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# Expected Precision of Higgs Boson Partial Widths within the Standard Model

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ABSTRACT

We discuss the sources of uncertainty in calculations of the partial widths of the Higgs boson within the Standard Model. The uncertainties come from two sources: the truncation of perturbation theory and the uncertainties in input parameters. We review the current status of perturbative calculations and note that these are already reaching the parts-per-mil level of accuracy for the major decay modes. The main sources of uncertainty will then come from the parametric dependences on $\alpha_s$, $m_b$, and $m_c$. Knowledge of these parameters is systematically improvable through lattice gauge theory calculations. We estimate the precision that lattice QCD will achieve in the next decade and the corresponding precision of the Standard Model predictions for Higgs boson partial widths.

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# Contents

# 1 Introduction

After the discovery of the Higgs boson by the ATLAS and CMS collaborations [1,2], much attention has been given to the measurement of the properties of this particle. In principle, accurate measurements of the Higgs properties can tell us whether the corresponding Higgs field is the sole source of mass for quarks, leptons, and gauge bosons, and whether there are new particles that also receive mass from this field. The report [3] for the 2013 Snowmass Community Summer Study reviews the current status of measurements of the Higgs boson couplings and projections of the capabilities of future collider programs.

However accurately the couplings of the Higgs boson are measured, though, these measurements are useful only if combined with comparably accurate predictions from the Standard Model (SM). New physics associated with the Higgs boson appears as deviations from the SM predictions. The report [3] gives many examples of new physics effects that alter the Higgs boson couplings at the few-percent level. The discovery of these effects will require both the measurements and theory of these couplings to have uncertainties below the percent level. If deviations cannot be discerned because of intrinsic uncertainty in the the theoretical predictions, the goal of the program of precision measurements on the Higgs boson will be frustrated.

In particular, the proposed experiments at the International Linear Collider have demonstrated the capability of measuring individual Higgs boson couplings in a model-independent way to the level of parts per mil [4,5]. This seems to us an important goal, but it is only important if the SM predictions for Higgs boson couplings can be given with similar accuracy.

Currently, the partial widths of the Higgs boson within the SM are generally agreed to be predicted to accuracies of a few percent. This situation is summarized in the work of the LHC Higgs Cross Section Working Group [6,7] and in a recent paper by Almeida, Lee, Pokorski, and Wells [8]. This latter paper presents a significant challenge:

> "... the SM uncertainty in computing $B(H \to b\bar{b})$ is presently 3.1% (sum of absolute values of all errors) and expected to not get better than 2.2%, with most of that coming from the uncertainty of the bottom Yukawa coupling determination ... Thus, without a higher-order calculation to substantially reduce this error, any new physics contribution to the $b\bar{b}$ branching fraction that is not at least a factor of two or three larger than 2% cannot be discerned. Thus, a deviation of at least 5% is required of detectable new physics." [8]

We agree with the general conclusions of [6–8] as far as the current situation is concerned, and we will often refer to these useful papers in our analysis below.

However, we believe that the quote in the previous paragraph, which applies the current uncertainties to experiments that will be done a decade from now and draws pessimistic conclusions, is seriously misleading. Most importantly, it underestimates the power of lattice QCD to give us precision knowledge of the $b$ quark mass and of its renormalization to the Higgs boson mass scale. We will argue here that the SM predictions for the Higgs boson partial width to $b\bar{b}$, and for the other dominant decay modes, will be improved to the parts-per-mil level on a time scale that matches the needs of the High-Luminosity LHC and ILC experimental programs.

This paper is organized as follows: In Section 2, we develop basic notation for our study of Higgs partial width uncertainties. In Section 3, we review perturbative computations of the partial widths to the dominant SM decay modes and the uncertainties that they imply. Our conclusion is that it is within the current state of the art to reduce the uncertainties from missing terms in perturbation theory to the parts-per-mil level. For many of the Higgs boson partial widths, this is already achieved.

In Section 4, we discuss the determination of the most important input parameters — $\alpha_s$ and the $b$ and $c$ quark masses — from lattice gauge theory. Data from lattice QCD simulations can be used to determine the QCD parameters in several different ways. The most straightforward method to describe is to compute the spectrum of heavy-quark mesons, adjust the parameters of the lattice action to fit the measurements, and then convert these parameters to a continuum definition (for example, $\overline{\text{MS}}$ subtraction). This method is typically limited by the accuracy of existing lattice QCD perturbation theory calculations. An alternative and more promising method is to use lattice simulations to predict continuum quantities such as QCD sum rules that can be readily interpreted using continuum QCD calculations. It is worth noting that almost all of the highest-precision determinations of $\alpha_s$ and many of the highest-precision determinations of $b$ and $c$ mass reported by the Particle Data Group [9] use this strategy. In Section 4, we illustrate this approach with the lattice calculation of the moments of $b$ and $c$ quark pseudoscalar current correlation functions. These correlation functions were used in [10] to provide measurements of $\alpha_s$, $m_b$ and $m_c$ with accuracy at the current state of the art. Using toy Monte Carlo calculations, we estimate how much the uncertainties computed in [10] could be decreased over the next decade using the increased computer resources that should become available over this time. Section 5 gives our conclusions.

## 2 Structure of Higgs boson partial widths

In this paper, we will quote uncertainties using the Higgs boson partial widths. Our common coin will be the relative theoretical uncertainty $\delta_A$ on the extracted coupling of the Higgs boson to $A\overline{A}$, which we will take uniformly to be $\frac{1}{2}$ of the uncertainty on the the corresponding partial width.

$$\delta_A = \frac{1}{2}\frac{\Delta\Gamma(h \to A\overline{A})}{\Gamma(h \to A\overline{A})} . \tag{1}$$

For definiteness, we set the Higgs boson mass to $m_h = 126.0$ GeV throughout this paper.

More generally in this paper, we will use the symbol $\Delta$ to denote an absolute uncertainty on a measurable quantity, and $\delta$ to denote the relative uncertainty,

$$\delta X = \frac{\Delta X}{X} \ . \tag{2}$$

In this notation, $\delta_A = \frac{1}{2}\delta\Gamma(h \to A\overline{A})$.

There are two contributions to the $\delta_A$. The first is the *theoretical error* due to the fact that the perturbation theory is computed only up to a certain order. As we will see, theoretical errors for the $\delta_A$ are, in almost all cases, already at the few parts-per-mil level. The second is the *parametric error* due to the uncertainties of needed input parameters. These parametric errors will have most of our attention in the paper.

In [6] and [8], uncertainties are quoted for the prediction of Higgs branching ratios. We prefer to work with partial widths, because these are more primitive objects. Branching ratios are composites that depend on all of the partial widths, through

$$BR(h \to A\overline{A}) = \frac{\Gamma(h \to A\overline{A})}{\sum_C \Gamma(h \to C\overline{C})} \ , \tag{3}$$

where the sum over $C$ runs over all decay modes. This can potentially lead to some confusion. For example, in Table IV of [6], the authors quote an uncertainty of 2% in the branching ratios $BR(h \to \tau^+\tau^-)$ and $BR(h \to WW^*)$ for a 120 GeV Higgs boson due to parametric dependence on the $b$ quark mass. This comes entirely from the dependence on $\Gamma(h \to b\overline{b})$ in the denominator of (3) and has nothing to do with the Higgs couplings to $\tau^+\tau^-$ or $WW$. This impression is rectified in the presentation in Table 1 of [7]. We note that the complete program of Higgs boson measurements planned for the ILC allows the absolutely normalized partial widths to be extracted in a model-independent way [4].

A Higgs boson partial width typically has the structure

$$\Gamma(H \to A\overline{A}) = \frac{G_F}{\sqrt{2}}\frac{m_h m_A^2}{4\pi} \cdot \mathcal{F} \tag{4}$$

where $\mathcal{F}$ is a scalar function of coupling constants and mass ratios. The factor $m_A^2$ arises from the fact that the Higgs coupling to $A\overline{A}$ is proportional to $m_A$. It is often the case that the dominant contribution to the parametric uncertainty in $\Gamma(H \to A\overline{A})$ comes from this term.

The contributions to $\delta_A$ from the first two terms of the prefactor are

$$\begin{aligned} \delta_A &= \frac{1}{2}\delta G_F \oplus \frac{1}{2}\delta m_h \\ &= (3 \times 10^{-7}) \oplus (1.2 \times 10^{-4}) \ , \end{aligned} \tag{5}$$

where the first term uses the current uncertainty [9] and the second term assumes a Higgs boson mass measurement with an uncertainty of 30 MeV, as expected at the ILC [4]. The dependence on the Higgs mass is larger in the $h \to gg, \gamma\gamma, \gamma Z$ partial widths, which are proportional to $m_h^3$. However, it is only non-negligible for the partial widths to $WW^*$ and $ZZ^*$, which depend strongly on the available phase space. The uncertainty in (5) coming from $m_A$ depends on the particle species in question. For $\tau$, $W$, and $Z$, there are well-defined on-shell values which are known quite accurately [9]:

$$\delta m_\tau = 9 \times 10^{-5} \qquad \delta m_W = 1.9 \times 10^{-4} \qquad \delta m_Z = 2.3 \times 10^{-5} \ . \tag{6}$$

These estimates give the impression, which is also correct in the complete theory, that the uncertainties in Higgs couplings due to the uncertainties in these input parameters are negligible.

For quark and gluon final states, the situation is quite different. Well-defined on-shell states are not theoretically accessible, and so we must rely on QCD perturbation theory, which potentially brings in sizable parametric uncertainties.

QCD perturbation theory is best behaved if one evaluates Higgs partial widths using the $\overline{\text{MS}}$ mass evaluated at the Higgs boson mass scale. However, the masses of the quarks are usually quoted at a much lower scale, either as the perturbative pole masses or as the $\overline{\text{MS}}$ masses at scale near the quark threshold. The conversion of these mass values to $\overline{\text{MS}}$ masses at $m_h$ is often a dominant uncertainty in the prediction of the Higgs boson couplings.

QCD sum rules measure off-shell quark masses at momenta of the order of $2m_Q$. The Higgs boson couplings are obtained most accurately by directly extrapolating these values to $m_h$. We will use the $\overline{\text{MS}}$ masses $m_b(10.0\text{ GeV})$ and $m_c(3.0\text{ GeV})$ as our basic inputs. The conversion of a mass value at $2m_Q$ to a pole mass brings in a substantial QCD uncertainty, and there is an additional uncertainty in converting the pole mass back to an $\overline{\text{MS}}$ value at $m_h$. The papers [6–8] use the pole masses as inputs. This leads to a stronger dependence on the input mass and $\alpha_s$ values than what we quote below and, consequently, an overestimate of the uncertainty.

The QCD theory of the evolution of mass parameters is nicely reviewed by Chetyrkin, Kühn, and Steinhauser in [11], with a computer code `RunDec` implementing their prescriptions with terms up to NNNLO also provided. The uncertainty in the conversion from low scale masses to $m_Q(m_h)$ due to the truncation of perturbation theory is small: for example, the NNNLO terms in the series give a relative correction of $0.8 \times 10^{-4}$. In the following, and in our later discussion of QCD effects,

$$a(\mu) = \frac{\alpha_s^{\overline{MS}}(\mu)}{\pi} \tag{7}$$

Using the notation of [11], the parametric uncertainties in $m_b(m_h)$ are proportional to the derivatives

$$\begin{aligned} \frac{m_b(10)}{m_b(m_h)} \frac{dm_b(m_h)}{dm_b(10)} &= 1 \\ \frac{\alpha_s(m_Z)}{m_b(m_h)} \frac{dm_b(m_h)}{d\alpha_s(m_Z)} &= a(m_Z) \cdot \frac{\gamma_m(a(m_h)) - \gamma_m(a(10))}{\beta(a(m_Z))} = -0.38 \ . \end{aligned} \tag{8}$$

The numerical values are computed using 5-flavor running and the current PDG value $\alpha_s(m_Z) = 0.1185$. Note that the derivative in the first line is reduced by taking a fixed renormalization point of 10.0 GeV for $m_b$ rather than one that depends on $m_b$. If we took $m_b(m_b)$ as a reference, this coefficient would be 1.19; for the pole mass, this coefficient is 1.28. We will also need the conversion factor

$$\frac{\alpha_s(m_Z)}{\alpha_s(m_h)} \frac{d\alpha_s(m_h)}{d\alpha_s(m_Z)} = \frac{\alpha_s(m_Z)}{\alpha_s(m_h)} \frac{\beta(a(m_h))}{\beta(a(m_Z))} = 0.95 \ . \tag{9}$$

For the input variable $m_c(3)$, we need to take into account 4-flavor running between $m_b$ and the reference point. The bulk of the effect is accounted in

$$\begin{aligned} &\frac{\alpha_s(m_Z)}{m_c(m_h)} \frac{dm_c(m_h)}{d\alpha_s(m_Z)} = \\ &\quad a(m_Z) \cdot \frac{\gamma_m(a(m_h)) - \gamma_m(a(m_b)) + \gamma_m^{(4)}(a^{(4)}(m_b)) - \gamma_m^{(4)}(a^{(4)}(3))}{\beta(a(m_Z))} = -0.91 \ . \end{aligned} \tag{10}$$

There is also a small dependence of $m_c(m_h)$ on the position of the matching point $m_b$, given approximately by

$$\frac{m_b(m_b)}{m_c(m_h)} \frac{dm_c(m_h)}{dm_b(m_b)} = 2(\gamma_m(a(m_b)) - \gamma_m^{(4)}(a^{(4)}(m_b))) = 0.004 \ . \tag{11}$$

Finally, there are tiny discontinuities between the 5-flavor and 4-flavor formulae that sightly change the dependences given in these two equations. We quote the final result in (12).

In all, we find that the term $m_Q^2$ in (4), for the cases of $Q = b$ or $c$, gives a contribution to the uncertainty from the parametric dependence on quark masses and on $\alpha_s$. This dependence is given by

$$\delta m_b(m_h) = 1.0 \cdot \delta m_b(10) \oplus (-0.38) \cdot \delta\alpha_s(m_Z)$$
$$\delta m_c(m_h) = 1.0 \cdot \delta m_c(3) \oplus (-0.90) \cdot \delta\alpha_s(m_Z) \oplus (0.006) \cdot \delta m_b(10) \ . \tag{12}$$

The coefficients in this expression are of order 1, so it is already clear that very accurate values of the parameters on the right-hand side are needed to predict Higgs partial widths to part-per-mil accuracy.

## 3 Perturbation theory for Higgs boson partial widths

With the orientation given in the previous section, we now review the status of perturbative computations of the partial width for the major decay modes of the SM Higgs boson. A detailed overview of SM Higgs decay modes is given in Djouadi's review paper [12]. That discussion has been updated in [7,8]. In particular, Table 3 of [8] gives the parametric dependence of the predictions for the full set of input parameters. However, since we are using a different scheme of inputs,we must revisit the dependence on the most important parameters $m_b$, $m_c$, and $\alpha_s$.

### 3.1 $h \to b\bar{b}$

The corrections to the partial width $\Gamma(h \to b\bar{b})$ can be grouped as (i) QCD corrections to the correlation function of scalar currents $b\bar{b}$, (ii) additional QCD corrections involving flavor singlet intermediate states, (iii) electroweak corrections and mixed QCD/electroweak corrections. All terms are proportional to $m_b^2(m_h)$. The dominant corrections are of the type (i).

The corrections of type (i) are known to $\mathcal{O}(\alpha_s^4)$ through a very impressive calculation of Baikov, Chetyrkin, and Kühn [13]. They evaluate to

$$\begin{aligned}\tilde{R} &= 1 + 5.667a + 29.15a^2 + 41.76a^3 - 825.7a^4 \\ &= 1 + 0.2037 + 0.0377 + 0.0019 - 0.0013 \ ,\end{aligned} \tag{13}$$

so that the series seems to be converging, with a residual error at the part-per-mil level in $\delta_b$ [14]. The parametric dependence of (13) on $\alpha_s$ is obtained as

$$\frac{\alpha_s(m_h)}{\tilde{R}} \frac{d}{d\alpha_s(m_h)} \tilde{R} = 0.22 \tag{14}$$

This must be combined with the dependence of the prefactor given in (12).

The corrections (ii) begin in $\mathcal{O}(a^2)$, are known to $\mathcal{O}(a^3)$, and are less than 1% corrections to $\delta\Gamma_b$ [15,16].

For the corrections of type (iii), the complete $\mathcal{O}(\alpha)$ results is known [17–19], but at the 2-loop level only the leading terms of $\mathcal{O}(\alpha a m_t^2/m_h^2)$ [20,21] and $\mathcal{O}(\alpha^2 m_t^4/m_h^4)$ [22] have been computed. Numerically, these three terms are, respectively,

$$\delta\Gamma = 0.3\% - 0.02\% + 0.05\% \tag{15}$$

Thus, the theoretical understanding of this decay is already close to the part-per-mil level in $\delta_b$. The parametric dependence on the most important parameters is

$$\delta_b = 1. \cdot \delta m_b(10) \oplus (-0.28) \cdot \delta\alpha_s(m_Z)\ . \tag{16}$$

In [5], it was estimated that the $hb\bar{b}$ coupling would be measured to 0.3% at the ILC in its late stages.

### 3.2 $h \to c\bar{c}$

The theoretical calculation of the partial width $\Gamma(h \to c\bar{c})$ is essentially the same as that for $h \to b\bar{b}$. In particular, the qualitative picture that the theory is close topart-per-mil accuracy continues to hold. The parametric uncertainty, combining (12) and (14), is

$$\delta_c = 1. \cdot \delta m_c(3) \oplus (-0.80) \cdot \delta\alpha_s(m_Z)\ . \tag{17}$$

In [5], it was estimated that the $hc\bar{c}$ coupling would be measured to 0.7% at the ILC in its late stages.

### 3.3 $h \to \tau^+\tau^-$

The theoretical calculation of the partial width $\Gamma(h \to \tau^+\tau^-)$ is very similar to that for $h \to b\bar{b}$, except that there are no QCD corrections except for universal ones. We see no issue here in obtaining a precise SM prediction. In [5], it was estimated that the $h\tau^+\tau^-$ coupling would be measured to 0.7% at the ILC in its late stages.

### 3.4 $h \to gg$

The theoretical prediction for the partial width $\Gamma(h \to gg)$ begins in $\mathcal{O}(a^2)$. The series of QCD corrections has been computed to $\mathcal{O}(a^4)$ by Schreck and Steinhauser [23], with each term given as a series in $\tau = m_h^2/4m_t^2$. Baikov and Chetyrkin [24] and Moch and Vogt [25] have also obtained the leading term at $\mathcal{O}(a^5)$. If $\Gamma_0$ is the leading-order result for $m_t \gg m_h$, the series evaluates to

$$\begin{aligned}\frac{\Gamma}{\Gamma_0} &= 1.0671 + 19.306a + 172.76a^2 + 467.68a^3 \\ &= 1.0671 + 0.6942 + 0.2234 + 0.0217\end{aligned} \tag{18}$$

The parametric dependence of (18) on $\alpha_s$ is obtained as

$$\frac{\alpha_s(m_h)}{\Gamma}\frac{d}{d\alpha_s(m_h)}\Gamma = 2.6 \tag{19}$$

There is also an electroweak correction of +5%, known only to the leading order (which is already $\mathcal{O}(\alpha a^2)$), computed by Actis, Passarino, Sturm, and Uccirati [26]. At the 1% level, some final states produced by the $hgg$ coupling contain $b\bar{b}$ due to gluon splitting. It should be clarified through simulation to what extend these final states will be classified by the experiments as $h \to b\bar{b}$ rather than $h \to gg$ decays.

We find that the uncertainty from theory in prediction of the $hgg$ coupling is now at the 1% level. This situation is improvable, though the calculation by computing additional orders of perturbation theory will be challenging. The important parametric dependence of the SM prediction is

$$\delta_g = 1.2 \cdot \delta\alpha_s(m_Z) \ . \tag{20}$$

In [5], it was estimated that the $hgg$ coupling would be measured to 0.6% at the ILC in its late stages.

### 3.5 $h \to \gamma\gamma$

For $\Gamma(h \to \gamma\gamma)$, the leading term is $\mathcal{O}(\alpha^2)$. The electroweak correction of order $\mathcal{O}(\alpha^2)$. has been computed by Passarino, Sturm, and Uccirati [27], and the QCD corrections of $\mathcal{O}(\alpha\alpha_s^2)$ and $\mathcal{O}(\alpha\alpha_s^3)$ have been computed by Maierhöfer and Marquard [28]. The relative sizes of the corrections are, respectively,

$$-1.6\% + 1.8\% + 0.08\% \ . \tag{21}$$

The uncertainty in the prediction of the $h\gamma\gamma$ coupling is, then, at the parts-per-mil level, and there is no significant parametric uncertainty. In [5], it was estimated that the $h\gamma\gamma$ coupling would eventually be measured to 0.8% using a combination of LHC and ILC results.

### 3.6 $h \to WW^*$, $h \to ZZ^*$

The situation for the decays $h \to WW^*$ and $h \to ZZ^*$ is somewhat more complicated, and beyond the scope of this paper to explain in full. The decay involves color-singlet particles in leading order, so the radiative corrections are at the percent level. The complete $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha)$ corrections have been computed by Bredenstein, Denner, Dittmaier, and Weber [29]. These authors find corrections of, for example, 1% for $h \to e^+e^-\mu^+\mu^-$, 3% for $h \to \nu_e e^+\mu^-\overline{\nu}_\mu$, 7% for $h \to \nu_e e^+ q\overline{q}$, and 10% for

$h \to q\bar{q}q\bar{q}$. Quite consistently, the difference between the full radiative corrections and those of the Improved Born Approximation (IBA), in which the two off-shell vector bosons are treated separately, is 1%. Additional corrections to the IBA are known, including corrections of $\mathcal{O}(\alpha a m_t^2/m_h^2)$, $\mathcal{O}(\alpha a^2 m_t^2/m_h^2)$, and $\mathcal{O}(\alpha^2 m_t^4/m_h^4)$. These corrections are reviewed in [30]; they bring the calculation of this approximation to the part-per-mil. A full 2-loop analysis without the IBA approximation will be more difficult.

These partial widths have no important parametric uncertainty due to $\alpha_s$ or $m_b$, but they do depend strongly on the mass of the Higgs boson. From [8] (based on [29]),

$$\delta_W = 6.9 \cdot \delta m_h \ , \quad \delta_Z = 7.7 \cdot \delta m_h \ . \tag{22}$$

That is, a measurement of the Higgs boson mass to 30 MeV precision would lead to a 0.2% theoretical uncertainty in these partial widths.

In [5], it was estimated that the $hWW$ and $hZZ$ couplings would each be measured to 0.2% at the ILC in its late stages. It seems within the state of the art for theory to match this level of accuracy, though it will be a challenge.

## 4 Improved parameters from lattice QCD

One of the implications of the previous section is that the SM predictions for several of the Higgs boson partial widths depend strongly on $\alpha_s$, $m_b$, $m_c$, and $\alpha_s$. The current values of these parameters are

$$\begin{aligned} \alpha_s(m_Z) &= 0.1185 \pm 0.0006 \qquad (\pm 0.5\%) \\ m_b(10) &= 3.617 \pm 0.025 \text{ GeV} \qquad (\pm 0.7\%) \\ m_c(3) &= 0.986 \pm 0.006 \text{ GeV} \qquad (\pm 0.6\%) \end{aligned} \tag{23}$$

The first of line of (23) is the current Particle Data Group value [9]. The second and third lines are lattice gauge theory determinations, from [10]. These are consistent with the PDG averages, with a slightly larger error for $m_b$ and a somewhat smaller error for $m_c$. Using the values in (23), assuming that the errors are uncorrelated and that it is correct to combine errors in quadrature, we find the parametric components of the uncertainty in Higgs coupling predictions to be

$$\delta_b = 0.7\% \ , \quad \delta_c = 0.7\% \ , \quad \delta_g = 0.6\% \ . \tag{24}$$

This is already quite impressive accuracy, but the future program of precision Higgs measurements will require that we do better.

In the rest of this section we will describe how one uses lattice QCD (LQCD) to extract the $\overline{\text{MS}}$ coupling and masses. We will illustrate these ideas with a simple example, and use that example to explore what improvements will be possible over the next decade. Finally we will briefly survey other approaches from LQCD that are likely to contribute over that period.

### 4.1 Lattice QCD

In LQCD, continuous space and time are replaced by a discrete mesh of lattice sites with a lattice spacing $a$ that is typically of order 0.15 fm or less. The path integral describing QCD becomes an ordinary multidimensional integral in this approximation, with the lattice functioning as the ultraviolet regulator. Lattice simulations integrate the path integral numerically, using Monte Carlo methods, to obtain Monte Carlo estimates for vacuum expectation values of a wide variety of operators from which physics is extracted.

Having chosen a value for the bare coupling, the first step in an LQCD simulation is to tune the bare quark masses and the lattice spacing to values that reproduce physical results from the real world. The masses are typically adjusted to reproduce experimental results for particular, well-measured hadron masses: for example, $m_\pi$, $m_K$, $m_{\eta_c}$, and $m_{\eta_b}$. The lattice spacing is set using some other well-measured quantity, such as the pion decay constant $f_\pi$. Once one has tuned these parameters, renormalized matrix elements from a LQCD simulation will agree with the corresponding matrix elements from continuum QCD up to errors of $\mathcal{O}(a^2)$. Simulations are generally performed at multiple values of $a^2$ and results extrapolated to $a = 0$.

The quark masses and the QCD coupling constant are specified quite accurately by the tuning process, but they are defined for the lattice regulator, not the $\overline{\mathrm{MS}}$ regulator typically used in continuum calculations. In principle, bare lattice masses and couplings can be converted into $\overline{\mathrm{MS}}$ quantities using perturbation theory– see, for example, [31] and [32]. In practice, however, the precision of this approach has been limited by the difficulty involved in calculating the conversion formulae, which require high-order perturbative calculations using the (very complicated) lattice regulator.

A different approach that has proven quite successful is to use lattice simulations to generate nonperturbative values for renormalized short-distance quantities, such as matrix elements of current-current correlators at short distances. Renormalized quantities are regulator independent, and therefore values obtained from LQCD simulations can be analyzed using ordinary continuum $\overline{\mathrm{MS}}$ perturbation theory once they have been extrapolated to zero lattice spacing. Such analyses can be used to extract values for the $\overline{\mathrm{MS}}$ coupling and masses in the same way that values are extracted from experimental data.

### 4.2 An example

One quantity that can be used to compute all three of our important input parameters $m_b$, $m_c$ and $\alpha_s$ is the current-current correlator

$$G(t) \equiv a^3 \sum_{\mathbf{x}} m_{0Q}^2 \langle 0 | j_{5Q}(\mathbf{x}, t) \, j_{5Q}(0,0) | 0 \rangle \tag{25}$$

where $j_{5Q} \equiv \overline{\psi}_Q \gamma_5 \psi_Q$ is the pseudoscalar density for a heavy quark $Q$ (either $c$ or $b$) [10]. This correlator is a close relative of the vector current-current correlators, which may be obtained from $e^+e^-$ heavy-quark production. The data for these vector correlators, analyzed with continuum perturbation theory, currently provide the most precises determinations of the heavy quark masses [33]. We consider only the connected correlator, where both currents are on the same quark line, since this simplifies both the lattice simulation and the continuum analysis. The factors of the LQCD bare quark mass $m_{0Q}$ in (25) make $G(t)$ ultraviolet finite. Then the lattice and continuum versions are equal up to finite-lattice spacing corrections:

$$G_{\text{cont}}(t) = G_{\text{lat}}(t) + \mathcal{O}(a^2). \tag{26}$$

Low-$n$ moments of $G(t)$,

$$G_{2n} \equiv a \sum_t t^{2n} G(t) = (-1)^n \frac{\partial^{2n}}{\partial E^{2n}} G(E=0), \tag{27}$$

are perturbative, since the energy $E = 0$ at which they are evaluated is far below the threshold $E \approx 2m_Q$ for heavy quarks. Moments with $2n \geq 4$ are ultraviolet finite. Consequently, in perturbation theory, we obtain

$$G_{2n} = \frac{g_{2n}(\alpha_{\overline{\text{MS}}}(\mu))}{m_Q(\mu)^{2n-4}} \tag{28}$$

where $\mu$ is the renormalization scale and $g_{2n}(\alpha_{\overline{\text{MS}}})$ is a perturbation series known through third order for $2n = 4$, 6, 8, and 10 [34–38]. The scale $\mu$ should be taken close to $2m_Q$ to avoid large logarithms. One then adjusts the values of the $\overline{\text{MS}}$ coupling and quark masses so that the (continuum) perturbative expressions agree with the nonperturbative values for the moments $G_{2n}$ generated by LQCD.

A detailed LQCD analysis of correlator moments is given in [10]. It replaces the moments $G_{2n}$ by reduced moments $R_{2n}$ in order to suppress systematic errors caused by the simulation; statistical errors from the Monte Carlo are insignificant. The values obtained for $m_c(3\,\text{GeV})$ and $\alpha_{\overline{\text{MS}}}(M_Z)$ are accurate to 0.6%, while $m_b(10\,\text{GeV})$ is accurate 0.7%. The dominant source of uncertainty in the first two quantities is the lack of $4^{\text{th}}$-order perturbation theory. The dominant error in $m_b(10\,\text{GeV})$ comes from the finite lattice spacing, which matters more for the $b$ quark because of its larger mass.

An LQCD simulation offers several advantages over experiment as a source for nonperturbative results, beyond the obvious fact that it is easier to instrument a simulation than an experiment. Here, for example, we can produce results for $m_Q = m_c$ and $m_Q = m_b$, but also for several quark masses in between $m_c$ and $m_b$. This allows us to vary the value of $\alpha_{\overline{\rm MS}}(\mu)$, since $\mu \sim 2m_Q$, and therefore to use the simulation data to estimate and bound perturbative corrections beyond third order. The result is a much more reliable estimate of the perturbative errors than comes from the standard procedure of replacing $\mu$ by $\mu/2$ and $2\mu$. Varying the quark mass also allows us to probe and fit the leading nonperturbative behavior, from the gluon condensate. The Operator Product Expansion implies that

$$G_{2n} = G_{2n}^{\rm short-distance} \left\{ 1 + d_{2n}(\alpha_{\overline{\rm MS}}) \frac{\langle \alpha_s G^2/\pi \rangle}{(2m_Q)^4} + \cdots \right\} \tag{29}$$

In practice the condensate correction turns out to be negligible compared to other uncertainties, because it is suppressed by $1/(2m_Q)^4$.

LQCD simulations also allow us for the first time to determine ratios of quark masses nonperturbatively [10,39]. These ratios, which can be determined quite accurately, provide a highly non-trivial check on values obtained from perturbative methods, and can be used to leverage a precise determination of one mass into precise determinations of other masses.

### 4.3 Projections

The LQCD analysis described above has yielded the heavy quark masses and the QCD coupling constant with precisions that are already below 1%. A detailed study of those results shows that the most important limiting factors are the lack of higher-order perturbation theory ($4^{\rm th}$-order) and the finite lattice spacing [10]. Both sources of error can potentially be reduced. It seems feasible, given time, to compute the next term in perturbation theory for the correlator moment, bringing these to $4^{\rm th}$ order in $\alpha_s$. The extent to which the lattice spacing can be reduced depends upon further reductions in the cost of computing. Figure 1 shows that computing costs have fallen by roughly a factor of 100 since 2005 at the USQCD facilities at Fermilab and Jefferson Lab. Similar reductions are expected over the next 10–15 years. Simulation costs scale roughly as $1/a^6$, and so we expect that the smallest lattice space used for this simulation could be reduced by about a factor of two, from 0.045 fm used in [10] to 0.023 fm.

There are more issues to face, beyond securing adequate hardware, if one wishes to achieve per-mil accuracy in LQCD simulations. Some, like the inclusion of isospin

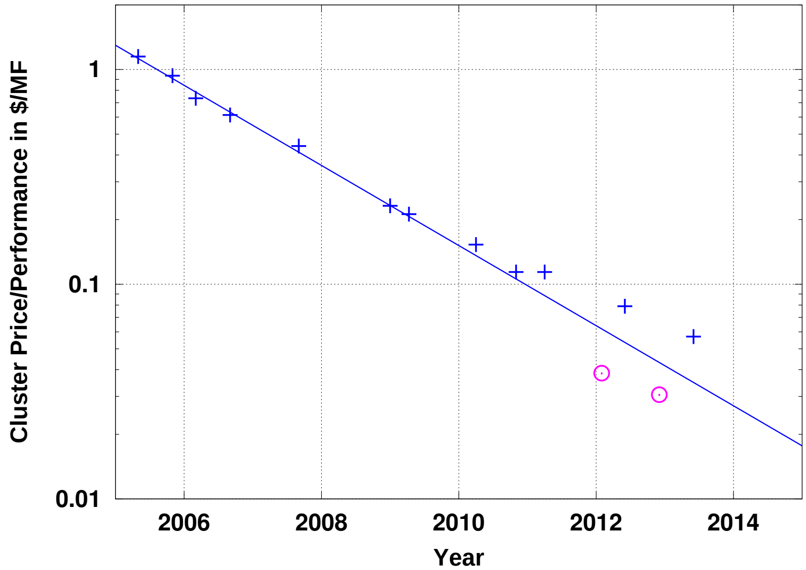


Figure 1: Measured cost per megaflop of lattice QCD computing on the USQCD cluster facilities at Fermilab and Jefferson Lab, plotted versus year. The exponentially improving price/performance of conventional cluster hardware (blue crosses) that was observed through 2011 has fallen off somewhat in the last few years. This has been mitigated by the introduction, where possible, of GPU-accelerated clusters (magenta circles) for lattice calculations.

violation and electromagnetic corrections, we expect will be straightforward. It is possible that some may prove greater challenges. For example, topological structure in the gauge field develops more slowly in simulations at smaller lattice spacing, which may pose problems at very small lattice spacing. Such issues might require new ideas, but the tremendous advances in LQCD over the past decade make us optimistic that any new obstacles will be overcome during the next 10–15 years.

The existing lattice analysis can be used to predict the impact of these improvements in the order of perturbation theory and the size of the lattice spacing on the precision with which we can determine the coupling constant and masses from the correlator moments. The current analysis compares results from multiple lattice spacings in order to determine the dependence of LQCD results on the lattice spacing. This allows us to extrapolate existing results to smaller lattice spacings. By adding realistic noise to these extrapolations, we create synthetic data for smaller lattice spacings that can be combined with existing LQCD data in a new analysis of the masses and coupling. The results tell us the extent to which smaller lattice spacings reduce errors on the masses and coupling. The impact of higher-order perturbation theory is also easily evaluated by adding fake $4^{\text{th}}$-order terms to the perturbation theory.

We have gone through this exercise starting from the analysis in [10]. Our results

| | $\delta m_b(10)$ | $\delta \alpha_s(m_Z)$ | $\delta m_c(3)$ | $\delta_b$ | $\delta_c$ | $\delta_g$ |
|---|---|---|---|---|---|---|
| current errors [10] | 0.70 | 0.63 | 0.61 | 0.77 | 0.89 | 0.78 |
| + PT | 0.69 | 0.40 | 0.34 | 0.74 | 0.57 | 0.49 |
| + LS | 0.30 | 0.53 | 0.53 | 0.38 | 0.74 | 0.65 |
| + LS$^2$ | 0.14 | 0.35 | 0.53 | 0.20 | 0.65 | 0.43 |
| + PT + LS | 0.28 | 0.17 | 0.21 | 0.30 | 0.27 | 0.21 |
| + PT + LS$^2$ | 0.12 | 0.14 | 0.20 | 0.13 | 0.24 | 0.17 |
| + PT + LS$^2$ + ST | 0.09 | 0.08 | 0.20 | 0.10 | 0.22 | 0.09 |
| ILC goal | | | | 0.30 | 0.70 | 0.60 |

Table 1: Projected fractional errors, in percent, for the $\overline{\text{MS}}$ QCD coupling and heavy quark masses under different scenarios for improved analyses. The improvements considered are: PT - addition of 4$^{\text{th}}$ order QCD perturbation theory, LS, LS$^2$ - reduction of the lattice spacing to 0.03 fm and to 0.023 fm; ST - increasing the statistics of the simulation by a factor of 100. The last three columns convert the errors in input parameters into errors on Higgs couplings, taking account of correlations. The bottom line gives the target values of these errors suggested by the projections for the ILC measurement accuracies.

are presented in Table 1. This table shows the percent errors we expect in the masses and coupling from the correlator analysis under various scenarios for improvements: PT denotes the effect of computing QCD perturbation theory through 4$^{\text{th}}$ order. LS denotes the effect of decreasing the lattice spacing to 0.03 fm. LS$^2$ denotes the effect of using lattices with 0.03 fm and 0.023 fm lattice spacing. We recall that the stage LS$^2$ corresponds to an increase in computing power by about a factor of 100. ST denotes the effect of improving the statistics by a factor of 100. We also show percent errors for the Higgs couplings to $b\bar{b}$, $c\bar{c}$, and $gg$, accounting for correlations among the errors in the determination of the parameters. The last line of the table gives, for comparison, the experimental uncertainties in the Higgs boson couplings expected after the ILC measurements [5].

We find that reducing the lattice spacing to 0.023 fm is sufficient to bring parametric errors for the Higgs couplings below the errors expected from the full ILC. Adding 4$^{\text{th}}$-order perturbation theory reduces the parametric errors further, to about half of the expected ILC errors. Adding statistics gives a relatively small further reduction in the errors.

These error estimates are likely conservative because they assume that there is no further innovation in LQCD simulation methods. There already are many alternative lattice methods for extracting the QCD coupling from LQCD simulations: see, for example, [32,40–43]. None of these methods involve heavy quark masses directly and so none have correlations between $\alpha_s$ and heavy quark masses. Small lattice spacings are important for an accurate $b$ mass because of $(am_b)^2$ errors; these can be avoided completely by using effective field theories such as NRQCD [31] or the Fermilab formalism [44] for $b$-quark dynamics in correlators, rather than (highly corrected) relativistic actions. Other renormalized lattice matrix elements, such as off-shell expectation values of $m_Q\overline{\psi}\gamma_5\psi$, can be used to compute masses [45]. There are many ideas that are likely to come into play over the next decade or so.

## 5 Conclusions

In this paper, we have surveyed the current status of the uncertainties in the Standard Model calculations of the Higgs boson partial widths. We have shown that the current theory of these partial widths is already accurate to better than 1%. We have also seen that both the perturbation theory and the parametric inputs to this theory can be improved, to yield predictions at an accuracy beyond even the high level expected for the experiments at the ILC. Lattice gauge theory has a crucial role to play in improving the determination of the most important input parameters.

There is much work to be done in the next decade to realize the program we have outlined. But the result will be that these calculations, combined with the results of precision experiments, will offer a powerful probe into the mysteries of the Higgs boson.

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