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Why the angular distribution of the top decay lepton is unchanged by anomalous tbW couplings

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We give a simple physical argument to understand the observation that the angular distribution of the top decay lepton depends only on the polarisation of the top and is independent of any anomalous tbW coupling to linear order.

The top quark is the heaviest known fundamental particle. Its average lifetime is about one order of magnitude smaller than the typical hadronisation time scale. This leads to decay of the top quark before the strong-interaction hadronisation process can wipe out its spin information. Thus, one can extract the top quark polarisation from the kinematical distributions of its decay products.

The polarisation of the t quark produced via Standard Model (SM) processes at hadron colliders is known. It is zero for the dominant QCD-induced $t\bar{t}$ production and is dominantly left-handed but calculable for the subdominant single-t production. The rigidity of these predictions allows us to use the t polarisation to probe for possible new physics contributions to these production processes. From simple angular momentum considerations, the angular distribution of a spin 1/2 decay product f of the t quark must take the form

$$\frac{1}{\Gamma_t}\frac{d\Gamma_t}{d\cos\theta_f} = \frac{1}{2}\left(1+\alpha_f P_t\ \cos\theta_f\right) \tag{1}$$

In the SM, one finds for the $t \to b\ell\nu$ decay the values $\alpha_b = -0.4$, $\alpha_\ell = 1$ and $\alpha_\nu = -0.32$ at tree level, and only small modifications of these values at the one-loop level. Using these values, the measurement of the t decay angular distributions can be used to obtain the t polarisation.

However, there is a possible problem. If new physics can modify the t quark production amplitudes, it can also modify the t quark decay amplitudes. Then we would expect new physics to modify the values of the α_i in Eq. (1). The measurement of the distributions gives only the combinations $\alpha_i P_t$, so if the α_i can be shifted by new physics effects, this method loses its power.

Thus it is noteworthy that, in a series of investigations on $\bar{t}t$ production at an e^+e^- collider [1–3] and a $\gamma\gamma$ collider [4, 5], it was observed that α_ℓ remains unchanged even after inclusion of anomalous tbW couplings, up to linear order in new physics parameters. This independence of α_ℓ (or "lepton decoupling") was also observed for more general processes of top-quark production [6, 7], suggesting that it is a property of the top quark decay and not of any specific production process. This would make the angular distribution of the decay lepton with respect to the top spin direction a very robust measure of the top polarisation.

It was also noted [6, 7] that this lepton decoupling follows because, for the SM, the full kinematic distribution of the decay lepton factorises into a term dependent on the lepton energy E_ℓ and another term dependent on the angular variables, and that this factorisation is maintained even in the presence of anomalous tbW couplings up to linear order. Actually, the dependence of the decay distributions on E_ℓ is modified by anomalous tbW couplings. Then it is possible to use the angular and energy distributions together to measure the polarisation of the t quark and in addition to probe for the presence of anomalous couplings in the decay vertex [6, 8, 9].

Both the lepton decoupling and the factorization do receive corrections at the quadratic order in anomalous couplings [5, 10]. But, in view of the already rather strong constraints on the tbW vertex [11], in which the least constrainted parameter f_{2R} is required to be less than about 0.1, lepton decoupling to linear order is quite sufficient for practical purposes.

In Ref. [12], Hioki has given an argument for lepton decoupling based on a physical picture. In this paper, we would like to present a more transparent derivation of this result.

Derivation: The key ingredient in our proof of lepton decoupling is the fact that, in the SM, the $b\nu_\ell$ system produced in $t \to b\ell\nu$ is in a $J = 0$ state. As a result of this, the entire spin of the top is transferred to the lepton. This can be seen by a Fierz transformation of the SM decay amplitude. Starting from this fact, we will show that lepton decoupling for anomalous terms in the tbW vertex follows from simple rotation algebra.

At the tree level in the SM, the amplitude for t decay is a product of matrix elements of left-handed currents.

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Using

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

and considering only the upper two components of the Dirac spinors, we can write the decay matrix element as

$$i\mathcal{M} = iG(p_W)\ u^\dagger(b)\bar{\sigma}^\mu u(t)\ u^\dagger(\nu)\bar{\sigma}_\mu v(\ell)\ , \qquad (2)$$

with

$$G(p_W) = \frac{g^2}{2} V_{tb} \frac{1}{p_W^2 - m_W^2}\ .$$

Then the Fierz identity

$$(\bar{\sigma}^\mu)_{ab}\ (\bar{\sigma}_\mu)_{cd} = 2\epsilon_{ac}\epsilon_{bd}$$

converts Eq. (2) into

$$i\mathcal{M} = 2i\ G(p_W)\ [u_a^\dagger(b)\epsilon_{ab}u_b^\dagger(\nu)]\ [u_c(t)\epsilon_{cd}v_d(\ell)]\ . \qquad (3)$$

Each bracket is a Lorentz-invariant. In particular, the $(b\nu)$ system is produced in a $J = 0$ state.

We can now use the result in Eq. (3) to compute the spin density matrix for the t quark in terms of the lepton orientation. We will do this first in the SM and then add anomalous tbW couplings to linear order.

The decay lepton produced in (an assumed SM) W decay is always right-handed. Hence the lepton direction is correlated with the lepton spin. We work in the rest frame of the decaying t quark. The t spin orientation is defined by a 2-component spinor ξ in the frame of the decay. Then we can best analyze the density matrix by choosing coordinates in which the lepton momentum is parallel to the $\hat{z}$ axis. The decay amplitude is a linear combination of the amplitudes for two configurations, those in which the t spin is parallel and antiparallel to the $\hat{z}$ axis. We show these two cases in Fig. 1.

In the SM, it follows directly from Eq. (3) that the decay amplitude for t spin $S_t^z = -\frac{1}{2}$ vanishes. Then the spin density matrix takes the form

$$\Gamma_t = F(E_\ell) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\ ,$$

Already here we see the factorization of the dependence on t spin and lepton energy. To obtain the density matrix for a general t spin orientation—or for a general orientation (θ_ℓ, ϕ_ℓ) of the lepton direction relative to the $\hat{z}$ axis—we perform a rotation and obtain

$$\Gamma_t = F(E_\ell)\ \begin{pmatrix} (1+\cos\theta_\ell) & \sin\theta_\ell e^{i\phi_\ell} \\ \sin\theta_\ell e^{-i\phi_\ell} & (1-\cos\theta_\ell) \end{pmatrix}\ . \qquad (4)$$

in accord with [3].

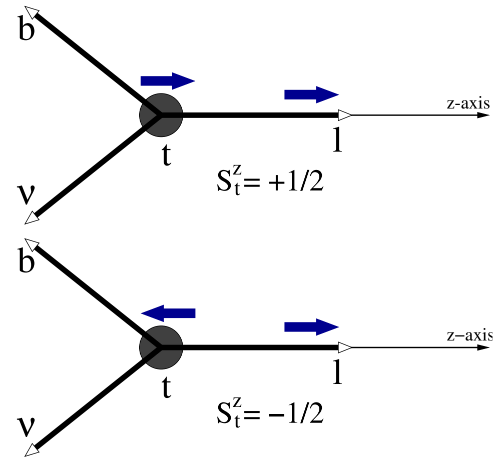

FIG. 1. The configurations with the lepton momentum along the top spin quantization axis, the z axis.

Now introduce a general set of form factors representing anomalous couplings in the tbW vertex,

$$\Gamma^\mu = \frac{-ig}{\sqrt{2}}\ [\gamma^\mu((1+f_{1L})P_L + f_{1R}P_R) - \frac{i\sigma^{\mu\nu}}{m_W}\ (p_t - p_b)_\nu\ (f_{2L}P_L + f_{2R}P_R)] \qquad (5)$$

The SM vertex is the case in which all coefficients f_i are zero.

Analyze this more general situation in the frame shown in Fig. 1. Now the decay matrix elements for $S_t^z = +\frac{1}{2}$ and $S_t^z = -\frac{1}{2}$ are both nonzero, and thus all four elements of the t spin density matrix receive nonzero contributions either linear or quadratic in the f_i. However, the new contributions to the decay amplitudes can depend on the angle between the plane containing the (b, ν) momentum vectors and the reference $(\hat{x}, \hat{z})$ plane. Call this angle ϕ_b. For $S_t^z = +\frac{1}{2}$, the (b, ν) system has $S^z = 0$ and so the decay amplitude is independent of ϕ_b. On the other hand, for $S_t^z = -\frac{1}{2}$, the (b, ν) system must carry away $S^z = -1$, and so

$$i\mathcal{M}(\phi_b) = i\mathcal{M}(\phi_b = 0) \cdot e^{i\phi_b}\ .$$

The density matrix then takes the form

$$\Gamma_t \propto \begin{pmatrix} 1+\mathcal{O}(f_i) & \mathcal{O}(f_i)\cdot e^{-i\phi_b} \\ \mathcal{O}(f_i)\cdot e^{+i\phi_b} & \mathcal{O}(f_i^2) \end{pmatrix}$$

To obtain the density matrix for the charged leptons, we integrate over the orientations of the other t decay products, keeping the lepton momentum fixed. This includes an integration over ϕ_b. After this integration, we find

$$\Gamma_t \propto \begin{pmatrix} 1+\mathcal{O}(f_i) & 0 \\ 0 & 0 \end{pmatrix}$$

up to terms of quadratic order in the f_i. Notice that the upper left matrix element of Γ_t can be modified by nonzero f_i, in a manner that depends on E_ℓ; however, the factorization between the dependences on t spin and E_ℓ is preserved. This is the result that we sought to prove. In this case, the density matrix for a general t spin orientation will be similar to Eq. 4, but with a different E_ℓ dependent factor.

It is useful to take stock of what we needed to assume, and what we did not need to assume, to achieve this result: *(i) Chiral lepton:* We treated the ℓ^+as massless, and, in accordance with the $V-A$ nature of the W boson decay, having strictly positive helicity. *(ii) SM spin correlation:* We needed the property of the SM amplitude that the t spin is completely correlated with the lepton spin. This property does not hold if we replace the charged lepton with either ν or b. So the result holds only for charged leptons, and for $T_3 = -1/2$ light quarks in hadronic decays of W^+-boson. *(iii) Partial averaging:* We needed to average over the azimuthal orientation of the b, ν vectors in the frame of the t decay. This would naturally be done if the t polarisation is measured from the inclusive lepton distribution.

On the other hand, we did not require the b quark to be massless or the W boson to be on-shell. For massive leptons, viz., τ's, the matrix element $\mathcal{M}(t_\downarrow \to \tau^+_\uparrow\, b\nu_\tau; \phi_b)$ does not vanish in the SM and we get $\alpha_\tau \neq 1$. This leads to a correction in α_τ at $\mathcal{O}(f_i)$, but this correction is suppressed by m_τ/m_t.

Conclusions: In this note, we have analyzed the robustness of the parameter α_ℓ associated with the t spin polarisation against the contributions from anomalous tbW couplings. We related this robustness to the factorisation of the energy and angle distributions for charged leptons. This factorisation emerges due to the SM property of the vanishing of the amplitude for the charged lepton with momentum in a direction opposite to the top spin. Further, the factorisation, demonstrated here in the rest frame of the decaying quark, remains true in the laboratory frame as well. Thus energy integrated angular distribution of the lepton produced in the decay of a polarised top quark does not receive any modifications from the anomalous tbW coupling, in the laboratory frame as well.

This analysis offers us insight into the effect of anomalous tbW couplings on the kinematic distributions of the charged lepton produced in the t decay. The same analysis applies, in fully hadronic W decays, to the angular and energy distribution of the $T^3 = -\frac{1}{2}$ quark in the final state [13, 14]. The robustness of the independence of the angular distribution from the anomalous couplings, to linear order, offers us the possibility of using these kinematic distributions to construct independent probes of both the top polarisation and the anomalous tbW couplings.

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[1] B. Grzadkowski and Z. Hioki, Phys. Lett. **B476**, 87 (2000) [arXiv:hep-ph/9911505].
[2] S. D. Rindani, Pramana **54**, 791 (2000) [arXiv:hep-ph/0002006].
[3] B. Grzadkowski and Z. Hioki, Phys. Lett. **B529**, 82 (2002) [arXiv:hep-ph/0112361]; Phys. Lett. **B557**, 55 (2003) [arXiv:hep-ph/0208079]
[4] K. Ohkuma, Nucl. Phys. Proc. Suppl. **111**,285 (2002) [arXiv:hep-ph/0202126].
[5] R. M. Godbole, S. D. Rindani and R. K. Singh, Phys. Rev. D **67** (2003) 095009 [Phys. Rev. D **71** (2005) 039902] [hep-ph/0211136]; In Proceedings, International Workshop on physics and experiments with future electron-positron linear colliders, LCWS 2002, Seogwipo, Jeju Island, Korea, August 26-30, 2002, p. 350-356, hep-ph/0212191.
[6] R. M. Godbole, S. D. Rindani and R. K. Singh, JHEP **0612**, 021 (2006), [arXiv:hep-ph/0605100]; Pramana **69** (2007) 915; In 'Electroweak Interactions and Unified Theories: Proceedings, 42 Recontres de Morinod, La Thuile, Italy, March 10-17, 2007', p. 137-141.
[7] R. M. Godbole, K. Rao, S. D. Rindani and R. K. Singh, JHEP **1011** (2010) 144 [arXiv:1010.1458 [hep-ph]]; AIP Conf. Proc. **1200** (2010) 682 [arXiv:0911.3622 [hep-ph]].
[8] A. Prasath V, R. M. Godbole and S. D. Rindani, Eur. Phys. J. C **75**, no. 9, 402 (2015), [arXiv:1405.1264 [hep-ph]].
[9] J. A. Aguilar-Saavedra and R. V. Herrero-Hahn, Phys. Lett. B **718** (2013) 983 [arXiv:1208.6006 [hep-ph]].
[10] J. A. Aguilar-Saavedra, J. Carvalho, N. F. Castro, F. Veloso and A. Onofre, Eur. Phys. J. C **50** (2007) 519 [hep-ph/0605190].
[11] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
[12] Z. Hioki, Int. J. Mod. Phys. A **31** (2016) no.04n05,

1650011 [arXiv:1508.01250 [hep-ph]].

[13] E. Ruiz Morales and M. E. Peskin, In Physics and experiments with future linear e+ e- colliders, Proceedings of the 4th Workshop, LCWS'99, Sitges, Spain, April 28-May 5, 1999, 461-466, [hep-ph/9909383].

[14] B. Tweedie, Phys. Rev. D **90**, no. 9, 094010 (2014),[arXiv:1401.3021 [hep-ph]].