Leptons as a probe of hadronic structure*<br>Frederick J. Gilman<br>Stanford Linear Accelerator Center

## I. Introduction

In these lectures we shall discuss some of the theoretical ideas involved in electroproduction, neutrino production, and electron-positron annihilation into hadrons. In doing so, we unavoidably will be studying simultaneously both the short distance behavior of products of currents and hadron structure.

Surely, the subjects under discussion here, like the quark parton model, scaling, and (quark) light cone algebra, are familiar to many these lectures will reach. A number of items which are explicilty discussed in some detail are therefore of a reference nature. However, I hope that some of the topics are not so familiar and that they are treated in sufficient detail to permit an easy grasp of forthcoming data. Also, as much as possible deep inelastic $e N, \mu N, \nu N$, and $\bar{N} N$ scattering, as well as $e^{+} e^{-}$annihilation are treated together, with emphasis on what kinds of behavior follow from the same theoretical assumptions for these different processes.

## II. Some Basics

For inelastic $e \mathbb{N}, \mu \mathbb{N}, \nu \mathbb{N}$, or $\mathrm{V} N$ scattering the assumption of a current-current form of the interation of leptons and hadrons results

[^0]in the laboratory double differential cross section ${ }^{1)}$
\[

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega^{\prime} d E^{+}} \propto L_{\mu \nu} W_{\mu \nu} \tag{I}
\end{equation*}
$$

\]

where $I_{\mu \nu}$ arises from a trace of lepton associated Dirac matrices and $W_{\mu \nu}$ arises from the hadronic matrix elements of the weak or electromagnetic current. In the particular case of $e N \rightarrow e+$ anything or $\mu \mathbb{N}+\mu+$ anything $^{2)}$ :

$$
\begin{align*}
W_{\mu \nu}= & \left|\frac{1}{2 \pi e^{2}}\right| \sum_{\text {spin }} \int d^{4} x e^{-i q x}\langle p|\left[J_{\mu}(x), J_{\nu}(0)\right]|p\rangle \\
= & W_{1}\left(\nu, q^{2}\right)\left(\delta_{\mu \nu}-q_{\mu} q_{\nu} / q^{2}\right)  \tag{2}\\
& +W_{2}\left(\nu, q^{2}\right)\left(p_{\mu}-p \cdot q q_{\mu} / q^{2}\right)\left(p_{\nu}-p \cdot q q_{\nu} / q^{2}\right) / M_{\mathbb{N}}^{2}
\end{align*}
$$

with $v=-p \cdot q / M_{N N}$ and $q^{2}$ the four-momentum squared carried by the virtual photon.

Defining the quantities

$$
\begin{equation*}
x=\frac{1}{\omega}=-\frac{q^{2}}{2 p \cdot q}=+\frac{q^{2}}{2 M v} \tag{3}
\end{equation*}
$$

and

$$
y=\frac{v}{E}=\frac{p \cdot q}{p \cdot k_{\text {incident }}}
$$

and neglecting terms of order $M / V$ and $M / E$, the cross section may be rewritten in terms of scaled variables,

$$
\begin{equation*}
\frac{d^{2} \sigma}{d x d y}=\left(\frac{4 \pi \alpha^{2}}{q^{4}}\right)\left(2 M_{N} E\right)\left[(1-y) v W_{2}+\left(\frac{1}{2} y^{2}\right)\left(2 M_{N} x\right) W_{1}\right] \tag{4}
\end{equation*}
$$

For neutrinos (and antineutrinos) we have an additional structure function, $W_{3}\left(\nu, q^{2}\right)$, involving one vector and one axial-vector current.

In scaled variable form, with the same approximations as before,

$$
\begin{equation*}
\frac{d^{2} \sigma\left(\frac{v}{v}\right)}{d x d y}=\left(\frac{G^{2} M_{\mathbb{N}} E}{\pi}\right)\left[(1-y) \nu W_{2}+\left(\frac{1}{2} y^{2}\right) 2 M_{N} x W_{1} \mp y\left(1-\frac{1}{2} y\right) x v W_{3}\right] . \tag{5}
\end{equation*}
$$

- Recall ${ }^{1)}$ also that instead of the structure functions $W_{1}, W_{2}$, $W_{3}$ one can define positive semi-definite cross sections; e.g., in electroproduction

$$
\begin{align*}
& W_{1}=\frac{K}{4 \pi^{2} \alpha} \sigma_{T}  \tag{6}\\
& W_{2}=\frac{K}{4 \pi^{2} \alpha} \frac{q^{2}}{q^{2}+v^{2}}\left(\sigma_{T}+\sigma_{L}\right),
\end{align*}
$$

where $K=v-q^{2} / 2 M_{M_{N}}$, and the transverse and longitudinal cross sections $\sigma_{T}$ and $\sigma_{L}$ again depend on $q^{2}$ and $v$.

If we consider the process $e^{+} e^{-} \rightarrow$ hadron( $p$ ) + anything, then formally much is the same: the differential cross section is proportional to a contraction of a leptonic and hadronic tensor ${ }^{3)}, \bar{W}_{\mu \nu}$, with

$$
\begin{align*}
\bar{W}_{\mu \nu} & \left.\left.=\left(\frac{1}{2 \cdots e^{2}}\right) \sum_{\text {spin }} \int d^{4} x e^{-i q x}\langle 0| J_{\mu}(x) \right\rvert\, p+\text { any }\right\rangle \cdot\langle p+\text { any }| J_{\nu}(0)|0\rangle\left(\frac{E_{h}}{M_{h}}\right)  \tag{7}\\
& =\bar{w}_{1}\left(\nu, q^{2}\right)\left(\delta_{\mu \nu}-q_{\mu} q_{\nu} / q^{2}\right)+\bar{w}_{2}\left(\nu, q^{2}\right)\left(p_{\mu}-p \cdot q q_{\mu} / q^{2}\right)\left(p_{\nu}-p \cdot q q_{\nu} / q^{2}\right) / w_{h}^{2}
\end{align*}
$$

Again, $v=-p \cdot q / M_{h} \geq 0$ if $M_{h}$ is the hadron mass. We will use $Q^{2}=-q^{2}>0$ in the time-like region. The scaling variable is

$$
\begin{equation*}
\bar{\omega}=\frac{2 p \cdot q}{q^{2}}=\frac{2 E_{\text {hadron }}}{\sqrt{Q^{2}}}=\frac{E_{\text {hadron }}}{E_{\text {beam }}} . \tag{8}
\end{equation*}
$$

Recall that $1 \leq \omega<\infty$ for $\omega$ in deep inelastic scattering, while here

$$
\begin{equation*}
0 \leq \frac{2 M_{h}}{\sqrt{Q^{2}}} \leq \bar{\omega} \leq 1 \tag{9}
\end{equation*}
$$

In terms of this scaled energy variable, the differential cross section for $e^{+} e^{-} \rightarrow$ hadron $(p)+$ anything is 3 )

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega} \frac{d \bar{\omega}}{}=\frac{3}{4 \pi} \sigma_{\mu \mu} \beta \bar{\omega}\left[M_{h} \bar{W}_{1}+\frac{\beta^{2} \bar{\omega}}{2} v \bar{W}_{2} \sin ^{2} \theta \sin ^{2} \phi\right] \tag{10}
\end{equation*}
$$

where $\beta=\left|\vec{p}_{h}\right| / F_{h}=\left(I-M^{2} / F_{h}^{2}\right)^{I / 2}, M_{h} \nu=-p_{h} \cdot q, \sigma_{\mu \mu}=4 \pi \alpha^{2} / 3 Q^{2}$ is the (point) cross section for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$, and $\theta, \phi$ are polar and azimuthal angles with respect to the beam and plane of the $e^{+} e^{-}$storage ring, respectively. Equation (10) applies to the case of $100 \%$ polarized (perpendicular to the plane of the ring) beams ${ }^{4}$ ). The unpolarized beams case can be obtained by replacing $\sin ^{2} \phi$ by $1 / 2$. Integrating over angles and neg.lecting $M / E_{h}$ terms

$$
\begin{equation*}
\frac{d \sigma}{d \bar{\omega}}=3 \sigma_{\mu \mu} \bar{\omega}\left[M_{h} \bar{W}_{1}\left(\bar{\omega}, Q^{2}\right)+\left(\frac{\bar{\omega}}{\sigma}\right) \nu \bar{W}_{2}\left(\bar{\omega}, Q^{2}\right)\right] \tag{II}
\end{equation*}
$$

Note that $V \bar{W}_{2}$ is not necessarily positive, as was $V W_{2}$ in electroproduction (see Eq. (6)). To see why this is the case, let us go back and derive $d^{2} \sigma / d \Omega$ $\alpha \bar{\omega}$ in terms of positive (semidefinite) cross sections for an arbitrary degree of beam polarization (perpendicular to the plane of the storage ring).

Consider a virtual photon decaying to a hadron, $h$, plus anything. There are three amplitudes, $g_{\lambda}$, for $\lambda=1,0,-1$, labelled by the net helicity of the final state along $\overrightarrow{\mathrm{p}}_{\mathrm{h}}$. By parity $\left|g_{+1}\right|=\left|g_{-1}\right|$. If $\beta$ is the angle between $\overrightarrow{\mathrm{p}}_{\mathrm{h}}$ and the polarization vector, $\vec{\epsilon}$, of the virtual photon, then the angular distribution of $h$ is

$$
\begin{align*}
\frac{d \sigma}{d \Omega} & \propto\left|g_{+1}\right|^{2}\left|\alpha_{10}^{1}(\beta)\right|^{2}+\left|g_{0}\right|^{2}\left|g_{0}\right|^{2}\left|a_{00}^{1}(\beta)\right|^{2}+\left|g_{-1}\right|^{2}\left|d_{-10}^{1}(\beta)\right|^{2}  \tag{12}\\
& =\left|g_{1}\right|^{2} \sin ^{2} \beta+\left|g_{0}\right|^{2} \cos ^{2} \beta .
\end{align*}
$$

(Recall that the helicity of the virtual photon is zero along the direction of $\vec{\epsilon}$.) With $100 \%$ polarized electron and positron beams, the virtual photon is formed in a linear polarization state, with $\vec{\epsilon}$ perpendicular to the plane of the ring. Let $\theta$ and $\phi$ be the polar and azimuthal angles of $\overrightarrow{\mathrm{p}}_{\mathrm{h}}$ relative to the beam (as z-axis), and the plane of the ring (as $\mathrm{x}-\mathrm{z}$ plane). Then $\cos \beta=\sin \theta \sin \phi$, and in these new coordinates,

$$
\begin{equation*}
\frac{d \sigma}{d \Omega} \propto\left|g_{+1}\right|^{2}+\left(\left|g_{0}\right|^{2}-\left|g_{+1}\right|^{2}\right) \sin ^{2} \theta \sin ^{2} \phi \tag{13}
\end{equation*}
$$

Now let us define

$$
\begin{aligned}
& \sigma_{I}\left(E_{h}, Q^{2}\right) \propto\left|g_{0}\right|^{2} \\
& \sigma_{T}\left(E_{h}, Q^{2}\right) \propto\left|g_{I}\right|^{2}
\end{aligned}
$$

which are manifestly positive, so that for $100 \%$ polarized beams

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{\text {pol }}=\sigma_{T}-\left(\sigma_{T}-\sigma_{L}\right) \sin ^{2} \theta \sin ^{2} \phi \tag{14}
\end{equation*}
$$

For unpolarized beams, $\left\langle\sin ^{2} \phi\right\rangle=1 / 2$, and

$$
\begin{equation*}
\left(\frac{\partial \sigma}{d \Omega}\right)_{\text {unpol }}=\sigma_{T}-\left(\sigma_{T}-\sigma\right) \frac{1}{2} \sin ^{2} \theta \tag{15}
\end{equation*}
$$

In general, if the polarization of each beam is $P(0 \leq P \leq 1)$, then

$$
\begin{align*}
\frac{d \sigma}{d \Omega} & =\left(1-P^{2}\right)\left(\frac{d \sigma}{d \Omega}\right)_{\text {unpol }}+P^{2}\left(\frac{d \sigma}{d \Omega}\right)  \tag{16}\\
& =\frac{1}{2}\left(\sigma_{T}+\sigma_{L}\right)\left\{I+\frac{\sigma_{T}-\sigma_{L}}{\sigma_{T}+\sigma_{L}} \cos ^{2} \theta+P^{2} \frac{\sigma_{T}-\sigma_{L}}{\sigma_{T}+\sigma_{L}} \sin ^{2} \theta \cos 2 \phi\right\}
\end{align*}
$$

where $\sigma_{T}$ and $\sigma_{L}$ depend in general on $Q^{2}$ and $E_{h}$. In the case where $P=0$, the angular distribution is $I+\alpha \cos ^{2} \theta$ with

$$
\begin{equation*}
-I \leq \alpha=\frac{\sigma_{T}-\sigma_{I}}{\sigma_{T}+\sigma_{I}} \leq+1, \tag{17}
\end{equation*}
$$

an old result.
Note that in the sense that the $\theta$ distribution determines $\sigma_{T}$ and $\sigma_{L}$ when $P=0$, the $\cos 2 \emptyset$ dependence induced by polarized beams tells us nothing new! In practice, because of complete SPEAR detector acceptance in $\emptyset$ and not $\theta$, it is much easier to separate $\sigma_{T}$ and $\sigma_{I}$ using polarized beams. If $P^{2}$ is known (say from $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$) the coefficient of $\cos 2 \emptyset$ has a unique $\theta$ dependence and its coefficient determines. $\left(\sigma_{T}-\sigma_{L}\right) /\left(\sigma_{T}+\sigma_{I}\right)$.

The connection to the structure functions $\bar{W}_{1}$ and $\bar{W}_{2}$ is easily made. It is

$$
\begin{align*}
& \bar{W}_{I} \propto \sigma_{T}  \tag{18}\\
& \bar{W}_{2} \propto \frac{Q^{2}}{Q^{2}-v^{2}}\left(\sigma_{T}-\sigma_{L}\right)
\end{align*}
$$

This should be compared with Eqs. (6) relating $W_{1}$ and $W_{2}$ to $\sigma_{T}$ and $\sigma_{L}$ in deep inelastic scattering. The sign in front of the positive quantity $\sigma_{L}$, has changed because the longitudinal "cross section,"
contains a hidden factor of $q^{2}$ (it must vanish at $q^{2}=0$ ), and hence changes sign on going from space-like to time-like. If we note that

$$
\begin{equation*}
\frac{Q^{2}}{\left(Q^{2}-v^{2}\right)}=-\frac{1}{\beta^{2}} \frac{M_{h}^{2}}{E_{h}^{2}} \tag{19a}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{\omega} v=\frac{2 \bar{E}_{h}^{2}}{M_{h}} \tag{19b}
\end{equation*}
$$

then we see that for $100 \%$ polarized beams,

$$
\begin{align*}
\frac{d \sigma}{d \Omega} & \propto M \bar{W}_{I}+\beta^{2}\left(\frac{\bar{\omega}}{2}\right) V \bar{W}_{2} \sin ^{2} \theta \sin ^{2} \phi  \tag{20}\\
& \propto M \sigma_{T}+\frac{I}{2} \beta^{2}\left(\frac{2 E^{2}}{M}\right)\left(-\frac{M^{2}}{\beta^{2} E^{2}}\right)\left(\sigma_{T}-\sigma_{L}\right) \sin ^{2} \theta \sin ^{2} \phi \\
& \propto \sigma_{T}-\left(\sigma_{T}-\sigma_{L}\right) \sin ^{2} \theta \sin ^{2} \phi,
\end{align*}
$$

which agrees exactly with Eq. (14). Therefore, all the extra factors in front of $\vec{W}_{1}$ and $\vec{W}_{2}$ in the angular distribution just serve to eliminate the factors connecting them to $\sigma_{T}$ and $\sigma_{I}$.

A formula completely analogous to Eq. (16) holds for the angular distribution of any three-vector quantity characteristic of the final state. In particular, the same formula holds for a jet axis. Of course the corresponding $\sigma_{T}$ and $\sigma_{I}$ then have no $E_{h}$ dependence, as they characterize the whole jet. The same remark holds for any particular final state. For example, $\pi \pi$ has only $\sigma_{I} \neq 0$, $\pi \rho$ has only $\sigma_{T} \neq 0$, while $\pi A_{I}$ has both $\sigma_{\mathrm{L}}$ and $\sigma_{\mathrm{T}} \neq 0$ in general. Of more current interest, $\gamma+0^{-}$or $\gamma+0^{+}$ has only $\sigma_{I^{\prime}}$ and hence the ganma distribution is $I+\cos ^{2} \theta$ (with unpolarized beams). On the other hand, electric dipole transitions to $r+I^{+}$ and $r+2^{+}$states have $\sigma_{I} / \sigma_{T}=2$ and $6 / 7$, respectively, so that their angular distributions are $1-\frac{1}{3} \cos ^{2} \theta$ and $1+\frac{1}{13} \cos ^{2} \theta$.
III. Currents at Short Distances

For deep inelastic electron or neutrino scattering as $v, q^{2} \rightarrow \infty$ or for the total cross section in electron-positron anninilation as $Q^{2} \rightarrow \infty$, the region of configuration space $x^{2} \simeq 0$ dominates ${ }^{5)}$ in

$$
\int a^{4} x e^{-i q \cdot x}\langle p|\left[J_{\mu}(x), J_{v}(0)\right]|p\rangle
$$

and

$$
\int d^{4} x e^{-i q \cdot x}\langle 0| J_{\mu}(x), J_{v}(0)|0\rangle
$$

respectively. It becomes relevant to consider the Wilson expansion for a product of currents at short distances. ${ }^{6)}$ :

$$
\begin{equation*}
\left[J_{\mu}(x), J_{\nu}(0)\right]=\sum_{n=0}^{\infty} S_{n}(x) \mathscr{O}_{\mu \nu \mu_{1}} \cdots_{\mu_{2 n}}(0) x_{\mu_{1}} \cdots x_{\mu_{2 n}}+\text { other terms } \tag{20}
\end{equation*}
$$

Here $S_{n}(x)$ are singular $C$-number functions and $\mathscr{O}_{\mu \nu} \mu_{I} \cdots \mu_{2 n}$ (0) are operators. Only the leading term contributing to $\nu W_{2}$ has been made explicit in Eq. (20) and internal symmetry indices have been dropped. Taking the spin averaged hadronic matrix element of Eq. (20) as $q^{2} \rightarrow \infty$ and putting it back in the expression for $W_{\mu \nu}$ in Eq. (2), one finds that the result can be expressed in terms of a series of moments:

$$
\begin{align*}
M_{n}\left(q^{2}\right) & =\int_{1}^{\infty} \frac{d \omega}{\omega^{2 n+2}} \nu w_{2}\left(v, q^{2}\right)  \tag{2I}\\
& =\int_{0}^{1} d x x^{2 n} \nu w_{2}\left(x, q^{2}\right)=c_{n} \tilde{S}_{n}\left(q^{2}\right)
\end{align*}
$$

for $n=0, l, 2, \ldots$, where $\tilde{S}_{n}\left(q^{2}\right)$ is related to the Fourier transform of $S_{n}(x)$ and the $C_{n}$ are constants proportional to the matrix elements of $\mathscr{O}_{\mu \nu \mu_{1}} \cdots_{\mu_{2 n}}(0)$.

The behavior of $\nu W_{2}$ as $q^{2} \rightarrow \infty$ then depends on that of the $\tilde{S}_{n}\left(q^{2}\right)$. This is true for all hadron targets since the $\tilde{S}_{n}\left(q^{2}\right)$ just depend on the behavior of the currents at short distances. The $C_{n}$ 's, on the other hand, are different for each hadron and give us information on hadronic structure.

As $q^{2} \rightarrow \infty$, some favorite behaviors of the moments are:
A. $M_{n}\left(q^{2}\right) \rightarrow C_{n}\left(\mu^{2} / q^{2}\right)^{d}{ }_{n}$ where the $d_{n}$ are called anomalous dimensions. One can show that $d_{n+1} \geq d_{n}$. Conventionally ${ }^{7}$ ) $d_{0}=0$, since a candidate for the corresponding operator, $0_{\mu \nu}$, is the energy momentum tensor which has canonical dimension (zero anomalous dimension).
B. $M_{n}\left(q^{2}\right) \rightarrow C_{n}\left[\frac{1}{\ln \left(q^{2} / \mu^{2}\right)}\right]^{A_{n}}$. This behavior is that deduced in asymptotically free gauge theories ${ }^{8)}$ of the strong interactions in which the coupling constant vanishes logarithmically as $q^{2} \rightarrow \infty$. The $A_{n}$ are known once the gauge group and fermion representation is chosen.
B. $\tilde{S}_{n}\left(q^{2}\right) \rightarrow$ finite, non-zero constants. This is the behavior of free field theory. It implies that ${ }^{2} W_{2}\left(\nu, q^{2}\right) \rightarrow W_{2}(x)$, i.e., Bjorken scaling ${ }^{9)}$.

To gain further results, we abstract the leading light cone ( $\mathrm{x}^{2}=0$ ) singularity from the free quark model ${ }^{5,10)}$ :

$$
\begin{aligned}
& {\left[V_{\mu}^{\alpha}(x), V_{v}^{\beta}(0)\right] \underset{x^{2}=0}{\sim}\left\{\frac{1}{4 \pi} \partial_{\lambda}\left[\epsilon\left(x_{0}\right) \delta\left(x^{2}\right)\right]\right\}} \\
& \cdot\left\{i f ^ { \alpha \beta \gamma } \left[\left(V_{\nu}^{\gamma}(x, 0)+V_{\nu}^{\gamma}(0, x) \delta_{\mu \lambda}+\left(V_{\mu}^{\gamma}(x, 0)+V_{\mu}^{\gamma}(0, x)\right) \delta_{\nu \lambda}\right.\right.\right. \\
& \left.-\left(V_{\lambda}^{\gamma}(x, 0)+V_{\lambda}^{\gamma}(0, x)\right) \delta_{\mu \nu}+i \epsilon_{\mu \nu \lambda \sigma}\left(A_{\sigma}^{\gamma}(x, 0)-A_{\sigma}^{\gamma}(0, x)\right)\right] \\
& +\alpha^{\alpha \beta \gamma}\left[\left(V_{V}^{\gamma}(x, 0)-V_{V}^{\gamma}(0, x)\right) \delta_{\mu \lambda}+\left(V_{\mu}^{\gamma}(x, 0)-V_{\mu}^{\gamma}(0, x)\right) \delta_{\nu \lambda}\right. \\
& \left.\left.-\left(V_{\lambda}^{\gamma}(x, 0)-V_{\lambda}^{\gamma}(0, x)\right) \delta_{\mu \nu}-i \epsilon_{\mu \nu \rho \sigma}\left(A_{\sigma}^{\gamma}(x, 0)+A_{\sigma}^{\gamma}(0, x)\right)\right]\right\} \quad .
\end{aligned}
$$

where $V_{\mu}^{\alpha}(x)$ and $A_{\mu}^{\alpha}(x)$ are vector and axial-vector currents with $\operatorname{SU}(3)$ index $\alpha$, and $V_{\mu}^{\alpha}(x, 0)$ and $A_{\mu}^{\alpha}(x, 0)$ are bilocal operators. Although Eq. (22) is for the commutator of two vector currents, similar expressions hold for two axial-vector currents or a vector and axial-vector current 5,10 ). The particular case of electron (or muon) scattering is realized with the identification

$$
\begin{equation*}
J_{\mu}^{\mathrm{em}}=e\left[V_{\mu}^{(3)}+(1 / \sqrt{3}) V_{\mu}^{(8)}\right] \tag{23}
\end{equation*}
$$

If the bilocal operators are expanded in a Taylor series in $x_{\mu}$, we recognize the coefficients as essentially the operators $0_{\mu \nu \mu_{]}} \cdots \mu_{2 n}(0)$ in the Wilson expansion, Eq. (20). The last term in Eq. (22) contributes to deep inelastic scattering of polarized electrons on a polarized target.

An examination of the quark light cone algebra as is partly displayed in Eq. (22), shows that it displays the following features:
(1) The singularity is a c-number characteristic of free field theory. Hence it yields scaling: $W_{1}, ~ \nu W_{2}$ and $\nu W_{3}$ should scale in deep inelastic. electron and neutrino scattering. Vector and axial-vector currents contribute equally to $W_{1}$ and $\nu W_{2}$. When applied to $e^{+} e^{-} \rightarrow$ hadrons, it predicts

$$
\frac{\sigma_{\text {total }}\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}=R\left(Q^{2}\right)=a \text { constant } .
$$

(?) The (Lorentz) tensor indices, due to the spin $1 / 2$ of the quarks, yield

$$
\begin{equation*}
(2 M x) W_{1}=\left(\frac{2 M}{\omega}\right) W_{1}=\nu W_{2}, \tag{24}
\end{equation*}
$$

i.e., $\sigma_{\mathrm{L}} / \sigma_{T} \rightarrow 0$, in deep inelastic electron and neutrino scattering.
(3) The $S U(3)$ indices on the $f^{\alpha \beta \gamma}$ and $d^{\alpha \beta \gamma}$ relate different processes involving electrons and neutrinos, as well as yield bounds like ${ }^{\text {ll) }}$

$$
\begin{equation*}
\frac{1}{4} \leq \frac{e n}{e p} \leq 4 \tag{25}
\end{equation*}
$$

(4) Matrix elements of the various bilocal operators determine the shape, as a function of $\omega$, of $W_{1}, \nu W_{2}$ and $\nu W_{3}$. Again, this is where all the hadronic structure information resides.
IV. The Quark Parton Representation

The quark light cone algebra in the last section has what is, at the very least, a very convenient representation in terms of quark partons ${ }^{12,13}$. In its usual derivation, one starts without mention of the light cone and short distance analysis, and instead pictures the hadron target as being composed of point, spin $1 / 2$, quarks (and antiquarks). When the hadron is boosted to an infinite momentum, the scattering at large $v$ and $q^{2}$ is viewed as taking place incoherently (in impusie approximation) and elastically off each quark. The variable $x=1 / \omega$ now also has the interpretation of the fractional longitudinal momentum carried by the struck parton.

The basic connection between the parton picture and the structure functions is given by

$$
\begin{equation*}
2 M x W_{1}(x)=\nu W_{2}(x)=\sum_{i} Q_{i}^{2} x f_{i}^{h}(x) \tag{26}
\end{equation*}
$$

where $f_{i}^{h}(x)$ is the distribution of partons of type $i$ in the hadron target $h$, and $Q_{i}$ is its (electromagnetic or weak) charge. It is straightforward to then write out the structure functions for various processes on nucleons, in terms of $u, \bar{u}, d, \bar{d}, s$, and $\bar{s}$ quark contributions:

$$
\begin{align*}
& F_{2}^{e p}=v W_{2}^{e p}=x\left[\frac{4}{9} f_{u}+\frac{1}{9} f_{d}+\frac{1}{9} f_{s}+\frac{4}{9} f_{\bar{u}}+\frac{1}{9} f_{\bar{d}}+\frac{1}{9} f_{\bar{s}}\right] \\
& F_{2}^{e n}=\forall W_{2}^{e n}=x\left[\frac{1}{9} f_{u}+\frac{4}{9} f_{d}+\frac{1}{9} f_{s}+\frac{1}{9} f_{\bar{u}}+\frac{4}{9} f_{d}+\frac{1}{9} f_{\bar{s}}\right] \\
& F_{2}^{\nu p}=\quad=2 x\left[f_{d}+f_{\bar{u}}\right]=F_{2}^{\bar{\nu} n}  \tag{27}\\
& \mathrm{~F}_{2}^{\bar{\nu} p^{-}}=\quad=2 x\left[f_{u}+f_{\bar{d}}\right]=F_{2}^{\nu n} \\
& -x F_{3}^{\nu p}=-x \nu W_{3}^{\nu p}=2 x\left[f_{d}-f_{\bar{u}}\right]=-x F_{3}^{\bar{\nu} n} \\
& -x F_{3}^{\bar{v} p}=-x \nu w_{3}^{\bar{\nu} p}=2 x\left[f_{u}-f_{\bar{d}}\right]=-x F_{3}^{\nu n}
\end{align*}
$$

In all these relations we have used the quark distribution functions, $f_{i}^{p}(x)$, for a proton. Where necessary those of the neutron were related using $f_{u}^{p}=f_{d}^{n}, f_{\bar{u}}^{p}=f_{\bar{d}}^{n}$, etc., which follow from isospin conservation. The extra factor of 2 in the neutrino structure functions arises because although quarks have unit "weak vector charge," there is a contribution from both vector and axial-vector currents. The anti-quarks contribue to $F_{3}$ with opposite sign because, being anti-fermions, they give an opposite sign to the V-A interference term which is involved in $W_{3}$. We have used the approximation of setting the Cabibbo angle to zero.

As long as the (positive semidefinite) functions $f_{i}(x)$ are not specified further, the expressions in Eq. (27) for the structure functions are perfectly general and predict no more or less than taking matrix elements of the quark light cone algebra between nucleon states. In particular, any relations between the structure functions derived from Eq. (27) can also be derived from quark light cone algebra manipulations. For example we have the local relation ${ }^{14}$

$$
\begin{equation*}
6\left(F_{2}^{e p}-F_{2}^{e n}\right)=2 x\left(f_{u}-f_{d}+f_{\bar{u}}-f_{\bar{d}}\right)=x\left(F_{3}^{v p}-F_{3}^{v n}\right) \tag{28}
\end{equation*}
$$

as well as the integral relations

$$
\begin{equation*}
\int_{0}^{1} \frac{d x}{x}\left(F_{2}^{\overline{v p}}-F_{2}^{\nu p}\right)=\int_{0}^{l} d x 2\left(f_{u}-f_{d}+f_{\bar{d}}-f_{\bar{u}}\right)=4\langle p| I_{z}|p\rangle=2, \tag{29}
\end{equation*}
$$

(the Adler ${ }^{15)}$ sum rule) and ${ }^{16)}$

$$
\begin{align*}
-\int_{0}^{I} d x\left(F_{3}^{\nu p}+F_{3}^{\nu n}\right) & =\int_{0}^{I} d x 2\left(f_{u}+f_{d}-f_{\bar{u}}-f_{\bar{d}}\right) \\
& =6 \cdot \frac{1}{3} \int_{0}^{1} d x\left(f_{u}+f_{d}+f_{s}-f_{\bar{u}}-f_{\bar{d}}-f_{\bar{s}}\right)  \tag{30}\\
& =6\langle p| B|p\rangle=6 .
\end{align*}
$$

In the last equation we used the constraint that

$$
\int_{0}^{1} d x\left(f_{s}-f_{\bar{s}}\right)=\langle p| s|p\rangle=0
$$

There are also easily derived and important inequalities like ${ }^{\text {ll) }}$

$$
\begin{equation*}
\frac{1}{4} \leq \frac{F_{2}^{e n}}{F_{2}^{e p}} \leq 4 \tag{31}
\end{equation*}
$$

and

$$
\begin{align*}
F_{2}^{v n}+F_{2}^{v p} & =2 x\left(f_{\bar{d}}+f_{u}+f_{\bar{d}}+f_{\bar{u}}\right) \\
& \leq \frac{18}{5} x\left(\frac{5}{9} f_{d}+\frac{5}{9} f_{u}+\frac{5}{9} f_{\bar{d}}+\frac{5}{9} f_{\bar{u}}+\frac{2}{9} f_{s}+\frac{2}{9} f_{\bar{s}}\right)  \tag{32}\\
& \leq \frac{18}{5}\left(F_{2}^{e p}+F_{2}^{e n}\right) .
\end{align*}
$$

For polarized scattering, there are other relations as well, such as the Bjorken sum rule ${ }^{17 \text { ) }}$ on the difference of proton and neutron spin dependent structure functions. ${ }^{18}$ 18)

This is as far as we can go without making assumptions which are outside the light-cone framework. A commonly made additional approximation in some $x$ regions, is to neglect the contribution of antiquarks, i.e., set $f_{\bar{u}}(x)=f_{\bar{a}}(x)=f_{\bar{s}}(x)=0$. It then follows that $f_{S}(x)=0$. In this case everything simplifies further. In addition to all the previous relations, we have:

$$
\begin{equation*}
2 M x W_{1}(x)=\nu W_{2}(x)=-x \quad \nu W_{3}(x) \tag{33}
\end{equation*}
$$

for each process. For neutrinos

$$
\begin{align*}
& \frac{d^{2} \sigma^{\nu N}}{d x d y}=\left(\frac{G^{2} M E}{\pi}\right) F_{2}^{\vee N}(x)  \tag{34a}\\
& \frac{d^{2} \sigma^{-\overline{V N}}}{d x \overline{d y}}=\left(\frac{G^{2} M E}{\pi}\right) F_{2}^{\overline{\nu N}}(x)(1-y)^{2}, \tag{34b}
\end{align*}
$$

and as a result:

$$
\begin{equation*}
\frac{\sigma^{\bar{\sim} \mathrm{N}}}{\sigma^{\nu \mathbb{N}}}=\frac{1}{3} . \tag{35}
\end{equation*}
$$

Also

$$
\begin{equation*}
F_{2}^{\nu p}+F_{2}^{\nu n}=F_{2}^{\overline{\nu p}}+F_{2}^{\overline{\nu n}}=\frac{18}{5}\left(F_{2}^{e p}+F_{2}^{e n}\right), \tag{36a}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma\left(\mathrm{F}_{2}^{\mathrm{ep}}-\mathrm{F}_{1}^{\mathrm{en}}\right)=\mathrm{F}_{2}^{-\bar{p}}-F_{2}^{\nu p} \tag{36b}
\end{equation*}
$$

so that measurement of ep and en deep inelastic scattering determines everything. Furthermore, the Adler sum rule, Eq. (29), can then be rewritten a.s

$$
\begin{equation*}
\int_{0}^{1} \frac{d x}{x}\left[F_{2}^{e p}-F_{2}^{e n}\right]=\frac{1}{3} \tag{37}
\end{equation*}
$$

Equation (37) actually holds using a somewhat weaker parton model assumption that the nucleon contains valence ( $u$ and d) quarks plus an isoscalar sea of $q \bar{q}$ pairs $\left(f_{\bar{u}}=\frac{f_{\bar{d}}}{}\right)$.

- For electron-positron annihilation we have the very simple result in the parton model that

$$
\begin{equation*}
R \equiv \frac{\sigma_{\text {total }}\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}=\sum_{i} Q_{i}^{2}, \tag{38}
\end{equation*}
$$

where the sum goes over each quark (not antiquark) type. If each quark also comes in three colors, then the sum is implicitly over color also. With colored $u, d$, and $s$ quarks, Eq. (38) predicts

$$
\begin{equation*}
R=3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right)=2 \tag{39}
\end{equation*}
$$

We are now in a position to state a zeroth order (to $\sim 20 \%$ ) picture of experiments on deep inelastic scattering in terms of the quark parton model: For values of the Bjorken scaling variable in the range $0.1 \leq \frac{1}{\omega} \leq 1$, only quark (not antiquark) partons are found in the nucleon and scaling holds for $q^{2}>1 \mathrm{GeV}^{2}$ and hadronic invariant masses $W>2 \mathrm{GeV}$. Some representative samples of the evidence is shown in Figures 1, 2, 3 and 4. In Figure 1 we have the results of inelastic scattering of electrons off hydrogen and deuterium ${ }^{19}$ ), showing scaling at SLAC energies. Figure 2 , taken from results of the CalTech-FNAL narrow band neutrino experiment ${ }^{20}$ ), confirms the scaling prediction that $v N$ and $\bar{\nu} N$ total cross sections should rise linearly with the beam energy. The integrated form of Eq. (36),

$$
\frac{\int_{0}^{1}\left[F_{2}^{e p}(x)+F_{2}^{e n}(x)\right] d x}{\int_{0}^{1}\left[F_{2}^{\nu p}(x)+F_{2}^{\nu p}(x)\right] d x}=\frac{5}{18}
$$



Fig. 1 The structure function $\nu W_{2}$ for $\theta=6^{\circ}$ and $10^{\circ}$ inelastic electron scattering on hydrogen and deuterium ${ }^{19}$.


Fig. 2 nucleons ${ }^{20)}$ as a function of incident energy.

Meon Square Charge of inferacting Constifuents $(S=0)$


Fig. 3 Comparison ${ }^{21)}$ of the ratio of integrated electron-nucleon to neutrino-nucleon structure functions to the value 5/18 expected from "quark charges."


Fig. 4 (a) Ratio of antineutrino to neutrino total cross sections; and (b) rise with energy of their sum ${ }^{21 \text { ). }}$
is compared with experiment ${ }^{21)}$ in Figure 3, and the prediction $\sigma_{T} \overline{\mathbb{N}} / \sigma_{T}^{2 \mathbb{N}}=1 / 3$ following from the absence of antiquark partons can be compared with the data ${ }^{\text {2I) }}$ in Figure 4.

- Within the stated errors, everything works rather well. The data for neutral current neutrino induced events will eventually be useful in testing this picture. At the moment, one usually works the other way and assumes the zeroth order picture in order to extract additional physics information from the present data.

In $e^{+} e^{-}$annihilation, the equivalent zeroth order picture only holds for $\sqrt{Q^{2}} \lesssim 3.5 \mathrm{GeV}$, data from $\operatorname{SPEAR}^{22)}$ for $2.4 \lesssim \sqrt{Q^{2}} \lesssim 3.4 \mathrm{GeV}$ are consistent with a constant value of $R$ lying between 2 and 3. Within errors, and given the possible approach of $R\left(Q^{2}\right)$ to its scaling limit from above, as in asymptotically free gauge theories 23,24 , such values are consistent with the value of 2 in Eq. (39) arising from colored $u$, $d$, and $s$ quarks.

## V. Scaling and Scaling Breakdown

Having summarized the state of experiments in deep inelastic scattering in a very simple way to an accuracy of $\sim 20 \%$, we shall now look at the evidence for corrections to this picture. As we will soon see, a variety of physical origins can be ascribed to these effects, and at the $10 \%$ level one encounters a confusing but exciting situation which is not yet sorted out.

Although it didn't seem that way a year ago, perhaps the clearest case is $e^{+} e^{-} \rightarrow$ hadrons. After the spectacular narrow resonances $\psi$ (3.1) and $\psi^{\prime}(3.7) \quad R\left(Q^{2}\right)$ rises rapidly near ${ }^{22)} \sqrt{Q^{2}}=4 \mathrm{GeV}$, and beyond $\sim 5 \mathrm{GeV}$ appears again ${ }^{25)}$ to have settled on a constant value between 5 and 6 . Presumably this signals passing the threshold for new physics and $\Sigma_{i} Q_{i}^{2}$
has increased appropriate to the number of new quarks which have become operative. This new physics very likely will also affect to some degree deep inelastic scattering, to which we now turn.

- Evidence for a scaling breakdown in deep inelastic scattering comes from three different experiments:

1. Cornell-Michigan State-UC Berkeley-UC San Diego; Experiment 26 at FNAL ${ }^{26,27 \text { ). }}$. Using 56 and 150 GeV muon beams at FINAL, this experiment tests scaling both by a relative comparison of the data at the two energies and by a comparison of the absolute rate with a Monte Carlo calculation based on a fit to SLAC data. As indicated in Figure 5, the latter comparison shows a fall of the data with increasing $q^{2}$ at small $\omega$ ( $\lesssim 5$ ), but a rise at large $\omega \quad(6 \lesssim \omega \lesssim 40)$.
2. Harvard-Pennsylvania-Wisconsin-FNAL; Experiment IA at FNAL ${ }^{28,29)}$. Using the broad band neutrino and antineutrino beam at FNAL, tests of scaling of $\sigma_{\text {total }}(E),\left\langle q^{2}\right\rangle, x$ and $y$ distributions, etc. are possible. While the claimed discrepancy has been presented in several different ways, Figure 6 is typical. For antineutrinos above 30 GeV , the y distribution disagrees with what is expected assuming scaling and quark partons. This is usually presented ${ }^{28)}$ as showing that there are too many events at large $y(y \approx 1)$ when $x<0.1$. Alternatively, there are too many events ${ }^{29)}$ in $\bar{\nu} \mathrm{N}$ scattering at large hadronic masses, $W$. However, it has been stressed by the Caltech group ${ }^{30 \text { ) that the discrepancy is really }}$ at low $y$. They claim that at large $y$ the ratio of $\bar{N}$ to $v N$ events is about what one expects with a reasonable amount of antiquark protons for $x<0.1$. It is at low $y$, where the distribution for $v N$ and $\bar{N}$ should be equal by charge independence, that the Harvard-PennsylvaniaWisconsin experiment has a depletion of events in $\overline{\mathrm{N}}$ compared to $\nu \mathbb{N}$.

Thus the "anomaly" is a low y effect. Further, the Caltech data ${ }^{30 \text { ) }}$ does not show such an effect, although the statistical errors are large enough so that an outright experimental disagreement can not be claimed af the present time.
3. SLAC Group A; Experiment $E 89^{31}$ ). Using large angle ( $50^{\circ}$ and $60^{\circ}$ ) electron scattering at SLAC, measurements have been made out to $q^{2} \simeq 30 \mathrm{GeV}^{2}$ on the structure function $W_{1}$. For fixed values of $\omega$ or $\omega^{\prime}(\leq 2.5)$, $W_{1}$ tends to drop with increasing $q^{2}$.

To what can these observations of apparent scaling violations be due?
Let us examine in turn the various possibilities.
(i) Radiative corrections, $\sigma_{\mathrm{I}} / \sigma_{\mathrm{T}}$ variation, A dependence on heavy nuclei. None of these should affect the claims based on neutrino experiments, at their present level of accuracy. Baring a major error, the radiative corrections should present no problem to anyone. Variation of $\sigma_{\mathrm{L}} / \sigma_{\mathrm{T}}$ within reasonable limits makes insignificant changes in the values of $W_{1}$ from SLAAC ${ }^{31)}$. It is possible that $\sigma_{L} / \sigma_{T}$ or $A$ dependence at large $\omega$ could vary in such a manner with $q^{2}$ so as to have a significant effect on the comparison of the results of the muon experiment at FINA with the SLAC results at lower $\omega$ and $q^{2}$ on deuterium.
(ii) Approach to Scaling. We have very little theoretical control over nonleading singularities on the light cone. An example is the use of the variables $\omega$ and $\omega^{\prime}=\omega+M_{N}^{2} / q^{2}$. Scaling is significantly better in $\omega^{\prime}$, but as $q^{2} \rightarrow \infty, \omega^{\prime} \rightarrow \omega$, so any differences involve the approach to scaling, i.e., non-leading terms in the light cone formalism. Particularly at small $\omega$, the choice of variable can be important at present energies ${ }^{32)}$. At large $\omega$, where the structure functions vary
slowly, it makes less of a difference. Unfortunately, it is at large (1) where the $q^{2}$ range is most limited and a definitive test of scaling over a large range of substantial $q^{2}$ values is not yet possible. For example, if one only tested for scaling beyond $q^{2}=2 \mathrm{GeV}^{2}$, it would make a considerable change in the magnitude of scaling violation deduced from Fig 5.at large $\omega$.
(iii) New Particle Production. As was noted earlier, the rise in $R\left(Q^{2}\right)$ for $e^{+} e^{-}$annihilation inspires one to consider new quarks and hence new particles. In particular, either heavy lepton or "charmed" particle production is a possible explanation of "anomalies" in neutrino or antineutrino scattering. If charm threshold is being passed at FNAL, one would in fact expect it to be a threshold in the hadron mass, $W$, exhibiting itself first as an excess at large $y$ (and small $x$ ) for a given incident energy $E$, and slowly propagating to small y as $E$ increases. In the case of charm or other new quantum numbers this could be accompanied by apparent charge symmetry violations 33). It could also involve $V+A$ hadronic weak currents ${ }^{34}$ ) which would change the $y$ distributions from these in Eq. (34). Whether this is what is going on in the Harvard-Pennsylvania-Wisconsin-FNAL experiment remains to be shown ${ }^{35)}$. The observation ${ }^{36)}$ of two muon events in the same experiment makes it clear that something new is happening at high energies with neutrinos--the only question is at what level it can be seen in $\sigma_{\text {total }}, y$ distributions, etc.

To an even greater extent this applies to the rise with $q^{2}$ at large $\omega$ in the inelastic muon scattering experiment ${ }^{27}$ ). A threshold in $W$ could exist for producing pairs of new particles, which first


Fig. 5 Ratioi ${ }^{27)}$ of observed to simulated event rate vs. $q^{2}$ for eight ranges of $\omega$. See text.
shows up at large $\omega$ for fixed $q^{2}$. As $q^{2} \rightarrow \infty$, it will then propagate to smaller $\omega$. It should be possible to explore this possibility in some detail in the next few years, both by doing inclusive deep inelastic experiments at larger $q^{2}$ and by looking inthe final state for the new particles.
(iv) Parton Size. The idea here is that scaling will break down if distances are probed which are less than $1 / \Lambda$, which characterizes a parton size ${ }^{37}$ ). In this manner, one parametrizes the data with a factor $1 /\left(q^{2}+\Lambda^{2}\right)^{2}$ and $\nu W_{2}$ falls uniformly as $q^{2} \rightarrow \infty$ for all $w$. The data does not show such a behavior ${ }^{27)}$, and $e^{+} e^{-}$annihilation, which originally was a motivation for this idea, shows scaling and no indication of such a propagator form.
(v) Scaling Breakdown in Field Theory. Ideally, one would like to have a complete set of moments,

$$
M_{n}\left(q^{2}\right)=\int_{0}^{1} d x x^{2 n} \nu W_{2}\left(x, q^{2}\right)
$$

and examine their behavior as $q^{2} \rightarrow \infty$. Unfortunately, data only exists over a finite kinematic range, and at any given value of $q^{2}$ the data only extend up to a value of $v$ (and hence of $\omega=2 M_{N^{v}} / q^{2}$ ) bounded by the machine energy. Thus there is always a: region of small $x=1 / \omega$ near zero which is unmeasured for a given $q^{2}$, and consequently $M_{n}\left(q^{2}\right)$ is never fully determined experimentally.

Nevertheless, the qualitative behavior of $\nu W_{2}\left(x, q^{2}\right)$ expected from field theoretic scale breaking is clear. If $d_{0}=0$ (i.e., if the leading term contributing to the zeroth moment comes from the energy momentum tensor with canonical dimension), then


Fig. 6 Experimental and calculated (solid lines) y distributions in neutrino and antineutrino scattering ${ }^{28)}$. The calculations assume scale invariance and only quark partons.

$$
\begin{equation*}
\int_{0}^{1} d x \sim W_{2}\left(x, q^{2}\right) \xrightarrow[q^{1} \rightarrow \infty]{ } \text { constant. } \tag{40}
\end{equation*}
$$

Furthermore, $d_{n+l} \geq d_{n}$, so higher moments, which are more sensitive to the structure function near $x=1$, fall faster with increasing $q^{2}$. Therefore, $\nu W_{2}\left(x, q^{2}\right)$ must drop for $x$ near 1 as $q^{2} \rightarrow \infty$. Since Eq. (40) says that the area under $\nu W_{2}$ is preserved in the same limit, it must rise near $x=0$ to compensate. This behavior is indicated schematically in Fig. 7. Of course, in what $q^{2}$ range this behavior sets in is unspecified a priori. In fact, since the constant on the right hand side of Eq. (40) should be the same for the neutron and proton, it is clear from the data that at present energies there are important contributions to at least the zeroth moment from other than the leading term arising from the energy momentum tensor.

In any case, it turns out to be possible to invert the moments 38), so that if one knows the anomalous dimensions, $d_{n}\left(M_{n} \propto\left(\mu^{2} / q^{2}\right)^{d}\right)$ or the $A_{n}\left(M_{n} \propto\left[1 / \ln \left(q^{2} / \mu^{2}\right)\right]^{A} n^{n}\right)$ in an asymptotically free theory, then $\nu W_{2}\left(x, q^{2}\right)$ may be calculated for all $q^{2}>q_{0}^{2}$ from $\nu W_{2}\left(x, q_{0}^{2}\right)$, where $q_{0}^{2}$ is a value of $q^{2}$ for which the behavior of the structure function moments are controlled by the leading terms. Thus $\psi W_{2}\left(x, q_{0}^{2}\right)$ from experiment, plus the $A_{n}$ 's from theory, and an assumption of relevance of the leading terms at $q^{2}=q_{0}^{2}$, allows a calculation of $\nu W_{2}\left(x, q^{2}\right)$ for $q^{2}>q_{0}^{2}$

There have been a number of calculations along this line ${ }^{39 \text { ). }}$ Recently Tung ${ }^{40)}$ has used the electron data at $q_{0}^{2}=4 \mathrm{GeV}^{2}$ and made extensive calculations of what one expects in either field theories with anomalous dimensions or asymptotically free theories. The results agree qualitatively with Fig. 7. The structure function decreases for


Fig. 7 Expected form of the change in $F_{2}\left(x, q^{2}\right)$ in theories with anomalous dimensions, or in asymptotically free gauge theories, for increasing $q^{2}: q_{2}^{2}>q_{1}^{2}$.
$x \gtrsim .25$ and increases for $x \lesssim .15$ as $q^{2}$ grows. It is difficult to tell the case of anomalous dimensions from that of asymptotically free theories with the $q^{2}$ range likely to be available in the near future. At least qualitatively this is also the behavior of the data $27,31,41$ ). However, at large $\omega(x \simeq 0)$ both the neutrino and muon experiments at FNAL may be encountering a threshold for new physics. Therefore, at the present moment the only significant evidence of a violation of scaling which must be ascribed either to non-leading terms (approach to scaling) or to true field theoretic scale breaking, is the SLAC experiment measuring $W_{1}$ at large $q^{2}$. A really quantitative examination of this data in a field theory framework has yet to be made.

In spite of the altered character of the light cone singularity leading to scaling breakdown in field theory, the other features of the light cone algebra discussed in Section III remain unaltered., For example, the $\operatorname{SU}(3)$ relations among structure functions remain, as does the connection between electron and neutrino structure functions. Sum rules, such as the Adler sum rule or the Bjorken sum rule for polarized lepton-mucleon scattering are still correct, although the approach in the latter case ${ }^{43 \text { ) }}$ should in asymptotically free theories only be logarithmic, rather than by a power of $q^{2}$. Similarly, as noted before, the approach of $R\left(Q^{2}\right)$ in $e^{+} e^{-}$annihilation to a constant should be logarithmic and from above, and $\sigma_{\mathrm{L}} / \sigma_{\mathrm{T}}$ in deep inelastic scattering should behave as $l / \ell n\left(q^{2} / \mu^{2}\right)$ in asymptotically free theories. The latter behavior is to be contrasted with scaling (in $\omega$ ) of $\nu \sigma_{\mathrm{L}} / \sigma_{T} \propto q^{2} \sigma_{\mathrm{L}} / \sigma_{T}$ expected from the quark light cone algebra ${ }^{44)}$ with a leading singularity characteristic of free field theory. It is important to use both the information ${ }^{45)}$ on the $q^{2}$ dependence of $\sigma_{L} / \sigma_{T}$ and that of the individual structure functions when checking quantitatively to see if field theoretic scale breaking is what is being observed.

## VI. Partons and Final State Hadrons

In order to treat the distribution of final state hadrons in either deep inelastic scattering or in $e^{+} e^{-}$annihilation, additional assumptions beyond those already employed, must be made. Within a parton model ${ }^{46 \text { ), a }}$ commonly used set is:
(i) The virtual photon (or weak current) interacts in impulse approximation with point, spin $1 / 2$ constituents, the quark partons.
(ii) Partons fragment into hadrons independently of how they were produced.
(iii) The distribution of hadrons fragmented from a given parton moving with
 of the transverse momentum of the hadron relative to the parton's momentum. The transverse momentum distribution of the hadrons is assumed to be limited, and as a result one should see jets along the parton direction. We shall use the set of functions $D_{i}^{h}(z)$ to be the probability of finding a hadron, $h$, with fractional momentum $z$, arising from a parton of type $i$ (integrating over $p_{\perp}$ ).

The first of these assumptions is just the one used previously to obtain scaling of the structure functions in deep inelastic scattering or of $R\left(Q^{2}\right)$ in $e^{+} e^{-}$annihilation. It describes the "hard" process of a current interacting with an existing quark or pair producing them out of the vacuum. The second and third assumptions are new. They describe the "soft" process of hadrons being produced by partons. Of course this process can not be too soft or the quark itself would appear in the final state. An intuitive description of how the quarks are kept from escaping in a universal way, but delicately enough so that assumption (ii) still holds, was discussed two years ago in the SLAC Summer Institute by Bjorken ${ }^{49 \text { ). }}$ A really quantitative model in four-dimensional space-time has yet to be presented.

In electron-positron annihilation as $Q^{2} \rightarrow \infty$, one has that $p^{\text {(parton) }} \rightarrow \sqrt{Q^{2}} / 2$ and $p^{\text {(hadron) }} \rightarrow E_{h}$, so that

$$
\begin{equation*}
z \rightarrow \frac{E_{h}}{\sqrt{Q^{2} / 2}}=\frac{2 p_{h} \cdot q}{q^{2}}=\bar{\omega} \tag{42}
\end{equation*}
$$

the scaling variable defined previously. Therefore, integrating over angles and over $p_{\perp}$ relative to the parton direction, we find that as $Q^{2} \rightarrow \infty$, and $E_{h} \rightarrow \infty$ (but $\bar{\omega}$ finite and non-zero), we have for $e^{+} e^{-} \rightarrow h+$ anything:

$$
\begin{equation*}
\frac{I}{\sigma_{\mu \mu}} \frac{d \sigma}{d \bar{\omega}}=\sum_{i} Q_{i}^{2}\left[D_{i}^{h}(\bar{\omega})+D_{\bar{i}}^{h}(\bar{\omega})\right] \tag{43}
\end{equation*}
$$

where the sum extends over quarks ( $u, d, s, \ldots$ ), and the two terms in brackets arise because both a quark and an antiquark are produced out of the vacuum in each event and each fragment into hadrons. Equation (43) predicts scaling of the inclusive hadron spectrum in $e^{+} e^{-}$annihilation, i.e., $\bar{W}_{1}\left(\nu, Q^{2}\right)=\bar{W}_{1}(\bar{\omega})$, and $\quad \nu \bar{W}_{2}\left(\nu, Q^{2}\right)=\nu \bar{W}_{2}(\bar{\omega})$. Further, with $\operatorname{spin} 1 / 2$ partons,

$$
\begin{equation*}
\frac{2 M_{h}}{\bar{v}} \bar{W}_{1}=\bar{W}_{2} \tag{44a}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\sigma_{L}}{\sigma_{T}} \rightarrow 0 \tag{44b}
\end{equation*}
$$

as $Q^{2} \rightarrow \infty$.
At small values of $\bar{\omega}$, one expects $D_{i}^{h}(\bar{\omega}) \propto I / \bar{\omega}$, leading to a
logarithmic rise in multiplicity. This follows, since the integral of Eq. (43) over $\bar{a}$ gives

$$
\begin{align*}
R\left\langle n_{h}\right\rangle & =\int_{2 M_{h} / \sqrt{Q^{2}}}^{1} \frac{1}{\sigma_{\mu \mu}} \frac{d \sigma^{h}}{d \bar{\omega}} d \bar{\omega}  \tag{45}\\
& =\sum_{i} Q_{i}^{2} \int_{2 M_{h} / \sqrt{Q^{2}}}^{I}\left[D_{i}^{h}(\bar{\omega})+D_{\bar{i}}^{h}(\bar{\omega})\right] d \bar{\omega}
\end{align*}
$$

The corresponding formula for deep inelastic production of hadrons in the current fragmentation region reads ${ }^{46}$ )

$$
\begin{equation*}
\frac{I}{\sigma_{T}} \frac{d \sigma_{T}}{d z}=\frac{\sum_{i} Q_{i}^{2} f_{i}(I / \omega) D_{i}^{h}(z)}{\sum_{i} Q_{i}^{2} f_{i}(I / \omega)} \tag{46}
\end{equation*}
$$

Here $z \rightarrow E_{h}^{I a b} / v$ as $v, q^{2} \rightarrow \infty$. In the same limit $z$ is the ratio of $\mathrm{p}_{\|}^{\text {C.M. }} / \mathrm{p}_{\max }^{\text {C.M. }}$ where $\mathrm{p}_{\|}$is defined along the incident current direction (i.e., z is asymptotically also equal to Feynman x). Note that Eq. (46) gives a "hybrid" scaling law, which involves scaling both with respect to the Bjorken variable $\omega$ and with respect to $z$. Furthermore, because of assumption (ii), the same functions $D_{i}^{h}(z)$ occur here and in $e^{+} e^{-}$ annihilation. In fact, if only one quark contributes to the deep inelastic scattering, its contribution cancels out on the right hand side of Eq. (46). Such is the case for deep inelastic $\nu \mathrm{p}$ scattering if there are only quarks in the nucelon: for then only $f_{d}(1 / \omega)$ enters the cross section and therefore

$$
\begin{equation*}
\frac{1}{\sigma^{\nu p}} \frac{d \sigma}{d z}(\nu p \rightarrow h+\cdots)=D_{u}^{h}(z) \tag{47}
\end{equation*}
$$

can be used to isolate individual $D_{u}^{h}(z)$ 's.
Again, note the analogy and the similar assumptions needed to get scaling of $R=\Sigma Q_{i}^{2}$ and Bjorken scaling, $F_{2}(I / \omega)=\Sigma_{i} Q_{i}^{2} f_{i}(I / \omega)$; and the additional assumptions needed to obtain Eq. (43) for $e^{+} e^{-}$annihilation
and its analog for deep inelastic scattering, Eq. (46). This is true in the light cone framework as well. While the first two scaling laws only need an assumption about the operator product of two currents, the latter two demand knowing ${ }^{48)}$ a four-fold product of operators which include two hadronic sources. No matter how one does it, these latter rather powerful results demand additional strong assumptions.

The data on inclusive hadron distributions in $e^{+} e^{-}$annihilations which is presently available is shown ${ }^{49)}$ in Fig. 8. The distributions of $Q^{2} d \sigma / d \overline{1} \propto(d \sigma / d \bar{\omega}) / \sigma_{\mu \mu}$ vs. $\bar{\omega}$ at $\sqrt{Q^{2}}=3.0,3.8$, and 4.8 GeV do show possible scaling when $\bar{\omega} \underset{\sim}{\sim} 0.5$. Of course, since $R\left(i . e ., \Sigma_{i} Q_{i}^{2}\right.$ ) is changing over this range of center of mass energies one does not expect scaling of the inclusive distribution either. Presumably, only the data at $\sqrt{Q^{2}}=3.0 \mathrm{GeV}$ is the result of $u, d$ and $s$ quarks fragmenting into hadrons, while the data at 3.8 and 4.8 GeV contains contributions from whatever is responsible for the rise in $R$ near 4 GeV .

Therefore any test of scaling of inclusive distributions must compare data only below $\sqrt{Q^{2}} \simeq 3.5 \mathrm{GeV}$ or only above $\sqrt{Q^{2}} \simeq 4.5 \mathrm{GeV}$, where $R$ seems to have settled on a constant value again. This will only be possible with data soon to be forthcoming from SPEAR. However, if we assume that scaling of the inclusives does hold above $\sqrt{Q^{2}} \simeq 4.8 \mathrm{GeV}$, then the scaling observed for $\bar{\omega} \gtrsim 0.5$ in Fig. 8 has added significance. For then the $u$, $d$, $s$ quarks may well be the principal contributors for $\bar{\omega} \gtrsim 0.5$ at all $Q^{2}$ and the new contribution to $R$ only affects the $\bar{\omega}<0.5$ region.

In electroproduction there is considerable data both from bubble chamber or streamer chamber experiments at DESY and SLAC and from counter experiments at DESY, CEA, Cornell and SLAC ${ }^{50 \text { ). These indicate the possi- }}$ bility that the inclusive pion distributions may possibly scale in both $\omega$


Fig. 8 Inclusive charged particle distributions ${ }^{49)}$ at $\sqrt{Q^{2}}=\sqrt{s}=3.0,3.8$, and 4.8 GeV . The variable $\mathrm{x}=2 \mathrm{p} / \sqrt{\bar{s}} \rightarrow \bar{a}=\mathrm{E}_{\text {hadron }} /{ }^{\mathbf{T}}{ }_{\text {beam }}$ for relativistic particles, and correspondingly $s d \sigma / d x \rightarrow Q^{2} d \sigma / d \bar{\omega}$.
and $z$, but the limited kinematic range presently available prevents the from making any strong conclusion ${ }^{32}$ ). The most striking result pointing toward partons is the ratio of positive to negative hadrons produced in the photon fragmentation region. Some of the data ${ }^{51}$ ) is shown in Fig. 9. The large value of the ratio has a natural explanation in the quark parton model where it is the $u$ quark in the proton which is predominantly struck by the virtual photon, and it should fragment into $\pi^{+}, K^{+}$and $p$ much more often than into $\pi^{-}, K^{-}$and $\bar{p}$.

This is also supported by the data from Gargamelle on neutrino-nucleon collisions ${ }^{52}$ ) where the ratio of $h^{+}$to $h^{-}$agrees with what is predicted from the $D_{u}^{h}(z)^{\text {'s }}$ s extracted from electroproduction. It will be very interesting to see the forthcoming results from the neutrino-exposure of the 15' bubble chamber at FNAL in this regard.

In all processes, the data shows a slow rise of the multiplicity with available energy--a rise which is consistent with being logarithmic and which is consistent with having the same coefficient of $\ln Q^{2}$ as that of $\ln \mathrm{s}$ in hadron-hadron collisions. Also, as indicated in Bjorken's lectures 53), the shape and magnitude (to within factors of two) of the hadron spectra in $e^{+} e^{-}$annihilation and in the current fragmentation region of electroproduction are consistent with one another. Much more detailed comparisons including separation of hadron species, should be made with the data that will soon be available.

Some further restrictions follow from the use of symmetry principles to relate the $D(z)$ 's. Most importantly, isospin and charge conjugation invariance yield

$$
\begin{equation*}
D_{u}^{\pi+}(z)=D_{d}^{\pi-}(z)=D_{\bar{u}}^{\pi-}(z)=D_{\bar{d}}^{\pi+}(z) \tag{48}
\end{equation*}
$$

and

$$
\begin{equation*}
D_{i}^{\pi}(z)+D_{i}^{+}(z)=2 D_{i}^{\pi^{-}}(z) \tag{49}
\end{equation*}
$$




Fig. 9 The ratio ${ }^{51)}$ of positive to negative hadrons in the range, $0.4<x<0.85$ electroproduced on proton and neutron targets as a function of $q^{2}$.
for $i=u, d, s, \ldots$. A consequence of Eq. (49) is that in $e^{+} e^{-}$ annihilation ${ }^{54}$ )

$$
\begin{equation*}
\frac{d \sigma^{+}}{d z}=\frac{d \sigma^{-}}{d z}=\frac{d \sigma^{0}}{d z} \tag{50}
\end{equation*}
$$

In electroproduction, there are sum rules ${ }^{46 \text { ) for current fragments like: }}$

$$
\begin{align*}
& \int_{0}^{1}\left[\left\langle n_{\pi^{+}}\right\rangle_{e n}-\left\langle n_{\pi^{-}}\right\rangle e_{e n}\right] W_{1}^{e n}(x) d x \\
& \quad=\frac{2}{7} \int_{0}^{1}\left[\left\langle n_{\pi^{+}}\right\rangle_{e p}-\left\langle n_{\pi^{-}}\right\rangle_{e p}\right] W_{1}^{e p}(x) d x \tag{51}
\end{align*}
$$

An even more speculative possibility is that of relating the probability of finding a quark with longitudinal momentum fraction $x$ in a hadron, $f_{i}^{h} \quad(x=I / \omega)$, to the probability of finding a hadron with longitudinal momentum fraction $z$ in a quark, $D_{i}^{h}(z)$. While such a relationship sounds like "crossing" in field theory, it is not. Nevertheless, some model calculations do yield such a "reciprocity relation" ${ }^{55}$ ):

$$
\begin{equation*}
D_{i}^{h}(z)=f_{i}^{h}\left(x=\frac{1}{\omega}=z\right) . \tag{52}
\end{equation*}
$$

Given Eq. (52), we immediately find that,

$$
\begin{align*}
\frac{\bar{\omega}}{\sigma_{\mu \mu}} \frac{d \sigma}{d \bar{\omega}}\left(e^{+} e^{-} \rightarrow h+\cdots\right) & =\sum_{i} Q_{i}^{2} \bar{\omega} D_{i}^{h}(\bar{\omega}) \\
& =\left.\sum_{i} Q_{i}^{2}\left(\frac{1}{\omega}\right) f_{i}^{h}\left(\frac{1}{\omega}\right)\right|_{I / \omega=\bar{\omega}}  \tag{53}\\
& =\left.v w_{2}^{e h}(\omega)\right|_{I / \omega=\bar{\omega}},
\end{align*}
$$

relating $e^{+} e^{-} \rightarrow h+\cdots$ to deep inelastic eh scattering. This is equivalent to the Gribov-Lipatov relations ${ }^{56}$ )

$$
\begin{align*}
& \bar{\omega} \bar{F}_{1}(\bar{\omega})=F_{1}\left(\omega=\frac{1}{\bar{\omega}}\right)  \tag{54}\\
& \bar{\omega}^{3} \overline{\mathrm{~F}}_{2}(\bar{\omega})=F_{2}\left(\omega=\frac{1}{\bar{\omega}}\right)
\end{align*}
$$

If one tries to test the reciprocity relation, or rather its consequence, Eq. (53), using the data available on $e^{+} e^{-} \rightarrow \bar{p}+\cdots$ and $e p \rightarrow e+\cdots$, then one finds a least rough agreement ${ }^{32)}$. However, if one then uses $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi+\cdots$ data to get $\mathrm{F}_{2}^{\mathrm{e} \pi}(\omega=1 / \omega)$, much too large a structure function results. Some caution is required here since for $\sqrt{Q^{2}} \gtrsim 3.5 \mathrm{GeV}$, one is presumably including those pions which are weak decay products of hadrons formed from the new quark or quarks which cause the rise in $R$. Even for $\sqrt{Q^{2}}=3.0 \mathrm{GeV}$, pions from the decay of $K_{s}^{\prime \prime} s$, $\Lambda^{\prime} s$, etc. are contaminating the sample. This also affects the comparison of the $D(z)$ 's extracted from $e^{+} e^{-}$annihilation with those from $e N$ and $\mathcal{N}$ deep inelastic processes. Whether the situation for $e^{+} e^{-} \rightarrow \bar{p}+\cdots$ and $e p \rightarrow e+\cdots$ is a fortuitous accident thus remains open. Further progress in comparison with experiment demands the separation of particle types in all reactions and pushing nearer $\bar{\omega}=1$ (and $z=1$ in $e \mathbb{N}$ and $\nu \mathbb{N}$ ) where contaimination by weak or other decays should be minimized.

## VII. Conclusion

Leptons have emerged, both because of deep inelastic scattering and electron-positron annihilation, as the probe of hadronic structure. In electron-positron colliding beams, measurement of $R\left(Q^{2}\right)$ gives the sum of the squares of the charges of the fundamental fermions and this tells which quarks are present to make up all hadrons. Deep inelastic scattering on a particular hadron then tells us the distribution of these quarks within
that hadron. The experiments so far done on nucleons with incident electrons, muons, neutrinos, and antineutrinos have yielded a very simple zeroth order picture of nucleon structure, as indicated in Section IV.

The exact nature and cause of scaling breakdown, as discussed in Section V, is difficult to ascertain. More detailed explorations of this question will go on for years. However, more important for our understanding of hadron structures is the fact of at least approximate scaling, which allows us to use deep inelastic processes as a stepping stone and tool for creating and exploring new kinds of quarks and corresponding hadrons. As such, electron-positron annihilation and deep inelastic neutrino scattering will very likely continue to be the prime way of gaining new information on hadronic structure for years to come.

## References

1. The kinematics of inelastic scattering are reviewed in F. J. Gilman, Phys. Reports 4C, 98 (1972). See references to other work therein. We use $E$ and $E^{\prime}$ for the initial and final lepton energies, and neglect lepton masses. See also the lectures of E.D. Bloom, this Institute.
2. The metric is $q_{\mu} q_{\mu}=q^{2}=\vec{q}^{2}-q_{0}^{2} \quad \alpha=e^{2} / 4 \pi \simeq 1 / 137$.
3. S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. Dl, 1617 (1970).
4. For a discussion of polarization in electron-positron colliding beams see V. N. Baier, XLVI Corso Scuola Int. di Fizica "Erico Fermi" (Academic Press, New York, 1971), p. 1. Also Y. S. Tsai, SLAC preprint SLAC-PUB-1623, 1975 (unpublished).
5. For a review of the subject of light cone and short distance singularities, see Y. Frishman, in Proceedings of the XVI International Conference on High Energy Physics, J. D. Jackson and A. Roberts, editors (National Accelerator Laboratory, Batavia, Illinois, 1972), p. 119. See references to previous work therein.
6. K. Wilson, Phys. Rev. 179, 1499 (1969).
7. This is true only if the internal symmetry indices allow the energy momentum tensor to contribute. Even if it does contribute, $d_{0}=0$ is just the smallest anomalous dimension--generally there is a sum of operators with differing values of $d_{0}$ on the right hand side of the short distance expansion, Eq. (20).
8. D. J. Gross and F. Wilczek, Phys. Rev. Letters 30, 1343 (1973); G. 't Hoof't (unpublished); H. D. Politzer, Phys. Rev. Letters 30, 1346 (1973).
9. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
10. H. Fritzsch and M. Gell-Mann, in Scale Invariance and the Light Cone (Gordon and Breach, New York, 1971), Vol. 2, p. 1.
11. O. Nachtmann, Nucl. Phys. B38, 397 (1972).
12. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969); and R. P. Feynman, Photon-Hadron Interactions (W. A. Benjamin, Reading, Máss., 1972).
13. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
14. C. H.Llewellyn Smith, Nucl. Phys. B17, 277 (1970) and Phys. Rev. D4, 2392 (1971).
15. S. L. Adler, Phys. Rev. 143, 1144 (1966).
16. D.J. Gross and C. H. Llewellyn Smith, Nucl. Phys. Bl4, 337 (1969).
17. J. D. Bjorken, 'Phys. Rev. 148, 1467 (1966).
18. For a discussion of spin-dependent deep inelastic scattering, see F. J. Gilman in Proceedings of the Summer Institute on Particle Physics, July 9-28, 1973, SLAC Report No. 167, Vol. I, p. 71 (1973).
19. J. S. Poucher et al., Phys. Rev. Letters 32, 118 (1974).
20. B. C. Barish et al., Caltech preprint CALT-460, 1975 (unpublished).
21. B. C. Barish et al., papers No. 586 and 588 submitted to the London Conference; see F. Sciulli in Proceedings of the XVII International Conference on High Energy Physics, J. R. Smith, ed. (Rutherford Laboratory, Chilton, Didcot, 1974), p. IV-105.
22. J. E. Augustin et al., Phys. Rev. Letters 34, 764 (1975).
23. T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973).
24. A. Zee, Phys. Rev. D8, 4038 (1973).
25. C. Morehouse, invited talk at this Institute, 1975; and V. Luth et al., contribution from the SLAC-LBL collaboration submitted to the Palermo Conference, June, 1975.
26. D. J. Fox et al., Phys. Rev. Letters 33, 1504 (1974).
27. Y. Watanabe et al., Cornell preprint CLNS-302, 1975 (unpublished).
C. Chang et al., Michigan state preprint MSU-CSL-23, 1975 (unpublished).
28. B. Aubert et al., Phys. Rev. Letters 33, 984 (1974).
29. A. Benvenuti et al., Phys. Rev. Letters 34, 597 (1975).
30. See F. Sciulli, invited talk at the Balaton Conference and Caltech preprint CALT 68-506, 1975 (unpublished).
31. W. Atwood, Stanford University thesis, 1975 (unpublished); R. E. Taylor, rapporteur talk at the Palermo Conference, June, 1975 and SLAC-PUB-1613, 1975 (unpublished).
32. See the discussion in F. Gilman, Proceedings of the XVII International Conference on High Energy Physics, J.R. Smith, ed. (Rutherford Laboratory, Chilton, Didcot, 1974), p. IV-149.
33. This possibility is discussed in some detail by A. De Rujula et al., Rev. Mod. Phys. 46, 391 (1974).
34. See, for example, the recent proposal of A. De Rujula et al., Harvard University preprint, 1975 (unpublished).
35. See the exploratory fits by V. Barger, T. Weiler and R. J. N. Phillips, University of Wisconsin preprints, 1975 (umpublished).
36. A. Benvenuti et al., Phys. Rev. Letters 34, 419 (1975).
37. M. Chanowitz and S. D. Drell, Phys. Rev. Letters 30, 807 (1973), and Phys. Rev. D9, 2078 (1974). Also K. Matumoto, Prog. Theoret. Phys. 47, 1975 (1973).
38. G. Parisi, Phys. Letters 43B, 207 (1973).
39. G. Parisi, Phys. Letters 50B, 367 (1974); D. J. Gross, Phys. Rev. Letters 32, 1071 (1974); A. De Rujula et al., Phys. Rev. D10, 2141 (1974).
40. W. K. Tung, University of Chicago preprints EFI 75/14 and EFI 75/36, 1975 (unpublished).
41. E. M. Riordan et al., Phys. Letters 52B, 249 (1974).
42. Note that this is generally not true if there are new quarks and thresholds for new physics in either muon, electron, or neutrino scattering.
43. See, for example, K. Sasaki, Kyoto preprint KUNN 318, 1975 (unpublished).
44. K. T. Mahanthappa and T. Yao, Phys. Letters 39B, 549 (1972); J. E. Mandula, Phys. Rev. D8, 328 (1973).
45. See, for example, D. V. Nanopoulos and G. G. Ross, CERN preprint TH. 2007, 1975 (unpublished).
46. R. P. Feynman, ref. 12; for detailed application to inclusive hadron production see M. Grounau, F. Ravndal, and Y. Zarmi, Nucl. Phys. B51, 611 (1973).
47. See J. D. Bjorken in Proceedings of the Summer Institute on Particle Physics, July 9-28, 1973, SLAC Report No. 167, Vol. I, p. 1 (1973), and references therein.
48. See, for example, J. Ellis, Phys. Letters 35B, 537 (1971).
49. B. Richter, in Proceedings of the XVII International Conference on High Energy Physics, J. R. Smith, ed. (Rutherford Laboratory, Chilton, Didcot, 1974), p. IV-37.
50. See the parallel session of the London Conference organized by H. Meyer in Proceedings of the XVII International Conference on High Energy Physics, J. R. Smith, ed. (Rutherford Laboratory, Chilton, Didcot, 1974), p. IV-57 and references therein.
51. J. T. Dakin et al., Phys. Rev. D10, 1401 (1974).
52. See the data presented by M. Hagenauer in Proceedings of the XVII International Conference on High Energy Physics, J. R. Smith, ed. (Rutherford Laboraory, Didcot, Chilton, 1974), p. IV-95.
53. J. D. Bjorken, lectures at this Institute, 1975.
54. But note that if the pions are not direct quark fragments, but tertiary products which are the result of weak decays, where isospin is not conserved, then neither Eq. (48) nor (50) need hold.
55. See the review of J. Sullivan in Proceedings of the Summer Institute on Particle Physics, July 9-28, 1973, SLAC Report No. 167, Vol. I, p. 289 (1973); also see Y. Eylon and Y. Zarmi, Nucl. Phys. B83, 475 (1974).
56. V. N. Gribov and L. N. Lipatov, Yad. Fiz. 15, 1218 (1972) [Sov. J. Nucl. Phys. 15, 675 (1972)].

[^0]:    ${ }^{*}$ Supported by the Energy Research and Development Administration.
    (Extracted from the Proceedings of Summer Institute on Particle Physics, SLAC Report No. 191, November 1975)

