

## Effective confining potentials for QCD

Arkadiusz P. Trawiński and Stanisław D. Głazek  
*Institute of Theoretical Physics, Faculty of Physics,  
 University of Warsaw, Hoża 69, 00-681 Warsaw, Poland*

Stanley J. Brodsky  
*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

Guy F. de Téramond  
*Universidad de Costa Rica, 1000 San José, Costa Rica*

Hans Günter Dosch  
*Institut für Theoretische Physik, Philosophenweg 16, Heidelberg, Germany*  
 (Dated: March 22, 2014)

We observe that the effective potentials obtained from lattice gauge theory, derived from string models of hadrons, and determined from models using front-form dynamics and light-front holography agree with each other at leading approximation, not only in their shape – which depends on the form of dynamics – but also in their numerical strength.

PACS numbers: 11.15.Kc, 12.38.Aw, 12.39.-x, 12.39.Pn,

One of the central questions in QCD is to understand the nonperturbative dynamics underlying the confinement of quarks and gluons from first principles [1–5]. For example, in lattice gauge theory one obtains numerical results for the shape and mass scale of the non-relativistic potential which confines pairs of infinitely heavy quarks from the calculation of a Wilson loop where quarks are represented by static color sources [5–8].

Another effective description of quark confinement in mesons is the string model for hadrons, where color-electric fields between two static color sources are squeezed into a thin, effectively one-dimensional, flux tube or vortex [9–12]. In the stochastic vacuum model string formation is a property of the gauge-invariant gluon field-strength correlator, which can be obtained by lattice simulations. It thus connects the lattice with the hadronic string picture [13, 14].

An operator approach to quark confinement in hadrons has also been developed in the “front form” (FF) Hamiltonian dynamic framework [15] which provides a rigorous frame-independent formalism for solving non-perturbative QCD in the FF Fock space.

The renormalization group procedure [16–18] is used to derive the FF QCD Hamiltonian eigenvalue equation which incorporates the effects of all the hadron’s quark and gluon Fock components.

Alternatively, one can write the FF equation of motion for mesons in the form of a single-variable relativistic eigenvalue equation analogous to the non-relativistic quark-antiquark radial Schrödinger equation [19]. The same equation for massless quarks arises from the light-front (LF) holographic mapping [20, 21] of the soft-wall model modification of AdS<sub>5</sub> space [22]

with any dilaton profile which breaks the maximal symmetry of AdS<sub>5</sub> space. It has also been shown that the effective Hamiltonian which follows from modified AdS<sub>5</sub> space with the specific profile  $e^{+\kappa^2 z^2}$  can be derived from a conformally invariant action in one dimension [23, 24].

A framework based on the renormalization group procedure for effective particles (RGPEP) [25], that is close in principle to the similarity renormalization group procedure introduced in Ref. [16], allows one to develop a relativistic quark model in which the effective mass of particles is different from zero. The FF potential is quadratic and its form agrees with the one obtained by applying the Ehrenfest [26] principle to the FF Hamiltonian theory [27] (see below).

One thus obtains, using several different descriptions, a quantum-mechanical wave equation which incorporates color confinement, relativity, and essential spectroscopic and dynamical features of hadron physics, such as linear Regge trajectories [28, 29]. Furthermore, an important prediction of LF-holography is a massless pion for zero quark mass [30, 31] and a unique effective potential [24].

It is important to note that the form of the effective potential in each formalism depends on the form of the dynamics. Our goal in this paper is to compare the physical descriptions, the effective potentials, and the mass scales controlling quark confinement from the lattice, string, and FF theories; we also will bridge the descriptions from light to heavy quarks. An important tool will be the use of the Wentzel-Kramers-Brillouin (WKB) [32] formalism which allows one to relate the maximum distance of separation between quarks within mesons as predicted by each theory.

We begin by recalling that the ‘‘instant-form’’ (IF) of the non-relativistic Schrödinger operator for a system made of two strongly interacting particles of identical mass  $m$  and momenta  $\vec{p}_q = \vec{p}$  and  $\vec{p}_{\bar{q}} = -\vec{p}$ , such as  $J/\psi$ ,  $\Upsilon$  or other mesons, is

$$\mathcal{M} = 2m + \frac{\vec{p}^2}{m} + V_{\text{eff}}, \quad (1)$$

where  $2\sqrt{m^2 + \vec{p}^2} \approx 2m + 2\frac{\vec{p}^2}{2m}$ . The eigenvalue of  $\mathcal{M}$  is the mass of the system. In contrast, the FF formulation of the theory of interacting particles is applicable to relativistic as well as non-relativistic constituents. It leads to an effective eigenvalue equation for the mass squared operator

$$\mathcal{M}^2 = \frac{k_{\perp}^2 + m^2}{x(1-x)} + U_{\text{eff}}, \quad (2)$$

instead of  $\mathcal{M}$  in the IF of dynamics, Eq. (1). The boost-invariant FF variables  $x$  and  $1-x$  are ratios of longitudinal FF momenta  $p^+ = p^0 + p^3$  of the constituents to the longitudinal FF momentum of the meson  $P^+ = P^0 + P^3$ . The term  $\frac{k_{\perp}^2 + m^2}{x(1-x)}$  is the LF kinetic energy as well as the invariant mass squared  $s = (p_q + p_{\bar{q}})^2$  of the  $q\bar{q}$  pair.

It will be convenient to define a relative three-vector momentum operator  $\vec{p}$  (in the constituent rest frame,  $\vec{p}_q + \vec{p}_{\bar{q}} = 0$  [33–35]), so that

$$\mathcal{M}^2 = \frac{k_{\perp}^2 + m^2}{x(1-x)} + U_{\text{eff}} \equiv 4m^2 + 4\vec{p}^2 + U_{\text{eff}}. \quad (3)$$

We identify  $p_{\perp}^2 = \frac{k_{\perp}^2}{4x(1-x)}$  and  $4m^2 + 4p_3^2 = \frac{m^2}{x(1-x)}$ , so  $p_3 = \frac{m}{\sqrt{x(1-x)}}(x - \frac{1}{2})$  has infinite range and is proportional to  $m$ . The conjugate variables are  $r_{\perp} = i\frac{\partial}{\partial p_{\perp}}$  and  $r_3 = i\frac{\partial}{\partial p_3}$ . Discussions of two-body systems in the FF as a model of mesons can be found in Refs. [36–39].

The central problem then becomes the derivation of the effective interaction  $V_{\text{eff}}$  or  $U_{\text{eff}}$ . We observe that nearly all considerations in the IF of the Hamiltonian dynamics lead to the conclusion that the potential between a quark and antiquark at large distances should be linear. A main point of this article is that the linear IF potential  $V_{\text{eff}}$  implies a quadratic FF potential  $U_{\text{eff}}$  at large  $q\bar{q}$  separation, as implied by Eqs.(1) and (3),

$$U_{\text{eff}} = V_{\text{eff}}^2 + 2\sqrt{\vec{p}^2 + m^2} V_{\text{eff}} + 2V_{\text{eff}}\sqrt{\vec{p}^2 + m^2}. \quad (4)$$

Thus, for a linear IF potential  $V_{\text{eff}}$ , the FF potential  $U_{\text{eff}}$  is a harmonic oscillator, predicting linear Regge trajectories in the hadron mass square for small quark masses.

#### Lattice

The static potential obtained in the quenched approximation of lattice QCD can be parameterized in the

form of the Cornell potential [40]; *i.e.* (up to a constant term)

$$V_{\text{eff}}^{(\text{lattice})}(r) = -\frac{A}{r} + \sigma r, \quad (5)$$

where  $r$  denotes the distance between an infinitely heavy quark and antiquark. The coefficients have been calculated in lattice QCD simulations for static and massive quarks [41–43]. For instance in Ref. [42] it is found that for quark’s mass 1.74(3) GeV the square root of string tension  $\sqrt{\sigma} = 394(7)$  MeV. Although our discussion concerns quarks with physical value of masses it should be mentioned that in the case of static quarks (infinite mass) the value of  $\sqrt{\sigma}$  is obtained around 460 MeV [42, 44].

#### String

One can also study the spectrum of hadrons in multi-dimension string models [45–47] with strings described by the Nambu-Goto action [48, 49]. This approach yields a string with a constant energy density per unit length and a static potential which rises linearly as a function of the distance  $r$ . In 4-dimensional space-time the potential is given (up to a constant term) by [47, 50]

$$V_{\text{eff}}^{(\text{string})}(r) = \sigma r \sqrt{1 - \frac{\pi}{6\sigma r^2}}. \quad (6)$$

From this, one can calculate the dependence of the meson spectrum on the internal angular momentum. By comparing with the empirical Regge trajectories, one finds that  $470 \text{ MeV} < \sqrt{\sigma} < 480 \text{ MeV}$  for pseudo-scalars ( $\pi$  and  $K$ ), while for other mesons the value of  $\sqrt{\sigma}$  varies between 424 and 437 MeV [51]. The string description applies for distances  $r \gg r_c = \sqrt{\pi/(6\sigma)}$  and  $r_c \simeq 0.33$  fm for  $\sqrt{\sigma} \simeq 430$  MeV. The string picture of confinement can be considered [7] as the strong coupling limit of the IF Hamiltonian formulation of lattice QCD. A review of the lattice and the string theories can be found in Ref. [51].

#### Stochastic vacuum model

The stochastic vacuum model [13] starts with the assumption that the nonperturbative (long-distance) part of the functional integral over the gluon field can be approximated by a Gaussian integration. Wilson loops can be evaluated easily and are determined by the gauge-invariant correlator of the gluon fields; for large loops one derives an area law signifying linear confinement. The resulting static potential begins quadratically and becomes linear at distances comparable to the correlation length of the gluon field. Confinement mechanism is due to the formation of a color-electric string between the (static) quark and antiquark [14]. The string tension is given by [13, 14]

$$\sigma = \frac{\pi}{48 N_c} \int dz^2 \langle g_s^2 F(z) F(0) \rangle \quad (7)$$

where  $\langle g_s^2 F(z)F(0) \rangle$  is the scalar function of the gauge invariant field correlator; it can be calculated on the lattice using the cooling method [52]. Using the numerical results of this lattice simulation [52] one obtains for the string tension  $\sqrt{\sigma} = 410(11)\text{MeV}$ .

#### LF-holography

Light-Front-holography connects AdS<sub>5</sub> space to the front-form in physical space-time. The result for the soft-wall model with the positive sign dilaton  $e^{+\kappa^2 z^2}$  [53] provides a meson equation of motion for zero quark mass. The fifth-dimension variable  $z$  becomes identified with the invariant  $q\bar{q}$  separation  $\zeta$  variable, where  $\zeta^2 = \frac{1}{4} r_\perp^2 = x(1-x)b_\perp^2$ , and  $b_\perp = i\frac{\partial}{\partial k_\perp}$  is the transverse distance between constituents [31]. The resulting FF wavefunctions satisfy a single-variable relativistic equation of motion with a harmonic oscillator potential

$$U_{\text{eff}}^{(LF)}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J-1), \quad (8)$$

where  $J$  is the total angular momentum of the  $q\bar{q}$  meson.

The harmonic oscillator form of the FF potential arises uniquely when one extends the formalism of de Alfaro, Fubini and Furlan [23] to light-front Hamiltonian theory [24]. The action of the effective one-dimensional quantum field theory remains conformal invariant, which reflects the underlying conformal invariance of the classical QCD chiral Lagrangian. The constant term  $2\kappa^2(J-1)$  is derived from spin- $J$  representations of dynamics in AdS space [21].

The mass parameter  $\kappa$  can be determined outside of QCD from a single measurement such as the pion decay constant. The consistency range from hadron spectroscopy places  $\kappa$  between 540 and 590 MeV.

Similar LF models of mesons have been recently discussed in Refs. [54, 55]. The universal value of  $\kappa$  is unchanged when short-range spin-dependent interactions are included. It will be interesting to extend these results retaining 3-dimensional rotation symmetry.

It is natural to replace  $\kappa^4 \zeta^2 = \frac{1}{4} \kappa^4 r_\perp^2$  in  $U_{\text{eff}}^{(LF)}$  by  $\frac{1}{4} \kappa^4 (r_\perp^2 + r_3^2)$  in the case of massive quarks [25]. Then  $U_{\text{eff}}^{(LF)}$  becomes a 3-dimensional oscillator. The corresponding wave function matches phenomenology, *e.g.*, see Ref. [56]. Thus, excitations in the transverse plane are paired with excitations in the 3-direction, and 3-dimensional rotational symmetry is restored in the massive case,  $m \neq 0$ .

#### RGPEP

An effective quark-antiquark FF potential can be introduced in the RGPEP framework as a function of a 3-dimensional quark-antiquark distance  $r$ , inspired by [57]

$$U_{\text{eff}}^{(RGPEP)}(r) = \left(\frac{\pi}{3} \varphi_{\text{glue}}\right)^2 r^2, \quad (9)$$

where  $\varphi_{\text{glue}}^2$  represents the gluon condensate inside hadrons. In the operator product expansion [58] the expectation value corresponding to gluon condensate can also refer to matrix elements inside hadrons rather than the vacuum [59–65].

The original value of  $\varphi_{\text{glue}}^2 = 0.012 \text{ GeV}^4$  obtained by Shifman, Vainshtein and Zakharov [66] has been updated by Narison [67], which in the case of in-hadron condensate implies

$$\varphi_{\text{glue}}^2 = \frac{1}{\pi} \frac{\langle G | \alpha_s G^{\mu\nu c} G_{\mu\nu}^c | G \rangle}{\langle G | G \rangle} \simeq 0.023(3) \text{ GeV}^4, \quad (10)$$

where  $\alpha_s$  is the QCD coupling constant, and  $|G\rangle$  represents the gluons condensed inside a meson.

#### Ehrenfest

One can apply the Ehrenfest theorem to quantum field theory in the sense of calculating expectation values which average quantities of interest over all Fock sectors and all constituents, keeping active just one constituent which moves in an effective potential generated by the remaining constituents. The potential  $U_{\text{eff}}$  in the FF effective Hamiltonian, Eq. (2), describes the motion of a constituent close to the minimum of the potential energy. The resulting potential around the minimum is quadratic,

$$U_{\text{eff}}^{(\text{Ehrenfest})}(r) \sim r^2. \quad (11)$$

Both the Ehrenfest equation and potential agree with the requirement of rotational symmetry, because all Fock sectors in the bound-state dynamics are included, *cf.* [68–70]; *i.e.* multiplets of the spectrum have the mass degeneracy allowed by the symmetry in 3-dimensions.

#### Using the WKB Method

In the IF models the confinement potential increases linearly at large distances between static quarks, as exemplified in Eqs. (5) and (6). Other terms contribute at small distances. In contrast, the Hamiltonian eigenvalue in the FF of dynamics is quadratic in the meson mass  $M$ : Eqs. (8), (9) and (11). Note that  $M^2 = (2m + \epsilon)^2 = 4m^2 + 4m\epsilon + \epsilon^2$  where  $\epsilon$  is the binding energy. It is essential to retain the  $\epsilon^2$ -term in the FF eigenvalue equation, Eq. (2), since the  $\epsilon^2$ -term contributes to the FF potential  $U_{\text{eff}}$  even if  $m$  is large. Thus, the FF potential in the non-relativistic limit for a large distance between quarks should be quadratic, if the IF potential is linear [27, 71]. This can be seen straightforwardly in the cases where the mass of constituents,  $m$  tends to zero.

In order to compare different descriptions of confinement we can adopt the WKB method. It defines the turning point  $r_{\text{max}}$  where the kinetic energy is completely turned into potential energy. One obtains

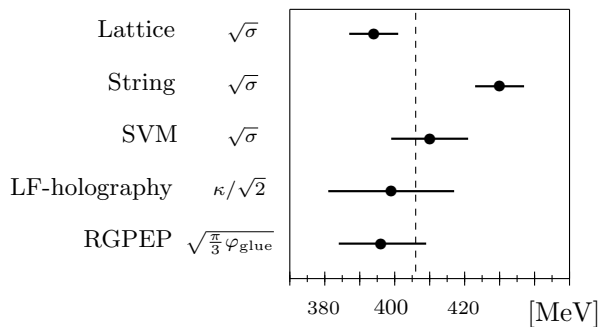


FIG. 1. Phenomenological results for the coefficient of  $r_{\text{max}}$  obtained using the WKB method (see the text). We compare the coefficients obtained from the lattice approach  $\sqrt{\sigma} = 394(7)$  MeV, string theory  $\sqrt{\sigma} = 430(7)$  MeV, the stochastic vacuum model  $\sqrt{\sigma} = 410(11)$  MeV, the LF holography approach  $\kappa/\sqrt{2} = 381 \sim 417$  MeV, and the in-hadron gluon condensate in the RGPEP approach  $\sqrt{\pi\varphi_{\text{glue}}/3} = 396(12)$  MeV. The dashed line is the average of these values.

$M = 2m + \sigma r_{\text{max}}$  in the lattice and string approach,  $M^2 = 4m^2 + (\frac{\pi}{3}\varphi_{\text{glue}})^2 r_{\text{max}}^2$  in the RGPEP approach, and  $M^2 = 4m^2 + \frac{1}{4}\kappa^4 r_{\text{max}}^2$  in the LF-holography approach. The last factor  $\frac{1}{4}$  comes from the fact that  $x = \frac{1}{2}$  at the WKB turning point, where  $p_{\perp}$  and  $p_3$  both vanish.

Figure 1 compares the phenomenological results for the coefficient of  $r_{\text{max}}$ . The values for the effective confinement scales derived from the WKB analysis in each model discussed above are sufficiently close to each other that one can argue that the various confinement models describe the same effective two-body system in the IF and in the FF. In particular the linear confining potential of the IF is consistent with the quadratic confining potential in the FF.

APT wants to express his gratitude for the hospitality extended to him at SLAC where this article was written. This work was supported by the Polish-U.S. Fulbright Commission and the Foundation for Polish Science International Ph.D. Projects Programme, co-financed by the EU European Regional Development Fund.

[1] M. Gell-Mann, *Phys.Rev.* **125**, 1067 (1962).  
[2] M. Gell-Mann, *Phys.Lett.* **8**, 214 (1964).  
[3] T. Appelquist and H. D. Politzer, *Phys.Rev.Lett.* **34**, 43 (1975).  
[4] T. Appelquist, A. De Rujula, H. D. Politzer, and S. Glashow, *Phys.Rev.Lett.* **34**, 365 (1975).  
[5] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. Lane, and T. M. Yan, *Phys.Rev.Lett.* **34**, 369 (1975).  
[6] K. G. Wilson, *Phys.Rev.* **D10**, 2445 (1974).  
[7] J. B. Kogut and L. Susskind, *Phys.Rev.* **D11**, 395

(1975).  
[8] W. A. Bardeen and R. Pearson, *Phys.Rev.* **D14**, 547 (1976).  
[9] A. Abrikosov, *Sov.Phys.JETP* **5**, 1174 (1957).  
[10] H. B. Nielsen and P. Olesen, *Nucl.Phys.* **B61**, 45 (1973).  
[11] G. 't Hooft, *Nucl.Phys.* **B72**, 461 (1974).  
[12] A. A. Migdal, *Phys.Rept.* **102**, 199 (1983).  
[13] H. G. Dosch and Y. Simonov, *Phys.Lett.* **B205**, 339 (1988).  
[14] A. Di Giacomo, H. G. Dosch, V. Shevchenko, and Y. Simonov, *Phys.Rept.* **372**, 319 (2002), arXiv:hep-ph/0007223 [hep-ph].  
[15] P. A. Dirac, *Rev.Mod.Phys.* **21**, 392 (1949).  
[16] S. D. Glazek and K. G. Wilson, *Phys.Rev.* **D48**, 5863 (1993).  
[17] K. G. Wilson, T. S. Walhout, A. Harindranath, W.-M. Zhang, R. J. Perry, *et al.*, *Phys.Rev.* **D49**, 6720 (1994), arXiv:hep-th/9401153 [hep-th].  
[18] S. D. Glazek and K. Wilson, *Phys.Rev.* **D49**, 4214 (1994).  
[19] G. F. de Téramond and S. J. Brodsky, *Phys.Rev.Lett.* **94**, 201601 (2005), arXiv:hep-th/0501022 [hep-th].  
[20] G. F. de Téramond and S. J. Brodsky, *Phys.Rev.Lett.* **102**, 081601 (2009), arXiv:0809.4899 [hep-ph].  
[21] G. F. de Téramond, H. G. Dosch, and S. J. Brodsky, *Phys.Rev.* **D87**, 075005 (2013), arXiv:1301.1651 [hep-ph].  
[22] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, *Phys.Rev.* **D74**, 015005 (2006), arXiv:hep-ph/0602229 [hep-ph].  
[23] V. de Alfaro, S. Fubini, and G. Furlan, *Nuovo Cim.* **A34**, 569 (1976).  
[24] S. J. Brodsky, G. F. De Tramond, and H. G. Dosch, *Phys.Lett.* **B729**, 3 (2014), arXiv:1302.4105 [hep-th].  
[25] S. D. Glazek, *Acta Phys.Polon.* **B42**, 1933 (2011).  
[26] P. Ehrenfest, *Z. Physik* **45**, 455 (1927).  
[27] S. D. Glazek and A. P. Trawiński, *Phys.Rev.* **D88**, 105025 (2013), arXiv:1307.2059 [hep-ph].  
[28] T. Regge, *Nuovo Cim.* **14**, 951 (1959).  
[29] T. Regge, *Nuovo Cim.* **18**, 947 (1960).  
[30] M. Gell-Mann, R. Oakes, and B. Renner, *Phys.Rev.* **175**, 2195 (1968).  
[31] S. J. Brodsky and G. F. de Téramond, *Phys.Rev.* **D77**, 056007 (2008), arXiv:0707.3859 [hep-ph].  
[32] For the original work, see J. Liouville, *J. de Math.* **2**, 16, 418 (1837); Lord Rayleigh, *Proc. Roy. Soc.* **A86**, 207 (1912); H. Jeffreys, *Proc. London Math. Soc.*(2) **23**, 428 (1923); G. Wentzel, *Zeits. f. Physik.* **38**, 518 (1926); H. A. Kramers, *Zeits. f. Physik.* **39**, 828 (1926); L. Brillouin, *Comptes Rendus* **183**, 24 (1926).  
[33] P. Danielewicz and J. Namyslowski, *Phys.Lett.* **B81**, 110 (1979).  
[34] V. Karmanov, *Nucl.Phys.* **B166**, 378 (1980).  
[35] S. D. Glazek, *Acta Phys.Polon.* **B15**, 889 (1984).  
[36] H. Leutwyler and J. Stern, *Phys.Lett.* **B69**, 207 (1977).  
[37] H. Leutwyler and J. Stern, *Phys.Lett.* **B73**, 75 (1978).  
[38] H. Leutwyler and J. Stern, *Nucl.Phys.* **B133**, 115 (1978).  
[39] H. Leutwyler and J. Stern, *Annals Phys.* **112**, 94 (1978).  
[40] E. Eichten, K. Gottfried, T. Kinoshita, K. Lane, and T.-M. Yan, *Phys.Rev.* **D17**, 3090 (1978).  
[41] S. Aoki *et al.* (PACS-CS Collaboration), *Phys.Rev.* **D79**, 034503 (2009), arXiv:0807.1661 [hep-lat].  
[42] T. Kawanai and S. Sasaki, *Phys.Rev.* **D85**, 091503

- (2012), [arXiv:1110.0888 \[hep-lat\]](#).
- [43] International Lattice Data Grid/Japan Lattice Data Grid.
- [44] Y. Koma and M. Koma, PoS **LATTICE2012**, 140 (2012), [arXiv:1211.6795 \[hep-lat\]](#).
- [45] M. Luscher, G. Munster, and P. Weisz, *Nucl.Phys.* **B180**, 1 (1981).
- [46] M. Luscher, *Nucl.Phys.* **B180**, 317 (1981).
- [47] M. Luscher, K. Symanzik, and P. Weisz, *Nucl.Phys.* **B173**, 365 (1980).
- [48] T. Goto, *Prog.Theor.Phys.* **46**, 1560 (1971).
- [49] Y. Nambu, *Phys.Rev.* **D10**, 4262 (1974).
- [50] J. Arvis, *Phys.Lett.* **B127**, 106 (1983).
- [51] G. S. Bali, *Phys.Rept.* **343**, 1 (2001), [arXiv:hep-ph/0001312 \[hep-ph\]](#).
- [52] M. D'Elia, A. Di Giacomo, and E. Meggiolaro, *Phys.Lett.* **B408**, 315 (1997), [arXiv:hep-lat/9705032 \[hep-lat\]](#).
- [53] S. J. Brodsky, G. F. de Teramond, and A. Deur, *Phys.Rev.* **D81**, 096010 (2010), [arXiv:1002.3948 \[hep-ph\]](#).
- [54] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys.Rev.* **D82**, 074022 (2010), [arXiv:1008.0268 \[hep-ph\]](#).
- [55] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys.Rev.* **D87**, 056001 (2013), [arXiv:1212.5196 \[hep-ph\]](#).
- [56] S. J. Brodsky, T. Huang, and G. P. Lepage, (1980), [SLAC-PUB-2540](#), C80-07-17-21.
- [57] S. D. Glazek and M. Schaden, *Phys.Lett.* **B198**, 42 (1987).
- [58] K. G. Wilson, *Phys.Rev.* **179**, 1499 (1969).
- [59] A. Casher and L. Susskind, *Phys.Rev.* **D9**, 436 (1974).
- [60] P. Maris and C. D. Roberts, *Phys.Rev.* **C56**, 3369 (1997), [arXiv:nucl-th/9708029 \[nucl-th\]](#).
- [61] P. Maris, C. D. Roberts, and P. C. Tandy, *Phys.Lett.* **B420**, 267 (1998), [arXiv:nucl-th/9707003 \[nucl-th\]](#).
- [62] S. J. Brodsky and R. Shrock, *Phys.Lett.* **B666**, 95 (2008), [arXiv:0806.1535 \[hep-th\]](#).
- [63] S. J. Brodsky and R. Shrock, *Proc.Nat.Acad.Sci.* **108**, 45 (2011), [arXiv:0803.2541 \[hep-th\]](#).
- [64] S. J. Brodsky, C. D. Roberts, R. Shrock, and P. C. Tandy, *Phys.Rev.* **C82**, 022201 (2010), [arXiv:1005.4610 \[nucl-th\]](#).
- [65] L. Chang, C. D. Roberts, and P. C. Tandy, *Phys.Rev.* **C85**, 012201 (2012), [arXiv:1109.2903 \[nucl-th\]](#).
- [66] M. A. Shifman, A. Vainshtein, and V. I. Zakharov, *Nucl.Phys.* **B147**, 385 (1979).
- [67] S. Narison, *Phys.Lett.* **B387**, 162 (1996), [arXiv:hep-ph/9512348 \[hep-ph\]](#).
- [68] R. J. Perry, (1994), [arXiv:hep-th/9407056 \[hep-th\]](#).
- [69] M. M. Brisudova, R. J. Perry, and K. G. Wilson, *Phys.Rev.Lett.* **78**, 1227 (1997), [arXiv:hep-ph/9607280 \[hep-ph\]](#).
- [70] M. M. Brisudova and R. Perry, *Phys.Rev.* **D54**, 1831 (1996), [arXiv:hep-ph/9511443 \[hep-ph\]](#).
- [71] S. D. Glazek, *Phys.Rev.* **D69**, 065002 (2004), [arXiv:hep-th/0307064 \[hep-th\]](#).