SLAC-PUB-15974

Short Bunch Wake Potentials for a Chain of TESLA Cavities

Alexander Novokhatski [1]

SLAC National Accelerator Laboratory, Menlo Park,CA, 94025, U.S.A.

Alban Mosnier

Institute for Research into the Fundamental laws of the Universe, CEA - Saclay IRFU 91191 Gif sur Yvette Cedex France

Abstract

The modification of wake fields from a single cavity to a quasi periodic structure of cavities is of great concern, especially for applications using very short bunches. We extend our former study [1]. A strong modification of wake fields along a train of cavities was clearly found for bunch lengths lower than 1 mm. In particular, the wakes induced by the bunch, as it proceeds down the successive cavities, decrease in amplitude and become more linear around the bunch center, with a profile very close to the integral of the charge density. The loss factor, decreasing also with the number of cells, becomes independent of bunch length for very short bunches and tends asymptotically to a finite value. This nice behavior of wake fields for short bunches presents good opportunity for application of very short bunches in Linear Colliders and X-ray Free Electron Lasers.

Keywords: wake field potentials, accelerating structure, loss factor,

Submitted to Nuclear Instruments and Methods in Physics Research A

Work supported in part by US Department of Energy under contract DE- AC02- 76SF00515.

1. Introduction

The wake fields, excited by very short bunches in accelerating structures, are of major concern in the design of new projects for Linear Colliders and X-ray Free Electron Laser, because they can give rise to large energy spreads and transverse instabilities. Wake fields calculations performed for single TESLA cavities [2] showed dramatic bunch length dependence, with amplitude scaling approximately inversely with square root of bunch length. In the TESLA-ILC linac [3] and especially in the European XFEL [4] such wake field behaviors could lead to relatively large uncompensated energy spreads. It is worthwhile noting that the relevant parameter for wake field studies, the ratio of iris radius over the bunch length, is about 120 for the TESLA-ILC linac and even 1460 for the XFEL parameters. Some analytical and qualitative results (see, for example [5]) predict the behavior of fields induced by short bunches in a single cavity and periodical structure. For relatively long bunches, the same wake field description can be used for a cavity and for a multi-cavity structure. However, for very short bunches, wake potentials are quite different: they scale inversely with the square root of distance and become infinite (in ultra-relativistic case) at vanishing distance for a single cavity, whereas they have a finite bound for periodical structures. It is then of outstanding importance to know when a chain of cavities behaves like a periodical structure or like a single cavity and what is happening in the "transient region". There are many difficulties about wake potential calculations in the TESLA-type cavities : the elliptical shape of the TESLA cavity, for example, is far from the simple rectangular shape, which is usually assumed for deriving analytical expressions; the beam tube diameter is larger than the iris hole; an accelerating section consists of a cascade of short 9-cell and non corrugated beam tubes and is far from a pure periodic structure. We analyze the transition dynamics of wake fields in TESLA cavities using numerical calculations for a bunch length down to 25 micrometers. We use code [6], a numerical program for solving Maxwell's equations in time domain. This code provides such a possibility because its algorithm is free of numerical dispersion. Results are compared with analytical formulas established for a single cavity and for periodical structures. In addition to the loss factor, which gives the global energy loss of the bunch, the profile of the wake function, which is of major interest for bunch energy spread calculations, has been also carefully studied.

2. Single-cell approach

First, we consider the single-cell approximation, where the different cells of a cavity are assumed to be independent. For relatively long bunches, this approximation gives results with good accuracy and can help in understanding the wake potential evolution with bunch length. Wake fields are usually described by the wake function $w(s)$ and the bunch wake potential $W(s)$, where the former is the wake potential of a point-like charge (which may be called a Green's function) and the latter is the wake potential of a bunch with arbitrary charge density. Once the Green's function is known, then the wake potential for a bunch with longitudinal charge density $q(s)$ and total charge Q can be obtained from a convolution integral

$$W(s) = \frac{1}{Q}\int_{-\infty}^{s} q(s')w(s-s')ds' \tag{1}$$

The loss factor k, which represents the bunch energy loss, is expressed in terms of W(s) by

$$k = \frac{1}{Q}\int_{-\infty}^{+\infty} q(s)W(s)ds \tag{2}$$

Two parameters can be used to describe the "smallness" of the bunch length: the aperture radius a and the wake field parameter s_0, which is defined for a cell with gap length g as

$$s_0 = \frac{a^2}{2g} \tag{3}$$

In the case of a single cell with simple rectangular shape, for the distances smaller than the aperture radius $s < a$, it is possible to derive a good analytical estimation of the Green's function [7]

$$w(s) = \frac{Z_0 c}{a\pi^2}\{\frac{s+g}{\sqrt{s(s+2g)}}F(\frac{\sqrt{s(s+2g)}}{a}) - F(\frac{s}{a})\} \tag{4}$$

where the function $F(x)$ is given by

$$F(x) = \begin{cases} (2/x)\arcsin(x/2) & \text{if } x \leq 2 \\ (\pi/x) & \text{if } x > 2 \end{cases}$$

The bunch wake potential $W(s)$ induced by a Gaussian bunch with a bunch length $\sigma = 0.5$ mm passing through a TESLA-like cell, with rectangular shape (a=35 mm, g=90 mm and b=103.3 mm), has been calculated from this formula and is shown on Fig.1. The result of time-domain computations is also shown

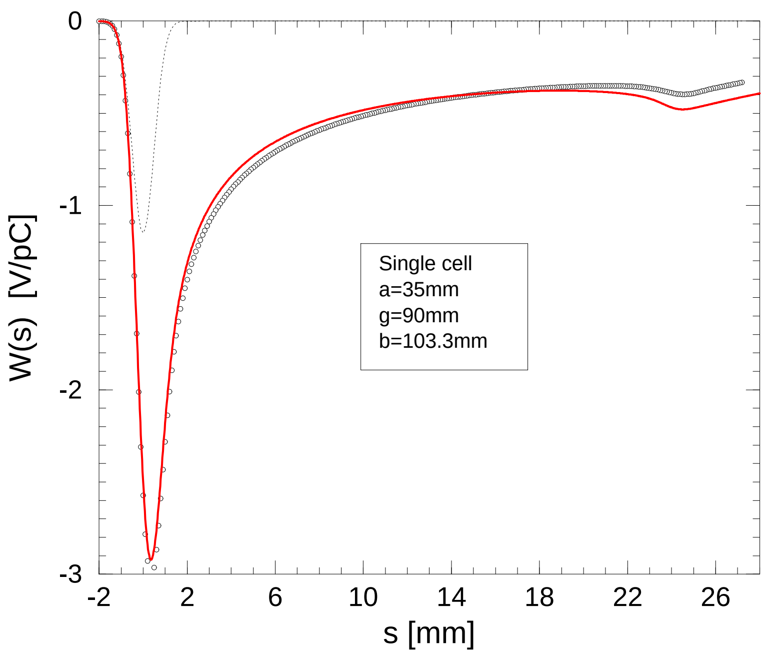

Figure 1: Wake potential of a TESLA type cell: numerical results (circles) and analytical results (solid line).

for comparison and is in very good agreement with the analytical expression. For much shorter bunches, $s << s_0$ ($s_0 = 6.8$ mm for an equivalent TESLA cell with the assumption that a=35 mm and g=90 mm), formula (4) may be reduced to a very simple form

$$w(s) = \frac{Z_0 c}{2\pi^2} \frac{1}{\sqrt{s_0 s}} \tag{5}$$

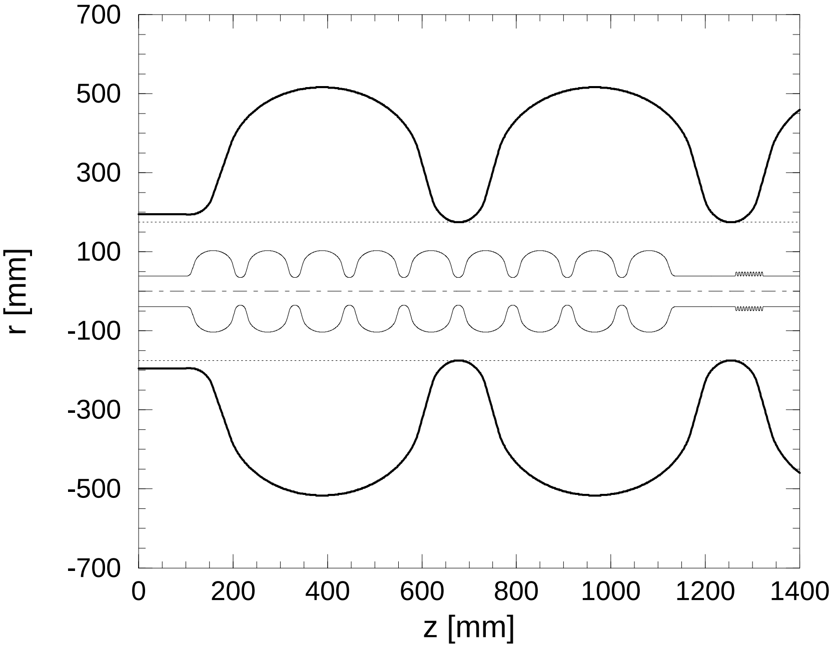

Figure 2: Geometry of a TESLA cavity with its beam tube and details of the end-cells and inner-cells. The aperture radii are 35 mm and 39 mm for the inner-cells and outer-cells, respectively. The cell length is equal to the half of the RF wavelength and the total length of the cavity, including the beam tube with a bellows, amounts to 6 wavelengths.

which is very close to the result of M.Sands [5]. For a single cell, the Green's function or the bunch wake potential varies inversely as the square root of the distance or the bunch length and the cavity gap. However, a cell of a TESLA cavity has an elliptical shape (the drawing of a cavity is represented in Fig.2), and the dependence can change due to the change of the "equivalent" gap size with bunch length. This effect can be observed in Fig.3, which shows the normalized wake potential, product of the wake potential with the square root of the bunch length, for different bunch lengths. It is worth noting that, for longer bunches, the wake potentials decrease more quickly than the square root of bunch length. After a comparison of the loss factors obtained with time-domain computations and analytical results, it turns out that the "equivalent" gap size increases from 88 mm to 100 mm when the bunch length is reduced from 1 mm to 100 μm.

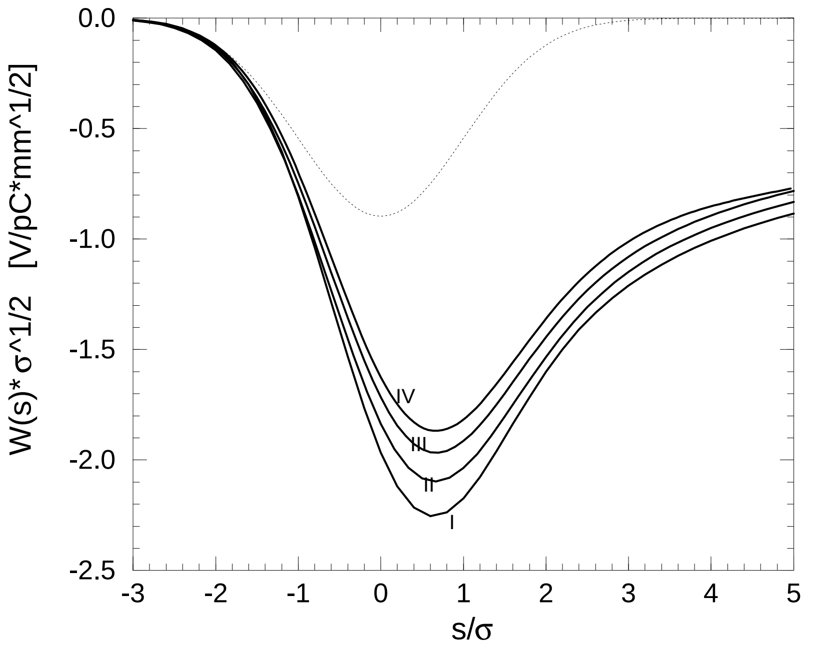

Figure 3: Normalized wake potentials of a TESLA type cell for different bunch lengths: I - 0.05 mm, II - 0.2mm, III - 1 mm and IV - 4 mm. Dotted curve shows charge density distribution.

3. Multi-cell approach

In the single-cell approximation, the bunch is assumed to keep the same field pattern around it before and after crossing the cell. The radiation process is then identical in every cell and the total energy loss in multi-cell structures is simply the sum of the energy losses in the individual cells. However, for short bunches, the field pattern is tremendously changed after the passage through a cell. The electric field lines left by the bunch in the exit beam tube of a single-cell cavity are represented on Fig.4 for two bunch lengths $\sigma = 2$ mm and $\sigma = 100$ μm. The bunch position with respect to the end of the cell is approximately the same for both cases. The electric field lines are almost radial for the longer

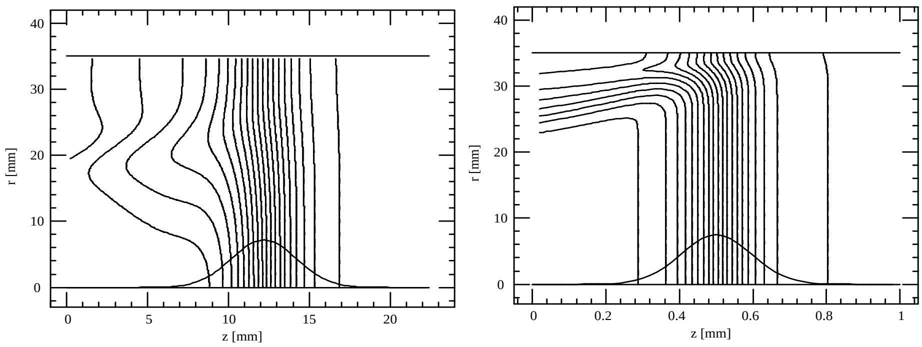

Figure 4: Electric field lines of the bunch field in the tube after first cell of TESLA cavity: left - for σ=2 mm, right - for σ=0.1 mm (the cell is on the left side of bunch)

bunch, as they were before the cell, whereas a lot of lines lie horizontal and do not touch the wall of the beam tube in the vicinity of the shorter bunch. The next cell will then be excited by the bunch of "extended length" and the radiation energy will be smaller than in the previous cell. After several cells, the electric field lines in the iris region reach a "steady state regime", as if they were induced by an equivalent longer bunch. The number of cells N needed to achieve this periodic behavior depends on the bunch length σ: the more the shorter the bunch, the larger the required number of cells. This number can be estimated from a simple geometrical model of the radiation process. At first, the electric field lines of the bunch "head" touch the iris and then, at some distance D, the field from the charge and current of this iris will catch the "tail" of the bunch

$$D = \frac{a^2}{2\sigma} \tag{6}$$

If a period of a structure or a cell length is L then the number of cells is

$$N = \frac{D}{L} = \frac{s_0}{\sigma} \tag{7}$$

where the wake field parameter s_0, which was earlier defined for a cavity(3), but now with the change of g to $L = \lambda_0/2$ takes the form

$$s_0 = \frac{a^2}{2L} = \frac{a^2}{\lambda_0} \tag{8}$$

For the parameters of the TESLA structure (λ_0 =230.6 mm and a=35 mm) this wake field parameter s_0=5.3 mm. The above expression of N, which gives the number of required cells is good when the loss factor is approaching a constant value. For example the number o cells needed for a 0.2 mm bunch is approximately 27. For a 50 μm bunch the required number of cells increases to 106. However the required number of cells to converge the wake potentials may be several times larger. In this paper we will make comparison between the prediction of this formula and the results of computer calculations of the wake potentials in many cells.

The parameter s_0 with some coefficient can be considered as a minimum wavelength λ_{min} for a periodic accelerating structure, which defines the frequency limit, above which no waves can be excited by a very short bunch. Also it means that for distances less then λ_{min} the wake potential of a point charge is nearly a constant. To determine this parameter, one can compare the wake potential gained in one cell and normalized per unit length with known wake potentials at a vanishing distance for infinite structures, as a dielectric canal [8] or a resistive wall tube [9]. Assuming that the wake potential amplitudes are identical for infinite structures, then in the range of small distances the wake is approximately constant, around the value

$$w(s) = \frac{Z_0 c}{\pi a^2} \tag{9}$$

And the minimum wavelength is

$$\lambda_{min} = s_0/\pi^2 \tag{10}$$

The value of the wake potential amplitude is in agreement with the predictions of other authors [10],[11],[12].

In order to understand the transformation of the wake potential in a semi-finite structure we did calculations for the periodic structure, consisting of the regular TESLA inner-cells. The wake potentials induced in each cell are shown on Fig.5.

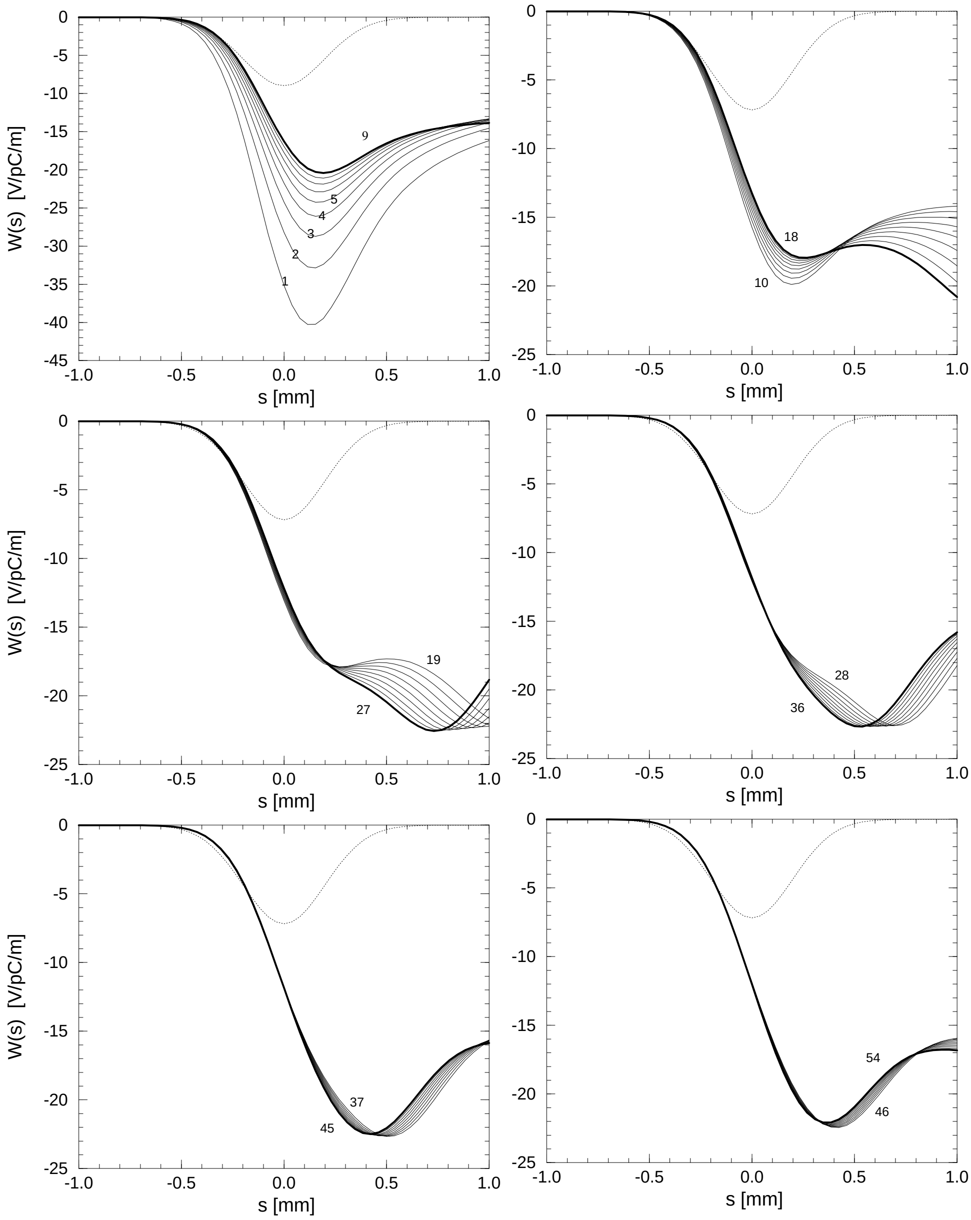

Figure 5: Wake potential of a 0.2 mm long bunch in the periodic structure formed of regular TESLA cells: first nine cells - up left, second nine cells - up right (scales are changed), third nine cells - center left, fourth nine cells - center right, fifth nine cells - down left, six nine cells - down right. Thick solid curves are for cells 9, 18, 27, 36, 45, 54. Dotted curve shows the charge density distribution.

The bunch length is 0.2 mm in this calculations. The results are shown for 9 consecutive cells on each plot. Wake potentials are normalized per unit length. The change in amplitude and shape with increasing number of cells can be clearly seen. The peak value of the wake potential is decreasing during about the first ten cells. After a larger number of cells, a new maximum appears at the tail of the bunch, which slowly moves to the bunch center. In the region after the bunch the wake potentials have damped oscillations, which converge very slowly. More than 50 cells are needed for the stabilization of the wake potential, the quantity of primary interest for calculating the final bunch energy spread.

Analyzing the behavior of the wake potentials calculated for different bunch lengths we found that these potentials have a mutual part. This part can be very well seen if we plot all these potentials on the same plot, as we did in Fig. 6. We found that the mutual part can be very well approximated by an exponential function.The argument of this function is the square root of the distance. We

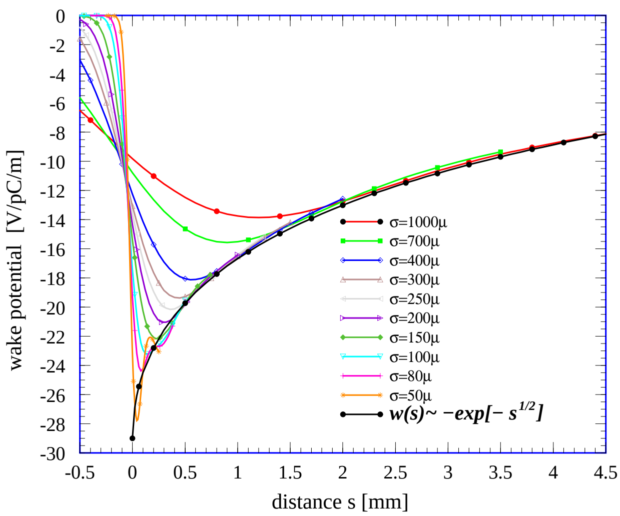

Figure 6: Wake potentials for different bunch lengths and an exponential function.

assume that this exponential function can be a part of the approximate Green's function for the wake potentials in the periodical structure. We did more careful analysis of the wake potentials and the loss factors and found the expression, which gives the best fit of the numerical data. This approximate expression has the following form

$$w(s) = \frac{Z_0 c}{\pi a^2}((1+\beta)\exp(-\sqrt{\frac{s}{s_0}}) - \beta) \tag{11}$$

Parameter β was found to be 0.16 and s_0 is defined as before by the expression(8). For a=35 mm the amplitude is

$$\frac{Z_0 c}{\pi a^2} = 29.4V/pC/m \tag{12}$$

We can note that the number of cells needed to converge the wake potential in the region of the bunch ($\pm\sigma$) corresponds to the number of cells needed to converge the loss factor. Just for a 0.2 mm bunch only 20 cells are needed for the loss factor to converge, as we can see in Fig.7, where the loss factor is plotted as a function of the number of cells. The loss factor also has oscillations with the number of cells, but this oscillation are damped very quickly. We may compare the needed number of cells with the prediction of the formula (7) that gives 27 cells. The calculations for shorter bunch length show that, after a large enough

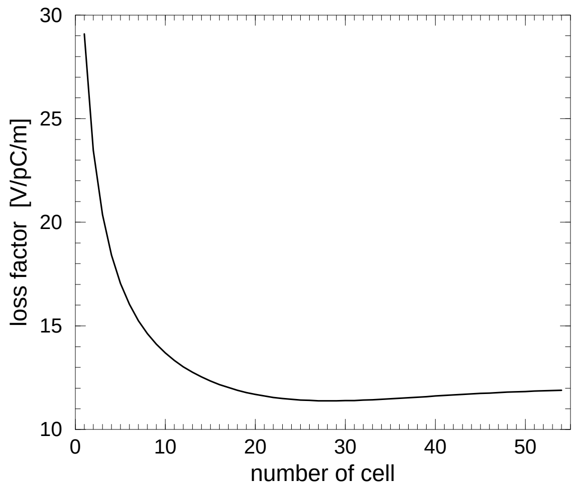

Figure 7: Loss factor vs number of cells in the periodic structure formed of regular TESLA cells.

number of cells, the loss factor tends to a finite constant value. In the region of small bunch lengths, the loss factor nearly does not depend on the bunch length and is in good agreement with the formula, which can be also derived from the estimation of the wake potential (9).

$$k = \frac{Z_0 c}{2\pi a^2} \tag{13}$$

The finite number of the loss factor for the very short bunches opens large possibility of the superconducting accelerator in the application to the X-ray Free Election Lasers, which need bunches of the micron length.

4. Multi-periodic structure

The multi-cell approach gives a good estimation of the wakes induced in periodic structures. However, the linacs made of TESLA cavities are not pure periodic structures, but are composed of a long chain of cryo-modules. The periodicity is indeed broken at two levels: 1) the cavity is not composed of a series of identical cells but includes end-cells with beam tubes of larger aperture; 2) the linac is not composed of regularly spaced cavities but includes longer drift tubes linking the last cavity of a cryo-module and the first cavity of the next cryo-module.

It is also worth noting that, as the tube diameter is larger than the iris hole of the cavity, field must be created to fill in the space enclosed between the two different radii of 35 and 39 mm when the bunch goes out of the cavity and enter the drift tube. In Fig. 8 are shown the electric field lines, linking with the tube wall, that are generated by a 50 μm long bunch, at the exit of a first 9-cell cavity (left plot) and at the exit the last eighth cavity of the cryo-module (right plot). The fields chasing the bunch need some time (or distance) to come closely to the bunch. The superpositions of all fields generated in the previous cavities forms some distribution, which becomes constant after some transient distance. The distribution of the field lines on the right plot can be considered to be a "steady state distribution" or to be very close to it.

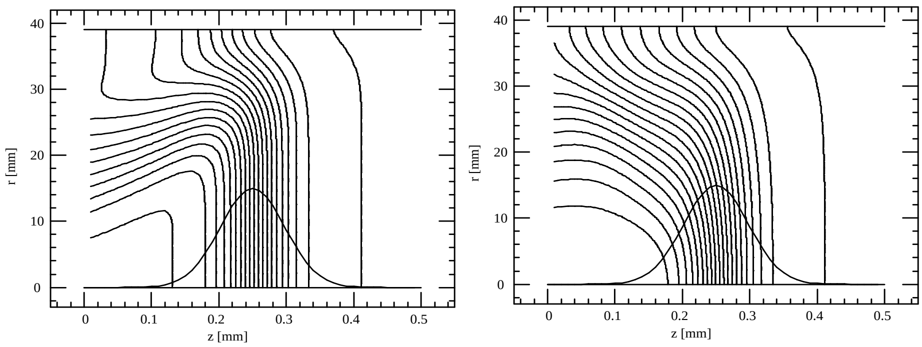

Figure 8: Electric field lines in the tube just after the first TESLA cavity, before the next one (left) and after the eight cavity (right). The bunch length is 50 μm. The distribution of the field lines on the right plot can be considered to be a "steady state distribution".

4.1. Effect of end-cells with beam tubes

The wake potentials of the TESLA cryo-module, housing eight 9-cell cavities and beam tubes, were calculated for different bunch lengths. The successive wake potentials, induced in each individual cavity with a part of beam tube, by a bunch of length $\sigma = 0.2$ mm, are shown on Fig.9 (left side). In the same way, the wake potentials were calculated for the periodic structure, just composed of regular TESLA inner-cells, and without spacing between cavities, for comparison (right side). The thick solid black curves represent the wake potential induced in the last 8th cavity. We note that the wake functions have the same shape, and the amplitudes per cavity are higher in the real multi-periodic TESLA structure, but smaller per unit length.

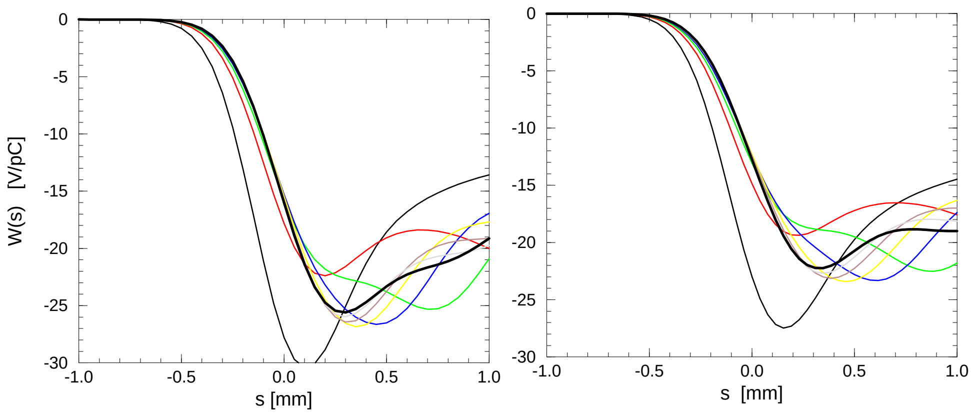

Figure 9: Wake potentials induced by a 0.2 mm long bunch in the cavities of a TESLA cryomodule, for the real geometry (left) and for a periodic structure made of nine regular cells (right). The thick solid curves represent the wake potentials in the last 8th cavity.

In the same way as for the periodical structure, the Green's function of the multi-periodic structure of the TESLA cavities can be well approximated by an expression similar to (11), but with two additional parameters: A and α

$$w(s) = A\frac{Z_0 c}{\pi a^2}((1+\beta)\exp(-\alpha\sqrt{\frac{s}{s_0}}) - \beta) \tag{14}$$

The parameters $A = 0.94$, $\alpha = 1.33$ and $\beta = 0.18$ were found from the fit of the loss factors, computed for short bunches. Fig.10 shows the loss factor of the 8th cavity for different bunch lengths, computed from time-domain simulations and calculated with the expression (14).

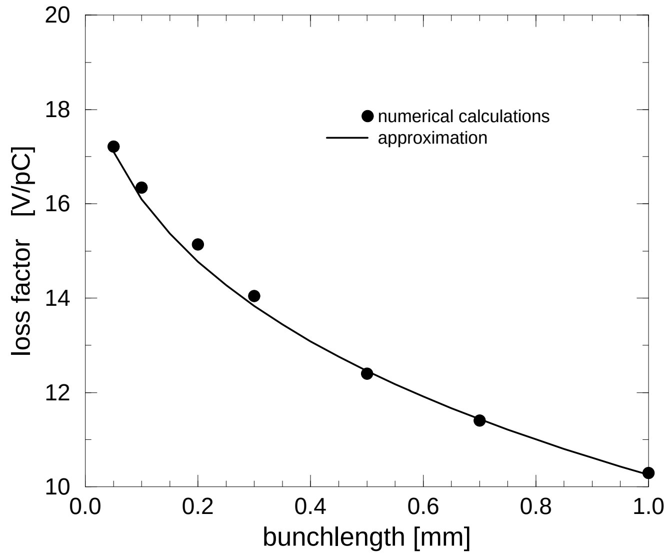

Figure 10: Loss factor of the last cavity in the TESLA cryo-module as a function of the bunch length,computed from time-domain simulations (circles) and calculated with the approximated wake function.

The approximate wake function or the Green's function may not give a good description of the wake potentials, which are not yet converged. As we discussed before eight cavities are enough for the wake potential of a 0.2 mm bunch to be converged, when the wake potential of a 50 μm needs much more cavities.This means that the wake potentials of shorter bunches in one TESLA cryo-module

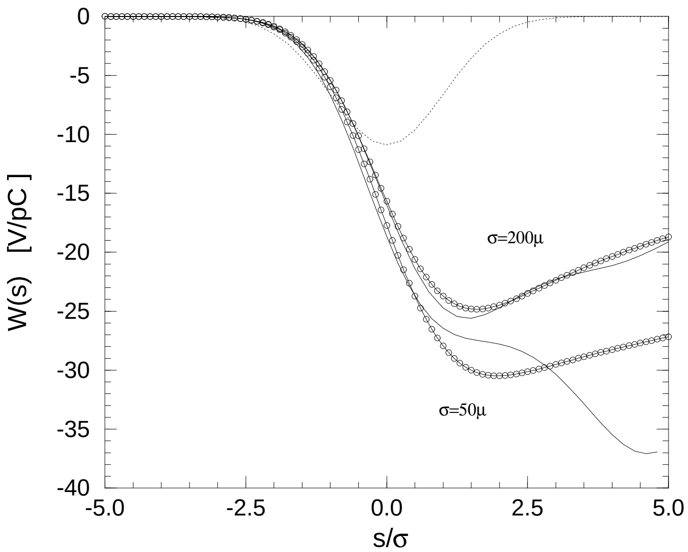

Figure 11: Wake potentials excited in the 8th cavity of the TESLA cryo-module for a bunch length of a 0.2 mm and a 50 μm and analytical approximations (circles)

still have a transient character . In Fig.11, the wake potentials of the 8th cavity of the TESLA cry-module excited by these bunches are plotted together with the analytical approximations.There is a very good approximation of the wake potential of a longer bunch. But for a shorter bunch at the distances above one sigma we have disagreement. However in the main part of the bunch region we have much better agreement. As the calculation of the loss factor needs only this region (expression (2)) then we get a good agreement for the loss factors.

We may continue to calculate wake potentials of very short bunches in the next cryo-modules but we need to know the distance between cry-modules.These distances may be different along the linac. As we mentioned above the periodic solution of the wake fields could not be realized because the accelerating structure is not a periodic and contains different other elements like collimator, bunch compressor chicanes or beam diagnostic regions between cryo-modules.

4.2. Effect of drift tubes between cryo-modules

In order to study the impact of the cavity periodicity break, the wake fields were computed first for the 10 cavities in a row, then for the sequence of 10 cavities but with the 9th cavity replaced by a beam tube, this latter case being close to a real linac. Fig. 12 shows the wake potentials excited by a 50 μm long bunch in the last cavity for both cases. We conclude that the periodicity break results in a significant increase of the wake potentials (about 20 % per active length in this case). Besides, there are different devices along the beam

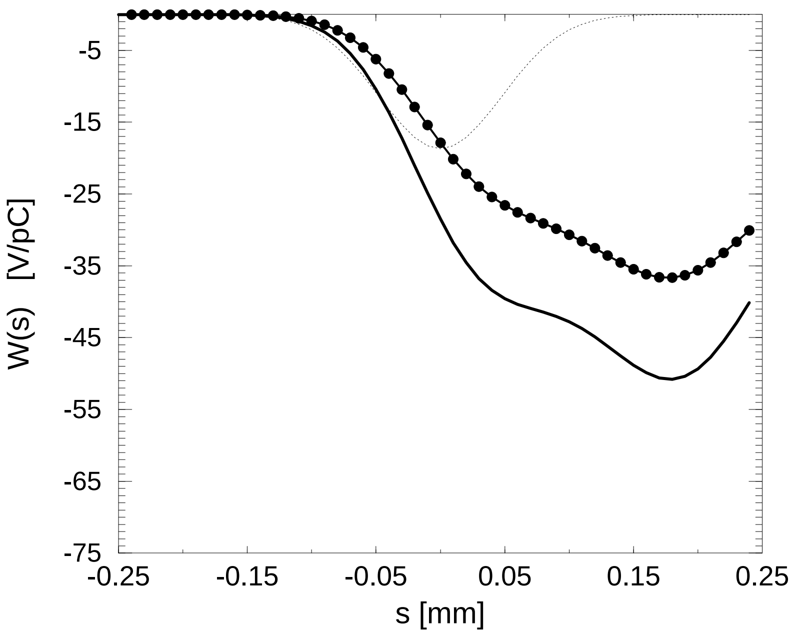

Figure 12: Wake potentials of the 10th cavity excited by a 50 μ m long bunch in the case of periodic structure (circles) and in the real case, when the 9th cavity is replaced by a beam tube(solid line)

tube (as couplers, BPMs, bellows, ...), that do not break bi-periodicity, but can give additional energy loss. Calculations were carried for the train of TESLA cavities including bellows, as shown in geometry of the TESLA cavity in Fig.2.

The first element consists also of bellows. The wake potential excited in the bellows changes with the number of units (cavity plus bellow) in the same way as the wake potentials excited in the cavitie Fig.13 shows the wake potentials excited by a 50 μ m long bunch in the first bellow and the sixth bellow. For comparison, wake potentials excited in the first cavity and the fifth cavity are also shown. These bellows are usually made from a stainless steel, which is not a superconducting material and may absorb some part of the wake fields. The precise estimate of the fractional power deposited in the bellows would require a more quantitative analysis.

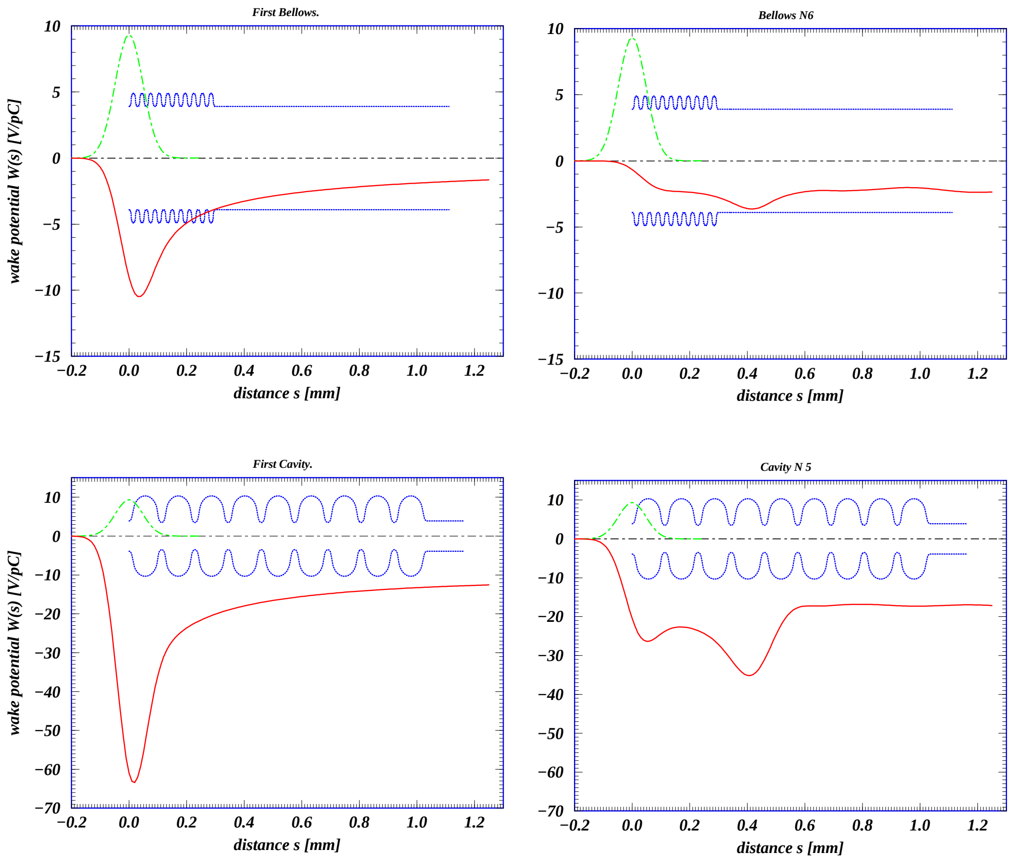

Figure 13: Wake potentials excited by a 50 μ m long bunch in a first and sixth bellows (upper plots) and wake potentials excited in the fist and the fifth cavities (bottom plots) The dashed (green) curves shows the charge density distribution.

5. Loss factor and power loss

The loss factors of the TESLA cry-omodule excited by a 0.7 mm and 50 μ m are shown in Fig.14. From the comparison between the computations with bellows (circles) and without bellows (solid lines), we can conclude that long bunches feel an additional energy loss, approximately proportional to the length of the bellows, whereas short bunches do not feel the presence of bellows.

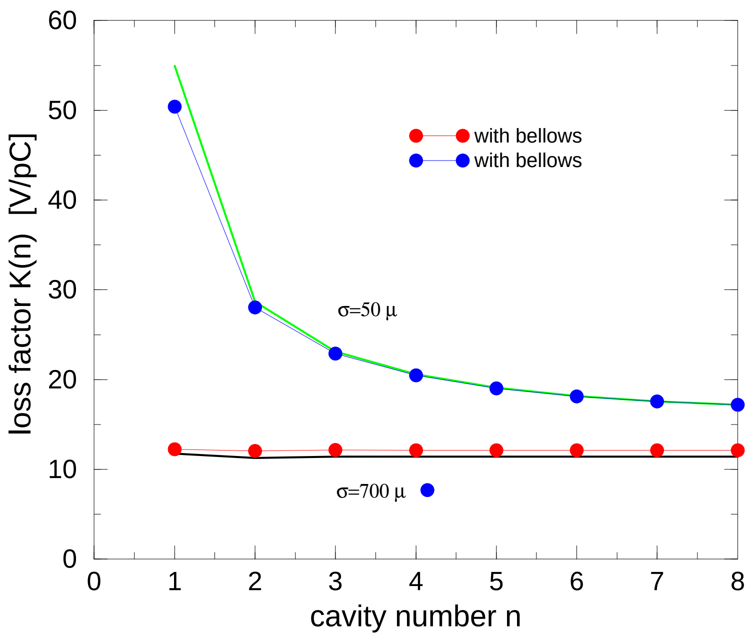

Figure 14: Loss factor of the of TESLA cryo-module with bellows (circles) and without (solid lines) for a bunch length of 0.7 mm and 50 μ m

If we know the loss factor we may also calculate the average power of loss by a train of bunches, with a bunch spacing τ and average current I_{av}

$$P = kI_{av}^2\tau \tag{15}$$

As an example for an average current of 0.2 mA and a bunch spacing of $1\mu sec$, typical parameters envisaged for future high repetition rate FEL, and a loss factor of 18 V/pC/m the power loss can reach a level of 0.72 W/m or 8 W in one cryo-module.

6. Conclusion

A strong modification of wake fields along the train of cavities was clearly found for bunch lengths lower than 1 mm. Starting from the first cavity, the wake fields decrease in amplitude, and the shape becomes more linear around the bunch center, very close to the integral of the charge density.

The loss factor also decreases with the number of cells, becoming independent of the bunch length. The limit for the wake potential of a point charge has been calculated.

The wake potentials in a long chain of TESLA-like cavities can be approximated by an analytical expression. This nice behavior of wake fields for short bunches opens a good opportunity for the application of very short bunches in superconducting linacs based Linear Colliders and Free Electron Lasers.

Nevertheless, the transient behavior of the wake fields must be taken into account because the periodic solution of the wake fields can not be achieved. Actually, the accelerating structure is not fully periodic and contains various elements like bellows, collimators, bunch compressor chicanes or beam diagnostic regions. Every time the periodic structure breaks, the radiation of the field is increasing, resulting in more losses in the next first cavities. A longer breaking distance provides higher losses and the energy loss will not be uniform along the cryo-module. Stainless steel bellows may absorb some part of these losses - low frequency part - but the very high frequency part (above 600 GHz) will be absorbed in the cavities.

Acknowledgment

The authors would like to thank Mike Sullivan and Olivier Napoly for fruitful discussions. Work supported by Department of Energy Contract No. DOE-AC03-76SF00515

[1] A. Novokhatski and A. Mosnier,"Wakefield Dynamics in Quasi Periodic Structures", Proc. of PAC 1997, Vancouver, Canada, p. 467.

[2] A. Mosnier, "Longitudinal and Transverse Wakes for the TESLA Cavity", TESLA report 93-11, DESY Print, May 1993.

[3] C. Adolphsen et al (eds.), "International Linear Collider Tehnical Design Report 2013", Volume 3: Accelerator.

[4] M. Altarelli et al (eds.), "European X-Ray Free-Electron Laser Tehnical Design Report", DESY 2006-097.

[5] Special Issue "Impedance beyond cutoff", Particle Accelerators, Vol.25, Num 2-4 (1990).

[6] A. Novokhatski, "The Computer Code NOVO for the Calculation of Wake Potentials of the Very Short Ultra-relativistic Bunches", SLAC-PUB-11556, Dec 2005.

[7] A.V.Novokhatski "On the estimation of the wake potential for an ultrarelativistic charge in an accelerating structure", Preprint INP 88-39, Novosibirsk, 1988.

[8] A.V.Burov and A.V.Novokhatski "Wake potential of dielectric canal", Proc. of the XVth Int. Conf. on High Energy Accelerators, Hamburg, July 1992, p.537

[9] K.Bane and M. Sands, "The short Range Resistive Wall Wake fields", SLAC-PUB-95-7074, December 1995.

[10] S.A.Heifets and S.A.Kheifets, "High-frequency limit of the longitudinal impedance of an array of cavities", Phys. Rev. D 39, 960 (1989).

[11] R.L.Gluckstern, "Longitudinal impedance of a periodic structure at high frequency", Phys. Rev. D 39, 2780 (1989).

[12] G. Dome, L. Palumbo, V. G. Vaccaro, L. Verolino , "The Frequency Behavior of the Longitudinal. Coupling. Impedance of a Periodic Array of Diaphragms", Proc. of EPAC 1992, Berlin, Germany, p.925.