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MULTI-DIMENSIONAL OPTIMIZATION OF A TAPERED FREE ELECTRON LASER*

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Abstract

Energy extraction efficiency of a free electron laser (FEL) can be increased when the undulator is tapered. In this paper, we report a multi-dimensional optimizer to maximize the radiation power in a tapered FEL by searching for an optimal taper profile as well as a reasonable variation in electron beam radius. This code has the advantage that it is not necessary for the users to have strong background knowledge of the tapering-related physics. Applications of the proposed multi-dimensional optimization to the terawatt-level, hard X-ray, tapered FELs with LCLS-II like electron beam parameters are presented.

INTRODUCTION

Recent results on single pulse coherent diffraction imaging of proteins [1] and viruses [2] using an X-ray free electron laser (FEL) show that the resolution can be improved by both increasing the number of the coherent photons and simultaneously reducing the pulse duration to about 10 femtosecond (fs) or less, thus requiring a peak power of one terawatt (TW) or larger compared to the present values of 20 to 50 GW available at saturation from the self-amplified spontaneous emission (SASE) mode. Theoretical work done at DESY [3] and SLAC [4] shows that one way to increase the peak radiation power of a SASE X-ray FEL to the TW level is to use a tapered undulator, following a concept initially proposed by Kroll, Morton and Rosenbluth (KMR) [5], together with the self-seeding option [6].

To look insight the FEL process in a self-seeded tapered FEL, we develop an explicit physical model by extending one-dimensional (1D) KMR theory to include certain three-dimensional (3D) effects, *e.g.*, refraction, diffraction and the radial dependence of the radiation field and of the electron trapping in the ponderomotive well [7]. Based on the physical model, we propose a multi-dimensional optimization scheme, which allows us to explore the full potential of a high-peak-power FEL not only by tapering the undulator parameters in longitudinal dimension but also by optimizing the transverse effects.

The paper is arranged as follows. In Sec. II we introduce the multi-dimensional optimization scheme; in Sec. III we introduce the framework of the optimization code; in Sec. IV we present an example of optimization for a self-seeded, hard X-ray, tapered FEL.

TAPERING-RELATED PHYSICS AND MULTI-DIMENSIONAL OPTIMIZATION SCHEME

In Ref. [7], we develop a modified 1D physical model for tapered FEL (especially with self-seeding option), and give a general description of the FEL process. From the analysis, we find that the decreasing of refractive guiding is the major cause of the efficiency reduction, particle detrapping, and then saturation of the radiation power. To maximize the radiation power in an actual tapered FEL requires sophisticated control of the decrease of refractive guiding and particle detrapping along the undulator. To this purpose we propose a multi-dimensional optimization scheme.

To illustrate the optimization scheme specifically, we formulate the taper profile as

$$a_w(z) = a_w(z_0)\times[1-c\times(z-z_0)^d], \qquad (2)$$

where z_0 indicates the taper start-point, d is the taper profile order, and c is the scale coefficient which is related to the taper ratio ξ by $c = \xi/(L_w-z_0)^d$, with L_w the total undulator length. Empirically we find that it is best to start the taper slightly before *initial saturation* and use a moderate taper profile order with $d \approx 2$. The optimal ξ for the maximum radiation power varies with the undulator length and various initial electron and radiation beam parameters.

We find that a reasonably varied transverse focusing can, to some extent, further enhance the energy extraction efficiency [7]. Thus, we introduce a three-segment variation of the quadrupole field strength,

$$K_q = \begin{cases} K_q(z_1), & 0 < z \le z_1, \\ K_q(z_1)\times[1-f\times(z-z_1)], & z_1 < z \le z_2, \\ K_q(z_2)\times[1-g\times(z-z_2)], & z_2 < z \le L_w, \end{cases} \qquad (3)$$

where $K_q(z)$ is the quadrupole field strength; z_1 indicates the first K_q-variation start-point, which is usually around the *initial saturation* location; z_2 indicates the second K_q-variation start-point, which is usually around the location where a_{s0} reaches a maximum value; f and g are related to $K_q(z_1)$, $K_q(z_2)$ and $K_q(L_w)$ by $K_q(z_2) = K_q(z_1)\times(1-f\times(z_2-z_1))$ and $K_q(L_w) = K_q(z_2)\times(1-g\times(L_w-z_2))$; f can be either positive or negative, while g is usually negative. In thin-lens approximation, the average beta function $\beta_{\text{aver}} \propto 1/K_q(z)$ for periodic transverse focusing lattice cells. According to the relation $r_b = (2\varepsilon_n\beta_{\text{aver}}/\gamma)^{1/2}$, a linearly decreasing K_q approximately corresponds to r_b increasing with square root of z, and vice versa.

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With the above specification for functional dependences of the a_w taper and transverse focusing, we can obtain the maximum radiation power for a tapered FEL with specific electron beam, radiation seed and undulator properties by performing multi-dimensional scans with GENESIS single-frequency simulations over the following eight parameters, z_0, d, ξ, z_1, z_2, $K_q(z_1)$, $K_q(z_2)$ and $K_q(L_w)$.

PROGRAM SETUP AND ACTION

We program a multi-dimensional optimization code [8] in Matlab environment and on Windows system. For each set of eight parameters, z_0, d, ξ, z_1, z_2, $K_q(z_1)$, $K_q(z_2)$ and $K_q(L_w)$, the code generates corresponding input file, invokes GENESIS simulation via Cygwin, and reads GENESIS output file. The parameter scan is performed by varying one parameter while fixing the other seven. The variation of the radiation power with respect to a specific parameter can be obtained and the optimal value is then found by means of data analysis. After the optimal values of all eight parameters are obtained, the code will output the optimal taper profile and transverse focusing variation and the corresponding radiation power.

This code consists of several Matlab M-files, whose names and functions are:

- **optimization_STF_standard.m**: to initialize the global parameters via intrinsic or external definition and call the main optimization function;
- **STF_OPT_1.m**: the main optimization function, to start multi-dimensional parameter scans and output the optimization results;
- **generate_taper_input*.m**: to generate the GENESIS input file for each set of eight parameters;
- **runandread.m**: to invoke the GENESIS single-frequency simulations and then to call other functions to read the GENESIS output file.
- **read_genesis_mod*.m**: to read the GENESIS output file.

The "*" symbol stands for any number of characters.

To set up the code requires installing Cygwin (a Linux emulator) [9] on Windows. The folder for Cygwin installation is recommended to be C:\cygwin. Make sure there is tcsh.exe in the folder of C:\cygwin\bin, and put GENESIS executable file to the same folder. Then copy the above M-files into another folder, and include the folder to the Matlab search path.

To start the optimization, one should specify 14 global parameters, within or outside the main optimization function, optimization_STF_standard.m, to clarify the initial electron and radiation beam parameters as well as the undulator property. Those 14 global parameters are:

- **E0**: E_0, average energy of the initial electron beam, in unit of eV;
- **DELE**: σ_E, energy spread of the initial electron beam, in unit of eV;
- **EMITX**: $\varepsilon_{x,n}$, normalized horizontal emittance of the initial electron beam, in unit of m.rad;
- **EMITY**: $\varepsilon_{y,n}$, normalized vertical emittance of the initial electron beam, in unit of m.rad;
- **CRUPEARK**: I_p, peak current of the initial electron beam, in unit of ampere;
- **PRAD0**: P_{in}, power of the radiation seed, in unit of watt;
- **XLAMD**: λ_w, undulator period, in unit of meter;
- **AW0**: a_{w0}, dimensionless, initial vector potential of the undulator which, together with E_0 and λ_w, defines the resonant radiation wavelength λ_r by the relation $\lambda_r = \lambda_w(1+ a_{w0}^2)/2\gamma^2$, with $\gamma = E_0/mc^2$ and mc^2 the rest mass energy of electron;
- **NWIG**: N_u, number of undulator periods within a single undulator section;
- **NGAP**: N_g, break length of a single undulator section in unit of undulator period;
- **NSEC**: N_s, number of sections of the undulator;
- **LU**: L_u, undulator length for optimization, in unit of meter, which should be smaller than $N_s*(N_g+N_u)*\lambda_w$;
- **IWITYP**: undulator type, 0 for planar undulator and 1 for helical undulator;
- **NPro**: number of processors used by the optimization code, default value is 1.

There are six options to call the multi-dimensional optimization code optimization_STF_standard.m,

- [out] = optimization_STF_standard;
- [out, Pamx, Lmax] = optimization_STF_standard;
- [out, Pmax, Lmax, z0op, oop, rop, k0op, k1op, k2op, z1op, z2op] = optimization_STF_standard;
- [out] = optimization_STF_standard(in);
- [out, Pamx, Lmax] = optimization_STF_standard(in);
- [out, Pmax, Lmax, z0op, oop, rop, k0op, k1op, k2op, z1op, z2op] = optimization_STF_standard(in).

In the first three options the global parameters are specified within the M-file, while in second three options the parameters are defined by an external Matlab structure in. One can obtain the final GENESIS output data out, the maximum radiation power Pmax and its *z*-location Lmax, and the optimal values of eight parameters, z_0, d, ξ, z_1, z_2, $K_q(z_1)$, $K_q(z_2)$ and $K_q(L_w)$, which determine the taper profile and transverse focusing variation via Eqs. (2) and (3). In addition, during the parameter scanning, the code will generate figures for each parameter scan, and save the intermediate and final optimization results to result*.txt (where * indicates the time which is in the form of _year_month_day_hour_minute). The GENESIS input files (2.in and taper.in) for the maximum radiation power are also saved.

Table 1: Main Parameters of a Hard X-ray, Tapered FEL with Linearly Polarized Undulator

Parameters	**Value**	**Unit**
E-beam energy	13.64	GeV
E-beam current	4000	Ampere
Normalized emittances $\varepsilon_{x,n}$/ $\varepsilon_{y,n}$	0.3/0.3	µm-rad
Energy spread	1.3	MeV
E-beam pulse length (FWHM)	10	fs
Normalized undulator parameter a_{w0}	2.3832	
Undulator period λ_w	32	mm
NWIG/NGAP	138/0 106/32	
NSEC	46	
Undulator length L_w	200	m
Radiation wavelength λ_r	1.5	Angstrom
Peak radiation input power P_{in}	5	MW

APPLICATION OF THE MULTI-DIMENSIONAL OPTIMIZATION

To illustrate the multi-dimensional optimization, here we present two examples. One is the optimization for a hard X-ray FEL with 200 m undulator without break sections, and the other with the same parameter setting but with nonzero-length break sections. The main parameters are listed in Table 1.

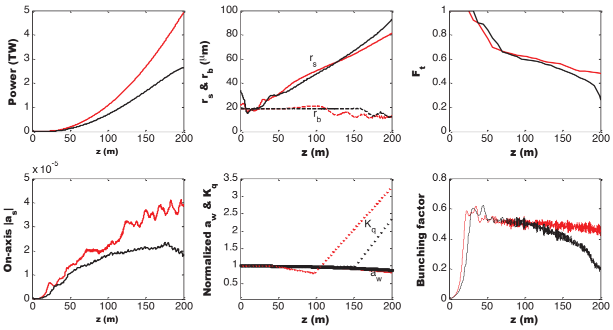

Figure 1: Multi-dimensional optimization results a 200-m, hard X-ray, tapered FEL with (black lines) and without (red lines) break sections.

Figure 1 presents the optimization results for these two cases. The optimal values for z_0, d, ξ, z_1, z_2, $K_q(z_1)$, $K_q(z_2)$ and $K_q(L_w)$ are shown in Table 2. In the case with break sections, the maximum radiation power is 2.65 TW, 54% of that in the case without break sections. The reduction in the available maximum power is partially due to 23% decrement of the magnetic length, which leads to a smaller value for the optimal taper ratio (0.12 vs. 0.19). Another important cause is the vacuum diffraction of radiation in break sections, which causes a further increase in r_s and decrease in a_{s0}. These changes cascade into a further increase of on-axis Ψ_r and decrease in F_t, and hence even less efficiency in energy extraction compared to the case without break sections.

Table 2: Optimal Taper and Transverse Focusing Parameters

Parameters	**Without break sections**	**With break sections**
z_0	11.5 m	15.5 m
d	2.03521	2.03281
ξ	0.2015	0.1321
z_1	30 m	50.2 m
z_2	111.7 m	155.1 m
$K_q(z_1)$	30.6 T/m	23.2 T/m
$K_q(z_2)$	19.1 T/m	22.9 T/m
$K_q(L_w)$	65.5 T/m	56.9 T/m
P_{max}	4.91 TW	2.65 TW

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