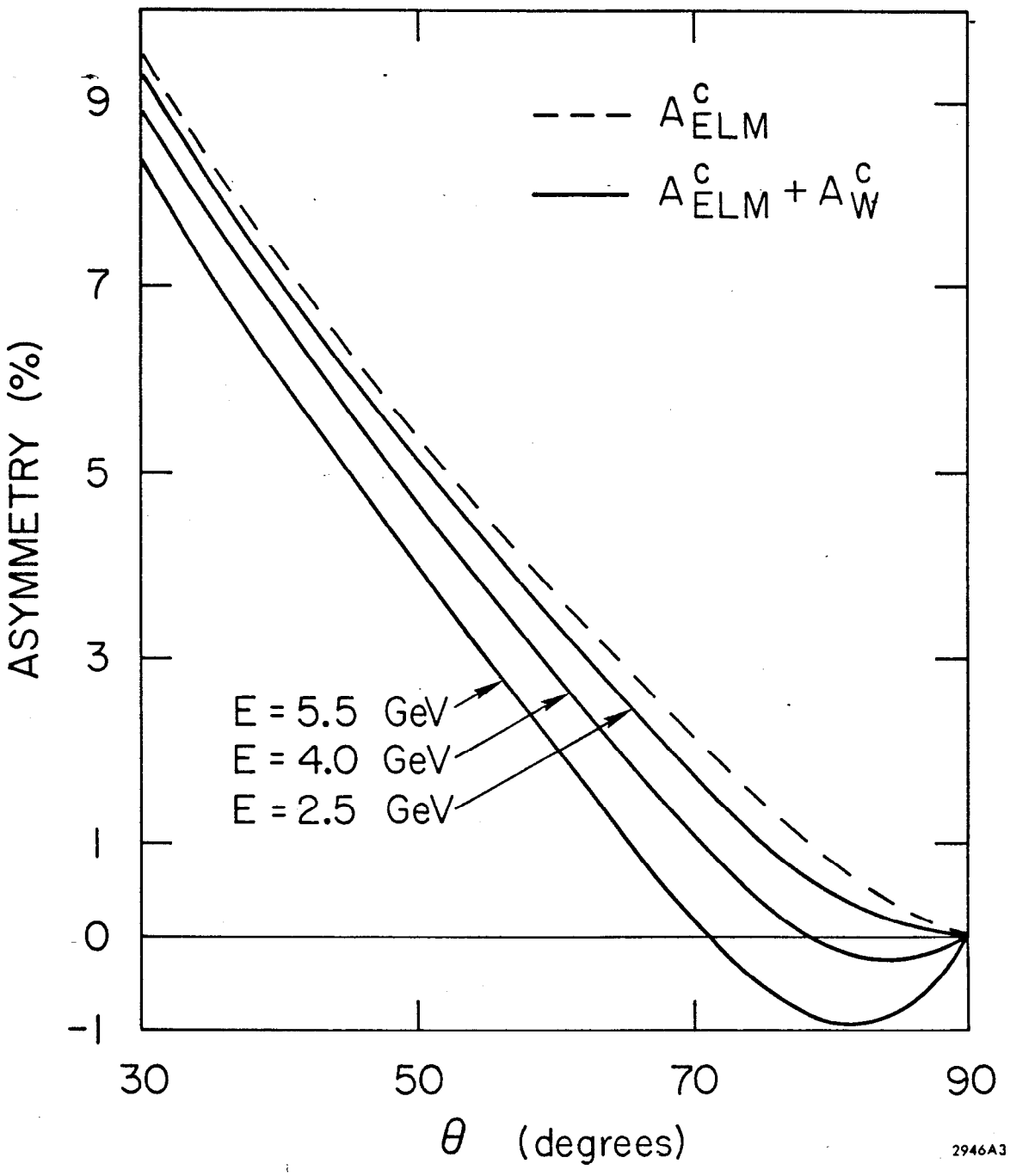
ASYMMETRY (%)
9
7
5
3
1
0
-1
30
50
70
90
θ (degrees)
$A^c_{ELM}$
$A^c_{ELM} + A^c_W$
E = 5.5 GeV
E = 4.0 GeV
E = 2.5 GeV
2946A3

Fig. 3

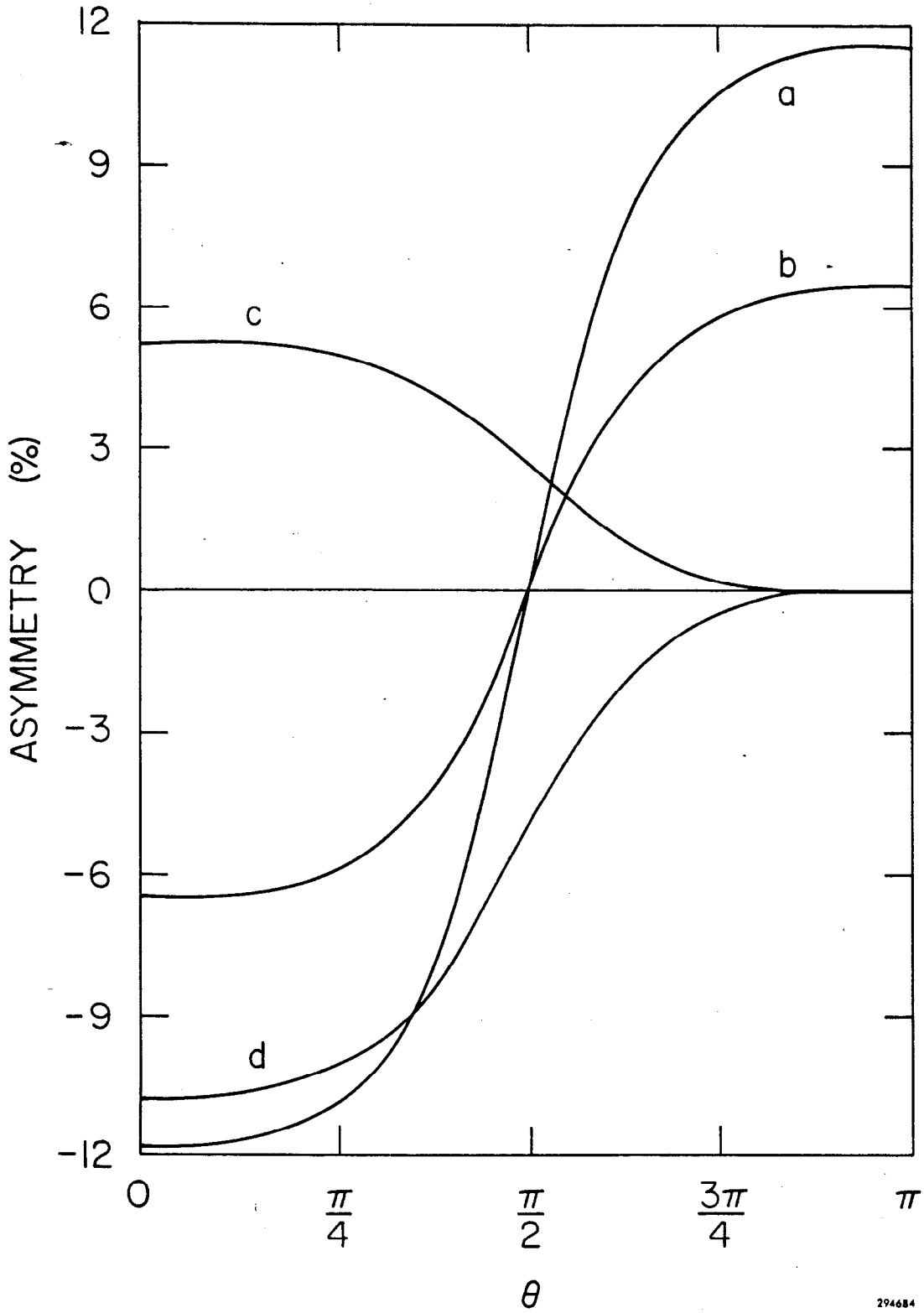


Fig. 4

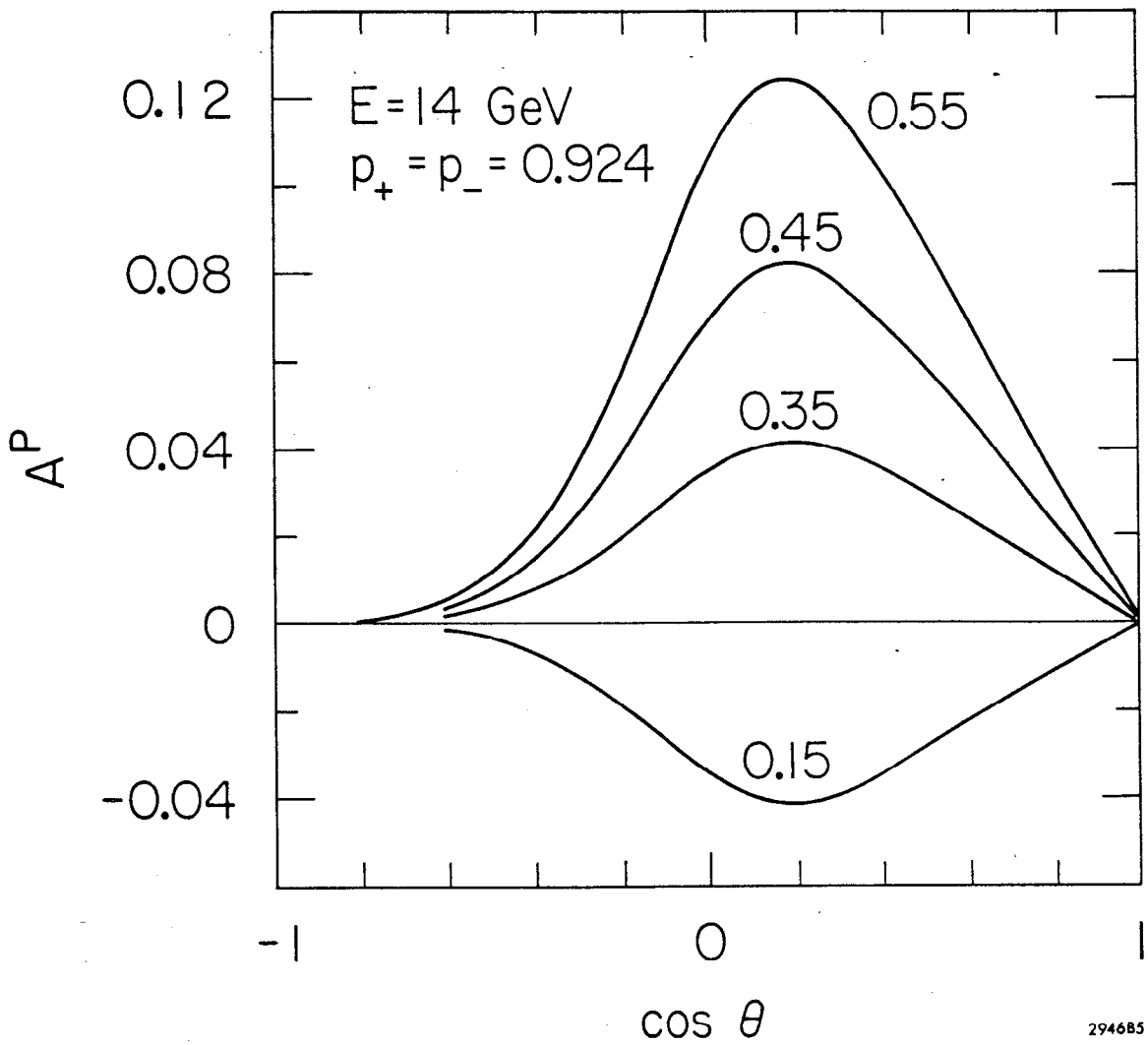
E=14 GeV
p+ = p- = 0.924
0.55
0.45
0.35
0.15
0.12
0.08
0.04
0
-0.04
A^P
-1
0
1
cos θ
2946B5

Fig. 5

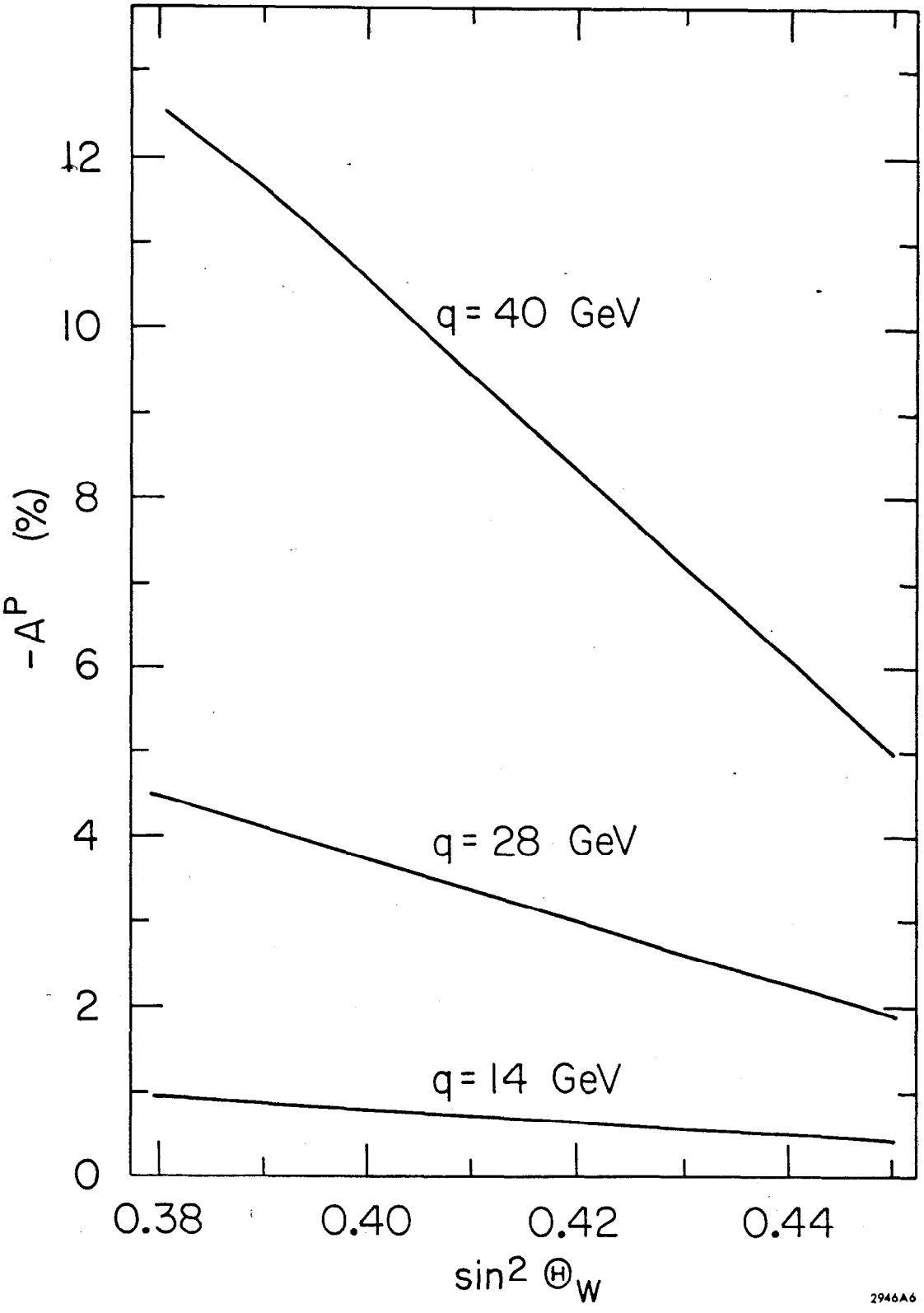
12
10
8
6
4
2
0
$-A^P$ (%)
q = 40 GeV
q = 28 GeV
q = 14 GeV
0.38
0.40
0.42
0.44
$\sin^2 \Theta_W$
2946A6

Fig. 6

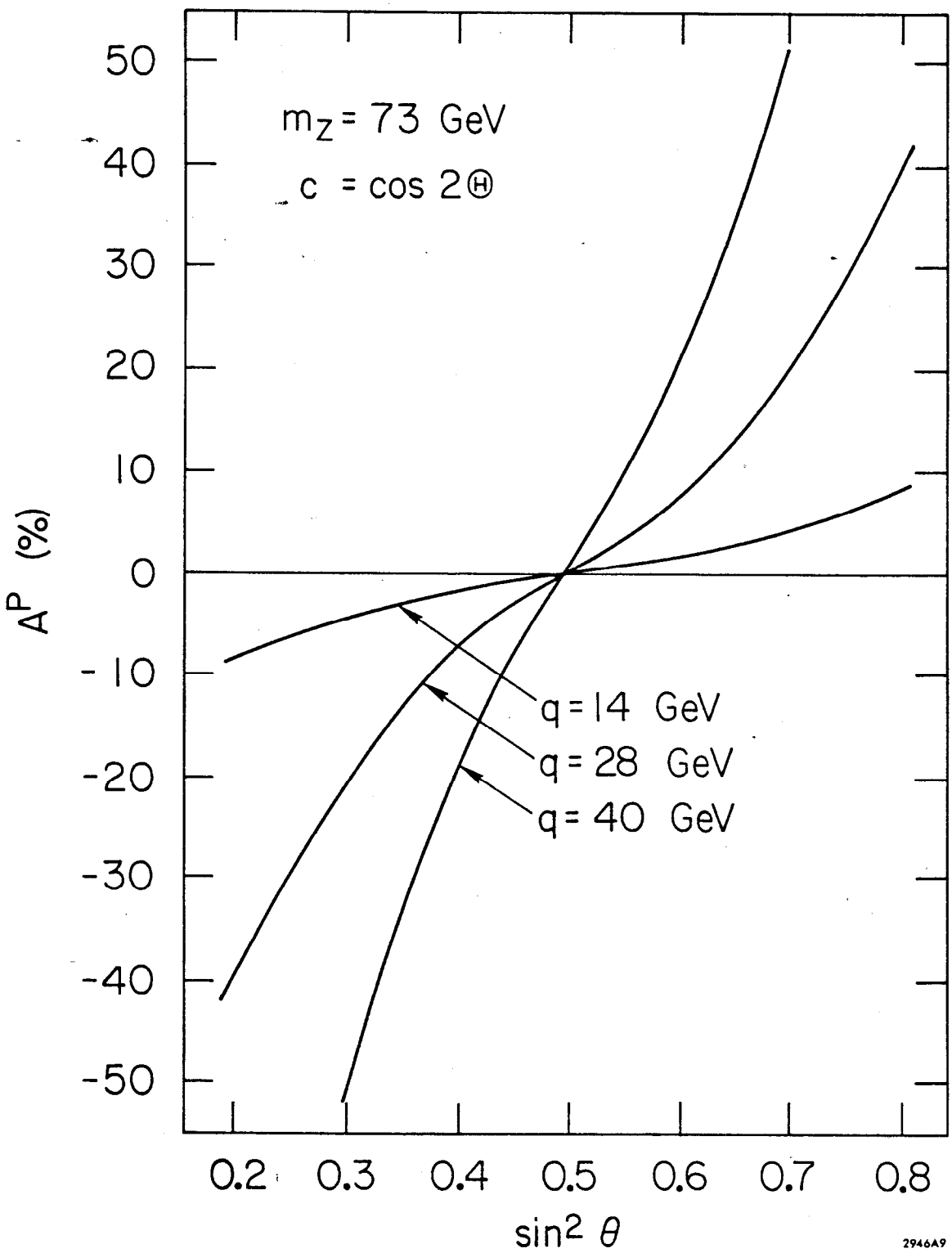
$m_Z$ = 73 GeV
c = cos 2Θ
q = 14 GeV
q = 28 GeV
q = 40 GeV
$A^P$ (%)
50
40
30
20
10
0
-10
-20
-30
-40
-50
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\sin^2\theta$
2946A9

Fig. 7

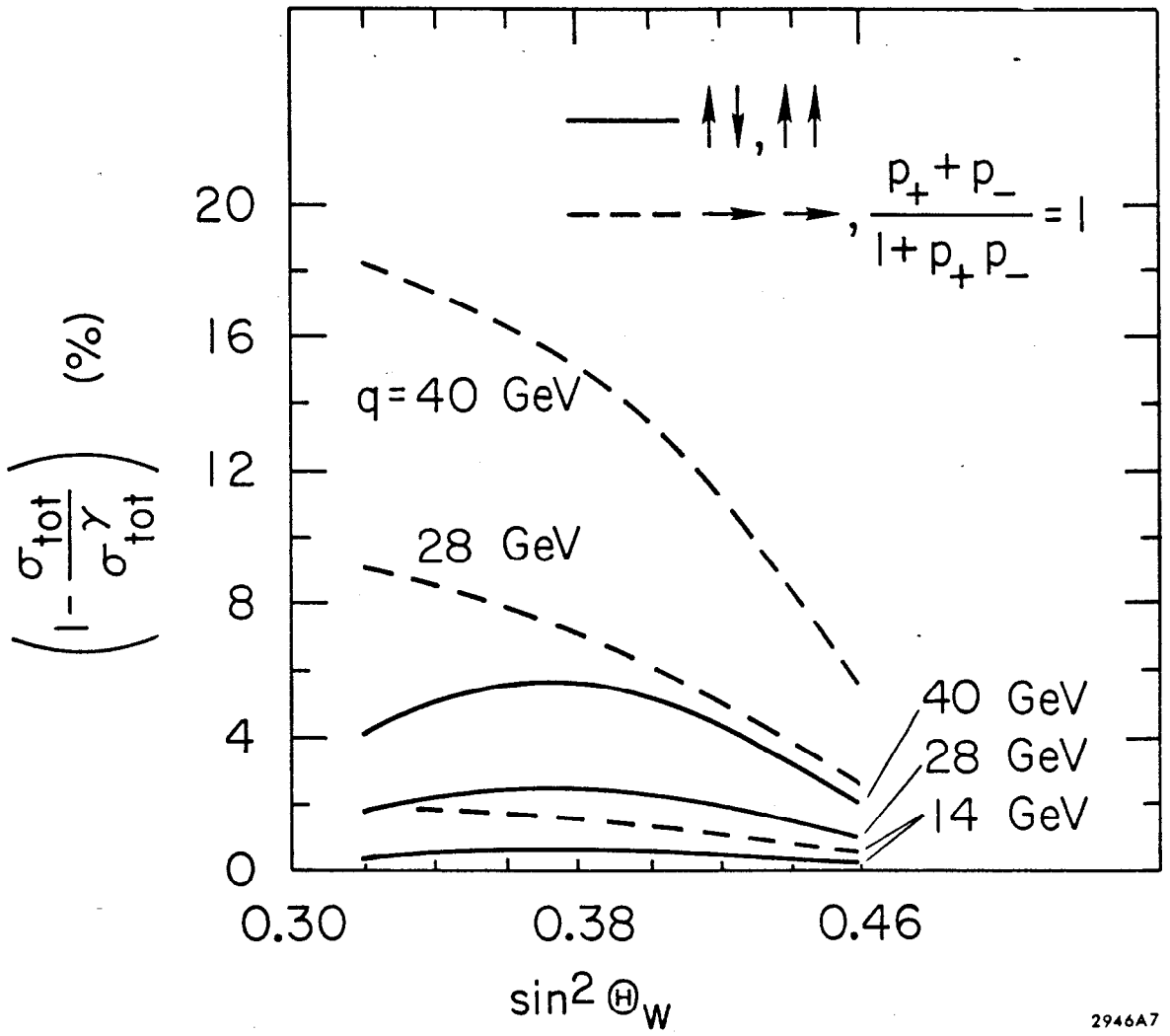
20
16
12
8
4
0
(%)
$\left(1-\frac{\sigma_{tot}}{\sigma^{\gamma}_{tot}}\right)$
$\frac{p_+ + p_-}{1+p_+p_-} = 1$
q = 40 GeV
28 GeV
40 GeV
28 GeV
14 GeV
0.30
0.38
0.46
$\sin^2\Theta_W$
2946A7

Fig. 8

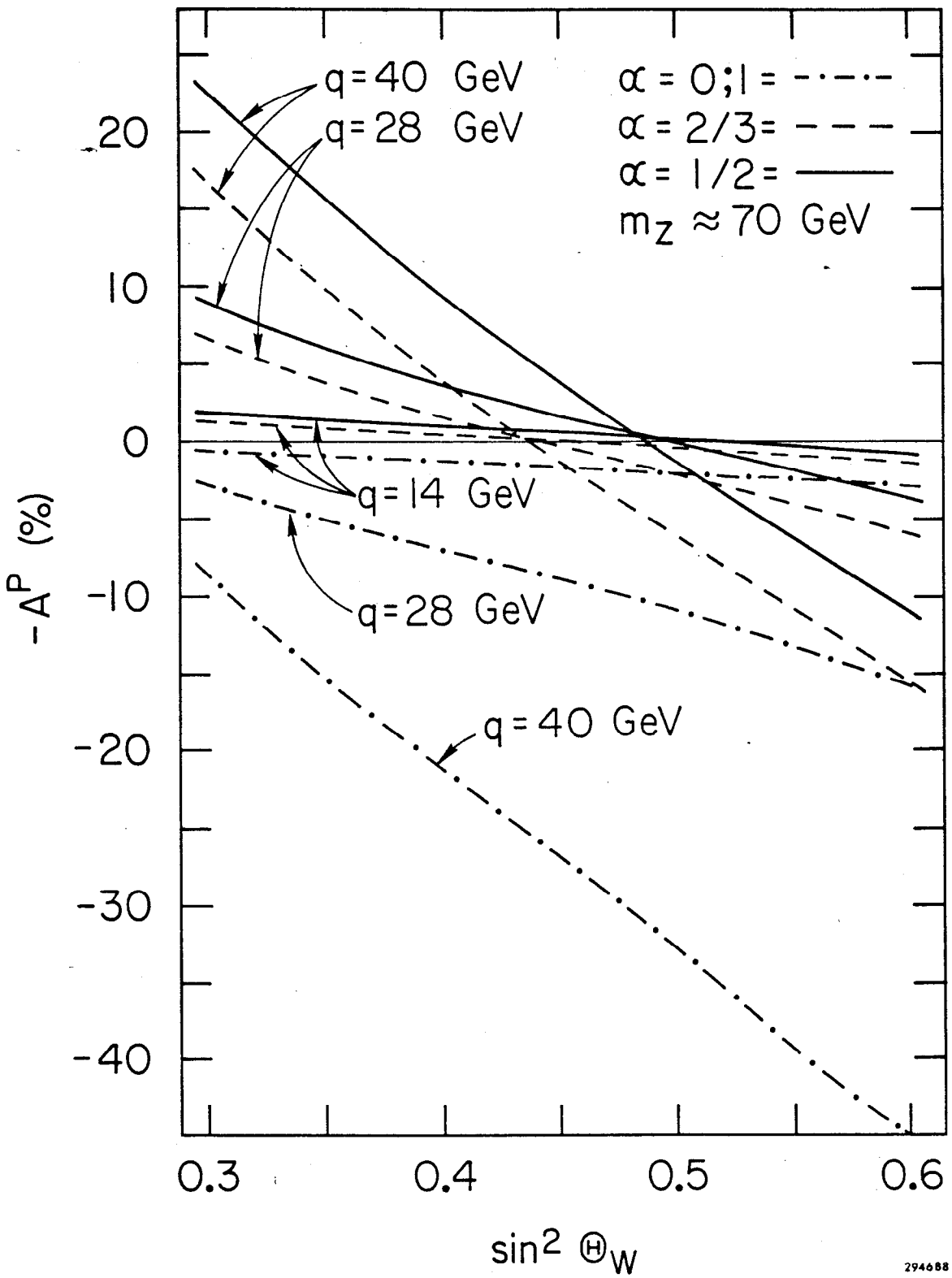
q=40 GeV
q=28 GeV
α = 0;1 = -·-·-
α = 2/3= - - -
α = 1/2= ——
$m_Z \approx 70$ GeV
q=14 GeV
q=28 GeV
q=40 GeV
20
10
0
-10
-20
-30
-40
$-A^P$ (%)
0.3
0.4
0.5
0.6
$\sin^2 \Theta_W$
294688

Fig. 9