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MULTIPLE SCATTERING CALCULATIONS IN πd ELASTIC SCATTERING*

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From a theoretical point of view, the πd system is in a privileged position as compared to other π-nuclei systems, as the Faddeev equations provide the basis for an exact formulation of three body problems. As we have heard in the report by A. Thomas, efforts have been made to solve Faddeev equations for the πd system, and the success obtained with these first results is stimulating. Unfortunately, the attempts to obtain solutions of Faddeev equations face limitations of practical nature, as soon as the energy goes above the very low energy limit (let us say 100 MeV), due to the large number of coupled angular momentum states involved.

At this point the multiple scattering method comes into play. At energies which are not very low, the rather simple multiple scattering calculations, without appeal to model-dependent calculations, are able to give a fairly good description of the πd elastic scattering process.

Applications of the multiple scattering method to evaluate πd scattering processes have been made by several authors.[1] Technical details of the computations are not uniformly treated in the several papers, however, which makes it difficult to develop a critical feeling for the value and the limitations of the method. Some of the effects which are or are not accounted for in some of these computations (such as the fermi-motion dependence of the amplitudes, presence of D wave component in deuteron wavefunction, nucleon recoil) may have important consequences in the results, such as in large angle scattering. Besides, in each application only one or a limited range of values of the energy have been considered, and, comparing the results, we note that the performance of the calculations varies strongly with the energy.

The existing data are scarce, and of low statistics, and must be used all as a whole if one wishes to learn about the applicability of the method. We here discuss results which, although they do not complete the analysis, represent an effort in this direction.[2]

We find that special attention must be given to a point which has been overlooked in most calculations, which is that of the indetermination in the values of the kinematical variables entering in the evaluation of the two particle amplitudes. One deals essentially with off-the-energy shell matrix elements of two-body transition operators. In general, these matrix elements are not known, and the values to be used have to be guessed, following some chosen prescription, from the on-the-energy values which are obtained from direct two-body experiments.

KINEMATICAL AMBIGUITIES. SOME SELECTED PRESCRIPTIONS.

The kinematical arbitrariness which is characteristic of this calculation can lead to very different predictions for the processes studied. We specify below three ways that can be taken to solve the indeterminacy. For easy future reference we call them prescriptions A, B, and C.

i. <u>Prescription A. Faddeev equations and reduction from three body to two body operators</u>. The exact three body amplitude for πd scattering given by

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Faddeev equations can be expanded in terms of the two-body collision operators, in the form of a multiple scattering series. In the explicit evaluation of the terms of the expansion, care must be taken when expressing the matrix elements of operators defined in three-particle Hilbert space in terms of the usual two-body matrix elements. We can then see that the value of the energy parameter to be used becomes uniquely determined. Attention to this point has been called by Thomas.[3]

Let the three particles be labeled by the indices 1, 2, and 3 with momenta $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ in the lab system of reference. Let us select a pair (2,3), and treat the particle 1 separately. We define the new momentum variables

$$\vec{K} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3\ ,\ \vec{k}_1 = (m_3\vec{p}_2 - m_2\vec{p}_3)/(m_2 + m_3)$$

$$\text{and } \vec{q}_1 = [(m_2 + m_3)\vec{p}_1 - m_1(\vec{p}_2 + \vec{p}_3)]/(m_1 + m_2 + m_3) \tag{1}$$

where $\vec{K}$ is the total momentum of the system, $\vec{k}_1$ is the internal momentum in the (2,3) pair relative to its center of mass, and $\vec{q}_1$ is the momentum of particle 1 with respect to the center of mass of the whole system. Defining $\mu_1 = m_2m_3/(m_2 + m_3)$ and $M_1 = m_1(m_2 + m_3)/(m_1 + m_2 + m_3)$, the kinetic energy in the c.m. system ($\vec{K} = 0$) can be written

$$H_0 = (k_1^2/2\mu_1) + (q_1^2/2M_1) \tag{2}$$

The selected pair of particles can be any of the three possible choices, and new (not independent) sets of variables can be defined for each case. Each choice is called a channel. Let us call v_1 the potential acting between particles 2 and 3, v_2 the potential acting between 1 and 3, and so on. An important concept is that of the channel Hamiltonian

$$h_\alpha = (k_\alpha^2/2\mu_\alpha) + (q_\alpha^2/2M_\alpha) + v_\alpha \tag{3}$$

where there appears interaction only between the two particles forming the pair in channel α. The channel resolvent is

$$g_\alpha(z) = (z - h_\alpha)^{-1} \tag{4}$$

Channel operators depend on the relative coordinates of only two particles, and their matrix element between three free particle states can be expressed in terms of operators defined in the two-body Hilbert space. Let us call

$$\hat{h}_\alpha = k_\alpha^2/2\mu_\alpha + v_\alpha \tag{5}$$

the <u>two-body</u> Hamiltonian in channel α, and

$$\hat{g}_\alpha(z) = (z - \hat{h}_\alpha)^{-1} \tag{6}$$

the corresponding two-body resolvent. We can then reduce a three-body channel matrix element writing

$$\langle\vec{k}_\alpha\vec{q}_\alpha|g_\alpha(z)|\vec{k}'_\alpha\vec{q}'_\alpha\rangle = \delta(\vec{q}_\alpha - \vec{q}'_\alpha)\langle\vec{k}_\alpha\left|\left[\left(z - \frac{q_\alpha^2}{2M_\alpha}\right) - \hat{h}_\alpha\right]^{-1}\right|\vec{k}'_\alpha\rangle$$

$$= \delta(\vec{q}_\alpha - \vec{q}'_\alpha)\langle\vec{k}_\alpha\left|\hat{g}_\alpha\left(z - \frac{q_\alpha^2}{2M_\alpha}\right)\right|\vec{k}'_\alpha\rangle \tag{7}$$

The displacement in the value of the argument of the resolvent is very important here.

The three body transition matrix T(z) is written as a sum $T=T_1+T_2+T_3$, where T_1, T_2, and T_3 satisfy the coupled equations $T_1 = t_1 + t_1 g_0(T_2 + T_3)$ and similarly for T_2 and T_3. Here $g_0(z) = (z-H_0)^{-1}$ is the resolvent for three free particles, and

$$t_\alpha(z) = v_\alpha + v_\alpha g_\alpha(z) v_\alpha \tag{8}$$

are channel α transition operators acting in the three particle Hilbert space, and satisfying a reduction relation analogous to Eq. (7).

The Faddeev version of the multiple scattering series is obtained in an obvious way by iterating the coupled integral equations written above. For the elastic scattering of particle 1 by the (2,3) bound pair the transition operator T(z) can be expanded in the form of a multiple scattering series

$$T(z) = t_2(z) + t_3(z) + t_2(z)g_0(z)t_3(z) + t_3(z)g_0(z)t_2(z) + \dots \tag{9}$$

where the interpretation of the terms is the usual one, and all operators are defined in the three-particle Hilbert space. The reduction to matrix elements of two body operators is made with the appropriate shift corresponding to the energy of the particle which, in each term, does not participate in the process. Let E be the value of the total kinetic energy of the particle-deuteron system in the center-of-mass system, $\vec{P}$ the nucleon (particle 1) lab momentum, and $\vec{p}(\vec{p}')$ the initial (final) meson (particle 3) momentum in the lab system. For the term with particle 2 as spectator,

$$\langle \vec{P}', -\vec{P}', \vec{p}' | t_2(E) | \vec{P}, -\vec{P}, \vec{p} \rangle = \delta(\vec{q}_2' - \vec{q}_2)\delta(\vec{K}' - \vec{K}) \langle \vec{k}_1' | \hat{t}_2\left(E - \frac{q_2^2}{2M_2}\right) | \vec{k}_1 \rangle \tag{10}$$

where $\vec{K}(\vec{K}')$ is the total initial (final) momentum of the three particles, $\vec{q}_2$ $(\vec{q}_2')$ is the initial (final) momentum of the spectator with respect to the center of mass, $\vec{k}_1$ $(\vec{k}_1')$ is the initial (final) momentum of the meson relative to the center of mass of the interacting meson-nucleon system, and M_2 is given by $M_2 = m_N(m_N + m_\pi)/(2m_N + m_\pi)$.

The argument of the two body transition operator $\hat{t}_2$ then reads

$$E - q_\alpha^2/2M_2 = E - [(2m_N + m_\pi)\vec{P} + m_N\vec{p}\,]^2/[2m_N(m_N + m_\pi)(2m_N + m_\pi)] \tag{11}$$

In the evaluation of the double scattering terms, one introduces complete sets of three free particle states between the operators, and the reduction to the two-body operators takes place in a manner analogous to that described above.

ii. Prescription B. The meson collides with an on-shell physical nucleon. If fermi-motion effects are taken into account, for each value and each direction of the nucleon momentum inside the deuteron, a different value is used for the relative energy between the incident particle and the nucleon.

iii. Prescription C. The spectator nucleon is treated as an on-shell physical nucleon. Experiments in which there is a breakup of the deuteron, and where an identification can be made between the spectator and the nucleon which was hit by the incident particle, show that the spectator nucleon recoils with a momentum distribution which is, in good approximation, the same as expected from the deuteron wavefunction. We are thus led to the assumption that the spectator nucleon behaves from beginning to end as an on-shell particle. The nucleon which participates in the collision must then be treated as an unphysical particle in the initial and final states. To fulfill energy conservation, the energy of the participant nucleon is equal to the deuteron mass m_d minus the energy $m_N + P^2/2m_N$ carried by the spectator nucleon, where P is the fermi-motion momentum. Thus the participant nucleon behaves as having an effective mass m_{eff} such that

$$m_{eff} + P^2/2m_{eff} = m_d - m_N - P^2/2m_N$$

In a certain sense, prescriptions B and C exchange the roles of the spectator and the participant nucleons. At zero fermi momentum the two prescriptions almost coincide, as then $m_{eff} = m_d - m_N \approx m_N$.

Prescription B has been often used in multiple scattering calculations of πd processes.[1] Prescription C was used in the analysis of pion deuteron breakup scattering.[4]

While prescription B seems to be intuitively more appealing, and prescription C has some kind of experimental support, prescription A has a safer theoretical basis. As off-energy-shell matrix elements are not intuitive quantities, we should rather rely on the more formal approach. The nucleons are not free physical particles inside the deuteron, and prescription A tells us how to take partially into account the effect in our calculation of the presence of two particles in the target nucleus.

In Fig. 1 are shown the values of the kinetic energy (excluded rest masses) in the center-of-mass system of the two colliding particles, as a function of the fermi motion m momentum. The relative energy depends not only on the magnitude, but also on the direction of the fermi motion momentum, and the lines drawn represent the average value over all directions for a fixed magnitude P of the momentum. In prescription B, the value plotted for the energy does not depend much on the value of the fermi momentum, and remains almost constant, while in cases A and C the variation is strong. We can thus expect that fermi-motion effects may be stronger in cases A and C than in case B. These predictions have been confirmed by our calculations, covering the interval of energies from zero up to about 400 MeV. A

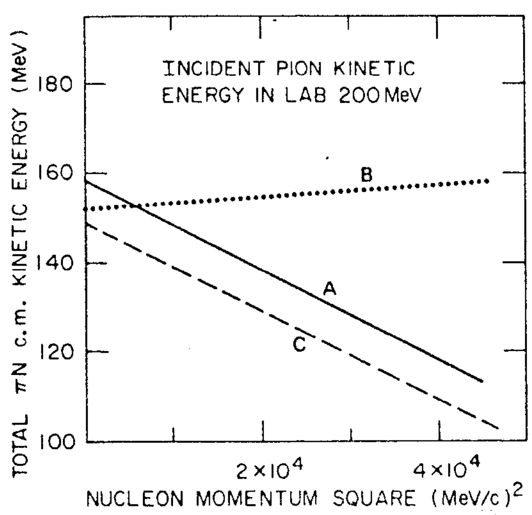


Fig. 1--Values of the total kinetic energy (rest masses excluded) in the πN c.m. system, according to prescriptions A, B, and C, against fermi-motion momentum squared. The energy values are averaged over all directions for a given magnitude of fermi momentum.

main observation is that fermi motion effects are extremely important for the correct evaluation of large angle scattering, because the strong cancellations which occur in the evaluation of the cross sections are sensitive to the proper account of the variation of the values of integrand as a consequence of these effects. A factor of up to four in the differential cross section can appear in the backward angles as the fermi motion effect is switched off.

We may expect that in the cases of prescriptions A and C the calculations are more sensitive to changes in the large momentum tail of the deuteron wavefunction than they are in case B.

OFF THE ENERGY SHELL BEHAVIOR OF AMPLITUDES

For each partial wave we must evaluate an off-shell amplitude $\langle k'|f_\ell(y)|k\rangle$ where k, k' are the initial and final relative momenta of the colliding pair, and y is the energy parameter defined according to each of the prescriptions adopted. These three quantities are not related among themselves through the usual on-shell relations. Integration is made over all initial and final values of the nucleon momentum and the values of k, k', and y vary rather disconnectedly. We must define the matrix element as a function of these variables.

A simple recipe consists in writing $\langle k'|f_\ell(y)|k\rangle = (kk')^{-\frac{1}{2}}\sin\delta_\ell(y)e^{i\delta_\ell(y)}$, where $\delta_\ell(y)$ is the physically measured πN phase shift at energy y. We have observed in the evaluation of πd cross section that, due to the smoothing caused by the integrations over k and k', it makes almost no difference to write $(kk')^{\frac{1}{2}}$ or simply k in the equation above.

Another possible specification for the off-shell extrapolation consists in defining a separable potential for each partial amplitude.[3]

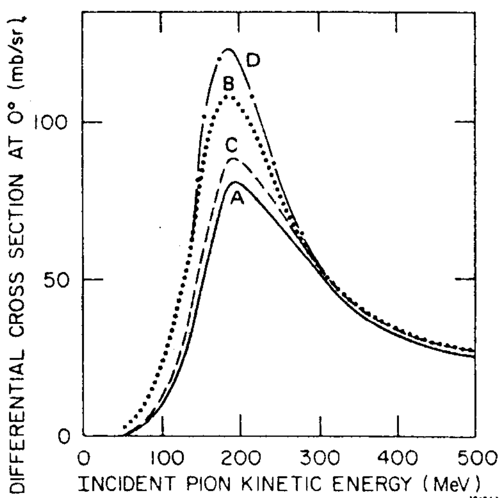


Fig. 2--Forward differential cross section for πd elastic scattering with Coulomb interaction switched off, comparing results obtained with kinematical descriptions A, B, C described in the text. Curve D shows results obtained neglecting fermi-motion effects. The peak in Curve A is displaced about 6 MeV towards higher energies as compared to the other curves.

FORWARD SCATTERING

In forward scattering, as in the value of the total cross section, fermi motion effects are not so important, unless we are near the dominant and resonant wave. In the case of πd scattering near the P33 resonance the influence of the fermi motion effect can be about 35% in the forward cross section in the case of prescription C and 15% in the case of prescription B. This behavior can be seen in Fig. 2 where we plot the forward differential cross section for πd elastic scattering as a function of the meson incident energy, comparing the three different prescriptions and the usual calculation without account for fermi motion effects (prescriptions B and C coincide in this case). Of course the Coulomb interaction has not been taken into account.

We see in Fig. 2 that the position of the peak due to the P33 resonance is nearly the same in all cases, with a shift of about 6 MeV towards higher values of the energy observed in the case of prescription A. This is an important, although rather obvious, result as we should expect a displacement to occur in the position of the peak as a consequence of the shift in the value of the energy caused by the reduction from three-body to two-body operators. This result, which is shown in Fig. 2 for the nuclear (non-Coulomb) interaction for zero angle scattering, is also true of the total cross section, as the elastic πd scattering is almost completely forward. It is interesting to remark that larger shifts are expected to occur in the scattering by heavier nuclei.

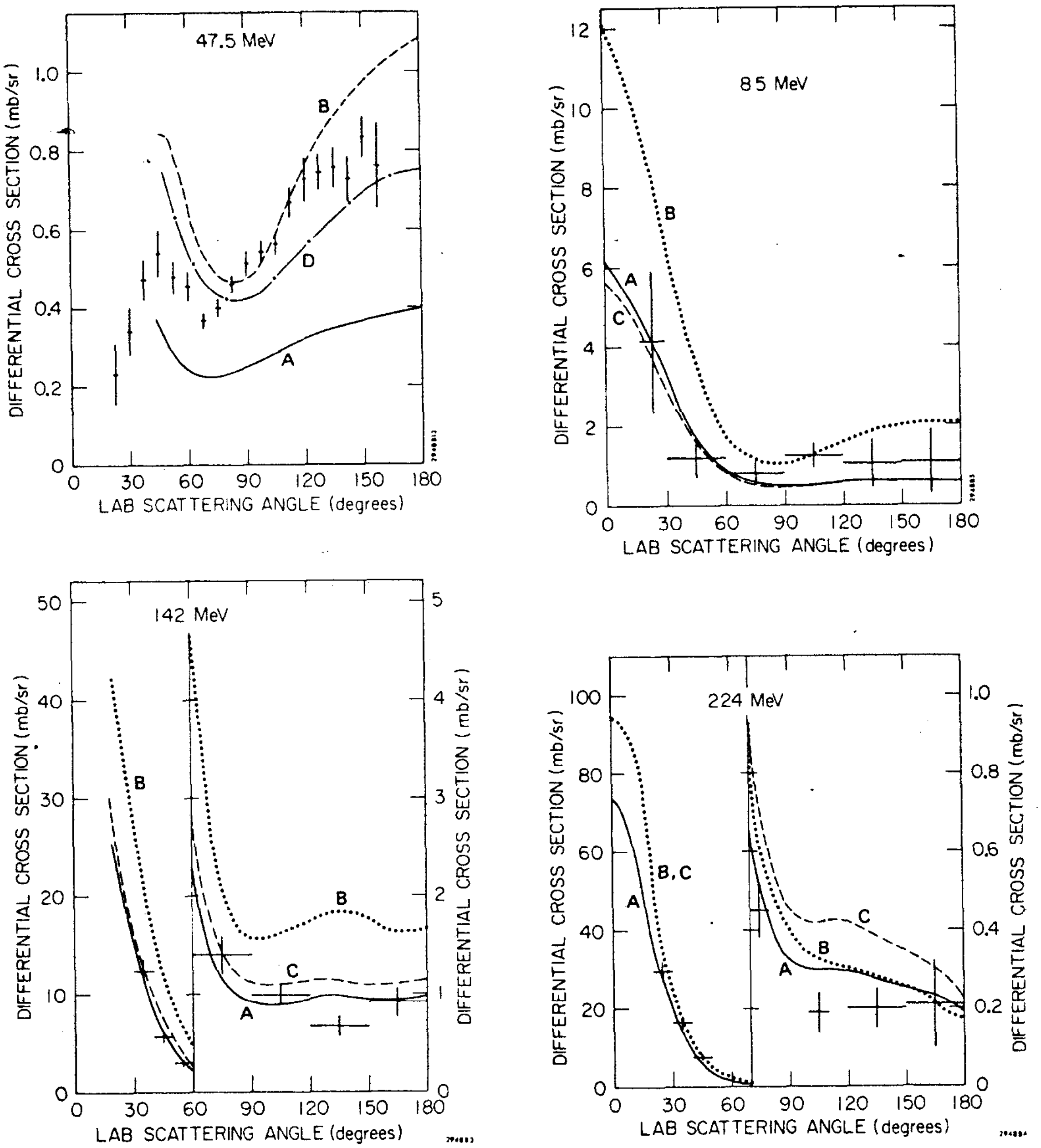
We must call attention to the result, shown in the figure, that the values of the total and forward cross sections, evaluated with prescription A in the resonance region, are remarkably lower than the values obtained in the other two cases.

EXPERIMENTAL RESULTS AND THEORETICAL CALCULATIONS

It is hoped that the chronic scarcity of data on πd scattering will change soon, as already indicated by the recent experiment at 47.5 MeV by D. Axen et al.,[5] and the expected results of the measurements at 347 MeV/c (234.4 MeV kinetic energy) and 443 MeV/c (324.9 MeV kinetic energy) performed by a collaboration of the groups at the University of Virginia and at Los Alamos Scientific Laboratory.[6] There are reported experimental results on the elastic πd differential cross section for incident pions at 61 MeV,[7] 85 MeV,[8] 140 MeV,[9,10] 182 MeV,[11] 224 MeV,[12] 256 MeV,[13] 300 MeV,[14] and 330 MeV.[15] For large angle scattering, between 140 and 180 degrees in the laboratory system, there are results obtained by Schroeder et al.[16] at 375.7, 412.4, and 469.6 MeV. The work of Gabathuler et al.[13] also includes measurements of the backward cross section at 160 degrees lab scattering angle for incident pions of 141, 163, 185, and 208 MeV.

In Figs. 3, 4, 5, and 6 (see also Ref. 2) we confront those data with results of multiple scattering calculations, comparing the different prescriptions for the kinematical variables used in the evaluation of the two-body amplitudes. The calculations include double scattering terms, allowing for nucleon recoil, and including both the delta function and the principal value parts originated from the pole in the propagator. Corrections to the differential cross section arising from the double scattering terms never amount to more than 20 percent in the whole range of angles and of energies here considered. It is thus unnecessary to include fermi-motion dependence in the double scattering terms, which brings an important simplification in the numerical computations. The comparatively small contribution obtained for the double scattering terms makes us confident that higher order terms of the series can be neglected. The calculations account for fermi motion effects in the single scattering terms, and are made with Moravcsik wavefunction, with 7 percent d-wave component.

As shown in Fig. 3, the experimental results obtained at 47.5 MeV are reasonably well fitted by a multiple scattering calculation with the most usual treatment of the two-body kinematics, namely, prescription B. Fermi motion effects do not seem to contribute substantially to improve the quality of this theoretical curve. The other two prescriptions perform badly at this energy. However, we must remark that such observations should not be taken on their own as a basis of judgment about the method of calculation. In fact,



Figs. 3, 4, 5, and 6--Data on πd elastic scattering and theoretical curves representing results of multiple scattering calculations. Labels A (solid), B (dotted), and C (dashed) refer to the kinds of kinematical prescription described in the text. Curve D (dot-dashed) at 47.5 MeV is obtained without account for fermi motion. Notice the enlarged scale used for large angles at 142 and 224 MeV. The solid curves best fit the data, if 47.5 MeV is considered too low energy for this kind of theoretical calculation.

we can see that the situation becomes very different at slightly higher energies. Thus, at 85, 142, 182, 224, and 256 MeV, as exemplified in Figs. 4, 5, and 6 (see also Ref. 2), prescription A seems to describe the data better. From the inspection of the figures, we are led to believe that these theoretical calculations make sense at these energies. The poor results obtained with prescription A at 47.5 MeV should thus be taken as demonstrating that this energy is too low for a multiple scattering calculation involving only single and double scattering terms. Binding corrections, off-the-energy shell extrapolations, or complicated three-body mechanisms might play important roles at such low energies.

Above 230 MeV the experimental cross sections at large scattering angles gain a structure which is not reproduced by these theoretical calculations. At small angles up to about 70 degrees in lab system the calculated values are reasonable, but at large angles the calculations are wrong by a factor of about two. (See also Ref. 2 for the energy behavior of large angle cross sections.)

We may speculate on what may be the cause of this discrepancy. We notice that the strong reduction in the large angle experimental cross section, as compared to the calculated values, occurs suddenly as the energy goes above 230 MeV. At this energy some new dynamical phenomenon may have started to play a role. We may think for example that pion production and consequent reabsorption by the other nucleon may have started to contribute significantly. At these energies, which are above the threshold for pion production, this essentially three-body mechanism could eventually be responsible for a change in the dynamics of the process.

Another possible explanation for the observed discrepancy is that we may have entered in a range of momentum transfer where the effects of our insufficient knowledge of the deuteron structure may have started to affect the calculations. A change in the large momentum tail in the deuteron wavefunction may substantially change the value of the integral over internal fermi momentum in the expression of the differential cross section. As an example, we mention that the introduction of the d wave component in deuteron wavefunction causes an increase by a factor 2 in the calculated cross section at large angles in the energies of Schroeder experiment (375.7 MeV and over).

These effects due to changes in the deuteron structure or in meson-nucleon interaction might be expected to be small at first sight. However we must notice that the value calculated for the πd differential cross section at large angles is several orders of magnitude smaller than the forward cross section, due to strong cancellations occurring in the integration procedure. The results obtained after such cancellations have a delicate and strong dependence on the quantities in the integrand.

The extreme sensitivity of the backward πd elastic cross section at large angles provides an excellent ground to study the deuteron structure and properties of the meson-deuteron and meson-nucleon interaction.

We find that more and more accurate experiments on πd scattering should be performed as soon as possible. The region of energies around and above 200 MeV should be carefully studied, as important changes in the process seem to take place in this region.

On the other hand, it is obvious that the theoretical effort must also be increased, both in the calculations with multiple scattering method and in direct solutions of Faddeev integral equations. A combination of the two methods, joining the nice features of each, may be an interesting and rewarding program of investigation.

REFERENCES

1. Multiple scattering calculations of πd elastic scattering: R. M. Rockmore, Phys. Rev. 105, 256 (1957); A. Ramakrishnan, V. Devanathan, K. Venkatesan, Nucl. Phys. 29, 680 (1962); H. N. Pendleton, Phys. Rev. 131, 1833 (1963); C. Carlson, Phys. Rev. C 2, 1224 (1970); D. S. Beder, Nucl. Phys. B 34, 189 (1971); W. R. Gibbs, Phys. Rev. C 3, 1127 (1971); R. L. Landau, Nucl. Phys. B 35, 390 (1971); J. M. Wallace, Phys. Rev. D 5, 1840 (1972); K. Gabathuler and C. Wilkin, Nucl. Phys. B 70, 215 (1974); E. Ferreira, L. P. Rosa, and Z. D. Thomé, Nuovo Cimento 20A, 277 (1974), and Nuovo Cimento Lett. 9, 707 (1974); M. A. Braun and V. B. Senyushkin, Sov. J. Nucl. Phys. 21, 147 (1975).
2. E. M. Ferreira, L. P. Rosa, and Z. D. Thomé, paper contributed to this conference (to be published).
3. A. W. Thomas, Proc. Int. Conf. on Few Body Problems in Nuclear and Particle Physics, Quebec, 27-31 Aug 1974 (Les Presses de L'Universite Laval, Quebec, 1975), p. 287.
4. E. M. Ferreira, L. P. Rosa, and Z. D. Thomé, Nuovo Cimento 21A, 187 (1974).
5. D. Axen, G. Duesdieker, L. Felawka, Q. Ingram, R. Johnson, G. Jones, D. Lepatourel, M. Salomon, W. Westlund, L. Robertson, Nucl. Phys. A 256, 387 (1976).
6. University of Virginia and Los Alamos Scientific Laboratory Collaboration - Preliminary Data, Los Alamos Report LA-6156-R, and private communication by J. McCarthy.
7. A. M. Sachs, H. Winick, and B. A. Wooten, Phys. Rev. 109, 1733 (1958).
8. K. C. Rogers and L. M. Lederman, Phys. Rev. 105, 247 (1957).
9. E. Arase, G. Goldhaber, and S. Goldhaber, Phys. Rev. 90, 160 (1953).
10. E. G. Pewitt, T. H. Fields, G. B. Yodh, J. G. Fetkovich, and M. Derrick, Phys. Rev. 131, 1826 (1963).
11. J. H. Norem, Nucl. Phys. B 33, 512 (1971).
12. J. L. Acioli, Ph.D. thesis, University of Chicago, 1968 (unpublished).
13. K. Gabathuler, C. R. Cox, J. J. Domingo, J. Rohlin, N. W. Tanner, C. Wilkin, Nucl. Phys. B 55, 397 (1973).
14. L. S. Dul'kova, I. B. Sokolova, and M. G. Shafranova, Sov. Phys. - JETP 35, 217 (1959).
15. G. Brunhart, G. S. Faughn, V. P. Kenney, Nuovo Cimento 24, 1162 (1963).
16. L. S. Schroeder, D. G. Crabb, R. Keller, J. R. O'Fallon, T. J. Richards, R. J. Ott, J. Trischuk, and J. Va'vra, Phys. Rev. Lett. 27, 1813 (1971).