RESONANCE MODEL FOR $e^+e^- \rightarrow HADRONS^*$

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ABSTRACT

A model is presented in which e^+e^- annihilation proceeds through the process $e^+e^- \rightarrow \pi R$, where R is a resonance. The contribution to the single pion inclusive distribution from resonance decay products is calculated. Scale-breaking in sd σ /dx and σ _{tot}(s) is attributed to a threshold for production of a new class of resonances.

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Recent results for e^+e^- annihilation from SPEAR¹ have confirmed that R, the ratio of hadronic to μ -pair cross sections, rises (except for the high energy side of resonances) in the range 3 GeV $\leq \sqrt{s} \leq 5$ GeV, and may be continuing to rise slightly up to 7.8 GeV. In the parton model,² this behavior is commonly interpreted as due to a threshold for production of one or more pairs of heavy pointlike objects, in addition to the "old" light hadronic constituents. Unfortunately, this picture reveals little about the single particle inclusive spectrum, which depends on the probability distribution function for finding the hadron in a quark.³ In contrast, thermodynamic or statistical⁴ models naturally provide the inclusive distribution but cannot predict the behavior of the total cross section.

In the present work, we explore a model which shares many features of the statistical, 4 resonance, 5 and vector dominance 6 models of e^+e^- annihilation, and which affords a satisfactory description of both the total hadronic cross section and the details of single-pion production. We have in mind a picture in ' which the inclusive distribution results from the buildup of resonances in the missing-mass channel. ⁷ This may be motivated in the following way. We assume, a la Bloom and Gilman, ⁸ that resonances dominate inelastic electroproduction in the Bjorken limit. Therefore, the Compton amplitude is of the form ⁹

$$T_{2}(s, 0, q_{1}^{2}, q_{2}^{2}) = \sum_{n} \frac{G_{n}(q_{1}^{2}) G_{n}(q_{2}^{2})}{s - M_{n}^{2} + i M_{n} \Gamma_{n}} , \qquad (1)$$

where $G_n(q^2)$ is the excitation form factor of the n'th resonance. The structure function for the process $e^+e^- \rightarrow \pi(p) + hadrons$ is given by ¹⁰

$$\overline{W}_{2}(M^{2}, s) = \frac{1}{\pi} \Delta_{M^{2}} T_{2}(M^{2}, 0, s + i\epsilon, s - i\epsilon')$$
$$= \frac{1}{\pi} \sum_{n} \Delta_{M^{2}} \frac{G_{n}(s + i\epsilon) G_{n}(s - i\epsilon')}{M^{2} - M_{n}^{2} + iM_{n}\Gamma_{n}} , \qquad (2)$$

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where now $s = q^2$ and $M^2 \equiv (q-p)^2$. Therefore, $e^+e^- \rightarrow \pi X$ is dominated by resonances in the missing-mass channel (poles in M^2).⁷ This is depicted in Fig. 1a.

As it stands, the above picture is somewhat inconsistent in that it takes no account of the possibility that the <u>observed</u> pion can be a decay product of the resonance, as is shown in Fig. 1b. In fact, diagrams such as Fig. 1b should contribute due to the presence of anomalous singularities in the virtual Compton amplitude. ¹¹ There remains the question of whether their inclusion constitutes double-counting. Such is the case in a pure dual-resonance picture, where the inclusive cross section is entirely accounted for by Fig. 1a. More generally, some fraction of the contribution from Fig. 1b is already accounted for by Fig. 1a, so that there is partial double-counting. However, for want of information on the amount of double-counting involved, we shall assume that pion production is given by the sum of the diagrams in Fig. 1.

According to this picture, upon neglecting interference between the diagrams of Fig. 1, we have

$$E \frac{d\sigma_{i}}{d^{3}p} = E \frac{d\sigma_{i}^{0}}{d^{3}p} + \sum_{j} \int \frac{d^{3}p'}{E'} E' \frac{d\sigma_{j}^{0}(p')}{d^{3}p'} E \frac{dN_{ij}(p,p')}{d^{3}p} , \qquad (3)$$

where $E d\sigma_i^o/d^3 p$ is the "bare" cross section for production of a pion of charge i corresponding to Fig. 1a. Also, $E dN_{ij}(p, p')/d^3 p$ is the probability distribution for observing a pion of momentum p and charge i from the decay of a resonance of charge -j, as in Fig. 1b. Henceforth, we assume that the bare cross sections for positive, negative and neutral pions are identical, and that $E dN_{ij}/d^3 p$ is independent of j. Furthermore, we shall specialize to negative pions and therefore suppress the index i, giving

$$E \frac{d\sigma}{d^3 p} = E \frac{d\sigma^0}{d^3 p} + 3 \int \frac{d^3 p'}{E'} E' \frac{d\sigma^0(p')}{d^3 p'} E \frac{dN(p, p')}{d^3 p} .$$
(4)

The probability distribution satisfies the sum rules¹²:

$$\int E \frac{dN}{d^3p} \frac{d^3p}{E} = \frac{n}{2}$$
(5)

and, in the resonance rest frame

$$\int E \frac{dN}{d^3p} d^3p = \beta m_R .$$
 (6)

Here \bar{n}_c is the charged multiplicity, β is the fraction of resonance energy carried away by negative pions, and m_R is given by

$$m_R^2 = s + m_\pi^2 - 2E! \sqrt{s}$$
 (7)

To compare (4) with experiment, we need to know the form of $E dN/d^3p$ and $E d\sigma^0/d^3p$. For the latter, we assume the Callan-Gross relation¹³

$$\overline{F}_{1}^{O}(\mathbf{x},\mathbf{s}) = -\mathbf{x}\overline{F}_{2}^{O}(\mathbf{x},\mathbf{s}) , \qquad (8)$$

which gives

$$E \frac{d\sigma^{0}}{d^{3}p} = \frac{2\alpha^{2}}{s^{2}} \left[1 - \frac{1}{2} \left(1 - \frac{4m_{\pi}^{2}}{x^{2}s} \right) \sin^{2} \theta \right] \bar{F}_{1}^{0}(x, s) , \qquad (9)$$

where $x \equiv 2p \cdot q/s = 1 - (M^2 - m_{\pi}^2)/s$. Furthermore, to connect electroproduction to annihilation, we assume that in the scaling region the Gribov-Lipatov reciprocity relation¹⁴ holds

$$\overline{F}_{2}^{0}(x) = -x^{-3} F_{2}(\omega = 1/x) , \qquad (10)$$

where $F_2(\omega)$ is the structure function for electroproduction off pions. Since R is not constant at SPEAR energies, \overline{F}_1^0 cannot scale there. However, the existence of the narrow resonances $^{15} \psi(3, 1)$ and $\psi'(3, 7)$ suggests, in our resonance dominance picture, that a threshold for production of a new class of resonances occurs at $\sqrt{s} \approx 3.5$ GeV. Assuming that the contribution from each class of resonances scales separately, and taking a square-root type effective threshold behavior for the new resonances, we get

$$\bar{F}_{1}^{0}(x,s) = \bar{F}_{10}^{0}(x) + \theta(M-3.5 \text{ GeV}) \left(1 - M^{2}/(3.5 \text{ GeV})^{2}\right)^{1/2} \bar{F}_{1N}^{0}(x) , \qquad (11)$$

where $\bar{F}_{10}^{o}(x)$ and $\bar{F}_{1N}^{o}(x)$ are the scaling functions for the "old" and "new" resonances, respectively. By (10), there exist electroproduction structure functions $F_{20}(\omega)$ and $F_{2N}(\omega)$ corresponding to $\bar{F}_{10}^{o}(x)$ and $\bar{F}_{1N}^{o}(x)$, and for these we take

$$F_{2O}(\omega) = C F_{2N}(\omega) = C_1 \frac{(\omega-1)}{\omega^2} , \qquad (12)$$

where C and C₁ are constants. The threshold behavior of (12) corresponds to a pion form factor which behaves as $F_{\pi}(t) \propto t^{-1}$ at large t. The ω^{-1} -type asymptotic behavior of (12) is that given by an effective Regge pole exchange with $\alpha_{eff}(0) = 0$. The highest-lying Regge trajectory (aside from the Pomeron) contributing to $\gamma\pi$ scattering is the P', ⁷ which probably has $0 \leq \alpha_{P'}(0) \leq 0.5$, so we consider (12) to represent the contribution of the resonances dual to <u>all</u> nondiffractive exchanges. Assuming (8) and (10) to apply to F_{2O} and F_{2N} separately, we have from (12)

$$\overline{F}_{10}^{0}(x) = C \overline{F}_{1N}^{0}(x) = C_{1}(1-x)/x$$
 (13)

Note that, in this case, (10) agrees with the simple crossing relation¹⁶

$$\bar{F}_{2}^{0}(x) = F_{2}(x)$$
 (14)

For resonance decay, we assume an isotropic distribution inspired by the thermodynamic $model^4$

$$E \frac{dN}{d^{3}p} = A e^{-bE} \text{ (rest frame)} . \tag{15}$$

The (m_R -dependent) constants A and b are determined by (5) and (6). In (5), we take

$$\bar{n}_{c}(m_{R}) = 1.9 + \ln m_{R}^{2}$$
, (16)

which is consistent with the charged multiplicity measured in $\overline{p}p$ collisions.¹⁷ For β , we take $\beta_{O}=0.3$ for the old resonances as is found in $\overline{p}p$ scattering,¹⁷ while β for the new resonances is left as a parameter β_{N} . Furthermore, for simplicity we average β for both old and new resonances above the new resonance threshold so that

$$\beta(m_{\rm R}^2) = \frac{C \beta_{\rm O}^{+} \theta(m_{\rm R}^{-} - 3.5 \text{ GeV}) \left(1 - m_{\rm R}^2 / (3.5 \text{ GeV})^2\right)^{1/2} \beta_{\rm N}}{C + \theta(m_{\rm R}^{-} - 3.5 \text{ GeV}) \left(1 - m_{\rm R}^2 / (3.5 \text{ GeV})^2\right)^{1/2}}$$
(17)

for all values of m_R^2 .

By using (15), (13), (11) and (9), we may apply (4) to the data¹ for $e^+e^- \rightarrow \pi^- X$ and determine the free parameters. Upon performing the integration in (4) numerically, we find

$$\beta_{\rm N} = 0.15$$

 $C = 1$ (18)
 $C_1 = 2.3$.

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Note that $\beta_N = 0.15$ corresponds to 70% of the new resonances' energy being carried away by neutral pions, compared with 40% for the old resonances. Assuming that isospin is conserved in the resonance decay, and that the resonances have I=1, then the upper bound for the energy carried by neutral pions is 18

$$< E_{\pi 0} > /E_{tot} < (9 + \sqrt{41})/20 \approx 0.77$$
 (19)

if the resonances are neutral. For charged resonances there is no upper bound.¹⁸ Therefore, the decay of the new resonances satisfies the bound (19), although why so much of the resonance energy goes into neutrals remains somewhat mysterious.

The resulting cross section, integrated over angles, is shown in Fig. 2. Note that the bare cross section constitutes ~10% of the contribution from resonance decay at small x, while at large x the bare contribution dominates. Most of the resonance decay contribution comes from resonances with $m_R^2 \approx s$, that is, resonances almost at rest in the c.m. system. Consequently, the decay contribution to $E d\sigma/d^3 p$ is almost exponential in E, with a slope close to the value of b in (15) for resonances with m_R^2 =s. The resulting two-component picture for e^+e^- annihilation is similar to current ideas about purely hadronic collisions.¹⁹ The "bare" cross section, dominant at large p, corresponds to "hard" scattering in high-p_T hadronic interactions, while the resonance decay contribution resembles low-p_T hadronic physics.

Since we have taken the scaling limit in (13), no resonance peaks appear in the solid lines in Fig. 2. Therefore, we cannot reproduce the "bulge" in $s d\sigma/dx$ seen in the data at 4.2 GeV. Harari has suggested²⁰ that this bulge, and the general restriction of scale-breaking to x < 1/2 is due to the production of pairs of equal mass particles at rest. Above threshold, the decay products would sometimes reach the region x > 1/2, but the cross section for production of pairs far above threshold might be suppressed. In our picture, the pions accompanying the new resonances do not reach $x \ge 1/2$ until $\sqrt{s} \ge \sqrt{2 s_{th}} \approx 4.9$ GeV; however,

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the resulting scaling violation is masked somewhat by the presence of resonance decay products for $x \ge 1/2$.

We now turn to the angular distribution in our model. Since the bare cross section gives an angular distribution proportional to $1 + \cos^2 \theta$, while the resonance decay contribution is isotropic within 10%, the overall angular distribution is of the form $1 + \alpha(x, s) \cos^2 \theta$, where $0 \le \alpha \le 1$. In Fig. 3, we show our prediction for α plotted with data²¹ from SPEAR at 7.4 GeV. The qualitative shape of the distribution at 7.4 GeV is reproduced by the predicted curve (solid line), although it is too low for low values of x. The prediction for 3.0 GeV (dashed line) is considerably more isotropic at high x, while the 15 GeV curve (dashed-dotted line) shows little change from the 7.4 GeV curve.

To obtain the cross section for $e^+e^- \rightarrow X$, we use the sum rule¹²

$$\sigma_{\text{tot}}(s) = \frac{1}{2\beta_1(s)} \int_{2m_{\pi}/\sqrt{s}}^{1} x \frac{d\sigma}{dx} dx , \qquad (20)$$

where $\beta_1(s)$ is the fraction of e^+e^- energy carried away by negative pions. We take $\beta_1(s) = \beta(s)$, which should be a reasonable approximation for the overall process, to get the result for $R \equiv \sigma_{tot}/\sigma_{\mu}+\mu_{-}$ shown with the data¹ in Fig. 4. The predicted R continues to rise slightly through energies which will be available at PEP, and has the value 7.16 at $\sqrt{s} = 15$ GeV. Again, because we have taken the scaling limit for the contributions from the old and new resonances, the structure in R between 3.8 and 4.5 GeV is not reproduced. This is the region where the new resonances are beginning to be produced, so scaling violations might be expected here.

We conclude with several remarks:

(i) We note that in (12) we have neglected the diffractive contribution to electroproduction, which is expected to give $F_2(\omega) \rightarrow \text{const}$ as $\omega \rightarrow \infty$. Through

the Gribov-Lipatov relation, this term would imply $\overline{F}_1^0(x) \to \operatorname{const}/x^2$ as $x \to 0$, which is more singular at x=0 than (13). Since, in our model the resonance decay contribution dominates the bare contribution at small x, this effect may be hidden.

(ii) We have not discussed the nature and dynamics of the new resonances beyond the creation of a threshold in R. Should further classes of resonances be produced at higher energies, associated thresholds would appear, much as in the parton model. 2

(iii) Scale-breaking in $sd\sigma/dx$ at small x is due to the onset of production of the new resonances, and phase space available for pions from resonance decay. At large x, $sd\sigma/dx$ is approximately scale-invariant at energies up to 7.4 GeV, although there is a slight rise at the highest energies due to the possibility of producing the new resonances accompanied by high-x pions.

(iv) Evidence²² for jet structure in $e^+e^- \rightarrow hx$ is an important clue for the study of the dynamics of e^+e^- annihilation. Resonance models, such as ours, can lead to jets, ²³ although an investigation of such structure requires a more detailed picture than we have presented here.

(v) Since the major contribution to pion production comes from resonances almost at rest in the c.m. frame, the model is similar to vector dominance,⁶ in which resonances build up in the direct channel. However, some structure should remain in the missing-mass channel due to the contribution of Fig. 1a.

(vi) Finally, the inclusive data¹ are for production of all species of hadrons, and may be significantly contaminated by K's, p's, etc. at high x. This should be kept in mind when comparing the high-x behavior of our model with the data in Fig. 2.

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FIGURE CAPTIONS

- 1. Contributions to the process $e^+e^- \rightarrow \pi_i(p) X_i$ (a) π_i produced in conjunction with a resonance; (b) π_i resulting from resonance decay.
- 2. Data from Ref. 1 for s d σ /dx for charged pions plotted versus \sqrt{s} , where x = 2p/ \sqrt{s} . The solid lines are our fit, and the dashed lines are the "bare" contribution s d σ^{0} /dx.
- Data from Ref. 21 at √s = 7.4 GeV for α ≡ (σ_T σ_L)/(σ_T + σ_L) plotted versus x ≡ 2p/√s. Our predictions are for √s = 7.4 GeV (solid line),
 3.0 GeV (dashed line), and 15 GeV (dashed-dotted line).
- 4. Prediction for $R \equiv \sigma_{tot} / \sigma_{\mu^+\mu^-}$ plotted versus \sqrt{s} . The data are from Ref. 1.



Fig. 1

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Fig. 2



Fig. 3





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