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VERY HIGH ENERGY ELECTRON-POSITRON COLLIDING BEAMS FOR THE STUDY OF THE WEAK INTERACTIONS*

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ABSTRACT

We consider the design of very high energy electron-positron colliding-beam storage rings for use primarily as a tool for investigating the weak interactions. These devices appear to be a very powerful tool for determining the properties of these interactions. Experimental possibilities are described, a cost minimization technique is developed, and a model machine is designed to operate at centre-of-mass energies of up to 200 GeV. Costs are discussed, and problems delineated that must be solved before such a machine can be finally designed.

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1. INTRODUCTION

In the last two to three years, the pace of discovery in high-energy physics has been extraordinary. Experimenters have seen the birth of a new class of particles — the ψ and other related heavy particles; found a new interaction the weak neutral current interaction; found familiar particles in unexpected situations — the large transverse momentum leptons found in p-p collisions; found familiar particles in strange combinations — the electron-muon events observed in e⁺-e⁻ annihilation reactions, and neutrino interactions as well as the dimuon events observed in neutrino interactions, etc. The larger part of the most exciting work of the last few years has centred on studies involving reactions with electrons, muons, or neutrinos in either the initial or final state.

Indeed, the centre of interest of high-energy physics at the present moment is on the experimental results pouring from the e^+-e^- colliding-beam machines SPEAR at SLAC and DORIS at DESY, and from the high-energy neutrino reactions being studied at FNAL and soon to be studied with higher intensity and more refined apparatus at CERN with the start-up of the SPS. Lepton reactions are illuminating the structure of the hadrons, the weak interaction, and possible relations between particles and classes of interactions. The importance of this work is indicated by the funding of construction of two new e^+-e^- colliding-beam machines with c.m. energies of 30-35 GeV in both Europe and the U.S. These machines are primarily being built for the contribution they can make to our understanding of hadrons and quantum electrodynamics. They should begin to see the effects of weak interactions but these weak effects are expected to make only a small contribution to the processes that can be studied at the maximum energies of these machines.

The weak interaction can at present be studied directly only by neutrinoinduced reactions with c.m. energies (E^*) of at best 10-20 GeV. There is intense interest in the weak interactions, which is manifested both by the effort going into studies of these interactions with presently and soon to be available machines, and by the justifications given for new generations of very high c.m. energy proton machines. The weak interactions are thought possibly to hold the clue to a unified picture of the elementary particles, their structure and dynamics. It is our belief that a study of these interactions through processes where one can hope to understand them and at energies where their strength has become comparable to that of the other interactions can best be understood through a study of very high energy e^+-e^- collisions.

This paper is concerned with the possibility of building e⁻-e⁻ colliding beam machines with energies of several hundred GeV in the c.m. system. With such a , machine one can hope to understand the structure of the weak force itself by a

study of reactions that involve only leptons in the initial and final states in an energy region where the weak interactions dominate. Relations between different classes of particles can be studied by comparing the production of such particles through the weak interactions, and the relation between the weak and electromagnetic interaction (and possibly even the strong interaction) can be studied by investigating various kinds of particle production in the energy region where the weak and electromagnetic forces are comparable in strength.

There is little argument about the desirability of studying the weak interactions via e^+-e^- interactions, but there is considerable uncertainty about the feasibility of the necessary machines and the costs of such machines compared to other approaches. These other approaches include very high energy conventional proton accelerators, high-energy p-p colliding-beam machines, or e-p colliding-beam machines; they have been studied and their properties are known. We believe that given costs for e^+-e^- tools comparable to those of the other machines mentioned above for the same effective centre-of-mass energy¹⁾, the e^+-e^- is the most desirable tool for the light it can cast on the important questions.

In this paper, we consider the possibility of designing such e^+-e^- machines. We first briefly consider the physics, for a guide is required to specify the necessary interaction rate for such a machine. An optimization procedure is developed to determine the two critical parameters, radius and total power consumption, which, together with the interaction rate and energy, determine the rest of the machine design and the costs. A specific model machine with a c.m. energy of 200 GeV is designed with the aim of identifying questions which need to be answered before such a machine could be built with confidence. Some problems do turn up, and suggestions for solutions are presented.

Our conclusions are as follows:

- i) e^+-e^- machines with $E^* \approx 200$ GeV can be built with conventional technology with sufficient interaction rate to study the weak interactions.
- ii) Such machines have very large radii and uncertain costs. Using unit costs derived from much smaller machines, the cost of a machine with $E^* = 200 \text{ GeV}$ and luminosity $\mathscr{L} = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$ appears to be about one to three times the cost of the CERN SPS.
- iii) The cost uncertainties are primarily related to the large extrapolation from much smaller machines where unit costs are likely to be larger than those in a very big machine. Engineering studies can resolve many of these uncertainties, and give a reasonable idea of the costs of such a project.

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- iv) There are some questions of machine design that must be answered, the most critical involving the amount of free space required at the collision points for experimental physics. Work by both experimental physicists and accelerator_physicists will be required to resolve this.
- v) New technology, such as superconducting RF systems, will make a significant contribution only if the costs of such systems can be greatly reduced from present amounts. Since no working large-scale superconducting system exists, an investigation of the possibilities of superconductivity will require an extensive research and development effort. Superconducting magnets have no application in the structure of the machine itself, for the magnetic fields involved are extremely low and the power consumption in the magnets is negligible compared to the RF system.
- vi) There appear to be significant opportunities for re-cycling the energy used in such a project.

2. PHYSICS

In this section we consider the rates for some processes of interest in very high energy e^+-e^- interactions. Our aim is to indicate qualitatively the kinds of reactions that can be studied, and to make some rough calculations of expected counting rates. These counting rate estimates must be approximate, for there is no established weak-interaction theory with which to make predictions in the energy region of interest, and different models differ by large amounts in their predicted cross-sections. This is in contrast to the situation at the energies of present experiments, where the differences between models are small. In spite of the uncertainties, rate calculations must be made to establish the required luminosity (reaction rate per unit cross-section).

We will begin by calculating the cross-section for the simple process

$$e^+ - e^- \rightarrow \mu^+ - \mu^- \tag{1}$$

and consider both the electromagnetic and the weak contributions to reaction (1). The lowest-order Feynman diagrams that contribute to (1) are shown in Fig. 1. Since we are interested here in rates for reactions so that we can determine the required luminosity \mathscr{L} for a high-energy machine, only total cross-sections are given. There are interference effects between the weak and electromagnetic contributions to (1) that affect the total cross-section as well as the angular distribution. What we will give here are the cross-sections for one type of interaction in the absence of the other. This will be a good approximation in those energy regions where one or the other interaction dominates, but there can be significant errors in the regions where they are comparable.

The cross-section for (1) via one-photon annihilation is

$$\sigma(\gamma) = \frac{4\pi \alpha^2}{3s}$$
(2)

where α is the fine-structure constant, and s is the square of the c.m. energy. We consider two models. One is what might be called the "Fermi" model, and the other the Weinberg-Salam model. By the "Fermi" model, we mean the usual V - A interaction with a coupling constant for both V and A each equal to $G/\sqrt{2}$, and an infinite Z⁰ mass. The cross-section in this case is

$$\sigma_{\rm E} = G^2 s / 6\pi \quad . \tag{3}$$

This cross-section increases linearly with s and will eventually overtake the electromagnetic cross-section. The ratio of this lowest-order weak cross-section to the electromagnetic cross-section is

$$\varepsilon_{\rm F} = \frac{G^2 s^2}{8\pi^2 \alpha^2} = 2.7 \times 10^{-8} s^2 \,({\rm GeV}) \,, \tag{4}$$

where G has been taken to be $10^{-5}/m_p^2$. The c.m. energy where $\varepsilon_F = 1$ is ~ 80 GeV. Above this energy the weak interaction dominates, and below it the electromagnetic dominates.

This kind of calculation gives a weak cross-section that will eventually exceed the unitarity bound that for a given angular momentum state is

$$\sigma_{\max} \leq \frac{(2J+1) 4\pi}{s} . \tag{5}$$

For J = 1 which is appropriate for the V and A interactions, the value of s at which the cross-section of Eq. (3) passes this bound is $(1500 \text{ GeV})^2$. We could modify Eq. (3) by the addition of an *ad hoc* form factor

$$F^2 = [1 + (s/s_0)]^{-2}$$

which with $s_0 = (1500 \text{ GeV})^2$ would assure that (3) never exceeded the unitarity bound. Such a form factor would reduce the weak cross-section by $\sim 25\%$ at a c.m. energy of 200 GeV.

Most interest is now centred on gauge theories of weak interactions that handle the divergences of the old weak theories in a natural way and that might give a unified description of the weak and electromagnetic interactions. The by now classical gauge theory is the Weinberg-Salam model which predicted the existence of neutral currents and which gives the carrier of the neutral current, Z^0 , a mass of \approx 100 GeV. In the Weinberg-Salam model, the cross-section for reaction (1) is given by

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$$\sigma_{W} = \frac{\sigma_{F}}{16[1 - (s/m_{Z^{0}}^{2})]^{2}}, \qquad (7)$$

where we have included only the term due to the axial-vector coupling. The vector coupling strength depends on the Weinberg angle, and although the present determinations of this angle are crude, they indicate that the vector coupling would be smaller than the axial-vector coupling.

The yield versus c.m. energy for the μ -pair production reaction from Eqs. (2), (3), and (7), is given in Fig. 2 for $\mathscr{L} = 10^{32}$ cm⁻² sec⁻¹. In this plot we have assumed a Z⁰ mass of 100 GeV for the Weinberg model. The combined yield of the electromagnetic and weak interactions at E^{*} = 200 GeV ranges from 1/h in the Weinberg model to 35/h in the Fermi model. These yields are adequate for experiments, although the yield in the Weinberg model is not comfortable. However, σ_W increases rapidly as E^{*} decreases toward the Z⁰ mass of 100 GeV.

If the Z⁰ were larger, σ_W at $E^* = 200$ GeV would increase. Any model with a finite mass neutral current carrier will have the form of Eq. (7). We can find the maximum Z⁰ mass to which we are sensitive in μ -pair production by requiring

$$\eta = \frac{\sigma_{W}(m_{Z^{0}})}{\sigma_{U}(\infty)}$$
(8)

to be greater than some minimum value that will allow experiments to make statistically significant measurements. For s = $(200 \text{ GeV})^2$, the Z⁰ mass which makes $\eta = 1.25$ is about 600 GeV, while for $\eta = 1.05$ it is 1200 GeV. These experiments appear to be sensitive to the existence of finite-mass neutral current carriers of higher mass than experiments suggested for other proposed machines.

In addition to μ -pair processes, there is elastic electron scattering which in lowest order goes through the four diagrams of Fig. 3. We know of no complete calculation of this process, but weak effects should be comparable to or possibly greater than those in μ -pair production for the same momentum transfer.

The high-energy $e^+ - e^-$ machine is a new-particle factory of unparalleled versatility. Hypothesized particles such as the charged weak-current carrier (W^{\pm}), Higgs particles, leptons (sequential or gauge), point bosons, gluons, etc., all can be produced directly if their masses are less than 100 GeV for those which can be produced only in pairs, or less than 200 GeV for those that can be produced singly or in association with light particles such as electrons or muons. There is *no* way to produce most of these particles in an environment free from very large backgrounds other than by the use of e^+-e^- colliding beams.

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Hadron production can take place in lowest order through the diagrams of Fig. 4. The usual conserved current arguments would lead us to expect the ratio of hadron production to μ -pair production via the weak interaction (R_{W}) to be about the same as the ratio of hadron production to μ -pair production via the effectromagnetic interaction (R_{γ}). We know R_{γ} only to $E^{*} \approx 8$ GeV from the SPEAR experiments, where it is found to be \sim 5. Making a large extrapolation, we expect that hadron production by both the weak and electromagnetic interactions can be studied with these machines, the yield being about five times that shown in Fig. 2.

So far, the discussion has centred on the physics that can be done with neutral-current weak interactions in lowest order. Both the charged and neutral currents contribute to the production of all kinds of particles in the next higher order. Two of the multitude of diagrams in second order that result in lepton production are shown in Fig. 5. These diagrams individually are infinite in the case of infinite Z^0 and W masses. One of the most interesting possibilities of high-energy e^+-e^- colliding beams is the study of the effect of such higher-order processes. The effective coupling constant of the weak interactions -- the analogue of α in the electromagnetic interactions -- is $Gs/\sqrt{2}$, which, at $E^* = 200$ GeV, is about 1/4. With such a large coupling the effect of higher-order weak interactions should be observable. Some calculations are required to see what effect these higher-order terms might be expected to have in a variety of models.

We shall not discuss further the physics that could be done with these $e^+ - e^-$ machines, for our object in this section has been to give rough cross-sections to use in order to determine the required luminosity. It seems that a luminosity of 10^{32} cm⁻² sec⁻¹ is reasonable to use in our first look at the design of high-energy machines, and we shall use that value throughout the rest of this study. A more thorough calculation of many kinds of cross-sections is important, for such calculations might indicate that a different luminosity is required.

3. A FISRT LOOK AT MACHINE PARAMETERS

The design of a high-energy $e^+ - e^-$ colliding-beams machine is governed by a set of equations linking the luminosity, interaction region tune shift, RF power, beam energy, and radius². For a large class of lattices, these equations together with the betatron phase change per cell of the lattice, determine nearly all the properties of the machine. The basic equation is that linking the luminosity (\mathcal{L}), maximum tune shift (Δv_y), RF power delivered to both beams (P_B) to make up for radiation loss, the bending radius (ρ), the interaction region vertical β -function (β_v *), and beam energy (E)

$$\mathscr{L} = 1.23 \times 10^{33} \frac{\Delta v P_B(MW) \rho(m)}{E^3 (GeV) \beta_{..} * (m)} , \qquad (9)$$

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where the units of the various quantities are indicated in the equation. The luminosity and beam energy are the input parameters determined by the physics that one wants to do. The parameter Δv_y is related to the non-linear electromagnetic interaction between the two beams at the collision point. Experiments with many e^+-e^- storage rings have indicated that there is a maximum value of Δv_y above which the beam lifetime decreases sharply. This maximum value is about 0.06, and we shall use this in Eq. (9).

The parameter $\beta_{\nu} \star$ should apparently be as small as possible. As we shall see later, the cost of a very large machine does not seem to depend strongly on β_v^* . There is, however, a lower bound to β_v^* arising from two sources. The first is practical: a very low β at one point in the machine means a much larger value at another point. These very large values of β impose tight tolerances on the elements of the machine at the point where β is large, and also generate serious problems with the momentum dependence of the machine parameters. This momentum dependence in extreme cases can be sufficiently severe to make difficulties in injection, and even in containing the natural energy spread in the circulating beams. The second source is related to the maximum luminosity at the two-beam instability limit. Equation (9) is derived under the assumption that $\beta_{y} * > \sigma_{l}$, the beam bunch length. For $\beta_y\star < \sigma_{\varrho}$ the maximum luminosity decreases for a given RF power and machine radius. The natural bunch length in an electron storage ring is easily calculable and is expected to be about a centimetre. However, there is a phenomenon known in the machine trade as "bunch-lengthening", which arises from the interaction of the beam with the vacuum chamber and results in significant increases in the bunch length. This bunch-lengthening is understood well enough now to make a rough calculation of the size of the effect, and it is reasonable to expect that the maximum bunch length in a machine with high luminosity and a beam energy of \sim 100 GeV will be < 5 cm. We will use 5 cm for $\beta_y \star$ in Eq. (9), which now becomes

$$\mathcal{L} = 1.47 \times 10^{33} \frac{P_B \rho}{E^3} .$$
 (10)

We can use Eq. (10) to get a 0^{th} -order idea of the beam power and radius required for a very large electron machine. We can also get some idea of special problems that might turn up in the design. To begin, we shall simply scale ρ and P_B from the values used in the LBL-SLAC PEP storage-ring design. We will let P and ρ each vary as $E^{\frac{3}{2}}$, while we hold \mathscr{L} constant at 10^{32} cm⁻² sec⁻¹. As we shall see in the next section, the machine we get with this kind of scaling is very far from optimal in cost, but this exercise is useful to give some feeling for the machine parameters.

Table 1 gives a few of the gross parameters of a machine scaled in this way. One thing immediately apparent about this table is the relatively low values of RF beam power required for even the highest machine energies. The 100 GeV c.m. example has a beam power which is only about 60% greater than the beam power in the PEP design, although the energy is a factor of 3 higher. This is a consequence of the value of β^* of 5 cm used here, compared to the β^* of 20 cm used in the PEP design.

The radii of these machines are large and the magnetic fields are correspondingly low. The 200 GeV c.m. machine of Table 1 has a bending field of only 1.1 kG. Even with this low value of B, however, the energy loss per turn of the circulating beams from synchrotron radiation is very large, and reaches in the 2×100 GeV machine about 3% of the beam energy per turn. This large fractional energy loss per turn is unprecedented in circular accelerators and may give rise to special beam dynamics problems associated with the distribution around the ring of the RF system.

The magnitude of the energy loss per turn also poses special problems for a CW RF system. While conventional electron linacs would have no trouble with the required energy gain (360 m of the SLAC accelerator would be sufficient), they are pulsed devices and not CW. In a CW device we must supply power to make up for the normal cavity dissipation, which depends only on the voltage per unit length on the cavity and the shunt impedance (Z). The power dissipated in cavity walls is given by

$$P_{\rm D} = V^2/Z \ L \tag{11}$$

where V is the total voltage on the cavity system, and L is the total length of RF cavities. The voltage V will be greater than the radiation loss U_0 given in Table 1 by a factor of about 15% in this size machine because of the overvoltage required to get a sufficiently long lifetime against quantum fluctuations in the synchrotron radiation. For our first look at a machine design, we will use the value Z = 19 M\Omega per metre, which is the shunt impedance of the SPEAR and PEP cavities at an RF frequency of 350 MHz. We can get a rough idea of the total length of RF cavity required in these machines by simply setting the power dissipated in the cavities equal to the power supplied to make up for synchrotron radiation in the beams. If we do so, we get the values for cavity length given in Table 2. It is the large length of RF cavity required, coupled with the high cost per metre of RF cavities, that makes this entire scaling procedure impractical. We have, for example, arrived at a 200 GeV c.m. energy machine, which has a total circumference of bending magnets of roughly 20 km and a total length of RF cavity of 25 km. If we use the cost per unit length of PEP RF cavities, we find that the RF cavities alone would cost considerably more than \$2000 million. We have arrived at this position by using a scaling which did not take into account the relative costs of different kinds of components. In the next section we develop a more realistic scaling to arrive at a more appropriate set of parameters for a very large electron storage ring.

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4. MACHINE PARAMETERS, VIA A COST OPTIMIZATION

The scaling in power and energy used in the previous section was an arbitrary guess that resulted in a machine with an anomalously long RF system. In this section we shetch a more reasonable scaling procedure which uses some rough component cost figures, and include these costs in choosing a radius and RF system length for a high-energy e^+-e^- machine which minimize the total cost of the machine for a given energy and luminosity. Specifically, we shall use as an example a 200 GeV c.m. machine, and base the component costs on those in the PEP cost estimate³). This optimization will be crude because cost per unit length of components of a real machine will depend on the detailed design of the machine and on its location. Hopefully the size of a 200 GeV c.m. ring will introduce economies of scale not possible in as "small" a machine as PEP. All costs used here include an allowance over fabrication costs of 20% for engineering and design and a further 20% for contingencies.

The model machine whose costs we minimize consists of a housing of radius R (we ignore the difference between bending radius ρ and R; in practice, these differ by 10-15%), containing the bending components plus a long straight section containing the RF cavity system. We will not include in the minimization costs of experimental areas, laboratory buildings, shops, etc., for these are fixed costs that depend on the scale of the planned experimental programme and are not affected significantly by changes in circumference.

The unit costs we use are given in Table 3 4).

The cost equation is

$$C = 2\pi k_1 R + (P_{\rm R} + P_{\rm D})k_2 + L k_3 + (k_4/\epsilon) (P_{\rm R} + P_{\rm D}) + F , \qquad (12)$$

where k_1 , k_2 , and k_3 are the unit costs of the ring, RF power, and cavities, respectively; R is the machine radius; P_B is the power supplied to both beams to make up for synchrotron radiation; P_D is the power dissipated as cavity losses; L is the RF cavity total length. The next-to-last term represents the 10-year power costs of the total RF system; k_4 is the 10-year cost of a megawatt of power (counting only machine on-time), and ε is the RF system efficiency. Finally, F is the fixed cost for such things as roads, experimental areas, workshops, offices, etc. The cost minimum is found by setting the partial derivative of C with respect to R and $P_D = 0$.

 $P_{\rm B}$ and L can be written in terms of R and $P_{\rm D}.$ Defining δ \equiv $E_{\rm beam}/100$ GeV and γ \equiv $\beta_y^{\star}/5$ cm,

$$P_{\rm B} = \frac{68 \ \delta^3 \gamma}{R} \qquad (MW) \qquad (13a)$$

$$L = \frac{1.02 \times 10^5 \,\delta^8}{P_{\rm D} \,R^2 \,Z} \qquad (\rm km) , \qquad (13b)$$

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where Z is the cavity shunt impedance in M\Omega/m. Substituting (13) into (12), we get

$$C = 2\pi k_1 R + \frac{68 \delta^3 \gamma [k_2 + (k_4/\epsilon)]}{R} + \frac{1.02 \times 10^5 \delta^8}{P_D R^2 Z} k_3 + \frac{k_4}{\epsilon} P_D + F.$$
(14)

We now work out several examples of machines with $\mathcal{L} = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. In all these examples, δ is set to unity (200 GeV c.m.), ε is assumed to be 75%, and Z is taken as 19 M Ω /m from the SPEAR and PEP designs.

A) <u>PEP costs</u>, $\beta_{\star}^* = 5 \text{ cm}$. Power at \$0.03/kWh

At the cost mininum, we find

$$R = 5.5 \text{ km}$$

$$P_{D} = 69 \text{ MW}$$

$$P_{B} = 12.3 \text{ MW}$$

$$L = 2.6 \text{ km}.$$

The cost for the ring, klystron system, cavities, and 10 years of operating power are \$440. \$50, \$210, and \$190 million, respectively. To find the total cost of the facility, we must guess at F in Eq. (12). Taking F = \$200 million, the total construction cost of this version of a 200 GeV c.m. e^+-e^- ring is about \$900 million, about 1.8 times the cost of the CERN SPS in 1975 dollars.

B) <u>PEP costs</u>, $\beta_{\rm v}^{\star} = 5$ cm. Power at \$0.05/kWh

Power costs have inflated relative to other costs by a large amount in the past few years. Whether it will continue to increase is uncertain, and so in this example we assume a 70% relative increase in those costs. At the cost minimum we find

$$R = 6.2 \text{ km} \\ P_{D} = 50 \text{ MW} \\ P_{B} = 11 \text{ MW} \\ L = 2.8 \text{ km}.$$

The cost for the ring, the klystrons, the cavities, and 10-years operating power are \$500, \$35, \$225, and \$235 million, respectively. The total construction cost of this version of the 200 GeV c.m. ring will be about \$1000 million -- about twice the cost of the CERN SPS in 1975 dollars.

C) PEP costs and $\beta = 10$ cm. Power at 0.05/kWh

Increasing β_y^{i} has only a small effect on costs. The change occurs through a change in the (P_B R) product derived from the luminosity equation (9). The result of the minimization is

R	=	6.5	km
P D	=	47	MW
P _B	=	21	MW
L	=	2.7	km.

The total cost in this case is increased by about \$20 million over case (A).

D) $\beta_{\rm V} = 5$ cm and lower component costs. Power at \$0.05/kWh

PEP costs are for a relatively small machine, installed in a tunnel bored in sandstone on the SLAC site. We can examine the changes in machine parameters that would occur if we reduced various component costs to take account of different fabrication techniques or a different kind of site. To give an extreme example, we will reduce the ring costs per unit length by a factor of 2 and the cavity costs per unit length by a factor of 4 5 over those used in case (B). The low ring cost might be realized by sitting the machine on flat terrain where the housing could be very inexpensive, and by using different techniques to fabricate the very small cross-section, very long magnets used. Cavity costs might be reduced by using a cavity of higher shunt impedance per unit length or possibly by mass-production fabrication techniques. These cost reduction factors are only intended to give some idea of the sensitivity of the machine parameters and total cost to the cost assumptions. The parameters found for this case are

$$R = 6.5 \text{ km}$$

$$P_{D} = 24 \text{ MW}$$

$$P_{B} = 10 \text{ MW}$$

$$L = 5.3 \text{ km}.$$

The costs are \$260, \$20, \$106, and \$132 million for the ring, klystrons, cavity, and 10-year operating costs. This gives a machine costing about a factor of 2 less than that in case (A).

This optimization procedure can easily be carried out for any centre-of-mass energy. When this is done for a constant luminosity of 10^{32} cm⁻² sec⁻¹, we find we are in a region where the machine radius and costs scale like the square of the c.m. energy. Figure 6 gives the radius for a machine of any energy, optimized as described above. The curve is for our case (B). For case (D), that of reduced unit costs, the radii are 10% larger, and the costs are a factor of 2 smaller at a given energy.

The E^2 scaling is an unexpected result. It is like that expected of stationarytarget machines which scale roughly linearly in machine energy but quadratically as a function of c.m. energy.

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The optimization procedure used here is certainly crude, for it does not include the cost of capital, the effect of the choice of radius on the cost of machine components, etc. With the excursion made in power costs, β_y^* , and component costs, the radius of the resulting ring has only changed from 5.5 to 6.5 km, while the cost of the machine has changed by over a factor of 2. It seems reasonable to assume after these exercises that the radius of a 200 GeV c.m. machine will be about 6 km, and in the next section a detailed design of a model ring is made.

5. A 200 GeV c.m. DESIGN

5.1 General design

We use the methods of Ref. 2, with some modifications, to design a machine for 200 GeV in the centre of mass. The basic design equations link the beam size, the number of particles in each bunch, the machine energy, the interaction region β -function, the RF power, the bending radius, and the two-beam interaction tune shift. They are

$$\mathscr{L} = \frac{N^2 f}{4\pi b \sigma_x^* \sigma_y^*}$$
(15)

$$\Delta v_{\mathbf{y}} = \frac{N \beta^{*} \mathbf{r}_{e} \mathbf{m}_{e}}{2\pi \mathbf{b} \mathbf{E}(\sigma_{\mathbf{x}}^{*} + \sigma_{\mathbf{y}}^{*}) \sigma_{\mathbf{y}}^{*}}$$
(16)

$$\Delta v_{\mathbf{y}} = \frac{N \beta_{\mathbf{x}}^{*} \mathbf{r}_{\mathbf{e}} \mathbf{m}_{\mathbf{e}}}{2\pi \mathbf{b} \mathbf{E}(\sigma_{\mathbf{x}}^{*} + \sigma_{\mathbf{y}}^{*}) \sigma_{\mathbf{x}}^{*}}$$
(17)

$$P_{B}(MW) = 2.83 \times 10^{-20} \text{ N f } \frac{E^{4}(GeV)}{\rho(m)}$$
, (18)

where N is the total number of particles in one beam; f is the orbit frequency; b is the number of bunches into which each beam is divided; σ_x^* and σ_y^* are the horizontal and vertical standard deviations of the beam size at the interaction point, respectively; β_x^* is the horizontal β -function at the interaction region; and r_e and m_e are the classical electron radius and electron rest mass, respectively. Following Ref. 2, we rewrite these as

$$\mathscr{L} = 1.23 \times 10^{33} \frac{\Delta v_{y} P_{B}(MW) \rho(m)}{E^{3}(GeV) \beta_{y}^{*}(m)} \qquad (cm^{-2} sec^{-1})$$
(9)

$$\frac{\sigma_{\mathbf{x}}^{*}\sigma_{\mathbf{y}}^{*}}{\beta_{\mathbf{y}}^{*}} = \frac{8.08 \times 10^{2} P_{B}^{}(MW) \rho(m)}{b \Delta v_{\mathbf{y}} E^{5}(GeV) f(Hz)}$$
(cm) (19)

$$\frac{\sigma_{x}^{*2}}{\sigma_{x}^{*}} = \frac{8.08 \times 10^{2} P_{B}(MW) \rho(m)}{b \Delta v_{x} E^{5} (GeV) f(Hz)}$$
(cm) (20)

$$N = \frac{3.53 \times 10^{19} P_B(MW) \rho(m)}{E^4 (GeV) f(Hz)} .$$
 (21)

We have assumed $\sigma_x^* >> \sigma_y^*$, and set $\sigma_x^* + \sigma_y^* = \sigma_x^*$. This is always true in practical $e^+ - e^-$ machines.

As indicated earlier, there is a bound on Δv_y . Not so much is known about Δv_x , but theory indicates there should be a bound on it also, and we will assume both Δv_x and Δv_y are limited to ≤ 0.06 and use the limiting value of 0.06 in the following discussion.

Equations (9), (18), (19), and (20) relate the properties of the beams to the lattice parameters at the interaction point. In order to specify a machine design, these parameters must be related to the arcs of the machine that connect the interaction regions. To do this, we adopt a procedure different from that in Ref. 2 and assume that the magnet systems that bring the beams from the ends of the arcs to the collision points are such that the dispersion of the lattice is zero at the collision points. There are good reasons for the choice of zero-dispersion interaction regions for machines with large fractional energy losses per turn. These have to do with the excitation of synchro-betatron resonances and with the horizontal separation of the electron and positron beams at the interaction regions. The two machines now under construction (PEP and PETRA) have made different choices. PEP has interaction regions with finite dispersion, while the PETRA interaction regions have zero dispersion. The reasons for these different choices relate to the detailed design of each machine and to the placement of their respective RF systems. At their c.m. energies of 15-20 GeV per beam, either choice is satisfactory if the RF placement matches that choice. The effect of our choice of dispersion-free interaction region is to increase the required horizontal aperture of the magnets in the arcs, and in the sense that one gets a larger value of the machine aperture, this choice might be called conservative.

We will make the arcs of the machine from simple doublet cells. The entire machine will then consist of a series of doublet cells connecting long straight sections composed of quadrupoles that match the machine functions at the ends of the arcs to those at the interaction points. The straight sections are assumed to be sufficiently long to contain the RF, injection, beam-separation equipment, etc. We do not consider the design of these straight sections, for it is a straightforward process. A summary of the properties of doublet cells is given in Ref. 2.

In this kind of machine, the horizontal beam emittance at the interaction points and in the arcs is given by

$$\frac{\sigma_{x}^{\star 2}}{\beta_{x}^{\star}} = \frac{\sigma_{x}\beta^{2}}{\beta_{x}} = \frac{2R}{\nu^{3}} \left(\frac{\sigma_{E}}{E}\right)^{2}$$
(22a)

=
$$1.48 \times 10^{-4} \frac{E^2 (GeV) R}{v^3 \rho}$$
, (22b)

where $\sigma_{\rm E}$ is the energy spread in the beam, $\sigma_{\rm x\beta}$ is the betatron contribution to the beam size in the arcs, $\beta_{\rm x}$ is the β -function in the arc, R is the average radius of the arc, ν is the contribution to the tune of the arcs only, and σ and β on the left side of Eq. (22b) are in centimetres. Equation (22) gives the maximum value of the tune of the machine allowable if the horizontal incoherent two-beam limit is not to be exceeded. This maximum tune in turn gives the minimum horizontal aperture in the vacuum chamber of the arcs as well as the smallest allowed momentum compaction. Combining (22b) and (19), and using the maximum value of $\Delta\nu_{\rm x}$, we find

$$\rho_{\text{max}}^{3} = 1.10 \times 10^{-8} \frac{E'(\text{GeV}) R(\text{m}) b f(\text{Hz})}{\rho^{2}(\text{m}) P_{B}(\text{MW})}$$
 (23)

5.2 Lattice parameters

We can now set down most of the properties of the 200 GeV c.m. machine, and will use a bending radius and beam power consistent with those found in the previous section. The design parameters are as follows:

L	=	10 ³²	$(cm^{-2} sec^{-1})$
Е	=	100	(GeV each beam)
ρ	*	6.2	(km)
R	=	6.8	(km)
P _B	=	11	(MW, both beams)
Δv _x	=	$\Delta v_{\rm v} = 0.06$	
Ъ	=	4	(eight interaction regions)
f	=	6.4	(kHz)
β ູ*	=	5	(cm)
ท้	=	3.8×10^{12}	(particles, each beam)
U ₀	=	1.43	(GeV/turn synchrotron loss)
В	=	540	(gauss in bending magnets)
σ _e /ε	=	1.1×10^{-3}	
v	=	88 '	(in arcs)
v	=	80	(in arcs).

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Fixing the betatron phase shift per cell determines the horizontal beam size and the cell parameters. We give two examples:

μ	90	45	(degrees/cell)
n 🖛	320	640	(cells)
cell length	134	67	(m)
ß	228	130	(m)
β min	39	59	(m) ·
σ _v (max)	0.34	0.26	(cm)
σ (max)	0.20	0.15	(cm) (full coupling)
quadrupole			
focal length	49	44	(m)
n _{max}	1.78	1.34	(m) / ∆p/p.

In these examples, $\sigma_x(\max)$ is taken at the centre of the horizontal-focusing quadrupoles and includes both synchrotron and betatron contributions to the beam size; $\sigma_y(\max)$ is measured at the centre of the vertical-focusing quadrupoles and is the value for *full* horizontal-vertical betatron coupling. This full coupling is not required in normal operation of such a machine, but might be accidentally reached, for example, during energy changes after injection.

To complete the design, we need the aperture and β_x^* . We take the minimum full aperture (A) required in the cells to be $20\sigma + 1$ cm. Of this, 1 cm is allowed for residual orbit distributions, $\pm 6\sigma$ is required to achieve sufficient lifetime for Gaussian beams, and $\pm 4\sigma$ is assumed to be needed for the mismatch and consequent increase in β_{max} coming from the strong beam-beam interaction. The horizontal apertures given below are for zero horizontal-vertical coupling, while the vertical apertures are for 100% coupling.

 β_x^* is determined from β_y^* , v, and v_{max} , by the choice of coupling during normal colliding-beam operations. This coupling is much less than the full coupling assumed to derive the vertical aperture. The colliding-beam configuration coupling constant k need be only larger than the natural coupling in the machine. The coupling coefficient k is defined by

$$\frac{\sigma_{y}}{\beta_{y}} = k \frac{\sigma_{x}}{\beta_{x}}$$
(24)

We take k to be 17%, a value much larger than the natural coupling in typical colliding-beam machines, where it ranges from 5% to 10%. The rest of the lattice parameters are then

μ	90	45	(degrees/cell)
A x	7.8	6.2	(cm)

A	5.0	4.0	(cm) (full coupling)
k	0.17	0.17	
β *	1	- 1	(m)

5.3 Operation below peak energy

This model machine has been designed for a luminosity of 10^{32} cm⁻² sec⁻¹ at a c.m. energy of 200 GeV. With this machine one should be able to conduct physics experiments over a broad band of energies, and the machine's potential depends on both the energy dependence of the luminosity and the expected energy dependence of the cross-section of interest. The energy dependence of the cross-sections illustrated in Fig. 2 are very different. The one-photon process is proportional to E^{-2} , the Fermi weak process to E^2 , and the Weinberg weak process depends on whether one is above or below the Z^0 mass. There are many other cross-sections of interest, and the energy dependence of a good fraction of them is not in the literature. Some work is required by the theoretical community.

We can, however, discuss the expected energy dependence of the machine's luminosity. For a fixed aperture in the arcs, the luminosity \mathcal{L} of an e^+-e^- machine can be made to be

$$\mathscr{L} \propto E^2 \tag{25}$$

for operation below the design maximum energy. Several procedures are available to achieve this \mathcal{L} versus E dependence. A variable tune scheme is described in Ref. 2, and a scheme which uses "wiggler" magnets is described in the PEP Design Report³). To achieve a flatter E dependence than that given in Eq. (25), the machine aperture must be increased. This, of course, wastes space for high-energy operation. To give some idea of how much additional space is required in the aperture, we can take, as an example, 100 GeV c.m. operation with $\mathcal{L} = 10^{32}$ cm⁻² sec⁻¹. For this lower-energy operation, the horizontal full aperture increases from the 7.8 cm found for 90° phase shift per cell at 200 GeV c.m. to a value of 14 cm. The magnet configuration is of course identical, and the quadrupoles are excited to give a tune of 34 in the arcs.

We shall not go here into the general question of aperture versus E for specified \mathcal{L} versus E dependences, but we do want to make the point that the desired \mathcal{L} versus E curve is what will actually determine the aperture of the machine. It is important to study both the physics needs for high \mathcal{L} at low energy and the economic consequences of increased aperture.

Here, then, is a model machine that realizes our goal of a design for an e^+-e^- colliding-beam device that operates at c.m. energies of up to 200 GeV. In the process of designing this machine, we have not found anything obviously impossible, but there are some problem areas. In the next section, we discuss some of

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these areas and indicate where we believe further work is required before the physics community can proceed confidently with the construction of such a device. There are also significant opportunities for cost savings, and a few of these are mentioned in the discussion.

6. PROBLEMS AND OPPORTUNITIES

In this section we shall discuss some of the things omitted from the previous analysis, including a re-examination of the one parameter that might seem at first glance somewhat radical, β_v^* .

6.1 <u>Low β</u>*

The value of β_y^* of 5 cm used in the model machine poses problems both to the designers of the machine and to its users. The β -function will increase as we move from the collision point in the long straight section required for experiments toward the position of the first of the focusing elements in the machine. The value of β_y in this straight section is given by

$$\beta_{y}(D) = \beta_{y}^{*} + (D^{2}/\beta_{y}^{*}),$$
 (26)

where D is the position in the straight section measured with respect to the interaction point. If, for example, the first magnetic element of the lattice is located at a distance of 20 m from the collision point, the β -function at 20 m will be 8 km. This very large β implies extremely stringent tolerances on the magnetic field quality of the first quadrupoles and, in addition, gives large chromatic aberrations in the focusing structure that must be corrected elsewhere in the lattice. The tolerances on the magnetic field quality are probably not a problem, for with only eight interaction regions there exist 16 sets of lenses, and if necessary these can be equipped with multiple-harmonic correcting coils.

The problem of chromatic aberrations is much more serious. The aberrations cause a variation in the tune of the machine over the natural momentum spread in the beam that must not become so large as to move the tune for any particle within the beam to a resonance. In addition, the chromaticity (C \equiv the change in tune with momentum) is of the wrong sign for stability of the beams with respect to certain coherent oscillations. The sign of C must be reversed with a system of sextupoles distributed in the arcs, and the fact that C cannot be corrected where it is generated leads to restrictions on the momentum aperture of the machine. C depends on the ratio of the β -function of the quadrupoles to their focal length. The focal length of the interaction region quadrupoles is almost equal to their distance from the collision point D₀, and we find

$$C \approx k(D_0/\beta_y^*)$$
, (27)

where D_0 has been assumed to be very much greater than β_y^* , and k is approximately independent of machine design. In the PEP design, where chromaticity effects have been investigated in detail, $D_0 \approx 10$ m, and $\beta_y^* = 0.2$ m, giving C = 50 k. This value seems safe, but not by a large amount.

We must guess at D_0 for our model machine with 100 GeV beams. Existing machines from which we might scale are SPEAR ($D_0 = 2.5 \text{ m}$, for 4 GeV/c beams), PEP ($D_0 = 10 \text{ m}$ for 18 GeV/c beams), and the ISR ($D_0 = 8 \text{ m}$ for 26 GeV/c beams). It is unquestionably convenient to the experimenters to have long free spaces clear at the interaction region, and we might guess, given no other considerations, that a D_0 of 20 m might be suitable for 100 GeV beams. With this choice and our value of $\beta_y^* = 5 \text{ cm}$, C is 8 times that of the PEP lattice. It is not at all clear that a workable compensation system can be designed with such a large value of C, and this is one of the areas that needs further study.

An alternative possibility is to use very small diameter quadrupoles close to the interaction region, as indicated in Fig. 7. Such quadrupoles might be buildable with $\cos 2\theta$ current distribution⁶). A quadrupole with an outer radius of 10 cm beginning at 2.5 m from the interaction region can greatly reduce C at the cost of allowing only a short free space for the study of particles produced in the interaction at an angle less than 40 mrad. Since the e^+-e^- reactions of interest are in low angular momentum states, there will be no very strong forwardpeaking of the secondary particles, and this loss of long free space at very small angles might not be a serious problem. This is a subject that needs study by experimental physicists.

6.2 Tolerances

The standard tolerance calculations compute the effect of a randomly distributed set of field, position, and gradient errors on energy distributions and tune errors. These calculations compute the probability that the orbit or tune will lie within some allowed deviation with some specified probability. These calculations are extremely conservative in that they take no account of the difference between short- and long-range tolerances. They indicate that for a tune error of 0.01, the quadrupole gradients in the arcs must be as specified to a few parts in 10⁴. For orbit distortions of the order of 0.5 cm, the bending magnet fields must be accurate to a few parts in 10⁴, and the quadrupole positions must be accurate to about 0.1 mm. The bending field and gradient tolerances can be met with some difficulty, but the quadrupole position tolerances cannot be met in a machine of 40-50 km circumference. Good position tolerances can be achieved, however, over short sections of the circumference, and these coupled with steering coils appropriately distributed, can keep the beam centred in the machine aperture. The probability of the beam's making the first tune on the first try will be negligibly small, and the machine will have to be equipped with sensors to allow the measurement of the position and angle of the first injected pulse at many points on the azimuth so that the necessary initial set-up ot the steering coils can be made.

6.3 - Injection and site

Injection into a storage ring with a maximum beam energy of 100 GeV will probably have to be done at an energy no lower than about 20 GeV. Single-beam and coupled-beam instabilities are much more serious at low than at high energies, and low-energy injection schemes have given trouble at the existing high-luminosity storage rings, SPEAR and DORIS.

There is at present only one electron accelerator that can serve as an injector at these kinds of energies, the SLAC linac, but the SLAC site cannot accommodate a ring of the size required for 100 GeV beams. However, a 20 GeV synchrotron can be built at a very small fraction of the cost estimated for this project, and thus the availability of an injector should not be a determining influence on the choice of site. The design of a 20 GeV synchrotron is not a difficult problem, and will not be discussed further.

Since electron storage rings require considerably less shielding than proton machines, a flat site that does not require either tunnels or deep cuts may be advantage. It might be possible to reduce housing costs greatly by building on a flat terrain and covering the machine with about one metre of concrete or slightly more earth.

6.4 More on RF

There are many questions here that need further analysis. A few are listed.

6.4.1 Azimuthal distribution of RF

The synchrotron radiation losses around the arcs of the machine are continuous but the number of RF stations is finite. Thus the central energy of the circulating beam fluctuates coherently about the design energy and is larger than design on leaving an accelerating station and less than design on entering one. The electron and positron beams will coincide at the collision points in spite of this effect, for we have designed a lattice with zero dispersion in the interaction regions.

If the RF accelerating sections are placed in long straight sections equally spaced around the machine, the distance of the beam from the central orbit (Δx) between accelerating stations is given by

$$\Delta x \approx \pm \frac{\eta U_0}{2 n_A E_0} \left(1 - \frac{2 D_0}{\Delta} \right), \qquad (28)$$

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where η is the momentum dispersion, D is the position along the orbit measured between accelerating stations, and Δ is the separation between accelerating stations. The + (-) sign is for the e^- (e^+) beam for example, and n_A is the number of accelerating stations. For our model machine at 100 GeV, Δx in the arcs immedimetely before or after an accelerating station is

$$\Delta x = \pm 1.3/n_{A} \text{ cm}$$
(29)

for the machine with 90° phase shift per cell. With eight symmetrically placed interaction regions, it is probably desirable to have at least four accelerating sections, implying Δx of about ±3 mm maximum in the arcs. For comparison, in the PEP design, Δx is about ±0.5 mm. The effect of these systematic deviations of the orbit from the central orbit needs further investigation: they might excite synchro-betatron resonances.

6.4.2 Higher-mode losses

A beam with very large beam currents, as in this design, can lose energy by exciting various high-frequency electromagnetic modes in the vacuum chamber and in the guide field, in addition to the normal synchrotron radiation. These highermode losses require larger RF voltages and more RF power. These effects have not been estimated here, and do need calculation.

6.4.3 Exotic RF

The prototype machine uses a great deal of power to heat the walls of its conventional RF cavities. Two techniques might save power or money or both: superconducting RF cavities and pulsed RF systems. The costs for cavity systems plus 10-year cavity power for cases (A) and (C) of Section 4 are about \$400 million and \$200 million, respectively. If using superconducting or pulsed techniques can save a significant percentage of these funds, they would be of great benefit.

Superconducting RF systems have been discussed and worked on for many years, but there is to date no reliably operating system of any significant size. What is clear from the work done so far, is that at the frequencies of greatest interest to the circular-accelerator builders (100 to 1000 MHz), the maximum voltage gradient attainable in a superconducting structure seems to be limited by some fundamental surface phenomena in the cavities. The maximum voltage gradient attained reliably at HEPL is about 1.5 to 2 MeV/m at a frequency of about 1000 MHz ⁷). While large voltage gradients have been obtained at higher frequencies, the aperture allowed in higher frequency cavities is generally too small for circular machines. Even if a voltage gradient can be made as large at a few hundred MHz as has been obtained at 1 GHz, 1-2 km of cavity will be required. If a complete cryogenic RF system -- cavities, dewars, tuners, refrigeration, He distribution, etc. -- can be produced at a cost of the order of a few $\times 10^7$ dollars per km, it would be of great interest. Besides costs, there are several beam dynamics questions that must be answered in connection with superconducting RF systems. The cavities will have natural Q's of the order of 10° for unwanted modes as well as for their designed fundamental mode. These modes can be excited by the beam, which is capable of driving all harmonics of the orbit frequency up to something around 100 GHz. The effect on beam stability needs investigation, and a method will probably be required to lower the Q for these modes while preserving the high Q for the accelerating frequency. Both beam dynamics and engineering considerations need investigation before such a system can be constructed. A vigorous programme of hardware research and development and theoretical investigation must be pursued to decide the feasibility of such systems and to get some idea of the costs. Such a programme will require many years of effort.

Pulsed RF systems are a possibility that might be worth some work. The orbit frequency of the machine design in this paper is only 6 kHz, and with four equally spaced bunches in the machine, the bunch frequency near the interaction regions is only \sim 24 kHz. It might be possible to design a lower-duty cycle RF system that significantly cuts total power demand.

6.5 Energy use

The energy consumption of our machine is very large. With conventional RF systems, the total energy input for RF alone ranges from 45 to 90 MW. We have tried to treat energy as an economic entity by including the 10-years operating costs for energy in the optimization of the machine parameters. We have used prices higher than those currently charged for energy, but even with this price, energy costs might not be a true reflection of its value to society. The high-energy physics community would do well to consider ways to minimize energy consumption or to re-use the energy consumed in machine operations.

If our e^+-e^- machine is considered as an energy converter, it produces two products: i) the degraded energy that comes out as heat in the cavity walls, magnets, etc.; ii) a more ordered form of energy, the 10 MW of X-rays with an energy of about 0.25 MeV emitted by the beam. Both of these products are valuable resources, both can be used, and we should consider these possible uses in the design and sitting of the machine.

The use of degraded heat energy is the simplest and most obvious. In most accelerators, the maximum temperature of the cooling water at the outlet is usually designed to be about 50° C. This is low for effective secondary use of the heat, but there is no reason why cooling water outlet temperatures cannot be raised to the 85° - 95° C region with ease, or even higher with difficulty. At temperatures of 85° - 95° C, there are many uses for this thermal energy. Three of the most obvious are: i) heating and cooling of residential, industrial, and commercial

buildings; ii) drying of grain after harvest; and iii) heating of greenhouses. Many others will undoubtedly occur to the reader. Since the machine energy input is electrical, we can in principle recover a fraction of the primary fuel used to generate the electricity we consume that depends on the ratio of electrical generating efficiency to the efficiency of conversion of primary fuel to low-grade heat.

The second form of energy, the synchrotron X-rays, might be of even more value as a resource when used as X-rays rather than as another source of low-grade heat. Two examples of applications that require large amounts of X-rays are sterilization and materials modification. For example, there has been discussion in the past year of the creation of a world-wide "grain bank". Such a project *might* be economically more attractive if the grain can first be sterilized by X-ray bombardment.

There has been some work recently on greatly strengthening plastics by X-ray bombardment during polymerization. If such a material is useful, the high-energy e^+-e^- machine might be an ideal source of X-rays.

In either case (both of which might be wishful thinking), our "product", X-rays, is worth far more than the energy consumed in producing it. There might be other such uses, and such applications should be considered before making a final design and before choosing a site.

7. CONCLUSIONS

An e^+-e^- storage ring in the range of a few hundred GeV in the centre of mass can be built with present technology. There are a few questions of accelerator physics to be resolved, and a very important question of the free space required for experiments. With more detailed work on unit costs, the total cost of $e^+-e^$ rings can be defined well enough to allow a comparison with other projects that have been discussed as the next step in high-energy physics machines. It is likely that the e^+-e^- costs will be at the low end of the range found in this paper, and in that case the e^+-e^- machine at 200 GeV c.m. would seem to be the most useful project on the horizon.

Acknowledgements

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REFERENCES AND FOOTNOTES

- The other machines involve the collision of an electron, neutrino, or proton with another proton (in e-p colliding beams, stationary-target machines, or p-p colliding beams). In our present picture of proton structure, the proton is composed of constituents, and the basic collisions occur between these constituents and the electrons, neutrinos, or other proton constituents. This gives an effective c.m. energy for the basic collisions considerably lower than that calculated considering the proton as a whole.
- 2) J. Rees and B. Richter, SLAC Report No. PEP-69 or SPEAR-167, October 1973.
- 3) PEP Design Report, SLAC, 1976.
- 4) Certain items in the budget of Ref. 3 have been combined to arrive at the unit costs used here. The responsibility for any omissions or misinterpretation is the author's.
- 5) C. Zettler (private communication) reports the complete cost per unit length for the CERN SPS RF-cavity system to be about 30×10^6 /km.
- 6) The possibility of using small-diameter distributed-coil quadrupoles was suggested by A. Hofman, CERN.
- M.S. McAshan et al., Proc. 9th Internat. Conf. on High-Energy Accelerators, Stanford, 1974.

Table 1						
Parameters	for	the	zero-order	high-energy	e ⁺ -e ⁻	machines

Р_В е***** U o ρ (2 beams) (m) (MW) (MeV) 100 1030 4.1×2 507 150 7.5×2 1900 1400

200

÷

2930

Table 2

11.6 \times 2

2870

Cavity length for cavity power dissipation equal to synchrotron power required by the beams

c.m. energy	L	
(GeV)	(m)	
100	2260	
150	9410	
200	25600	

-

Τ	a	b	1	е	3

Unit cost in 1975 dollars of systems derived from the PEP cost estimates. Engineering and design, and contingency are included at 20% each

System	Cost
	(\$ million)
Main Ring	
* Magnets, supports, installation, cooling	2.9 per km
Housing	4.6
Vacuum system	2.2
Instrumentation and control (excluding computer)	1.7
Misc.	1.4
	12.8 per km
RF Power	
Klystrons, waveguide, control	0.43 per MW
Cooling water, a.c. connections, misc.	0.15
	0.58 per MW
RF cavity	81 per km
<u>a.c. Power</u> (at \$0.03/kWh)**	0.18 per MW-year

- *) This item is lower by a factor of 2 than the PEP cost, since magnets for this machine are much lower-field than PEP's.
- **) The power cost used is typical of CERN and Brookhaven, not of SLAC. I have also assumed a system on time of 6000 hours/year.

i

Figure captions

- Fig. 1 : Feynman diagrams for µ-pair production by one-photon exchange and by the weak neutral current.
- Fig. 2 : μ -pair production counting rate *versus* c.m. energy for a machine with $\mathscr{L} = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. The curves are explained in the text.
- Fig. 3 : Feynman diagrams for Bhabha scattering via one-photon exchange and via the weak neutral current.
- Fig. 4 : Feynman diagrams for hadron production via one-photon exchange and via the weak neutral current.
- Fig. 5 : Two of the many higher-order diagrams for lepton production by the weak interactions.
- Fig. 6 : The radius *versus* c.m. energy for an optimized machine using the costs of Case B.
- Fig. 7 : Schematic diagram of a distributed quadrupole to reduce the peak value of $\beta_{\rm v}.$



Fig. 1

i

μ-



Fig. 2







Fig. 3





Fig. 4







Fig. 6



Interaction point

i

