# Branching fraction measurement of $B^{+} \rightarrow \omega \ell^{+} \nu$ decays 

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#### Abstract

We present a measurement of the $B^{+} \rightarrow \omega \ell^{+} \nu$ branching fraction based on a sample of 467 million $B \bar{B}$ pairs recorded by the BABAR detector at the SLAC PEP-II $e^{+} e^{-}$collider. We observe $1041 \pm 133$ signal decays, corresponding to a branching fraction of $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)=(1.15 \pm 0.15 \pm 0.12) \times 10^{-4}$, where the first error is statistical and the second is systematic. The dependence of the decay rate on $q^{2}$, the momentum transfer squared to the lepton system, is compared to QCD predictions of the form factors based on a quark model and light-cone sum rules.


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## I. INTRODUCTION

Most theoretical and experimental studies of exclusive $B \rightarrow X_{u} \ell \nu$ decays have focused on $B \rightarrow \pi \ell \nu$ decays, while $B \rightarrow \rho \ell \nu$ and $B^{+} \rightarrow \omega \ell^{+} \nu$ [1] decays involving the vector mesons $\rho$ and $\omega$ have received less attention. Here $\ell$ is an electron or muon, and $X$ refers to a hadronic state, with the subscript $c$ or $u$ signifying whether the state carries charm or is charmless. Measurements of the branching fraction of $B \rightarrow \rho \ell \nu$ are impacted by an irreducible $B \rightarrow X_{u} \ell \nu$ background, typically the dominant source of systematic uncertainty. In studies of $B^{+} \rightarrow \omega \ell^{+} \nu$ that background can be suppressed to a larger degree, since the $\omega$ width is about 15 times lower than that of the $\rho$. Extractions of the CKM matrix element $\left|V_{u b}\right|$ from $B^{+} \rightarrow \omega \ell^{+} \nu$ and $B \rightarrow \rho \ell \nu$ decay rates have greater uncertainties than those from $B \rightarrow \pi \ell \nu$,

[^0]due to higher backgrounds and more complex form-factor dependencies. The persistent discrepancy between $\left|V_{u b}\right|$ measurements based on inclusive and exclusive charmless decays is a motivation for the study of different exclusive $B \rightarrow X_{u} \ell \nu$ decays [2, 3]. Measurements of $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ have been reported by Belle [4, 5]; a measurement by $B A B A R$ has been performed on a partial dataset [6]. In this analysis we use the full $B A B A R$ dataset to measure the total branching fraction $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ and partial branching fractions $\Delta \mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right) / \Delta q^{2}$ in five $q^{2}$ intervals, where $q^{2}$ refers to the momentum transfer squared to the lepton system.

The differential decay rate for $B^{+} \rightarrow \omega \ell^{+} \nu$ is given by [7]

$$
\begin{align*}
& \frac{\mathrm{d} \Gamma\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)}{\mathrm{d} q^{2}}=\left|V_{u b}\right|^{2} \frac{G_{F}^{2} q^{2} p_{\omega}}{96 \pi^{3} m_{B}^{2}} \\
& \quad \times\left[\left|H_{0}\right|^{2}+\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}\right] \tag{1}
\end{align*}
$$

where $p_{\omega}$ is the magnitude of the $\omega$ momentum in the $B$ rest frame, $m_{B}$ is the $B$ mass, and $G_{F}$ is the Fermi coupling constant. As described in a related BABAR paper [8], the three helicity functions $H_{0}, H_{+}$, and $H_{-}$can be expressed in terms of two axial vector form factors $A_{1}$ and $A_{2}$ and one vector form factor $V$, which describe
strong interaction effects,

$$
\begin{aligned}
& H_{ \pm}\left(q^{2}\right)=\left(m_{B}+m_{\omega}\right)\left[A_{1}\left(q^{2}\right) \mp \frac{2 m_{B} p_{\omega}}{\left(m_{B}+m_{\omega}\right)^{2}} V\left(q^{2}\right)\right] \\
& H_{0}\left(q^{2}\right)=\frac{m_{B}+m_{\omega}}{2 m_{\omega} \sqrt{q^{2}}} \times {\left[\left(m_{B}^{2}-m_{\omega}^{2}-q^{2}\right) A_{1}\left(q^{2}\right)\right.} \\
&\left.-\frac{4 m_{B}^{2} p_{\omega}^{2}}{\left(m_{B}+m_{\omega}\right)^{2}} A_{2}\left(q^{2}\right)\right]
\end{aligned}
$$

We compare the measured $q^{2}$ dependence of the decay rate with form factor predictions based on light-cone sum rules (LCSR) [9] and the ISGW2 quark model [10]. We also use these form factor calculations and the measured branching fraction to extract $\left|V_{u b}\right|$.

## II. DETECTOR, DATA SET, AND SIMULATION

The data used in this analysis were recorded with the BABAR detector at the PEP-II $e^{+} e^{-}$collider operating at the $\Upsilon(4 S)$ resonance. We use a data sample of $426 \mathrm{fb}^{-1}$, corresponding to $(467 \pm 5)$ million produced $B \bar{B}$ pairs. In addition, we use $44 \mathrm{fb}^{-1}$ of data collected 40 MeV below the $B \bar{B}$ production threshold. This off-resonance sample is used to validate the simulation of the non $-B \bar{B}$ contributions whose principal source is $e^{+} e^{-}$annihilation to $q \bar{q}$ pairs, where $q=u, d, s, c$. The PEP-II collider and $B A B A R$ detector have been described in detail elsewhere [11]. Charged particles are reconstructed in a fivelayer silicon tracker positioned close to the beam pipe and a forty-layer drift chamber. Particles of different masses are distinguished by their ionization energy loss in the tracking devices and by a ring-imaging Cerenkov detector. Electromagnetic showers from electrons and photons are measured in a finely segmented $\mathrm{CsI}(\mathrm{Tl})$ calorimeter. These detector components are embedded in a 1.5 T magnetic field of a superconducting solenoid; its steel flux return is segmented and instrumented with planar resistive plate chambers and limited streamer tubes to detect muons that penetrate the magnet coil and steel. We use Monte Carlo (MC) techniques [12, 13] to simulate the production and decay of $B \bar{B}$ and $q \bar{q}$ pairs and the detector response [14], to estimate signal and background efficiencies and resolutions, and to extract the expected signal and background distributions. The size of the simulated sample of generic $B \bar{B}$ events exceeds the $B \bar{B}$ data sample by about a factor of three, while the MC samples for inclusive and exclusive $B \rightarrow X_{u} \ell \nu$ decays exceed the data samples by factors of 15 or more. The MC sample for $q \bar{q}$ events is about twice the size of the $q \bar{q}$ contribution in the $\Upsilon(4 S)$ data. The MC simulation of semileptonic decays uses the same models as in a recent $B A B A R$ analysis [8]. The simulation of inclusive charmless semileptonic decays $B \rightarrow X_{u} \ell \nu$ is based on predictions of a heavy-quark expansion [15] for the differential decay rates. For the simulation of $B \rightarrow \pi \ell \nu$ decays we use the ansatz of [16] for the $q^{2}$ dependence, with the single
parameter $\alpha_{B K}$ set to the value determined in a previous $B A B A R$ analysis [17]. All other exclusive charmless semileptonic decays $B \rightarrow X_{u} \ell \nu$, including the signal, are generated with form factors determined by LCSR [9, 18]. For $B \rightarrow D \ell \nu$ and $B \rightarrow D^{*} \ell \nu$ decays we use parameterizations of the form factors [19, 20] based on heavy quark effective theory; for the generation of the decays $B \rightarrow D^{* *} \ell \nu$, we use the ISGW2 model [10].

## III. CANDIDATE SELECTION

In the following, we describe the selection and kinematic reconstruction of signal candidates, the definition of the various background classes, and the application of neural networks to further suppress these backgrounds. The primary challenge in studying charmless semileptonic $B$ decays is to separate signal decays from Cabibbofavored $B \rightarrow X_{c} \ell \nu$ decays, which have a branching ratio approximately 50 times larger than that of $B \rightarrow X_{u} \ell \nu$. A significant background also arises due to multi-hadron continuum events. Based on the origin of the candidate lepton we distinguish three categories of events: 1) Signal candidates with a charged lepton from a true $B^{+} \rightarrow \omega \ell^{+} \nu$ decay; 2) $B \bar{B}$ background with a charged lepton from all non-signal $B \bar{B}$ events; 3) Continuum background from $e^{+} e^{-} \rightarrow q \bar{q}$ events. The $\omega$ meson is reconstructed in its dominant decay, $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$. For each of the three categories of events we distinguish correctly reconstructed $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decays (true- $\omega$ ) from combinatoric- $\omega$ candidates, for which at least one of the reconstructed pions originates from a particle other than the $\omega$.

## A. Preselection

Signal candidates are selected from events with at least four charged tracks, since a $B^{+} \rightarrow \omega \ell^{+} \nu$ leaves three tracks and the second $B$ in the event is expected to produce at least one track. The magnitude of the sum of the charges of all reconstructed tracks is required to be less than two, helping to reject events with at least two undetected particles. The preselection places requirements on the reconstructed lepton, $\omega$ meson, and neutrino from the $B^{+} \rightarrow \omega \ell^{+} \nu$ decay. At this stage in the analysis we allow for more than one candidate per event. The lepton is identified as either an electron or muon. The electron identification efficiency is greater than $90 \%$ and constant as a function of momentum above 1 GeV , while the muon identification efficiency is between $65 \%-75 \%$ for momenta of $1.5-3 \mathrm{GeV}$. The pion misidentification rates are about $0.1 \%$ for the electron selector and $1 \%$ for the muon selector. The lepton is required to have a momentum in the center-of-mass (c.m.) frame greater than 1.6 GeV . This requirement significantly reduces the background from hadrons that are misidentified as leptons, and also removes a large fraction of true leptons from secondary decays or photon conversions and from $B \rightarrow X_{c} \ell \nu$ de-
cays. The acceptance of the detector for leptons covers momentum polar angles between $0.41 \leq \theta \leq 2.54 \mathrm{rad}$. For the reconstruction of the decay $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$, we require that the candidate charged pions are not identified as leptons or kaons. The reconstructed $\omega$ mass must be in the range $680<m_{3 \pi}<860 \mathrm{MeV}$, and the $\pi^{0}$ candidate is required to have an invariant mass of $115<m_{\gamma \gamma}<150 \mathrm{MeV}$. To reduce combinatoric $\omega$ background, we require minimum momenta for the three pion candidates, $p_{\pi^{ \pm}}>200 \mathrm{MeV}$ and $p_{\pi^{0}}>400 \mathrm{MeV}$, and also energies of at least 80 MeV for photons from the $\pi^{0}$ candidate. The charged lepton candidate is combined with an $\omega$ candidate to form a so-called $Y$ candidate. The charged tracks associated with the $Y$ candidate are fitted to a common vertex $Y_{\text {vtx }}$. This vertex fit must yield a $\chi^{2}$ probability $\operatorname{Prob}\left(\chi^{2}, Y_{\mathrm{vtx}}\right)>0.1$. To further reduce backgrounds without significant signal losses, we impose two-dimensional restrictions on the momenta of the lepton and $\omega$. Each $Y$ candidate must satisfy at least one of the following conditions on the c.m. momentum of the lepton and $\omega: p_{\omega}^{*}>1.3 \mathrm{GeV}$, or $p_{\ell}^{*}>2.0 \mathrm{GeV}$, or $p_{\ell}^{*}+p_{\omega}^{*}>2.65 \mathrm{GeV}$, where quantities with an asterisk refer to the c.m. frame. These requirements reject background candidates that are inconsistent with the phase space of the signal decay. The condition $\left|\cos \theta_{\mathrm{BY}}\right| \leq 1.0$, where $\cos \theta_{\mathrm{BY}}=\left(2 E_{B}^{*} E_{Y}^{*}-M_{B}^{2}-M_{Y}^{2}\right) /\left(2 p_{B}^{*} p_{Y}^{*}\right)$ is the cosine of the angle between the momentum vectors of the $B$ meson and the $Y$ candidate, should be fulfilled for a well-reconstructed $Y$ candidate originating from a signal decay [21]. The energy $E_{B}^{*}$ and momentum $p_{B}^{*}$ of the $B$ meson are not measured event-by-event. Specifically, $E_{B}^{*}=\sqrt{s} / 2$, where $\sqrt{s}$ is the c.m. energy of the colliding beams, and the $B$ momentum is derived as $p_{B}^{*}=\sqrt{E_{B}^{* 2}-m_{B}^{2}}$. To allow for the finite resolution of the detector, we impose the requirement $-1.2<$ $\cos \theta_{\mathrm{BY}}<1.1$. The neutrino four-momentum is inferred from the missing energy and momentum of the whole event: $\left(E_{\text {miss }}, \vec{p}_{\text {miss }}\right)=\left(E_{e^{+} e^{-}}, \vec{p}_{e^{+} e^{-}}\right)-\left(\sum_{i} E_{i}, \sum_{i} \vec{p}_{i}\right)$, where $E_{e^{+} e^{-}}$and $\vec{p}_{e^{+} e^{-}}$are the energy and momentum of the colliding beam particles, and the sums are performed over all tracks and all calorimeter clusters without an associated track. If all tracks and clusters in an event are well-measured, and there are no undetected particles besides a single neutrino, then the measured distribution of the missing mass squared, $m_{\text {miss }}^{2}=E_{\text {miss }}^{2}-p_{\text {miss }}^{2}$, peaks at zero. We require that the reconstructed neutrino mass be consistent with zero, $\left|m_{\text {miss }}^{2} /\left(2 E_{\text {miss }}\right)\right|<2.5 \mathrm{GeV}$, and the missing momentum exceeds 0.5 GeV . The polar angle of the missing momentum vector is also required to pass through the fiducial region of the detector, $0.3<$ $\theta_{\text {miss }}<2.2 \mathrm{rad}$. Other restrictions are applied to suppress $q \bar{q}$ background, which has a two-jet topology in contrast to $B \bar{B}$ events with a more uniform angular distribution of the tracks and clusters. Events must have $R_{2} \leq 0.5$, where $R_{2}$ is the second normalized Fox-Wolfram moment 22], determined from all charged and neutral particles in the event. We also require $\cos \Delta \theta_{\text {thrust }} \leq 0.9$, where $\Delta \theta_{\text {thrust }}$ is the angle between the thrust axis of
the $Y$ candidate's decay particles and the thrust axis of all other detected particles in the event. We require $\mathrm{£} 2<3.0 \mathrm{GeV}$, with $\mathrm{£} 2=\sum_{i} p_{i}^{*} \cos ^{2} \theta_{i}^{*}$, where the sum runs over all tracks in the event excluding the $Y$ candidate, and $p_{i}^{*}$ and $\theta_{i}^{*}$ refer to the c.m. momenta and the angles measured with respect to the thrust axis of the $Y$ candidate. We reject candidates that have a charged lepton and a low-momentum charged pion consistent with the $\pi_{\text {slow }}^{-}$from a $B^{0} \rightarrow D^{*-} \ell^{+} \nu, D^{*-} \rightarrow \bar{D}^{0} \pi_{\text {slow }}^{-}$decay as described in [23]. The kinematic consistency of the candidate decay with a signal $B$ decay is ascertained by restrictions on two variables, the beam-energy substituted $B$ mass $m_{\mathrm{ES}}$, and the difference between the reconstructed and expected energy of the $B$ candidate $\Delta E$. In the laboratory frame these variables are defined as $m_{\mathrm{ES}}=\sqrt{\left(s / 2+\vec{p}_{B} \cdot \vec{p}_{e^{+} e^{-}}\right)^{2} / E_{e^{+} e^{-}}^{2}-p_{B}^{2}}$ and $\Delta E=\left(P_{e^{+} e^{-}} \cdot P_{B}-s / 2\right) / \sqrt{s}$, where $P_{B}=\left(E_{B}, \vec{p}_{B}\right)$ and $P_{e^{+} e^{-}}=\left(E_{e^{+} e^{-}}, \vec{p}_{e^{+} e^{-}}\right)$are the four-momenta of the $B$ meson and the colliding beams, respectively. For correctly reconstructed signal $B$ decays, the $\Delta E$ distribution is centered at zero, and the $m_{\mathrm{ES}}$ distribution peaks at the $B$ mass. We restrict candidates to $-0.95<\Delta E<0.95 \mathrm{GeV}$ and $5.095<m_{\mathrm{ES}}<5.295 \mathrm{GeV}$.

## B. Neural Network Selection

To separate signal candidates from the remaining background we employ two separate neural networks (NN), to suppress $q \bar{q}$ background and $B \rightarrow X_{c} \ell \nu$ background. The $q \bar{q} \mathrm{NN}$ is trained on a sample passing the preselection criteria, while the $B \rightarrow X_{c} \ell \nu \mathrm{NN}$ is trained on a sample passing both the preselection and the $q \bar{q}$ neural network criteria. The training is performed with signal and background MC samples. These NN are multilayer perceptrons that have two hidden layers with seven and three nodes. The variables used as inputs to the $q \bar{q}$ NN are $R_{2}$, Ł2, $\cos \Delta \theta_{\text {thrust }}, \cos \theta_{\mathrm{BY}}, m_{\text {miss }}^{2} /\left(2 E_{\text {miss }}\right)$, $\operatorname{Prob}\left(\chi^{2}, Y_{\text {vtx }}\right)$, the polar angle of the missing momentum vector in the laboratory frame, and the Dalitz plot amplitude $A_{\text {Dalitz }}=\alpha\left|\vec{p}_{\pi^{+}} \times \vec{p}_{\pi^{-}}\right|$, with the $\pi^{+}$and $\pi^{-}$ momenta measured in the $\omega$ rest frame and scaled by a normalization factor $\alpha$. True $\omega$ mesons typically have larger values of $A_{\text {Dalitz }}$ than combinatoric $\omega$ candidates reconstructed from unrelated pions. The $B \rightarrow X_{c} \ell \nu$ NN uses the same variables, except for $\cos \Delta \theta_{\text {thrust }}$, which is replaced by $\cos \theta_{W \ell}$, the helicity angle of the lepton, defined as the angle between the momentum of the lepton in the rest frame of the virtual $W$ and the momentum of the $W$ in the rest frame of the $B$. The data and MC simulation agree well for the NN input variables at each stage of the selection. The NN discriminators are chosen by maximizing $\sqrt{\epsilon_{\mathrm{sig}}^{2}+\left(1-\eta_{\mathrm{bkg}}\right)^{2}}$, where $\epsilon_{\mathrm{sig}}$ is the efficiency of the signal and $\eta_{\text {bkg }}$ is the fraction of the background misidentified as signal. The selection efficiencies for the various stages of the candidate selection for the signal and background components are given in Table $\mathbb{I}$.

After the preselection and NN selection, $21 \%$ of events in data contribute multiple $B^{+} \rightarrow \omega \ell^{+} \nu$ candidates. The candidate with the largest value of $\operatorname{Prob}\left(\chi^{2}, Y_{\mathrm{vtx}}\right)$ is retained. For the remaining candidates, the reconstructed 3 -pion mass is required to be consistent with the $\omega$ nominal mass [24], $\left|m_{3 \pi}-m_{\omega}\right|<23 \mathrm{MeV}$. The overall signal efficiency is $0.73 \%$ when the reconstructed candidate includes a true $\omega$ and $0.21 \%$ when it includes a combinatoric $\omega$. The efficiencies of the $B \bar{B}$ and $q \bar{q}$ backgrounds are suppressed by several orders of magnitude relative to the signal.

TABLE I: Successive efficiencies (in \%) predicted by MC simulation for each stage of the selection, for true- and combinatoric- $\omega$ signal, and backgrounds from $B \bar{B}$ and $q \bar{q}$ events.

| Source | true- $\omega$ comb.- $\omega$ signal signal |  | $B \bar{B}$ | $q \bar{q}$ |
| :---: | :---: | :---: | :---: | :---: |
| Preselection | 1.9 | 4.8 | 0.0094 | 0.00073 |
| Neural nets | 43 | 17 | 7.9 | 11 |
| 3-pion mass | 88 | 26 | 24 | 30 |
| Total (product) | 0.73 | 0.21 | 0.00018 | 0.000024 |

## C. Data-MC Comparisons

The determination of the number of signal events relies heavily on the MC simulation to correctly describe the efficiencies and resolutions, as well as the distributions for signal and background sources. Therefore a significant effort has been devoted to detailed comparisons of data and MC distributions, for samples that have been selected to enhance a given source of background. Specifically, we have studied the MC simulation of the neutrino reconstruction for a control sample of $B^{0} \rightarrow D^{*-} \ell^{+} \nu$ decays, with $D^{*-} \rightarrow \bar{D}^{0} \pi_{s}^{-}$and $\bar{D}^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$. This final state is similar to that of the $B^{+} \rightarrow \omega \ell^{+} \nu$ decay, except for the addition of the slow pion $\pi_{s}^{-}$and the substitution of a $K^{+}$for a $\pi^{+}$. This control sample constitutes a high-statistics and high-purity sample on which to test the neutrino reconstruction. We compare data and MC distributions for the control sample and find good agreement for the variables used in the preselection and as inputs to the NN. We have also used this sample to study the resolution of the neutrino reconstruction and its impact on $q^{2}, m_{\mathrm{ES}}$, and $\Delta E$.

## IV. SIGNAL EXTRACTION

## A. Fit Method

We determine the signal yields by performing an extended binned maximum-likelihood fit to the observed three-dimensional $\Delta E-m_{\mathrm{ES}}-q^{2}$ distributions. Our fit


FIG. 1: Distribution of $\Delta E$ versus $m_{E S}$ for true- $\omega$ signal MC. The 20 bins into which the plane is divided for the fit histogram are overlaid.
technique [25] accounts for the statistical fluctuations of the data and MC samples. For this fit the $\Delta E-m_{\mathrm{ES}}$ plane is divided into 20 bins, as shown in Fig. 1 and the data are further subdivided into five bins in $q^{2}$, chosen to contain roughly equal numbers of signal events. The $q^{2}$ resolution is dominated by the neutrino reconstruction. It can be improved by substituting the missing energy with the magnitude of the missing momentum and by rescaling $\vec{p}_{\text {miss }}$ to force $\Delta E=0, q_{\text {corr }}^{2}=$ $\left[\left(E_{\ell}, \vec{p}_{\ell}\right)+\delta \cdot\left(p_{\text {miss }}, \vec{p}_{\text {miss }}\right)\right]^{2}$, where $\delta=1-\Delta E / E_{\text {miss }}$. This correction to $q^{2}$ is used in the fit. We describe the data sample by a sum of four components: $B^{+} \rightarrow \omega \ell^{+} \nu$ signal (both true- $\omega$ and combinatoric- $\omega$ ), true- $\omega B \bar{B}$, true- $\omega q \underline{q}$, and the sum of the combinatoric- $\omega$ background from $B \bar{B}$ and $q \bar{q}$ events. The parameters of the fit are scale factors for the signal and background yields, and are listed in Table III Predictions for the shape of the $\Delta E-m_{\mathrm{ES}}$ distributions are taken from simulation of both signal and background sources, separately for each $q^{2}$ bin. The branching fraction for the signal decay is obtained by multiplying the fitted value of the scale factor with the branching fraction that is implemented in the MC simulation. For the fit to the full $q^{2}$ range there are three free parameters, for signal and true- $\omega B \bar{B}$ and $q \bar{q}$ backgrounds. The fit with five $q^{2}$ bins uses eleven free parameters; since the true- $\omega q \bar{q}$ contribution is small in $q^{2}$ bins $2-5$, the scale factors for these components are fixed to the value obtained for true- $\omega q \bar{q}$ in bin 1 . While

TABLE II: Fit scale factors $p_{k}^{s}$ for the simulated source $s$ and $q^{2}$ bin $k$ for the five $q^{2}$ bins and all- $q^{2}$ fits.

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-10$ | $10-12$ | $12-21$ | $0-21$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{+} \rightarrow \omega \ell^{+} \nu$ | $p_{1}^{\omega \ell \nu}$ | $p_{2}^{\omega \ell \nu}$ | $p_{3}^{\omega \ell \nu}$ | $p_{4}^{\omega \ell \nu}$ | $p_{5}^{\omega \ell \nu}$ | $p^{\omega \ell \nu}$ |
| $B \bar{B}$, true- $\omega$ | $p_{1}^{B \bar{B}}$ | $p_{2}^{B \bar{B}}$ | $p_{3}^{B \bar{B}}$ | $p_{4}^{B \bar{B}}$ | $p_{5}^{B} \bar{B}$ | $p^{B \bar{B}}$ |
| $q \bar{q}$, true- $\omega$ | $p_{1}^{q \bar{q}}$ | fixed fixed | fixed | fixed | $p^{q \bar{q}}$ |  |
| Comb. $\omega$ bkgd. | fixed fixed fixed | fixed | fixed | fixed |  |  |

the $\Delta E-m_{\mathrm{ES}} \mathrm{PDF}$ distributions for the signal and true-
$\omega B \bar{B}$ and $q \bar{q}$ sources are taken from MC samples, we choose to represent the dominant combinatoric- $\omega$ background by the distributions of data events in the $m_{3 \pi}$ sidebands, thereby reducing the dependence on MC simulation of these backgrounds. The normalization of these background data is taken from a fit to the $3-\pi$ mass distribution in the range $0.680<m_{3 \pi}<0.880 \mathrm{GeV}$. To obtain a sample corresponding to the combinatoric- $\omega$ background from $B \bar{B}$ and $q \bar{q}$ events only, we subtract the MC simulated $m_{3 \pi}$ contribution of the small combinatoric- $\omega$ $B^{+} \rightarrow \omega \ell^{+} \nu$ signal sample. The resulting $m_{3 \pi}$ distribution is fitted to the sum of a relativistic Breit-Wigner convolved with a normalized Gaussian function, and the combinatoric background described by a second degree polynomial. The resulting fit to the $m_{3 \pi}$ distribution for the all $-q^{2}$ sample is shown in Fig. 2, The $\chi^{2}$ per number of degrees of freedom (NDF) for the fits are within the range expected for good fits. The fitted background function is used to determine the weights to apply to the upper and lower sidebands to scale them to the expected yield of combinatoric- $\omega B \bar{B}$ and $q \bar{q}$ background in the $m_{3 \pi}$ peak region. The peak and two sideband regions


FIG. 2: Fit to the distribution of $m_{3 \pi}$ for data from the all- $q^{2}$ sample, with MC combinatoric- $\omega$ signal subtracted. The dashed (red) and dotted (blue) curves describe the fitted peaking and combinatoric background functions, respectively, and the solid (black) curve is their sum. The peak and sideband regions are also indicated.
are chosen to have a width of 46 MeV , with gaps between them of 23 MeV , also shown in Fig. 2 Since the normalization of the combinatoric- $\omega$ signal contribution depends on the fitted signal scale factor, which is a priori unknown, this component is rescaled for multiple fit iterations until the signal scale factor converges.

## B. Fit Results

The fitting procedure has been validated on pseudoexperiments generated from the MC PDFs. We find no biases and the uncertainties follow the expected statisti-
cal distribution. The yields of the signal, true- $\omega B \bar{B}$, and true- $\omega q \bar{q}$ components obtained from the binned maximum-likelihood fit to $\Delta E-m_{\mathrm{ES}}$ are presented in Table IIIT The uncertainties for true- $\omega q \bar{q}$ in $q^{2}$ bins $2-5$ are derived as the same fractional uncertainty on $q \bar{q}$ in $q^{2}$ bin 1. The uncertainties on the combinatoric- $\omega$ background are calculated from the $m_{3 \pi}$ fit errors. The signal yield from the fit to the full $q^{2}$ range is consistent within the uncertainty with the signal yields summed over all five individual $q^{2}$ bins. However, the true- $\omega$ $q \bar{q}$ and $B \bar{B}$ yields for the single $q^{2}$ fit differ significantly from the sum of the five $q^{2}$ bins; this can be explained by the large anti-correlation between these two components. The fixed combinatoric- $\omega$ background yield accounts for $83 \%$ of all backgrounds. Projections of the fitted distributions of $m_{\mathrm{ES}}$ for the all $-q^{2}$ fit and for the five $q^{2}$ bins fit are shown in Fig. 3. The agreement between the data and fitted MC samples is reasonable for distributions of $\Delta E, m_{\mathrm{ES}}$, and $q^{2}$, as indicated by the $\chi^{2} / \mathrm{NDF}$ probabilities listed in Table III. The correlations among the parameters are listed in Table IV, The strongest correlation is between the $q \bar{q}$ and $B \bar{B}$ components, which have relatively similar $\Delta E-m_{\mathrm{ES}}$ shapes. Fixing the $q \bar{q}$ component leads to a larger correlation between signal and background in the five $q^{2}$ bins fit than in the all$q^{2}$ fit. The branching fraction, $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$, averaged over electron and muon channels, is defined as $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)=\sum_{i}\left(N_{i}^{\text {sig }} / \epsilon_{i}^{\text {sig }}\right) /\left(4 f_{ \pm} N_{B \bar{B}}\right)$, where $N_{i}^{\text {sig }}$ refers to the number of reconstructed electron and muon signal events in $q^{2}$ bin $i, \epsilon_{i}^{\text {sig }}$ is the reconstruction efficiency, $f_{ \pm}$is the fraction of $B^{+} B^{-}$decays in all $B \bar{B}$ events, and $N_{B \bar{B}}$ is the number of produced $B \bar{B}$ events. The factor of 4 comes from the fact that $\mathcal{B}$ is quoted as the average of $\ell=e$ and $\mu$ samples, not the sum, and the fact that either of the two $B$ mesons in the $B^{+} B^{-}$event may decay into the signal mode. The partial branching fractions are listed in Table V

## V. SYSTEMATIC UNCERTAINTIES

Table VI summarizes the contributions to the systematic uncertainty. The largest systematic uncertainties are related to the event reconstruction, and are determined with procedures similar to those described in a recent $B A B A R$ analysis [8]. These uncertainties are primarily a consequence of the neutrino reconstruction, which depends on the detection of all of the particles in the event. To assess the uncertainty from the track reconstruction, the analysis is repeated, eliminating tracks at random with a probability determined by the uncertainty in the tracking efficiency. Similarly, we evaluate the uncertainty from photon reconstruction efficiency by eliminating photons at random as a function of the photon energy. Since a $K_{L}^{0}$ leaves no track and deposits only a small fraction of its energy in the calorimeter, the reconstruction of the neutrino is impacted. The uncertainty on the $K_{L}^{0} \mathrm{MC}$ simulation involves the shower energy deposited by the

TABLE III: Yields from the five $q^{2}$ bins and all $-q^{2}$ fits, with the fit $\chi^{2} / \mathrm{NDF}$ and corresponding probability. Components listed in bold have scale factors that are fixed in the fit; the uncertainties on their yields are determined separately from the fit.

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-10$ | $10-12$ | $12-21$ | $0-21$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| all signal | $263 \pm 77$ | $185 \pm 56$ | $175 \pm 41$ | $224 \pm 40$ | $252 \pm 63$ | $1041 \pm 133$ |
| true- $\omega$ signal | 244 | 163 | 147 | 173 | 144 | 804 |
| comb.- $\omega$ signal | 19 | 22 | 27 | 51 | 108 | 237 |
| $B \bar{B}$ (true- $\omega$ ) | $111 \pm 110$ | $305 \pm 77$ | $124 \pm 63$ | $51 \pm 63$ | $530 \pm 102$ | $852 \pm 250$ |
| $q \bar{q}$ (true- $\omega$ ) | $394 \pm 144$ | $\mathbf{1 3 9}[ \pm 51]$ | $\mathbf{6 2}[ \pm 23]$ | $\mathbf{3 3}[ \pm 12]$ | $\mathbf{6 1}[ \pm 22]$ | $1048 \pm 280$ |
| comb.- $\omega$ bkgd. | $\mathbf{1 7 4 2}[ \pm 44] \mathbf{1 8 2 0}[ \pm 47] \mathbf{1 2 4 1}[ \pm 34]$ | $\mathbf{1 5 2 0}[ \pm 37]$ | $\mathbf{3 9 0 8}[ \pm 61] \mathbf{1 0 2 2 2}[ \pm 102]$ |  |  |  |
| Total | 2509 | 2449 | 1601 | 1827 | 4752 | 13163 |
| $\chi^{2} / \mathrm{NDF}$ | $21.7 / 17$ | $20.6 / 18$ | $12.7 / 18$ | $23.6 / 18$ | $19.4 / 18$ | $15.2 / 17$ |
| Prob $\left(\chi^{2}, \mathrm{NDF}\right)$ | $20 \%$ | $30 \%$ | $81 \%$ | $17 \%$ | $37 \%$ | $58 \%$ |



FIG. 3: Distributions of $m_{\mathrm{ES}}$ after the fit, for five separate $q^{2}$ bins and the full $q^{2}$ range, in the $\Delta E$ signal band, $-0.25<$ $\Delta E \leq 0.25 \mathrm{GeV}$, and the ratio of the data to the fitted MC prediction. The points represent data in the $m_{3 \pi}$ peak, while the histogram represents the sum of source components, signal (white), true- $\omega B \bar{B}$ (light gray), true- $\omega q \bar{q}$ (dark gray), and combinatoric- $\omega$ background (diagonal lines).
$K_{L}^{0}$ in the calorimeter, the $K_{L}^{0}$ reconstruction efficiency, and the inclusive $K_{L}^{0}$ production rate from $B \bar{B}$ events. We assign an uncertainty on the identification efficiency of electrons and muons, as well as on the lepton and kaon vetoes of the $\omega$ daughter pions. These uncertainties are derived from studies of the selector performance on data control samples. The uncertainty in the calculation of the LCSR form factors impacts the uncertainty on the branching fraction because the signal efficiency depends
on the predicted distribution in $q^{2}$. We assess the change in the signal efficiency as the form factors are varied within their uncertainties. We include the uncertainty on the branching fraction of the $\omega$ decay, $\mathcal{B}\left(\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=$ $(89.2 \pm 0.7) \times 10^{-2}[24]$. To evaluate the uncertainty from radiative corrections, candidates are reweighted by $20 \%$ of the difference between the spectra with and without PHOTOS [26], which models the final state radiation of the decay. The uncertainty on the true- $\omega$ backgrounds

TABLE IV: Correlations among the fit parameters introduced in Table III All empty entries are zero; there are no correlations between separate $q^{2}$ bins.

|  | $p^{B \bar{B}}$ | $p^{\omega \ell \nu}$ | $p_{1}^{B \bar{B}}$ | $p_{1}^{\omega \ell \nu}$ | $p_{2}^{\omega \ell \nu}$ | $p_{3}^{\omega \ell \nu}$ | $p_{4}^{\omega \ell_{\nu}}$ |
| :--- | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| $p_{5}^{q \bar{q}}$ | -0.74 | -0.49 |  |  |  |  |  |
| $p_{5}^{\omega \ell_{\nu}}$ |  |  |  |  |  |  |  |
| $p_{1}^{q \bar{B}}$ | 1.00 | 0.02 |  |  |  |  |  |
| $p_{1}^{B \bar{B}}$ |  |  | -0.73 | -0.51 |  |  |  |
| $p_{2}^{B \bar{B}}$ |  |  | 1.00 | -0.02 |  |  |  |
| $p_{3}^{B \bar{B}}$ |  |  |  |  | -0.67 |  |  |
| $p_{4}^{B \bar{B}}$ |  |  |  |  | -0.65 |  |  |
| $p_{5}^{B \bar{B}}$ |  |  |  |  |  | -0.59 |  |

TABLE V: Measured $B^{+} \rightarrow \omega \ell^{+} \nu$ branching fraction and partial branching fractions in bins of $q^{2}$ with statistical and systematic uncertainties.

| $q^{2}\left(\mathrm{GeV}^{2}\right)$ | $\Delta \mathcal{B}\left(\times 10^{-5}\right)$ |
| :---: | ---: |
| $0-4$ | $2.2 \pm 0.6 \pm 0.3$ |
| $4-8$ | $1.6 \pm 0.5 \pm 0.1$ |
| $8-10$ | $1.6 \pm 0.4 \pm 0.1$ |
| $10-12$ | $2.1 \pm 0.4 \pm 0.2$ |
| $12-21$ | $4.1 \pm 1.0 \pm 0.5$ |
| $0-12$ | $7.5 \pm 1.0 \pm 0.8$ |
| $0-21$ | $11.5 \pm 1.5 \pm 1.2$ |

has a small impact on the signal yield since these components represent a small fraction of the total sample. The uncertainty from fixing the $q \bar{q}$ yield in $q^{2}$ bins $2-5$ is determined by repeating the fit with the $q \bar{q}$ yield set to $50 \%$ or $150 \%$ of the MC prediction from $q^{2}$ bin 1 . To assess the uncertainty of the $\Delta E-m_{\mathrm{ES}}$ shapes of the true- $\omega q \bar{q}$ and true- $\omega B \bar{B}$ samples, the fit is repeated after the events are reweighted to reproduce the inclusive $\omega$ momentum distribution measured in $B \bar{B}$ and $q \bar{q}$ events. We also assess the uncertainty on the modeling of the semileptonic backgrounds by varying the branching fractions and form factors of the exclusive and inclusive $B \rightarrow X_{u} \ell \nu$ backgrounds [24] and $B \rightarrow X_{c} \ell \nu$ backgrounds [3] within their uncertainties. To assess the uncertainties that result from the MC prediction of the $m_{3 \pi}$ distribution of the combinatoric- $\omega$ signal, we use the uncorrected distribution, in which the combinatoric- $\omega$ signal is not subtracted from the $m_{3 \pi}$ sidebands, and the signal fit parameter is set to scale only the true- $\omega$ signal contribution. Twenty percent of the difference between the nominal and uncorrected results is taken as the systematic uncertainty; it is largest for $12<q^{2}<21 \mathrm{GeV}^{2}$ because the fraction of combinatoric- $\omega$ signal in this $q^{2}$ bin is large. The sideband weights determined from the $m_{3 \pi}$ fit have a statistical uncertainty, which is determined by varying the sideband weights within their fit errors. The uncertainty in the chosen $m_{3 \pi}$ ansatz is assessed by repeating the $m_{3 \pi}$ fits, replacing the nominal functions for the peak and background components. For the background component, we use a 3rd-degree polynomial instead of a 2 nd-
degree polynomial. For the peaking component, we use a Gaussian function in place of a relativistic Breit-Wigner convoluted with a Gaussian. The systematic error from the $m_{3 \pi}$ ansatz is taken as the sum in quadrature of the change in signal yield for each of these functional variations. The branching fraction depends inversely on the value of $N_{B \bar{B}}$, which is determined with a precision of $1.1 \%$ [27]. At the $\Upsilon(4 S)$ resonance, the fraction of $B^{+} B^{-}$ events is measured to be $f_{ \pm}=0.516 \pm 0.006$ [24], an uncertainty of $1.2 \%$.

TABLE VI: Systematic uncertainties in \% on the branching fraction.

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-10$ | $10-12$ | $12-21$ | $0-21$ |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| Event reconstruction |  |  |  |  |  |  |
| tracking efficiency | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 |
| photon efficiency | 5.2 | 5.2 | 5.2 | 5.2 | 5.2 | 5.2 |
| $K_{L}$ prod./interactions | 8.9 | 2.1 | 2.0 | 2.9 | 6.2 | 4.4 |
| lepton identification | 1.7 | 1.5 | 1.4 | 1.3 | 1.4 | 1.4 |
| $K / \ell$ veto of $\omega$ daughters | 1.7 | 1.7 | 1.7 | 1.7 | 1.8 | 1.7 |
| Signal simulation |  |  |  |  |  |  |
| signal form factors | 6.3 | 1.5 | 1.1 | 2.9 | 4.6 | 4.8 |
| $\mathcal{B}\left(\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)$ | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| Radiative corrections | 0.4 | 0.3 | 0.2 | 0.3 | 0.1 | 0.1 |
| True- $\boldsymbol{\omega}$ background |  |  |  |  |  |  |
| $q \bar{q}$ yield | - | 4.2 | 1.4 | 1.5 | 2.7 | - |
| $q \bar{q} \Delta E-m_{\text {ES }}$ shapes | 1.7 | 0.9 | 0.2 | $<0.1$ | 0.5 | 0.7 |
| $B \bar{B} \Delta E-m_{\text {ES }}$ shapes | 1.0 | 0.1 | 0.1 | 0.1 | 0.9 | 1.1 |
| $B \rightarrow X_{c} \nu \nu \mathcal{B}$ and FF | 0.4 | 0.7 | 0.3 | 0.4 | 0.9 | 0.2 |
| $B \rightarrow X_{u} \ell \nu \mathcal{B}$ and FF | 0.5 | 0.6 | 0.2 | 0.6 | 1.4 | 0.4 |
| Comb. $\omega$ sources |  |  |  |  |  |  |
| signal $m_{3 \pi}$ distribution | 0.6 | 0.0 | 1.0 | 0.6 | 3.1 | 0.5 |
| bkgd. yield, stat. error | 1.1 | 0.8 | 2.5 | 0.5 | 0.2 | 1.0 |
| bkgd. yield, ansatz error | 0.6 | 0.3 | 1.1 | 0.2 | 0.6 | 0.9 |
| $\boldsymbol{B}$ production |  |  |  |  |  |  |
| $B \bar{B}$ counting | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| $f_{ \pm} / f_{00}$ | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| Syst. uncertainty | 13.5 | 9.1 | 8.6 | 8.8 | 11.7 | 10.1 |
| Stat. uncertainty | 29.2 | 30.2 | 23.2 | 17.7 | 24.9 | 12.8 |
| Total uncertainty | 32.2 | 31.5 | 24.8 | 19.7 | 27.5 | 16.3 |

## VI. RESULTS AND CONCLUSIONS

We have measured the branching fraction,

$$
\begin{equation*}
\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)=(1.15 \pm 0.15 \pm 0.12) \times 10^{-4} \tag{2}
\end{equation*}
$$

where the first error is statistical and the second is systematic, based on $1041 \pm 133$ observed signal candidates. Here, $\ell$ indicates the electron or muon mode and not the sum over them. We have also measured partial branching fractions with respect to $q^{2}$, after correcting for the finite $q^{2}$ resolution, which are presented in Table V. The measured $\Delta \mathcal{B} / \Delta q^{2}$ are compared to the predictions from two form factor calculations in Fig. 4. These QCD predictions have been normalized to the measured branching fraction. Neglecting the theoretical uncertain-


FIG. 4: Partial branching fractions (points with error bars) with respect to $q^{2}$. The data are compared with the predictions from light-cone sum rules (LCSR) 9] and a quark-model calculation (ISGW2) [10]. The uncertainty band (shaded) is given for the LCSR calculation.
ties, the $\chi^{2} / \mathrm{NDF}$ of the measured distribution relative to the LCSR prediction [9] is $6.9 / 5$, corresponding to a $\chi^{2}$ probability of $23 \%$; relative to the ISGW2 prediction [10] the $\chi^{2} / \mathrm{NDF}$ is $7.9 / 5$, with a $\chi^{2}$ probability of $16 \%$. Within the large experimental uncertainties, both the LCSR and ISGW2 form factor calculations are consistent with the data. The uncertainties of the ISGW2 form factor calculation are not available. The uncertainties of the LCSR calculation were estimated by the authors to vary linearly as a function of $q^{2}$; i.e., $\sigma_{\mathrm{dB}} / \mathrm{d} q^{2} /\left(\mathrm{d} \mathcal{B} / \mathrm{d} q^{2}\right)=21 \%+3 \% \times q^{2} /\left(14 \mathrm{GeV}^{2}\right)$, for the $B \rightarrow \rho \ell \nu$ decays [28]. It is assumed that this estimate is also valid for $B^{+} \rightarrow \omega \ell^{+} \nu$ decays. The value of $\left|V_{u b}\right|$ can be determined from the measured partial branching fraction, the $B^{+}$lifetime $\tau_{+}=(1.638 \pm 0.011) \mathrm{ps}$ [24], and the integral $\Delta \zeta$ of the differential decay rate:

$$
\begin{align*}
\left|V_{u b}\right| & =\sqrt{\frac{\Delta \mathcal{B}\left(q_{\min }^{2}, q_{\max }^{2}\right)}{\tau_{+} \Delta \zeta\left(q_{\min }^{2}, q_{\max }^{2}\right)}} \\
\Delta \zeta\left(q_{\min }^{2}, q_{\max }^{2}\right) & =\frac{1}{\left|V_{u b}\right|^{2}} \int_{q_{\min }^{2}}^{q_{\max }^{2}} \frac{\mathrm{~d} \Gamma_{\text {theory }}}{\mathrm{d} q^{2}} \mathrm{~d} q^{2} \tag{3}
\end{align*}
$$

Table VII lists the values of $\Delta \zeta$ and $\left|V_{u b}\right|$ for LCSR and ISGW2 in different ranges of $q^{2}$. LCSR calculations are more accurate at low $q^{2}$, while ISGW2 predictions are more reliable at high $q^{2}$. Both form factor calculations arrive at very similar values for $\left|V_{u b}\right|$. These values of $\left|V_{u b}\right|$ are consistent with $\left|V_{u b}\right|$ determined from $B \rightarrow \rho \ell \nu$ decays [8], and smaller than those determined from $B \rightarrow \pi \ell \nu$ decays [29]. The value of $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ measured in this analysis supersedes the previous $B A B A R$ measurement [6] based on a smaller data sample. The two results are in close agreement; the principal difference

TABLE VII: $\left|V_{u b}\right|$, determined from two form factor calculations of $\Delta \zeta$, in different ranges of $q^{2}$. The first uncertainty is experimental (the sum in quadrature of statistical and systematic); the second uncertainty is from theory, and is only available for LCSR.

|  | $q^{2}\left(\mathrm{GeV}^{2}\right)$ | $\Delta \zeta\left(\mathrm{ps}^{-1}\right)\left\|V_{u b}\right\|\left(\times 10^{-3}\right)$ |  |
| :--- | :---: | :---: | :--- |
|  | $0-12$ | $7.8 \pm 1.8$ | $2.41 \pm 0.20 \pm 0.27$ |
| LCSR [9] | $12-21$ | $6.4 \pm 1.5$ | $1.99 \pm 0.27 \pm 0.24$ |
|  | $0-21$ | $14.2 \pm 3.3$ | $2.23 \pm 0.18 \pm 0.26$ |
|  | $0-12$ | 7.3 | $2.51 \pm 0.20$ |
| ISGW2 [10] | $12-21$ | 6.8 | $1.92 \pm 0.27$ |
|  | $0-21$ | 14.1 | $2.24 \pm 0.18$ |

between this analysis and the previous one is that the combinatoric- $\omega$ background is taken from the sideband of the data $m_{3 \pi}$ distribution rather than from MC simulation. Although the dominant systematic uncertainties from event reconstruction cannot be avoided, this procedure substantially reduces the reliance on the MC simulation to model the composition of this largest source of background. Currently, the QCD predictions of the form factors, and in particular their uncertainties, have limited precision for $B^{+} \rightarrow \omega \ell^{+} \nu$ and $B \rightarrow \rho \ell \nu$ decays. Furthermore, the form factor uncertainties impact $\left|V_{u b}\right|$ derived from $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$. Form factor calculations with reduced uncertainties combined with improved measurements would also enable tests and discrimination among different predictions as a function of $q^{2}$, and thereby improve measurements of $\left|V_{u b}\right|$.

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