The BlackHat Library for One-Loop Amplitudes

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Abstract. We present recent next-to-leading order (NLO) results in perturbative QCD obtained using the BLACKHAT software library. We discuss the use of n-tuples to separate the lengthy matrix-element computations from the analysis process. The use of n-tuples allows many analyses to be carried out on the same phase-space samples, and also allows experimenters to conduct their own analyses using the original NLO computation.

1. Introduction

Experimental studies of Standard-Model processes at the Large Hadron Collider (LHC), whether as standard candles, as backgrounds to new physics, or as part of a program of precision measurements, require a theoretical counterpart. The dominant contributions to theoretical predictions arise from perturbative quantum chromodynamics (QCD). In QCD, unlike the case for other components of the Standard Model, leading-order (LO) calculations, which rely solely on tree-level amplitudes in QCD, do not offer a quantitatively reliable prediction. This problem is a result of the uncompensated dependence of the prediction on the renormalization and factorization scales. At LO this dependence arises only via the running coupling and the evolving parton distribution functions. Next-to-leading order (NLO) calculations in QCD cure this problem, and are thus the the first order in perturbation theory to provide a quantitatively reliable estimate of backgrounds due to Standard-Model processes [1, 2].

Recent years have seen remarkable progress in NLO calculations, thanks to advances in computing the one-loop matrix elements which lie at their core [3, 4, 5, 6, 7, 8, 9, 10]. The BLACKHAT collaboration has developed its eponymous software [11] to implement computations of one-loop amplitudes in a numerical approach. For multiplicities beyond $W + 2$-jet production, only tree-level amplitudes and integrals are implemented in computer code directly from analytic formulas. The library implements so-called on-shell methods, which rely only on information

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present in amplitudes with on-shell external states in order to arrive at values for more complex amplitudes. This keeps the relative simplicity — compared to the factorial complexity one might have feared from a Feynman-diagrammatic expansion — manifest at every stage of a calculation. This simplification is particularly evident in high-multiplicity calculations, for which the frontier has moved from $W + 2$-jet production [4, 12, 13] a decade and a half ago to $W + 5$-jet production earlier this year [14].

2. N-Tuples

While numerical one-loop libraries have made previously-unattainable NLO calculations feasible, they are not the only ingredient needed. In addition to the tree-level matrix elements for the basic process, we also need tree-level matrix elements with an additional parton emission: a gluon emitted, or a gluon splitting to a quark–antiquark pair. We use the COMIX [15] library within SHERPA [16] to compute these matrix elements. Different contributions to an NLO calculation have infrared divergences: in the virtual corrections, they are present explicitly, in the form of poles in the dimensional regulator ϵ; in the real-emission corrections, they arise in phase-space integrals over squared matrix elements. These divergences cancel in physical quantities such as cross sections and distributions of infrared-safe observables. In order to make this cancellation manifest in an operationally-tractable manner, one subtracts local approximations to the squared real-emission matrix elements, and adds back the analytic integrals of these local approximations. These integrals have manifest singularities, again in the form of poles in ϵ, which cancel those in the virtual corrections. The local subtraction terms and the computation of the appropriate subtraction integrals (using analytic formulas) are also performed by the COMIX library. We thus have four types of contributions to integrate numerically: Born (B), virtual corrections (V), integrated subtraction (I), and subtracted real emission (R).

A typical calculation at high multiplicity will involve dozens of subprocesses (two-quark, four-quark, six-quark, etc. in all possible crossings). Each must be integrated over final-state phase space, subject to cuts simulating the experimental ones. We use SHERPA to manage the subprocesses and perform the required numerical integrations. We use the FASTJET library [17] to apply a jet algorithm to phase-space configurations of partons. The different contributions span a large numerical range; to focus computer time on the largest contributions, it is helpful to further split up each of the four types (B,V,I,R) into parts, corresponding to leading and subleading color, or to different initial states, etc.

This subdivision does exact a penalty, in the form of the increased complexity of managing a calculation, tracking all the different parts, and ensuring that each part was run correctly. In a physics study conducted using stand-alone NLO codes such as `NLOJET++` [18], one would write code for a set of observables, and obtain distributions in those observables by running the complete code which computes both squared matrix elements and observables. Computing new observables as a physics study evolves; varying renormalization and factorization scales; computing uncertainties due to our imprecise knowledge of parton distribution functions (PDFs), would all require rerunning the code from scratch. This would force recomputation of the short-distance matrix elements, which are much more expensive to compute than observables. It would also force us to confront the subprocesses' and subparts' management complexity anew each time.

It would simplify the calculation to separate the computation of the squared matrix elements (along with the generation of phase-space configurations) from the computation of observables. We do this by saving generated phase-space configurations, along with their weights, in a set of ROOT [19] n-tuple files. The weights are obtained by combining the squared matrix elements with the phase-space measure, parton-distribution values, and appropriate Jacobian factors. In addition, we include a set of auxiliary coefficients which allow the recomputation of the weights using different renormalization and factorization scales, or different PDF sets. The files

are quite voluminous, especially for higher multiplicities, so the compression offered by ROOT yields a significant reduction compared to a naive binary format both in disk-space usage and in transmission times. A lightweight analysis code scans the resulting files, and computes cross sections or histograms of observables. This code can be rerun as needed for new observables or for estimating scale-sensitivity of observables, at relatively low cost in computer time.

The use of n-tuple files has a bonus: we can hand over sets of n-tuple files to our experimenter colleagues, who can then run their own analyses on the events they contain. This avoids the experimenters having to learn how to run a somewhat complicated set-up, and avoids them having to adopt our code or framework for performing analyses. The analyses are of parton-level fixed-order events, so that only distributions [of infrared-safe observables] can be compared in a meaningful way to experimental data. (The n-tuples we have computed to date are not suitable as input to existing NLO-matched parton-shower codes.)
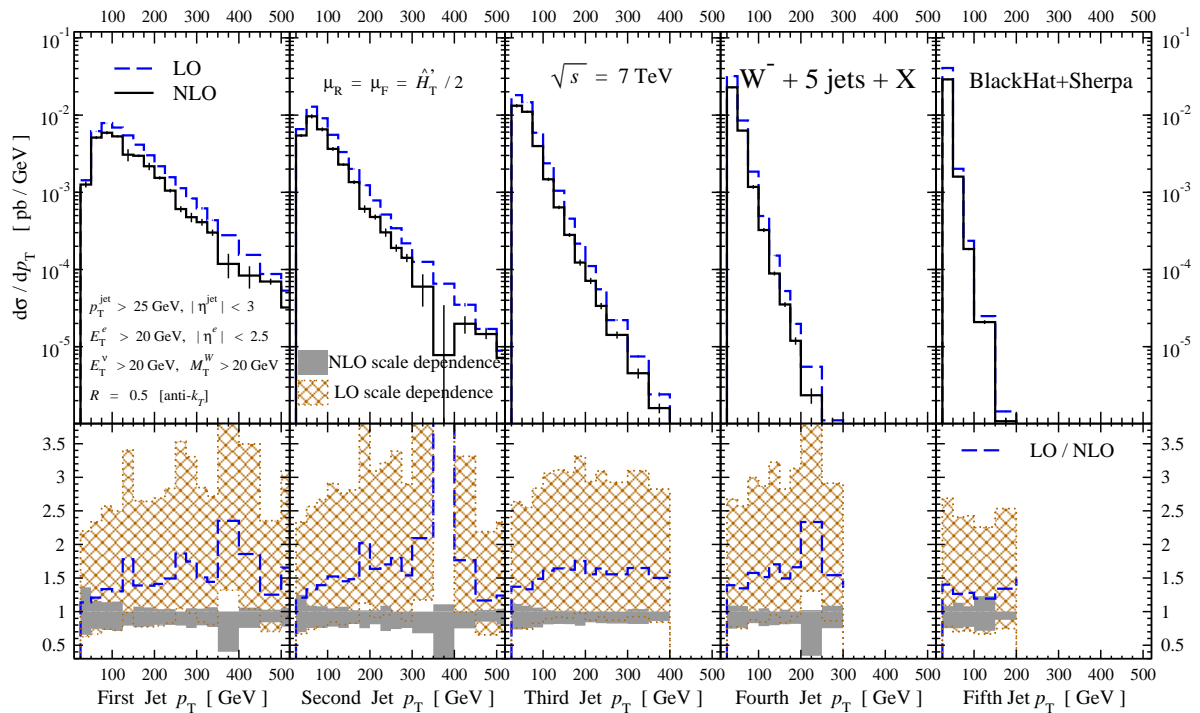
3. Vector Boson+Jets

Figure 1. The p_T distributions of the leading five jets in $W^- + 5$-jet production, ordered in decreasing p_T from left to right. In the upper panels, the blue (dashed) line shows the LO prediction, and the black (solid) line, the NLO one. The lower panels show the ratio of the central LO prediction to the NLO one, along with the LO scale variation band in orange-brown (hatched) and the NLO scale variation band in gray (solid).

As an example of recent studies we have performed using the BLACKHAT library running under SHERPA's control, and using n-tuple files, we show the distribution of the leading five jets in an NLO computation [14] of $W^- + 5$-jet+X production in fig. 1. Successive jet distributions fall more steeply. This reflects the fact that moving out on the distribution of, say, the third jet allows the fourth and fifth jet to remain at low p_T, thus forcing up the center-of-mass energy of the short-distance partonic collision less than moving out on the distribution of the fifth jet. As the center-of-mass energy grows, the short-distance cross section decreases rapidly, both because

of direct dependence in the squared matrix element and flux factors, and because the parton distributions decrease as well.

For the leading through next-to-softest jet (the fourth jet in this case), the NLO distributions fall somewhat more steeply, as reflected in the upward tilt of the LO/NLO ratio in the bottom panels of fig. 1. This continues a pattern seen at lower multiplicity, for example in the $W + 4$-jet calculation [20].

The scale-variation bands shrink dramatically in the NLO calculation as compared to the LO one. This fulfills one of the primary motivations for pursuing this calculation: we have achieved a 10–15% level of reliability, within expected experimental uncertainties for this high-multiplicity process.
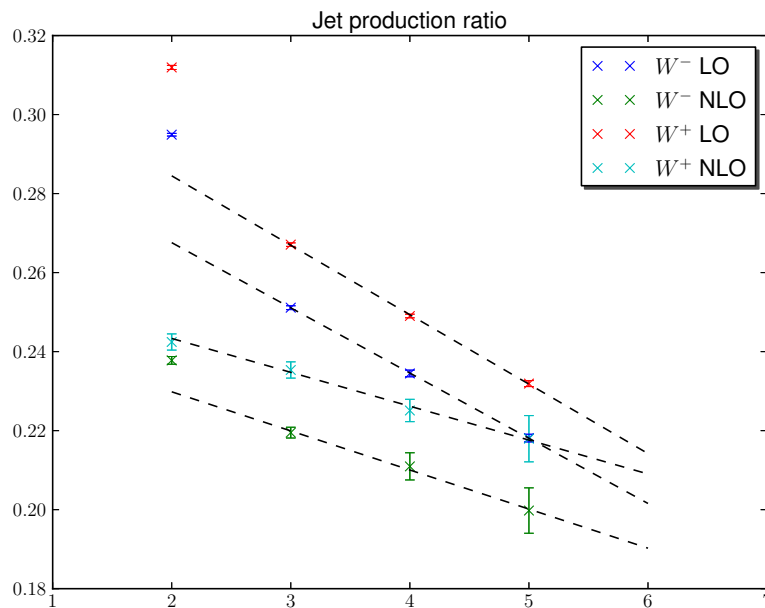

Figure 2. Ratios of $W + n$- to $W + (n-1)$-jet production at LO and NLO. The uppermost set of points shows the ratios for W^+ production at LO; the next set, for W^- production at LO; the third, for W^+ production at NLO; and the bottommost, W^- production at NLO.

There is an added bonus, if we consider ratios of cross sections for $W + n$- to $W + (n-1)$-jet production. This "jet-production" ratio [21] is sometimes called the "Berends" or "staircase" ratio. We should expect the $W + 1$-jet cross section to behave quite differently from cross sections with higher multiplicity, because some subprocesses (for example, gg-initiated ones) are missing at LO, and also because of kinematic constraints on phase space both at LO and at NLO. This in turn means that the $W + 2$- to $W + 1$-jet ratio should be expected to behave quite differently than higher-multiplicity ratios. There are also kinematic constraints evident in $W + 2$-jet production at LO, but these are largely relaxed at NLO; so we might expect to see signs of regularity or universality appearing with the $W + 3$- to $W + 2$-jet ratio at NLO. We consider the ratios of total cross sections with the cuts as shown in the leftmost panel of fig. 1. If we plot these ratios, as shown in fig. 2, regularity is exactly what we see: the ratios for $W + 3$-to $W + 2$-jet, $W + 4$- to $W + 3$-jet, and $W + 5$- to $W + 4$-jet production all lie on straight lines, both at LO and NLO. The former is a bit surprising, but it is the latter that is most significant for applications to LHC physics. It allows us to extrapolate linearly, and make a prediction for

$W + 6$-jet production. In this extrapolation, the $W + 5$-jet calculation plays a crucial role in establishing the reasonableness of a linear extrapolation. We find (see ref. [14] for details),

$$
\begin{aligned}
W^- + 6 \text{ jets} : \quad & 0.15 \pm 0.01 \text{ pb} \,, \\
W^+ + 6 \text{ jets} : \quad & 0.30 \pm 0.03 \text{ pb} \,,
\end{aligned}
\tag{1}
$$

using the same cuts as shown in fig. 1. The uncertainties are estimated using a Monte-Carlo procedure with synthetic data sets.

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