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Determination of $\Lambda_{\overline{MS}}$ at five loops from holographic QCD

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Abstract

The recent determination of the β–function of the QCD running coupling $\alpha_{\overline{MS}}(Q^2)$ at 5-loops, together with improvements in determining the holographic QCD nonperturbative scale parameter κ, allows a high accuracy computation of the perturbative QCD scale parameter $\Lambda_{\overline{MS}}$. We find $\Lambda^{(3)}_{\overline{MS}} = 0.339 \pm 0.019$ GeV for $n_f = 3$, in excellent agreement with the world average, $\Lambda^{(3)}_{\overline{MS}} = 0.332 \pm 0.019$ GeV. The convergence of the method used for this determination is discussed.

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1 Introduction

The strong coupling α_s is a central quantity for the study of Quantum Chromodynamics (QCD), the gauge theory of the strong interactions [1]. Traditionally, α_s –or equivalently, the perturbative QCD (pQCD) scale parameter Λ_s– has been determined from measurements of high momentum processes or from Lattice Gauge Theory. More recently, it has also been determined from nonperturbative dynamics using the light-front holographic approach to QCD (AdS/QCD) [2], an approach to color confinement that successfully describes both the hadronic spectrum and the bound-state light-front wave functions that control hadronic processes [3].

The method used to obtain Λ_s from AdS/QCD utilizes the effective charge α_{g_1}, defined from the Bjorken sum rule [4]. It has the specific nonperturbative form [5]:

$$\frac{\alpha_{g_1}^{AdS}(Q^2)}{\pi} = \exp\left(-\frac{Q^2}{4\kappa^2}\right). \tag{1}$$

Here Q is the momentum transfer in the spin-dependent nucleon structure functions appearing in the Bjorken sum rule, and κ is the fundamental AdS/QCD scale parameter determined from the light hadron spectrum. This prediction for $\alpha_{g_1}^{AdS}(Q^2)$ agrees remarkably well with experimental data for $\alpha_{g_1}(Q^2)$ in the domain $Q^2 \leq 1$ GeV^2 [6] where AdS/QCD is applicable, and displays an infrared fixed point. In the nonperturbative domain, the relations between $\alpha_{g_1}(Q^2)$ and $\alpha_s(Q^2)$ in other schemes, such as the $\overline{MS}$, MOM, or V schemes are given in Ref. [7]. Such relations are obtained by first assuming that α_s always has an infrared fixed point regardless of the scheme it is expressed in. Then, $\alpha_s(Q^2=0)$ is left as a free parameter to be determined by the matching procedure described below, but with Λ_s determined by the world data. The relations between couplings in different schemes are provided in the pQCD domain by "commensurate scale relations" [8], which are strict predictions of pQCD.

The effective charge α_{g_1} can be expressed at high momentum transfer as a perturbative expansion in the perturbative coupling $\alpha_{\overline{MS}}(Q^2)$, as defined by the $\overline{MS}$ renormalization scheme:

$$\alpha_{g_1}(Q^2) = \pi\left[\frac{\alpha_{\overline{MS}}(Q^2)}{\pi} + a_1\left(\frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\right)^2 + a_2\left(\frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\right)^3 \cdots\right], \tag{2}$$

with the a_i coefficients known up to a_4 [9] and a_5 having been estimated [10]. The normalization and evolution of α_{g_1} is then determined in the $\overline{MS}$ renormalization scheme by the QCD $\beta_{\overline{MS}}$-function and the mass scale $\Lambda_{\overline{MS}}$ [11]. Global hadron-parton duality [12] predicts that the nonperturbative description for $\alpha_{g_1}(Q^2)$ overlaps with the pQCD expression at intermediate values of Q^2. Matching the AdS/QCD and pQCD expressions of $\alpha_{g_1}(Q^2)$ and their derivatives then allows us to determine $\Lambda_{\overline{MS}}$ and the scale Q_0 characterizing the transition between the perturbative and non-peturbative descriptions. The comparison between $\Lambda_{\overline{MS}}$ obtained from light-front holographic QCD and the world data provides a key test of this novel approach to nonperturbative QCD.

It is usually argued that one determines the proton mass and other aspects of the QCD mass scale starting from a measurement of Λ_s in the pQCD domain. This ansatz is difficult to justify since Λ_s is renormalization scheme dependent, whereas masses or other physical observables are not. In fact, the procedure outlined above is the opposite: Λ_s is determined in any scheme starting from the fundamental –scheme independent– confinement scale κ of nonperturbative QCD. Since the QCD Lagrangian has no mass parameter in the limit where the quark masses are neglected, the magnitude of the mass parameter κ cannot be determined in fixed units by QCD itself. Actually, the units normally used for mass, GeV, are a convention. The key predictions are thus ratios such as Λ_s/κ. The value of κ determines all other mass scales in the chiral limit. Indeed, holographic QCD predicts the ratios of masses and mass times radius, etc. For example, it predicts m_p/Λ_s [2], m_ρ/m_p, $m_p \times R_p$ [3], etc. Thus κ is in a sense a "holding parameter", a scale which arises from color confinement and the breaking of conformal symmetry, but it cannot be determined in absolute units by QCD. In fact, the mechanism which sets the confinement scale in the limit of massless quarks is essentially unknown.

As shown in a remarkable article by de Alfaro, Fubini and Furlan (DAFF) [13], it is possible to generate a mass scale κ and a confinement potential while maintaining the conformal symmetry of the action. DAFF write the quantum mechanical evolution operator as a superposition of the generators of the conformal group $Conf\,(R^1)$: The generator of time translation H, the generator of dilatations D, and the generator of special conformal transformations K. Since the generators of $Conf\,(R^1)$ have different dimensions, a mass scale is introduced which in the present context plays a fundamental role, as initially conjectured in [13]. The resulting confining potential in the light-front Hamiltonian then has the unique form of a harmonic oscillator $\kappa^4\zeta^2$, and the soft wall dilaton, which encodes the breaking of conformal symmetry in the higher dimensional anti-de Sitter AdS$_5$ space, must have the form $e^{\kappa^2 z^2}$. The holographic variable z in the 5-dimensional classical gravity theory is identified with the invariant transverse separation ζ between the hadron constituents in the light-front quantization scheme [14, 15, 16].

The harmonic form of the confining light-front potential is equivalent to the familiar linear heavy quark $Q\bar{Q}$ potential in the instant form [17] and has been successful in reproducing essential non-perturbative QCD features, such as Regge trajectories and the Q^2-dependence of hadronic form factors [3].

Since the initial AdS/QCD determination of $\Lambda_{\overline{MS}}$ reported in Ref. [2], several new developments have occurred which allow us to significantly improve the comparison between light-front holographic QCD and the world data: 1) the AdS/QCD scale parameter κ has been determined with greater accuracy from a systematic analysis of the light-quark excitation spectra [18] in the context of a semiclassical superconformal approach unifying mesons and baryons [19]; 2) the running of $\alpha_{\overline{MS}}(Q^2)$ has been computed to five loops [20], that is the β–function is now known up to order β_4 in the $\overline{MS}$ renormalization scheme; and 3) the average world data for $\Lambda_{\overline{MS}}$ has been updated [21]. In this article, we improve our determination of $\Lambda_{\overline{MS}}$ from the light-front holographic QCD framework [2] utilizing these new developments. We also study the convergence of this determination. The pQCD approximants are asymptotic Poincaré series that converge up to an optimal order $\sim 1/a$, where $a = \alpha_s^{pQCD}/\pi$ is the expansion parameter of the series. We have shown in Ref. [7] that the transition between the AdS/QCD description of $\alpha_s(Q^2)$ and its pQCD description occurs at $Q_0^2 = 0.75 \pm 0.07$ GeV^2 in the $\overline{MS}$ scheme: The optimal order in the Poincaré series is thus $1/a(Q_0^2) \simeq 8$. Consequently, it is advantageous to use $\alpha_{\overline{MS}}^{pQCD}(Q^2 > Q_0^2)$ evaluated at five loops to obtain an accurate value of $\Lambda_{\overline{MS}}$ following the matching procedure with the nonperturbative regime described above.

2 Result for $\Lambda_{\overline{MS}}$

The perturbative series of the β function

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta\left(\alpha_s\right) = -\left(\frac{\alpha_s}{4\pi}\right)^2 \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^n \beta_n, \tag{3}$$

calculated up to order β_4 yields the five-loop expression of $\alpha_{\overline{MS}}^{pQCD}$ [22]:

$$\alpha_{\overline{MS}}^{pQCD}(Q^2) = \frac{4\pi}{\beta_0 t}\Bigg[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln(t)}{t} + \frac{\beta_1^2}{\beta_0^4 t^2}\left(\ln^2(t) - \ln(t) - 1 + \frac{\beta_2\beta_0}{\beta_1^2}\right)$$
$$+ \frac{\beta_1^3}{\beta_0^6 t^3}\left(-\ln^3(t) + \frac{5}{2}\ln^2(t) + 2\ln(t) - \frac{1}{2} - 3\frac{\beta_2\beta_0}{\beta_1^2}\ln(t) + \frac{\beta_3\beta_0^2}{2\beta_1^3}\right)$$
$$+ \frac{\beta_1^4}{\beta_0^8 t^4}\Bigg(\ln^4(t) - \frac{13}{3}\ln^3(t) - \frac{3}{2}\ln^2(t) + 4\ln(t) + \frac{7}{6} + \frac{3\beta_2\beta_0}{\beta_1^2}\left(2\ln^2(t) - \ln(t) - 1\right)$$
$$- \frac{\beta_3\beta_0^2}{\beta_1^3}\left(2\ln(t) + \frac{1}{6}\right) + \frac{5\beta_2^2\beta_0^2}{3\beta_1^4} + \frac{\beta_4\beta_0^3}{3\beta_0^4}\Bigg) + \mathcal{O}\left(\frac{\ln(t)^6}{t}\right)\Bigg], \quad (4)$$

with $t = \ln\left(Q^2/\Lambda_s^2\right)$ and

$$\beta_4 = 524.56 - 181.8\, n_f + 17.16\, n_f^2 - 0.22586\, n_f^3 - 0.0017993\, n_f^4, \quad (5)$$

for $N_C = 3$ [20]. The expressions for the lower order β_i can be found *e.g.* in [1, 7]. Here, we will set $n_f = 3$ and use the updated value of the holographic QCD scale parameter, $\kappa = 0.523 \pm 0.024$ GeV [18]. This value characterizes the mass scale of light-quark hadron spectroscopy and is compatible with the fit to the Bjorken sum data at low Q^2 [23] in the holographic QCD validity domain, which yields $\kappa = 0.496 \pm 0.007$ GeV [1]. The updated κ value is lower than –but compatible with– the value we used in [2]: $\kappa = M_\rho/\sqrt{2} = 0.548$ GeV [3], with M_ρ the ρ–meson mass. This value is also used in the study of hadronic form factors, which are expressed in terms of ρ mass poles and its radial recurrencies [3, 24].

As in Ref. [2], we compute $\alpha_{g1}^{pQCD}(Q^2)$ using the Bjorken sum rule [4] up to 5th order in $\alpha_{\overline{MS}}^{pQCD}$ [9]. At β_4 and $\left(\alpha_{\overline{MS}}^{pQCD}\right)^4$ orders, we obtain $\Lambda_{\overline{MS}} = 0.339 \pm 0.019$ GeV and $Q_0^2 = 1.14 \pm 0.12$ GeV^2, to be compared to the present world data, $\Lambda_{\overline{MS}}^{PDG} = 0.332 \pm 0.019$ GeV for $n_f = 3$ [21]. (The value of Q_0 is given in the g_1 scheme and is higher than that in the $\overline{MS}$ scheme [7].)

The uncertainties entering our determination stem from the uncertainty on κ (± 0.016 GeV), the uncertainty from the chiral limit approximation (± 0.003 GeV) and the truncation uncertainty on the Bjorken and $\alpha_{\overline{MS}}^{pQCD}$ series, Eqs. (2) and (4), respectively, ($\pm\, 0.010$ GeV). This uncertainty is taken, for order n, as the difference between the results at orders n and $n+1$, the uncertainty at the highest order being taken equal to that of the preceding order.

The total uncertainty has significantly improved compared to our previous determination, $\Lambda_{\overline{MS}} = 0.341 \pm 0.032$ GeV [2]. The updated prediction of the running coupling is shown in Fig. 1 together with the previous determination [2] and experimental data [6].

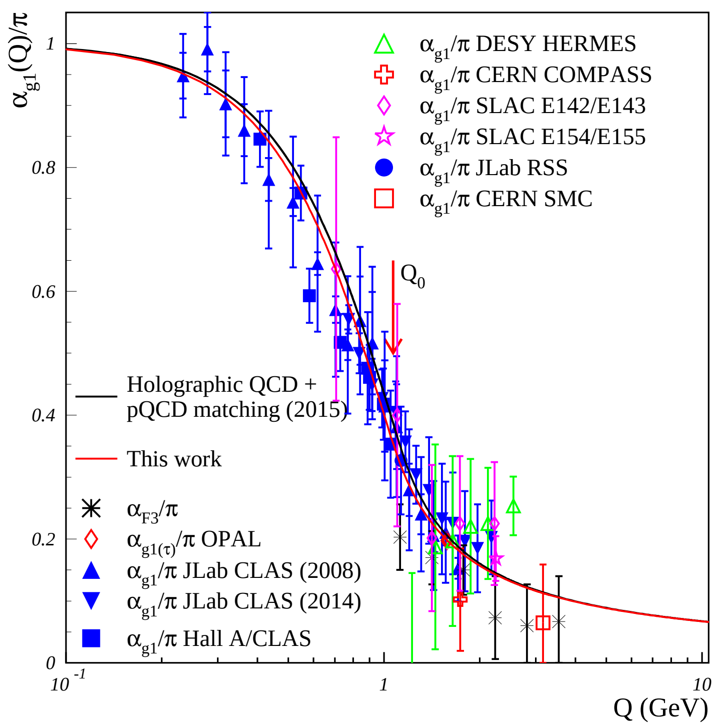

Figure 1: Running of $\alpha_{g_1}(Q)$ for $\kappa = 0.523$ GeV, $\Lambda_{\overline{MS}} = 0.339$ GeV (red line). Also shown are experimental data [6] and the earlier determination of $\alpha_{g_1}(Q)$ for $\kappa = 0.548$ GeV, $\Lambda_{\overline{MS}} = 0.341$ GeV (black line) [2]. The arrow marks the transition scale Q_0.

The result using the Bjorken sum rule coefficient a_5 in Eq. (2), which is assessed in Ref. [10], is $\Lambda_{\overline{MS}} = 0.317 \pm 0.019$ GeV. The uncertainty stems from the uncertainty on κ ($\pm\,0.015$ GeV), the uncertainty from the chiral limit approximation ($\pm\,0.003$ GeV), the truncation uncertainty on the Bjorken and $\alpha^{pQCD}_{\overline{MS}}$ series, Eqs. (2) and (4), respectively, ($\pm\,0.010$ GeV), and an estimate on the a_5 uncertainty (± 0.005 GeV). This latest contribution is assessed by rescaling a_5 by 175.7/130 and obtaining $\Lambda_{\overline{MS}}$ with this rescaled value. (The estimate of the Bjorken sum rule coefficient a_4 in [10] was 130 while the recent exact calculation yields $a_4 = 175.7$ [9]. The ratio 175.7/130 provides an indication of the uncertainty on a_5). We will not use the result for $\Lambda_{\overline{MS}}$ at order a_5 as our main result since a_5 has been only estimated rather than computed.

3 Convergence

The convergence with respect to the β-order is shown in Fig. 2 for the Bjorken series calculated at order $\left(\alpha_{\overline{MS}}^{pQCD}\right)^4$. This series oscillates but nevertheless converges well. The convergence with respect to the Bjorken series order is shown in Fig. 3 for $\alpha_{\overline{MS}}^{pQCD}$ calculated at order β_4. The overall convergence of our method is estimated with both the β- and Bjorken series calculated at the same order. This is also shown in Fig. 3. The convergence is slightly faster than the case when the β-series is kept at order β_4.

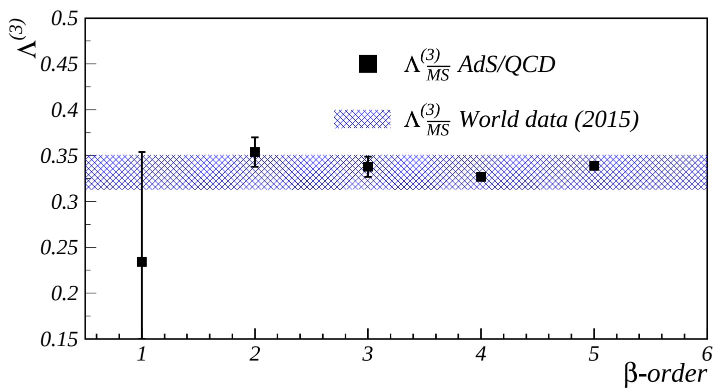

Figure 2: Convergence of our determination of $\Lambda_{\overline{MS}}$ (black squares) in function of the β-series order for $n_f = 3$. The pQCD series for the Bjorken sum rule is computed at order $\left(\alpha_{\overline{MS}}^{pQCD}\right)^4$. The error bars reflect only the uncertainty from the truncation of the β-series. The blue band gives the latest world data.

4 Conclusion

The updated prediction $\Lambda_{\overline{MS}} = 0.339 \pm 0.019$ GeV obtained from matching holographic QCD and pQCD at five loops, is in excellent agreement with the value from the present world data, $\Lambda_{\overline{MS}}^{\mathrm{PDG}} = 0.332 \pm 0.019$ GeV. Our method is applicable for setting the perturbative QCD scale Λ_s in any renormalization scheme. We have used the $\overline{MS}$ scheme since this has been the conventional choice for pQCD analyses. The convergence of the method is satisfactory overall, for both the β-series and the pQCD prediction for the Bjorken sum rule. The largest uncertainty stems from the truncation of the Bjorken sum pQCD series. A calculation of its next term –presently only estimated– and the application of the Principle of Maximum Conformality (PMC) [25, 26] would be valuable

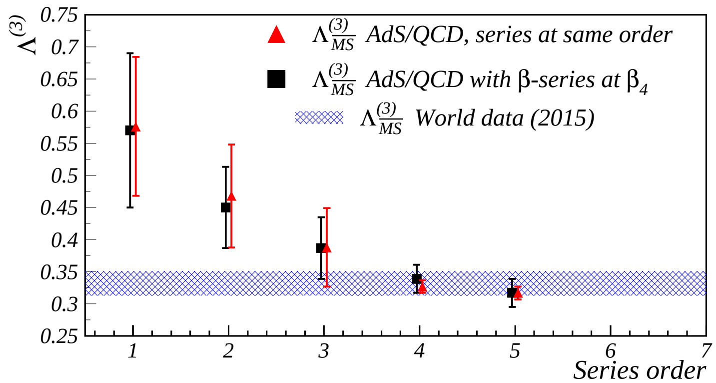

Figure 3: Convergence of our determination of $\Lambda_{\overline{MS}}$ in function of the Bjorken series order (squares). $\alpha^{pQCD}_{\overline{MS}}$ is computed at order β_4 and for $n_f = 3$. The triangles represent the results when both the β- and Bjorken series are computed at the same order. The error bars include only the uncertainty from the series truncation. The blue band gives the latest world data.

for further improving the accuracy of the method discussed here. The uncertainty from the determination of the mass scale κ from hadronic spectroscopy contributes similarly. Thus a reduction in the uncertainty of its value will provide an even more accurate holographic prediction for $\Lambda_{\overline{MS}}$. The excellent agreement between the light-front holographic prediction and the world data validates the relevance of the gauge/gravity approach to nonperturbative strong interaction phenomena.

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