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HADRON MULTIPLICITY IN COLOR GAUGE THEORY MODELS*

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ABSTRACT

The rising of hadron multiplicity in high energy reactions is attributed to the necessity of confining quark quantum numbers. Specifically, in a gauge theory model with $SU(3)_{color}$, each event in $e^+e^- \to$ hadrons begins with the separation of a 3 and $\overline{3}$ color pair which results in a specific, flavor-independent rising gluon multiplicity. This then leads to a final hadron multiplicity which is a function of the initial color current: $\langle n_{had} \rangle = n_{3\overline{3}}(s)$. In inelastic lepton scattering $\ell H \to \ell' X$, a separating $3,\overline{3}$ color current is also created, giving the same multiplicity $\langle n_{had} \rangle = n_{3\overline{3}}(W^2)$ - independent of Q^2 at fixed $W^2 = (p + q)^2$. We assume the masses of the 3 and $\overline{3}$ systems are constant. In addition, if hadron-hadron scattering is dominated by "wee" quark interactions the final state again consists of separated 3 and $\overline{3}$ of color, thus leading to a common jet structure and universal hadron multiplicity $\langle n_{had} \rangle = n_{3\overline{3}}(M_X^2)$ for $e^+e^- \to X$, $\ell H \to \ell H \to \ell' X$, $H_1H_2 \to X$, $\gamma H \to X$, and $H_1H_2 \to H_3X$, in agreement with phenomenological observations. In contrast, color gluon exchange models of the Pomeron give $\langle n_{had} \rangle \sim n_{88}(M_X^2) \sim \frac{9}{4}\, n_{3\overline{3}}(M_X^2)$ for $H_1H_2 \to X$. We also suggest $\langle n \rangle \sim \log^2 s$ (rising plateau in rapidity distributions) in analogy with the soft photon multiplicity in quantum electrodynamics.

The most straightforward applications of the quark parton model are to short-distance phenomena for which scaling and power law behaviors of cross sections can be derived. Predictions concerning the detailed structure of the final state are far more difficult, being closely related to the problems of quark confinement and large distance phenomena. Most approaches have been at a heuristic or descriptive level in terms of jet structure, short range rapidity correlation, etc. In principle, however, it should be possible to compute final state structure for any given underlying theory.

One of the few examples of computation from first principles occurs in quantum electrodynamics for which the multiplicity of soft photons emitted in charged particle scattering is well known[1]. The salient features of the results are:

a) soft photons arise via bremsstrahlung from initial or final charged particle lines. Neutral particles do not radiate.

b) the average multiplicity consists of a sum over charged particle pairs, each contribution depending only on the product of charges times a function which increases with the relative rapidity of the pair.

It is the purpose of this letter to discuss the implications of the analogous picture in a color gauge model such as quantum chromodynamics[2]. In this case, charge is replaced by color, and the hadrons, which are color singlets, do not radiate. Radiation of colored gluons occurs only when two colored objects (e.g. quarks) are separated in momentum so that their relative rapidity is non-zero. In addition, there is a natural infrared cut-off determined by the size, $R \sim O(1F)$, of the confinement region for color. The soft gluon calculations should be valid in a region where $E_{cm} \gg |\vec{k}| \gg R^{-1}$.

We presume that the radiated color gluons eventually materialize[3] as hadrons in such a way that the hadron multiplicity is a direct, monotonic (possibly linear) function of the rising gluon multiplicity, $\langle n_H \rangle = F(\langle n_g \rangle)$, and hence only depends on the separating color currents, i.e.: the underlying quark configuration. Two processes with the same initial color current configuration will thus produce the same multiplicity in the final hadronic state. Further, the gluon radiation is independent of the quark flavor for a given relative rapidity. The principal effect of quark flavor will be to influence the quantum numbers of the leading hadrons and to make changes of O(1) in the hadron multiplicity. Our underlying assumption is not unlike that made in computing widths of the ψ resonances in quantum chromodynamics[4]; in this case one identifies the gluon final states with the sum over all physical hadronic states. Thus the separation of color combined with the necessity of confining color naturally leads to a rising hadron multiplicity.

We will first consider the general implications of this picture for hadronic multiplicity in e^+e^- annihilation and then relate it to deep inelastic scattering, and hadron-hadron scattering. Our principal result is that because of the color group structure, it is natural for the plateaus in e^+e^- annihilation and deep inelastic scattering (as discussed, for instance, by Bjorken Ref. 3) to have a common structure. In addition the empirical fact noted by Albini et al.[5] that the hadron multiplicity has a universal parametrization for all of the above processes, has a simple interpretation in our approach.

In $e^+e^- \to X$, each event begins with the creation of separating 3 and $\overline{3}$ currents (see Fig. 1(a)), the radiation of gluons, and the eventual creation of the hadron multiplicity. Thus the average multiplicity $\langle n \rangle_{e^+e^- \to X} = n_{3\overline{3}}(s)$.

In analogy with quantum electrodynamics, the function $n_{3\overline{3}}(s)$ depends on the gluon coupling strength $\alpha_{3\overline{3}}$ to the $3\overline{3}$ system, the masses of the 3 and $\overline{3}$, the infrared cutoff R^{-1}, and increases monotonically with the initial separation in rapidity $y_{3\overline{3}}$. Different quark masses will not affect the leading logarithmic dependence, but the quark flavors would be expected to influence the quantum numbers of a finite number of hadrons and change the average multiplicity by a small fixed constant.

Because of the flavor-independence of the gluon coupling, the leading s-dependence of the multiplicity in $e^+e^- \to$ hadrons is predicted to be the same below and well above the threshold for heavy quarks. When $y_{3\overline{3}}$ is small, there can be a temporary reduction in $\langle n\rangle_{e^+e^- \to X}$[6]. Note also that the multiplicity at $s = m_\psi^2$ will be continuous with the background if the ψ decays via the production of a pair of normal quarks.

It is interesting to also consider the multiplicity in $e^+e^- \to H + X$ associated with the inclusive system X. For $z = 2p_H \cdot q/q^2$ near 1, it is clear that H should be considered as a fragment of one of the quarks. The recoil system begins with a separating 3 and $\overline{3}$ of SU(3) color with cm momenta $\vec{q}/2$ and $(1-z)\vec{q}/2$, thus leading to the multiplicity, $\langle n\rangle_X = n_{3\overline{3}}(W^2)$ where $W^2 = (q-p_H)^2 = q^2 - 2p_H \cdot q + m_H^2$. Notice that the multiplicity is predicted to be independent of $2p_H \cdot q$ at fixed W^2. Physically we expect that for $z \to 0$, the associated multiplicity is just the total multiplicity minus one. The fact that the form $\langle n\rangle_{e^+e^- \to HX} = n_{3\overline{3}}(W^2)$ satisfies this consistency check indicates that even in the "wee" $z \sim 0$ regime, this same underlying quark topology (with constant 3 and $\overline{3}$ masses) is applicable.

In deep inelastic lepton scattering $\ell H \to \ell' X$ (electromagnetic or weak), each event begins with the creation of separating 3 and $\overline{3}$ systems with momenta $(1 - x)p$ and $xp + q$ (see Fig. 1(b)) where $x = -q^2/2p\cdot q$ is the momentum fraction carried by the struck quark ($x = (k_0 + k_3)/(p_0 + p_3)$). In general the struck parton can be a quark in a non-valence component of the Fock space wavefunction of the target. The mass of the $(1 - x)p$ system at the instant of the scattering should still be of order of the target mass even for the states appropriate to the "sea" quarks. Thus as in $e^+e^- \to H + X$, we treat the effective masses of the 3 and $\overline{3}$ systems as constants.[7] The hadron multiplicity is thus predicted to be $\langle n \rangle_{\ell H \to \ell X} = n_{3\overline{3}}(W^2)$ with $W^2 = (1 - x)2p\cdot q = 2p\cdot q + q^2$. (Note that this is the natural continuation of the result obtained above for $e^+e^- \to HX$.) The connection between the multiplicity in e^+e^- annihilation and deep inelastic lepton scattering is immediate:

$$\langle n \rangle_{\ell H \to \ell X} = n_{3\overline{3}}(W^2) = \langle n \rangle_{e^+e^- \to X}\Big|_{s=W^2} \qquad (1)$$

The prediction that the multiplicity for $\ell H \to \ell' X$ is independent of q^2 for fixed W^2 is strikingly confirmed by experiment[8]. This represents a derivation of the result long advocated by Feynman,[9] and Bjorken[10]. In their language, the hadronic and $q\overline{q}$ "plateaus" are assumed to have the same height[11] (e.g. $C_{had} \log\frac{W^2}{|q^2|} + C_{e^+e^-} \log|q^2| = C_{e^+e^-} \log W^2$ where $\langle n \rangle^{e^+e^-} \sim C_{e^+e^-} \log s$). In our derivation there is no need to break up the multiplicity into two components and moreover the multiplicity is not constrained to be exactly a single power of log s. We also note that if a color octet part of the current becomes operative by changing either q^2 or W^2 then one would expect a corresponding change in the multiplicity. The relation

$$\langle n \rangle_{e^+e^- \to X} = \langle n \rangle_{\ell H \to \ell X}(W^2=s)$$

is still maintained however.

If q^2 is taken to zero in $\langle n \rangle_{\ell H \to \ell X} = n_{3\bar{3}}(W^2)$, then one predicts that the multiplicity for $\gamma + H \to X$ at cm energy $\sqrt{s}$ is the same as in deep inelastic scattering at $W^2 = s$. This suggests then that the multiplicity for any hadron scattering process $\langle n \rangle_{HH' \to X}$ is also of the universal form $n_{3\bar{3}}(s)$. However, a closer examination reveals that this result depends upon assumptions on the underlying quark topology. In particular, we will show that multiplicity universality holds if hadron collisions are dominated by "wee" quark exchange of the type indicated in Fig.1(c).

In Feynman's description[9] of hadron-hadron scattering the constancy of the totalcross section arises from the exchange of "wee" partons with a dx/x spectrum in the Fock space decomposition of the incoming hadrons' states. Taking the exchanged partons to be a quark, the interaction once again sets up a system with separating 3 and $\bar{3}$ color currents. As before, these spectator systems, which carry essentially all of the initial momenta, have a predetermined mass of order of the hadron masses. Thus we again obtain the universal form $\langle n \rangle_{H_1+H_2 \to X} = n_{3\bar{3}}(s)$. Similarly, the multiplicity in the recoil system for single particle inclusive reactions is given by $\langle n \rangle_{H_1+H_2 \to H_3+X} = n_{3\bar{3}}(M_X^2)$. ($H_3$ can be in the central or fragmentation region.)

In contrast to the above universal behavior, in the Low-Nussinov[12,13] model of the Pomeron, the initial interaction is via gluon exchange (see Fig.1(d)), which sets up a system with separated color octets near the ends of the rapidity axis. In this case there is no reason to assume the gluon multiplicity is the same as in the $3\bar{3}$ case. In particular, the coupling constant α_{88} of the gluon to separating color octets is 9/4 times the coupling constant to separating 3 and $\bar{3}$. Accordingly, one would also expect a significant rise at small $x = |q^2|/2M\nu$ in $\langle n \rangle_{eH\to eX}$ at fixed W^2, as the dominant interaction changes from quark exchange to gluon exchange in this model.

Similar effects also occur in standard multiperipheral model calculations.[14] The multiplicity associated with the non-planar Pomeron diagram is approximately twice that associated with planar dual diagrams appropriate to Reggeon exchange, e^+e^- annihilation, and valence-dominated deep inelastic scattering. The absence of any multiplicity rise at small x for fixed W^2 in the ep → eX data[8] is in dramatic conflict with both approaches to the Pomeron multiplicity.

One may clearly extend this general approach to the hard scattering models of high p_T reactions. An important result is that the final state multiplicities depend upon and thus can be used to discriminate between the different possible subprocesses, since they have dissimilar $3\bar{3}$ pair topologies. Elaboration will appear in a future publication.[15]

It is interesting to speculate on the form for the gluon multiplicity in non-Abelian gauge theories, using quantum electrodynamics as a guide.

In QED, the n soft photon cross section is[1]

$$\sigma_n = \frac{(2\,\alpha\,\tilde{B})^n}{n!}\, e^{-2\alpha B}\, \sigma_o \tag{2}$$

and hence

$$\langle n_\gamma \rangle = 2\,\alpha\,\tilde{B}\,(k_{max}) \tag{3}$$

where

$$2\alpha\tilde{B}(k_{max}) = -\sum_{i,j} \frac{\alpha}{4\pi^2} Q_i\, Q_j \eta_i \eta_j \int_{k_{min}}^{k_{max}} \frac{d^3k}{2k_o} \frac{p_i \cdot p_j}{p_i \cdot k\; p_j \cdot k} \tag{4}$$

The sum in i and j is over all incoming ($\eta_i = +1$) and outgoing ($\eta_i = -1$) charged lines. The factor $e^{-2\alpha\beta}$ gives the effect of the virtual gluons.

It is an open question whether there is a similar exponentiation in the non-Abelian theories[16]. Assuming there is an analogous form to Eq. (2), then we may calculate $\langle n_{gluons}\rangle$ from the lowest order diagrams. The only change is that α becomes $\frac{4}{3}\alpha_s$ in QCD for the $3\bar{3}$ currents[17]. Eq. (4) is formally infrared divergent in QED[18]. However, as assumed in the introduction, in the case of color confinement there is a maximum wavelength $1/k_{min}$ of hadronic scale (defined in the $3\bar{3}$ cm system) beyond which the color gluon can only resolve an overall neutral state. Then

$$\langle n_g\rangle = \frac{2\alpha_s}{3\pi}\frac{1}{\beta_{3\bar{3}}}\log\left(\frac{1-\beta_{3\bar{3}}}{1+\beta_{3\bar{3}}}\right)\log\frac{k_{max}^2}{k_{min}^2}$$

$$\sim \frac{4}{3}\frac{\alpha_s}{\pi}\log\frac{2p_i\cdot p_j}{m_3 m_{\bar{3}}}\left(\log\frac{2p_i\cdot p_j}{m_3 m_{\bar{3}}}\left(\log\frac{2p_i\cdot p_j}{k_{min}^2}+C_1\right)+C_2\right) \qquad (5)$$

(large $p_i\cdot p_j$) where $\beta_{3\bar{3}}$ is the relative velocity of the 3 and $\bar{3}$. The maximum phase space for the ij pair, $E_{ij}^{max} = \langle n_g\rangle k_{ij}^{max}$, scales with the available energy, $(p_i\cdot p_j)^{1/2}$. The parameters c_1 and c_2 have at worst log log variation in the kinematics. We thus obtain $\langle n_{had}\rangle \sim \langle n_g\rangle \sim \log^2 s$, assuming the proportionality of hadron and gluon multiplicities. If instead we had used an effective gluon mass λ (of order of hadron masses), such as might be established by the confinement mechanism, then the result (5) is unchanged except that m_3, $m_{\bar{3}}$ and k_{min} depend on λ^2.

The result (5) is consistent with flat distribution in rapidity. The final state multiplicity fills the rapidity gap because of the dk/k integration.[9] Further, the entire plateau dN/dy rises at any rapidity y because of the logarithmic angular integral which becomes increasingly singular due to the peaking (near the light cone) of the gluon distribution along the 3, $\bar{3}$ jet axes.

Recent experimental results[20] suggest that the plateau height is rising in hadronic collisions. The universal form of Albini et al.[5] (written in terms of $\sqrt{s_a} = \sqrt{s} - M_a - M_b$, with GeV units),

$$\langle n_{ch} \rangle^{a+b \to \text{hadrons}} = 2.50 + 0.28 \log\sqrt{s_a} + 0.53 \log^2\sqrt{s_a} \, , \quad (6)$$

is the best χ^2 fit to pp collisions and also fits $\pi p \to X, Kp \to X, \pi p \to \bar{p}_{slow} X$, $pp \to p_{slow} X$, $e^+e^- \to X$, and $ep \to eX$ multiplicities. If $\langle n_{had} \rangle = \frac{3}{2} \langle n_{ch} \rangle = \langle n_{gluon} \rangle$, then identifying the $\log^2 s$ coefficient with that in Eq. (6) gives $\alpha_s = 0.46$ which is not dissimilar from other determinations.

In conclusion, a universal result for the multiplicities in e^+e^-, lepton-hadron and hadron-hadron collisions does arise in a picture where hadron production occurs as a result of kinematically necessary separation of a 3 and $\bar{3}$ of $SU(3)_{color}$ followed by gluon bremsstrahlung and hadron materialization. This picture also implies that the jet structure and associated hadron multiplicities in the central region are the same in e^+e^- annihilation, deep inelastic scattering, and forward hadron collisions.

The crucial assumption in this explanation of universal rising multiplicity is that the materialization of hadrons only depends upon the SU(3) color topology initially established by the interaction. The specific form suggested by the gluon emission structure of QED should be regarded more cautiously but nonetheless appears consistent with the data and with expectations concerning the size of α_s.

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Footnotes and References

1. D. R. Yennie, S. C. Frautshi and H. Suura, Annals of Physics 13, 379 (1961). D. R. Yennie, Brandeis Summer Lectures in Theoretical Physics, Vol. I (1963).
2. For a review and references, see R. Dashen, Proc. of the 1975 Int. Symposium on Lepton and Photon Interactions at High Energies, Stanford (1975).
3. J. Bjorken, Proc. of the Summer Institute on Particle Physics, Vol. I, SLAC (1973). J. Kogut and L. Susskind, Physics Reports 8, 75 (1973). A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Letters 31, 792 (1973).
4. T. Appelquist and H. D. Politzer, Phys. Rev. Letters 34, 43, (1975).
5. E. Albini, P. Capiluppi, G. Giacomelli, and A. M. Rossi, Bologna preprint (1975). See also F. Takagi, Tohoku University preprint TU/76/140 (1976) and references therein.
6. These effects are probably compatible with the SPEAR data. See R. Schwitters, Proc. of the 1975 Int. Symposium on Lepton and Photon Interactions at High Energies, Stanford 1975.
7. Eventually the final mass of the (1-x)p system, including its radiation, will be of order $(\frac{m^2}{x})$.
8. See G. Wolf, Proc. of the Int. Symposium on Lepton and Photon Interactions at High Energies, 1975.
9. R. Feynman, Photon-Hadron Interactions, W. A. Benjamin, 1972.
10. J. Bjorken, Proc. of the Fifth Int. Symposium on Electron and Photon Interactions at High Energies, 1971.
11. J. Bjorken and J. Kogut, Phys. Rev. D8, 1341 (1973) argue that consistency with the Regge behavior of exclusive processes requires $C_{e^+e^-} = C_{had}$.
12. F. E. Low, Phys. Rev. D12 163 (1975).
13. S. Nussinov, Phys. Rev. Letters 34, 1286 (1975), and IAS preprint COO-2220-57 (1975).

14. H. Lee, Phys. Rev. Letters 30, 719 (1973), G. Veneziano, Phys. Letters 43B, 413 (1973). We wish to thank Dr. H. Miettinen for helpful conversations concerning this model.

15. S. Brodsky and J. Gunion (in preparation).

16. Such an exponentiation is indicated by the work of J. Cornwall and G. Tiktopoulos UCLA/75/TEP/21 and Phys. Rev. Letters 35, 338 (1975). For further discussion, see T. Appelquist, Harvard Preprint (1976), to be published in the proceedings of Orbis Scientiae, Coral Gables (1975).

17. Here $\alpha_s = g_s^2/4\pi$ where $L_{int} = g \sum_a \overline{\psi} \gamma_\mu \frac{\lambda_a}{2} \psi A_a^\mu$, $\mathrm{Tr}\, \lambda_a^2 = 2$.

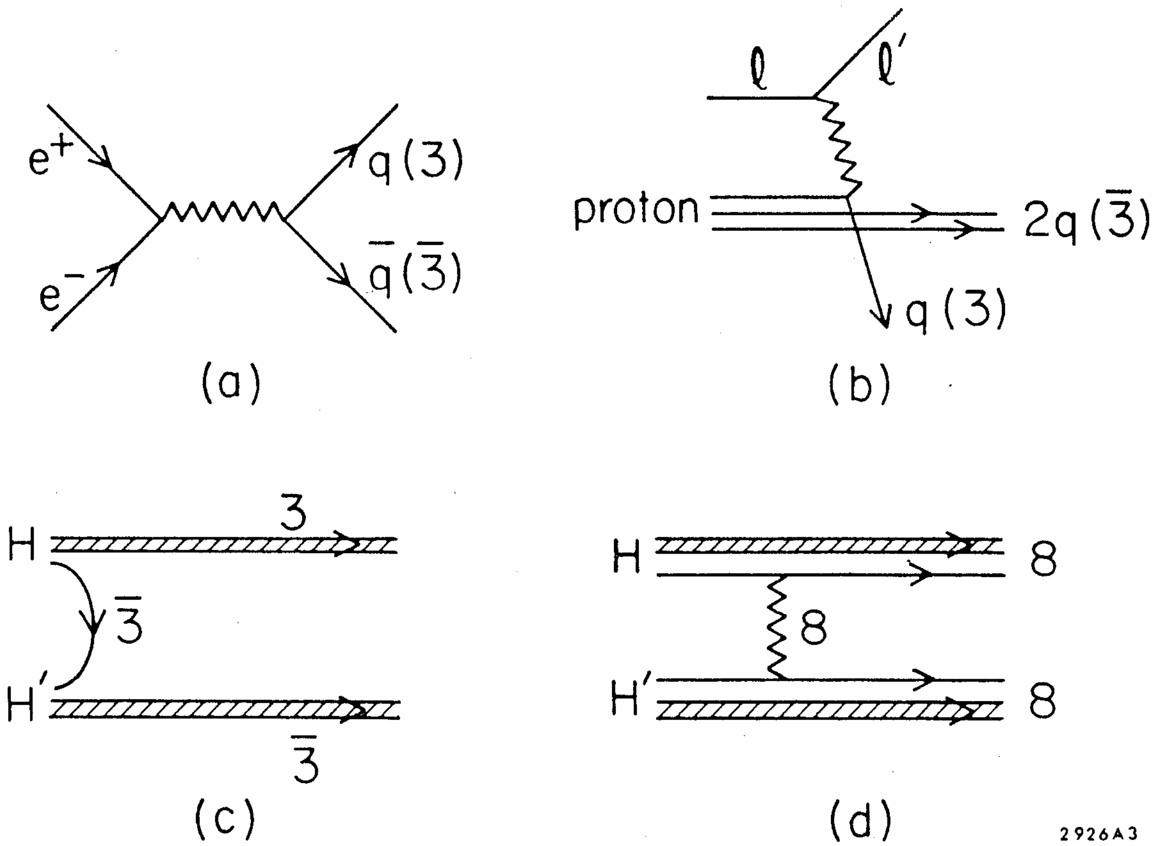
18. Except for Coulomb scattering σ_{tot} is always finite in QED. In fact σ_{tot} is even finite for $m_e \to 0$ when the incident particles are neutral. See T. Appelquist, ref 16, and references therein. However, the multiplicity still diverges for $m_e \to 0$.

19. L. Stodolsky, Phys. Rev. Letters 28, 60 (1972).

20. For references see R. Stroynowski, Sixth International Colloquium on Multiparticle Reactions, Oxford (1975).

Figure Captions

1. Separation of 3 and $\overline{3}$ of SU(3) color in (a) e^+e^- annihilation (b) $\ell p \to \ell' X$, and (c) hadron-hadron collisions. Figure (d) indicates the separation of right and left moving color octets following the exchange of a colored gluon.



e+
e−
q (3)
q̄ (3̄)
(a)
ℓ
ℓ′
proton
2q (3̄)
q (3)
(b)
H
3
3̄
H′
3̄
(c)
H
8
8
H′
8
(d)
2926A3