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Comparison of LHC and ILC Capabilities for Higgs Boson Coupling Measurements

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ABSTRACT

I estimate the accuracies on Higgs boson coupling constants that experiments at the Large Hadron Collider and the International Linear Collider are capable of reaching over the long term.

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1 Introduction

Now that a scalar boson of mass about 125 GeV has been discovered by the ATLAS and CMS experiments [1], the question of the hour is: Is this the Higgs boson? The key property of the Standard Model Higgs boson is that its coupling to each fermion and boson species are proportional to its mass. For a boson at 125 GeV, we can test this for a large number of Standard Model species.

How accurate must these tests be? Today, we would be pleased to achieve accuracies of 30-50% in the couplings. Agreement to this accuracy would make a strong case that the particle discovered at the LHC is indeed the Higgs boson.

However, there is a second interesting question that should be addressed. Many models with new physics beyond the Standard Model contain a light Higgs boson with properties very similar to the Higgs boson of the Standard Model. Haber has called attention to the 'Decoupling Limit' as a generic phenomenon in new physics models. In a large region of parameter space, the Higgs couplings in such models differ from those in the Standard Model by 10% or less [2]. Recently, Gupta, Rzehak, and Wells have illustrated this phenomenon in a wide variety of models [3].

As we look to the future in particle physics, it is a very interesting question how capably planned and proposed experiments will be able to probe the couplings of the Higgs boson at such a high level of precision. In this note, I will estimate the accuracies that can be achieved by experiments at Large Hadron Collider and at the International Linear Collider over the long term. It is challenging to predict the size of the error bars for experiments that will be carried out many years from now. I hope that this note will at least provide a plausible methodology, leading to estimates whose quality is straightforward to evaluate. Ultimately, the ATLAS and CMS experiments will need to make more definitive estimates of their ultimate sensitivities. But these are not yet available. This paper represents my attempt to fill this gap.

There have been many previous attempts to estimate the ultimate sensitivity of the LHC to the Higgs boson couplings. These include works of Zeppenfeld *et al* [4], Belyaev and Reina [5], and Dührssen *et al* [6–8]. I regard the work of Dührssen, Plehn, and collaborators as particularly important, and I will borrow many ideas from this work in the following. In general, the method here will be more simplistic but, I hope, more transparent than that used in [7] and [8].

2 Methodology

In this paper, I will always consider the couplings of a specific CP even scalar state to particle-antiparticle states. This state should be a scalar particle at a fixed mass. From here on, I will refer to this particle without further apology as 'the Higgs boson'. I will write the Higgs coupling to a particle A and its antiparticle as g(hAA). This coupling constant will be associated with a Lorentz structure in a canonical way that depends on the spin of the particle A. I will work directly in terms of the Higgs couplings g(hAA); the Higgs boson partial widths to $A\overline{A}$ are proportional to the squares of these quantities.

We should treat these Higgs boson couplings as completely unknown, to be determined by experimental measurements. Both for the LHC and for the ILC, the range of observables that will eventually be measured is broad enough to permit model-independent fits of to all couplings independently. Note that I will not assume any *a priori* relation between the tree-level couplings of the Higgs boson to fermions and massive vector bosons and the loop-level couplings to gg and $\gamma\gamma$. In the Standard Model, the loop-level couplings are related to the Higgs couplings to t and W; however, in a more general model, other heavy particles can contribute to the loop diagrams. A model-independent approach should fit the values of the loop diagrams separately from the direct couplings to t and W final states.

It is completely straightforward to measure Higgs boson couplings in a modelindependent way at the ILC. The ILC will make it possible to measure the cross section $\sigma(e^+e^- \rightarrow Z^0h^0)$ without reference to branching ratios of the Higgs by observing the recoil Z^0 at a fixed lab energy. Individual branching ratios can then be measured directly as the fractions of this total cross section in which the specific final state is observed.

For the LHC, it is less obvious that such a general analysis can be performed. However, it is possible to make fits to Higgs couplings that are almost model-independent through the use of a simple and very weak theoretical assumption. In the Lagrangian for $SU(2) \times U(1)$ gauge fields coupled to an arbitrary number of Higgs fields in arbitrary representations, it is always true that the various Higgs fields with vacuum expectation values make positive contributions to the W and Z masses. The Higgs couplings to WW and ZZ arise by differentiating these contributions with respect to the Higgs field vev, so it makes sense that these couplings are also positive terms whose sum is set by the W and Z masses. The precise version of this statement, derived by Gunion, Haber, and Wudka [9], is that, in a model with a CP-conserving Higgs sector in which only CP = +1 fields have nonzero vevs and in which couplings of doubly charged Higgs fields $g(W^+W^+\phi^{--})$ are absent,

$$\sum_{k} g^{2} (\phi_{k}^{0} WW)^{2} / g^{2} = (4m_{W}^{2} - 3\cos^{2}\theta_{w}m_{Z}^{2})$$
(1)

To high accuracy, the right-hand side can be replaced by m_W^2 . Then it follows that, for any individual neutral Higgs boson state h^0 ,

$$|g(hWW)| < g(hWW)|_{SM} , \qquad (2)$$

where the right-hand side is the value of the hWW coupling in the Standard Model. Similarly,

$$|g(hZZ)| < g(hZZ)|_{SM} , \qquad (3)$$

The importance of this constraint was recognized and first applied to the interpretation of LHC Higgs observables by Dührssen *et al* [6]. I follow their logic in the discussion below.

In some analyses, the constraints (2) and (3) are applied together with the constraint

$$g(hWW)/g(hZZ) = \cos^2 \theta_w \tag{4}$$

which is valid in models in which the Higgs boson is a linear combination of SU(2) singlets and doublets only. I do not apply that constrain here. The measurement of the ratio of the W and Z couplings is an important basic test of the nature of the Higgs boson and needs to be carried out however plausible the relation (4) might be.

Higgs boson observables at the LHC are either ratios of branching ratios or measured rates, proportional to cross sections times branching ratios. In the former case, the overall scale of the branching ratios cancels out. In the latter case, the quantity measured, for the observable $\sigma(A\overline{A} \to h)BR(h \to B\overline{B})$, is proportional to

$$\frac{g^2(hAA)\,g^2(hBB)}{\Gamma_T} \,\,,\tag{5}$$

where Γ_T is the total width of the Higgs. To determine the absolute magnitudes of the Higgs couplings, we must have some information about this total width. For a Standard Model Higgs boson at 125 GeV, the predicted width is 4 MeV. So the Higgs boson width is not expected to be directly measurable at any collider.

It is possible, though, to constrain the total width of the Higgs boson from the measurement of $\sigma \cdot BR$ observables at the LHC. Consider, for example, the measurement of the rate for WW fusion production of a Higgs boson which then decays to WW^* . Writing

$$\Gamma_T = \Gamma(h \to WW^*) / BR(h \to WW^*) , \qquad (6)$$

we see that the rate is proportional to

$$(g^2(hWW)) BR(h \to WW^*) . \tag{7}$$

The Higgs branching ratio to WW^* must be less than 1, so we obtain a lower bound on $g^2(hWW)$ and on Γ_T . We can improve this lower bound by adding in the branching ratios to other Higgs decay modes observed at the LHC, determined relative to $BR(h \to WW^*)$ from measurements of ratios of branching ratios. In fact, it is possible to observe at the LHC almost all of the significant decay modes of the Standard Model Higgs boson. Only for $h \to c\overline{c}$, a mode with a 2% branching ratio in the Standard Model, is there currently no strategy for observation. The decay $h \to gg$ is not directly observable, but the coupling of the Higgs boson to gg enters the analysis through the cross section for Higgs production from gluon fusion. So this lower bound on Γ_T could in principle be pushed up to 98% of the Standard Model value.

Still, there could in principle be other decay modes involving particles outside the Standard Model that are not observable at the LHC and could raise the value of Γ_T . Thus, to complete the analysis, we also need an upper bound on Γ_T . This is provided by the inequalities (2) and (3). In the Standard Model, the theoretical values of the upper and lower bounds are within 2% of one another and thus constrain the Higgs width Γ_T to an accuracy greater than the accuracy of the actual measurements that will be made.

We then proceed in the following way: Write the deviations from the Higgs couplings as

$$\frac{g(hAA)}{g(hAA)|_{SM}} = 1 + d(A) \tag{8}$$

I will include only one possible decay channel not included in the Standard Model, a decay to invisible decay modes, defining d for that channel by

$$d^{2}(\text{inv}) = BR(h \to \text{inv}) .$$
(9)

The invisible mode of Higgs decay can be observed at the LHC using the vector boson fusion process [10]. It is possible that there are additional non-Standard modes of Higgs decay that are not visible at the LHC. My fit includes the $c\bar{c}$ mode of Higgs decay, which is not visible at the LHC; other possible non-visible modes are taken into account here.

I will take the variables d(A), with flat priors, as the basic variables for this analysis.

In terms of d(A), deviations in the cross section are given by

$$\frac{\sigma(AA \to h)}{\sigma(A\overline{A} \to h)|_{SM}} = (1 + d(A))^2 .$$
(10)

Deviations in ratios of branching ratios are given by

$$\frac{BR(h \to A\overline{A})/BR(h \to B\overline{B})}{BR(h \to A\overline{A})/BR(h \to B\overline{B})|_{SM}} = \frac{(1+d(A))^2}{(1+d(B))^2} .$$
(11)

Deviations in rates are given by

$$\frac{\sigma(A\overline{A} \to h)BR(h \to B\overline{B})}{\sigma(A\overline{A} \to h)BR(h \to B\overline{B})|_{SM}} = \frac{(1+d(A))^2(1+d(B))^2}{D\Gamma}$$
(12)

where

$$D\Gamma = \left(\sum_{X} BR(h \to X\overline{X})|_{SM} \cdot (1 + d(X))^2\right) / (1.0 - d^2(\text{inv})) , \qquad (13)$$

where the expression on the right contains the Standard Model branching fractions for a Higgs boson of mass 125 GeV.s For the special case of invisible decays,

$$\frac{\sigma(A\overline{A} \to h)BR(h \to \text{invis})}{\sigma(A\overline{A} \to h)|_{SM}} = (1 + d(A))^2 (d(\text{invis}))^2 .$$
(14)

To estimate errors on the parameters d(A), I will work from a list of measurements that can be made at the LHC and the ILC. I will assume, somewhat ideally, that each measurement has the Standard Model value as its outcome and that the probability distribution for deviations from that outcome is Gaussian. This produces a likelihood function

$$\mathcal{L} = \prod_{i} exp[-\mathcal{D}_{i}^{2}/2\sigma_{i}^{2}] \cdot \mathcal{C} , \qquad (15)$$

where the variables \mathcal{D}_i are combinations of the form (10), (11), or (12) above, and \mathcal{C} is a product of theta functions implementing the constraints (2) and (3) and the constraints that (1 + d(A)) > 0 for all A (and d(inv) > 0). I integrated this likelihood function using VEGAS, formed the probability distribution for each variable, and computed for each the boundaries of the 68% confidence interval about the mean. These error intervals are tabulated for the various scenarios in Table 3 below. This approach can be described as 'naive Bayesian'. The results depend on the choice of a flat prior for the d(A); however, to the extent that the boundaries of the confidence intervals on the d(A) are close to 0, the results become independent of this choice.

In each scenario considered, the suite of measurements does produce for each variable d(A) a smooth probability distribution that decreases monotonically from a maximum close to 0. I see none of the pathologies described for these probability distributions for the more complex likelihood function studied in [7].

3 Inputs

In the previous section, I have explained a method that leads from input data in the form of a list of observables and their estimated relative errors to a set of confidence intervals for the variables d(A). We must now discuss what input data should be used.

On the LHC side, we would like to include the ultimate, systematics-limited errors on the measurements of Higgs observables. Unfortunately, there is no up-to-date study that reflects the current understanding of Higgs measurements or the measured capabilities of the ATLAS and CMS detectors. The best available study of the ultimate LHC capabilities for Higgs measurements is the 2003 Ph.D. thesis of Michael Dührssen [11]. This thesis estimated the expected errors on measurements of Higgs

observables by the ATLAS detector for 300 fb⁻¹ of data. By that point, these measurements will be systematics-limited. In Table 1, I report the conclusions of [11] for the important Higgs observables for a Higgs boson of mass 125 GeV. Measurements of $\sigma \cdot BR$ are also subject to significant theoretical systematic errors from the QCD calculations of the cross section. For specific measurements such as those for the final state WW, which requires jet vetos, and boosted Higgs measurements, which require a Higgs at high p_T , the selection of events lowers the precision of the QCD cross section by one order in α_s . I have guessed at these errors in the Table and added the experimental and theoretical errors in quadrature. I have assumed that the various Higgs production processes can be cleanly separated, with no correlations in the measurement errors. I have also ignored correlated theoretical errors in the estimation of common production cross sections. These two assumptions work to reduce the estimated errors resulting from the LHC measurements.

Dührssen's thesis must be used with some caution. First, Dührssen's thesis ignores complications from pileup. But, more importantly, his work assumed that the processes $pp \to W, Z + h$ and $pp \to t\bar{t}h$, with $h \to b\bar{b}$ would be straightforwardly observable. The small errors assumed for these processes played an important role in the optimistic conclusions of the fit [6]. The coupling q(hbb) plays a central role in the overall analysis, since the mode $h \rightarrow bb$ accounts for 60% of the total with of a 125 GeV Higgs boson in the Standard Model. The coupling q(hbb) is constrained only by the these two processes, and so the accuracy of $\sigma \cdot BR$ measurements for this processes have a crucial effect on the final results. In the mid-2000's, studies by the ATLAS and CMS experimental groups gave very pessimistic conclusions about the visibility of of these modes. More recently, there is optimism again, due to the development of methods for tagging boosted objects in the LHC environment [12,13]. However, this technique is still unproven, and it is not well understood how to estimate the efficiency of the event selection for the purpose of measuring a $\sigma \cdot BR$. In the Table, I have assigned these two processes a 25% experimental systematic error, which I believe is optimistic.

For the direct measurement of invisible decays of the Higgs, I estimate an experimental error of 20%, plus a theoretical systematic error estimated by Bai, Draper, and Shelton [14] to be 24%.

Truly persuasive estimates of the ultimate errors of the LHC measurements can come only from the ATLAS and CMS collaborations in analysis that reflect their best understanding of the capabilities of their detectors based on the detector performance in the 7 and 8 TeV runs. Many people would like to see the results of that analysis. I believe that the estimates given here are as valid as can be presented before that work is done.

For the ILC, error estimates for the $\sigma \cdot BR$ are reported in Table 2. I assume three stages of ILC operation: (1) an initial stage at 250 GeV near the maximum of the $e^+e^- \rightarrow hZ$ cross section, with 250 fb⁻¹ of data; (2) a stage at the ILC top energy of 500 GeV, with 500 fb⁻¹ of data; (3) a stage with an upgraded ILC at 1 TeV with 1000 fb⁻¹ of data. The nominal ILC program consists of stages 1 and 2 only. The errors quoted for each measurement are taken from the Physics Chapter of the ILC Detailed Baseline Design report [15]. Most of the quoted errors are supported by full-simulation studies with the ILD and SiD detectors, to be described in the ILC DBD. At the moment, the errors on $\tau^+\tau^-$ and $\gamma\gamma$ final states, and all errors quoted at 1 TeV, are preliminary. Some estimates for the 1 TeV stage are based on [16]. Hopefully, the numbers given here will be replaced by estimates from full-simulation study when the ILC DBD is completed later this year. I have ignored theoretical errors on the ILC measurements, since these are generally at the parts per mil level.

The 1 TeV ILC program will have other results that are interesting for Higgs physics, in particular, measurements of $g(h\mu\mu)$ and of the Higgs self-coupling at the 20% level [15]. Linear Collider measurements of the Higgs boson at higher energy are discussed in the CLIC Conceptual Design Report [17].

The fits to ILC data include the data from LHC. The fit to ILC data from the full program includes the measurements at 250 GeV. The fit to the extended ILC program at 1 TeV includes all previous measurements.

4 Results

In Table 3, I show the results of the various fits. The results are presented as 1σ confidence intervals on the parameters d(A). Note that, for invisible Higgs decays, d(inv) is the square root of the branching fraction.

In Fig. 1, I summarize the results of the fits for the ultimate LHC capabilities. The estimated errors are truly impressive. If the boson at 125 GeV indeed has properties close to those of the Standard Model Higgs boson, the LHC experiments will eventually be able to demonstrate this. However, it is unlikely that they will reach the level of 5% accuracy in model-independent Higgs coupling determinations needed to distinguish the Higgs boson of typical new physics models from the Standard Model Higgs boson Higgs boson. For this, we need another facility capable of higher precision Higgs boson measurements.

In Fig. 2, I summarize the results of the fits for the various stages of the ILC. The accuracy of the determinations increases progressively. The threshold measurements at 250 GeV should immediately attain a level of accuracy below 5% for many of the Higgs couplings. However, further significant improvements are possible with Higgs measurements at higher energies. The substantial decrease in the accuracy of the Higgs coupling to W between 250 GeV and 500 GeV results from the ability at the higher energy to measure the cross section for the WW fusion production of the Higgs



Figure 1: Capabilities of LHC for model-independent measurements of Higgs boson couplings. The plot shows 1 σ confidence intervals for LHC at 14 TeV with 300 fb⁻¹. No error is estimated for g(hcc). The marked horizontal band represents a 5% deviation from the Standard Model prediction for the coupling.



Figure 2: Comparison of the capabilities of LHC and ILC for model-independent measurements of Higgs boson couplings. The plot shows (from left to right in each set of error bars) 1 σ confidence intervals for LHC at 14 TeV with 300 fb⁻¹, for ILC at 250 GeV and 250 fb⁻¹ ('HLC'), for the full ILC program up to 500 GeV with 500 fb⁻¹ ('ILC'), and for a program with 1000 fb⁻¹ for an upgraded ILC at 1 TeV ('ILCTeV'). The marked horizontal band represents a 5% deviation from the Standard Model prediction for the coupling.

boson in the reaction $e^+e^- \rightarrow \nu \overline{\nu} h$. This increase in precision for g(hWW) is reflected generally in the quality of the fit. Separation of the gg and $c\overline{c}$ modes of Higgs decay becomes easier at higher energies, where the Higgs is more boosted. The accuracies for the measurement of the rare modes $\tau^+\tau^-$ and $\gamma\gamma$ increases progressively with higher statistics.

High accuracy for the Higgs coupling to t is realized only at the highest energies, well above the threshold for $e^+e^- \rightarrow t\bar{t}h$. However, there is a noticeable decrease in the error on g(htt) from the LHC to the 250 GeV ILC data set, despite the fact that there are no t quark measurements at 250 GeV. This results from the sharpening of other uncertainties in the fit to LHC couplings. This is a common phenomenon in studies of the complementarity of LHC and ILC, one well documented in [18].

The general conclusion is that the ILC can reach accuracies below 3% across almost the complete profile of Higgs boson couplings. The ILC will then allow us to reach beyond the capabilities of the LHC to explore for small deviations from the Standard Model predictions for Higgs couplings at the level at which these deviations are expected in models of new physics beyond the Standard Model.

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Observable	Expected Error (experiment \oplus theory)
LHC at 14 TeV with 300 fb^{-1}	
$\sigma(gg) \cdot BR(\gamma\gamma)$	$0.20 \oplus 0.15$
$\sigma(WW) \cdot BR(\gamma\gamma)$	$0.55 \oplus 0.10$
$\sigma(gg) \cdot BR(ZZ)$	$0.21 \oplus 0.15$
$\sigma(WW) \cdot BR(WW)$	$0.27 \oplus 0.10$
$\sigma(WW) \cdot BR(\tau^+\tau^-)$	$0.22 \oplus 0.10$
$BR(\tau^+\tau^-)/BR(ZZ)$	0.38
$BR(\gamma\gamma)/BR(ZZ)$	0.21
$BR(\gamma\gamma)/BR(WW)$	0.21
$\sigma(Wh) \cdot BR(b\overline{b})$	$0.25 \oplus 0.20$
$\sigma(Wh) \cdot BR(\gamma\gamma)$	$0.24 \oplus 0.10$
$\sigma(Zh) \cdot BR(b\overline{b})$	$0.25 \oplus 0.20$
$\sigma(Zh) \cdot BR(\gamma\gamma)$	$0.24 \oplus 0.10$
$\sigma(t\bar{t}h)\cdot BR(b\bar{b})$	$0.25 \oplus 0.30$
$\sigma(t\bar{t}h)\cdot BR(\gamma\gamma)$	$0.27 \oplus 0.20$
$\sigma(WW) \cdot BR(\text{invisible})$	$0.2 \oplus 0.24$

Table 1: Input data for the fits to Higgs couplings from LHC measurements.

Observable	Expected Error
ILC at 250 GeV with 250 fb^{-1}	
$\sigma(Zh)$	0.025
$\sigma(Zh) \cdot BR(b\overline{b})$	0.010
$\sigma(Zh) \cdot BR(c\overline{c})$	0.069
$\sigma(Zh) \cdot BR(gg)$	0.085
$\sigma(Zh) \cdot BR(WW)$	0.08
$\sigma(Zh) \cdot BR(ZZ)$	0.28
$\sigma(Zh) \cdot BR(\tau^+\tau^-)$	0.05
$\sigma(Zh) \cdot BR(\gamma\gamma)$	0.27
$\sigma(Zh) \cdot BR$ (invisible)	0.005
ILC at 500 GeV with 500 fb^{-1}	
$\sigma(Zh) \cdot BR(b\overline{b})$	0.016
$\sigma(Zh) \cdot BR(c\overline{c})$	0.11
$\sigma(Zh) \cdot BR(gg)$	0.13
$\sigma(Zh) \cdot BR(\tau^+\tau^-)$	0.07
$\sigma(Zh) \cdot BR(\gamma\gamma)$	0.36
$\sigma(WW) \cdot BR(b\overline{b})$	0.006
$\sigma(WW) \cdot BR(c\overline{c})$	0.04
$\sigma(WW) \cdot BR(gg)$	0.049
$\sigma(WW) \cdot BR(WW)$	0.03
$\sigma(WW) \cdot BR(\tau^+\tau^-)$	0.05
$\sigma(WW) \cdot BR(\gamma\gamma)$	0.28
$\sigma(t\bar{t}h)\cdot BR(b\bar{b})$	0.2
ILC at 1 TeV with 1000 fb^{-1}	
$\sigma(WW) \cdot BR(WW)$	0.01
$\sigma(WW) \cdot BR(gg)$	0.018
$\sigma(WW) \cdot BR(\tau + \tau -)$	0.02
$\sigma(WW) \cdot BR(\gamma\gamma)$	0.05
$\sigma(t\bar{t}h)\cdot BR(b\bar{b})$	0.12

Table 2: Input data for the fits to Higgs couplings from ILC measurements.

Program	1σ Confidence interval for $d(X)$			
LHC at 14 TeV with 300 fb^{-1}				
$h \to WW$	(-0.075, 0.000)			
$h \rightarrow ZZ$	(-0.087, 0.000)			
$h o b\overline{b}$	(-0.221, 0.067)			
$h \rightarrow gg$	(-0.129, 0.137)			
$h \to \gamma \gamma$	(-0.098, 0.052)			
$h \to \tau^+ \tau^-$	(-0.120, 0.151)			
$h \to c \overline{c}$				
$h \to t \overline{t}$	(-0.164, 0.139)			
$h \rightarrow \text{invisible}$	(-0.000, 0.232)			
ILC at 250 GeV with 250 fb^{-1}				
$h \to WW$	(-0.051, 0.000)			
$h \rightarrow ZZ$	(-0.009, 0.000)			
$h o b\overline{b}$	(-0.048, 0.026)			
$h \rightarrow gg$	(-0.076, 0.037)			
$h \to \gamma \gamma$	(-0.059, 0.044)			
$h \to \tau^+ \tau^-$	(-0.056, 0.033)			
$h \to c\overline{c}$	(-0.064, 0.038)			
$h \to t\bar{t}$	(-0.178, 0.096)			
$h \rightarrow \text{invisible}$	(-0.000, 0.058)			
ILC at 500 GeV with 500 fb^{-1}	· · · · · · · · · · · · · · · · · · ·			
$h \to WW$	(-0.006, 0.000)			
$h \rightarrow ZZ$	(-0.007, 0.000)			
$h o b\overline{b}$	(-0.002, 0.020)			
$h \rightarrow gg$	(-0.025, 0.025)			
$h o \gamma \gamma$	(-0.040, 0.049)			
$h \to \tau^+ \tau^-$	(-0.015, 0.026)			
$h \to c \overline{c}$	(-0.013, 0.028)			
$h \to t \overline{t}$	(-0.096, 0.066)			
$h \rightarrow \text{invisible}$	(-0.000, 0.047)			
ILC at 1000 GeV with 1000 fb^{-1}				
$h \to WW$	(-0.005, 0.000)			
$h \rightarrow ZZ$	(-0.007, 0.000)			
$h o b \overline{b}$	(-0.000, 0.020)			
h ightarrow gg	(-0.004, 0.023)			
$h o \gamma \gamma$	(-0.016, 0.032)			
$h \to \tau^+ \tau^-$	(-0.003, 0.023)			
$h \to c \overline{c}$	(-0.011, 0.029)			
$h \to t \overline{t}$	(-0.056, 0.041)			
$h \rightarrow \text{invisible}$	(-0.000, 0.046)			

Table 3: Results of the fits to Higgs couplings expressed as 1 σ confidence intervals on d(X).