# ANGULAR DEPENDENCE OF THE DIFFERENTIAL CROSS SECTION OF HADRONIC EXCLUSIVE PROCESSES AT ASYMPTOTIC ENERGIES* 

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#### Abstract

The angular dependence of the differential cross section is derived for meson-baryon and baryon-baryon elastic scattering and for some related processes, in the framework of the Constituent Interchange Model, assuming the validity of dimensional scaling laws and using the Mandelstam representation of the amplitude.


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[^0]The dimensional scaling laws

$$
\begin{equation*}
\frac{d \sigma}{d t}(a b \rightarrow c d) \sim s^{-n+2} f(t / s) \tag{1}
\end{equation*}
$$

for hadronic scattering at asymptotic energies and fixed angle derived by Brodsky and Farrar [1] and by Matveev, Muradyan, and Tavkhelidge [2] have been successfully confronted with experimental data. There have been some attempts recently to determine the angular dependence of the differential cross section in that same framework. We argue here that given some usual assumptions, the analytic properties of the amplitude determine its angular dependence. For that purpose we will assume with $T$. Uematsu [4] that the invariant amplitude obeys the Mandelstam Representation with subtraction. Some results related to the impulse approximation in the Constituent Interchange Model [3] will be used together with a requirement of nonexoticity. This paper is organized as follows:

In sect. 1 we present our method and emphasize the assumptions. Then we apply it to a simple case, namely the elastic scattering of two spinless particles. In sections 3 and 4 we consider the case of meson-baryon and baryon-baryon scattering taking carefully into account the spin of the fermions. Definite predictions are obtained in a number of cases. In sect. 5 we compare our results with other theoretical calculations.

## 1. Presentation of the Method

We will work in the framework of the Constituent Interchange Model [3]. Let us recall briefly the basic features of this approach that seems to describe successfully the physics of large transverse momentum processes: the baryons are described in the language of quark partons, the basic quark-quark interaction is governed by dimensionless coupling constants, and anomalous
dimensions are assumed to be absent. Furthermore, in hadronic reactions, processes of the quark interchange type (fig。1a) dominate those of the quark scattering (fig. 1b) type. For a more detailed description of the model and its phenomenological successes, we refer to the recent review of Sivers, Brodsky, and Blankenbecler [9]. Let us consider now an invariant amplitude A of a scattering process

$$
a+b \rightarrow c+d
$$

Defining as usual the Mandelstam variables $s$, $t$, and $u$ by

$$
\begin{equation*}
\left(p_{a}+p_{b}\right)^{2}=s \quad\left(p_{a}-p_{c}\right)^{2}=t \quad\left(p_{a}-p_{d}\right)^{2}=u \tag{2}
\end{equation*}
$$

we have in the high energy region, where all the masses can be neglected, the relations

$$
\begin{equation*}
\mathrm{t}=-(1-\mathrm{z}) \mathrm{s} / 2 \quad \mathrm{u}=-(1+\mathrm{z}) \mathrm{s} / 2 \tag{3}
\end{equation*}
$$

where $\mathrm{z}=\cos \theta_{\mathrm{c} . \mathrm{m}_{0}}$ 。 We will consider the fixed angle asymptotic region, $\mathrm{i}_{\mathrm{o}} \mathrm{e}_{\mathrm{o}}$,

$$
\begin{equation*}
s,-t,-u \rightarrow+\infty \quad t / s, u / s \text { fixed } \tag{4}
\end{equation*}
$$

We assume then $[1,2]$ that the amplitude scales with a power dependence in the energy

$$
\begin{equation*}
A=s^{-n} f(z) \tag{5}
\end{equation*}
$$

n being an integer. In asymptotically free theory, a logarithmic factor can modify this result. We will neglect here such modifications. Note that by continuity with the Regge domain $s \gg-t$, this dependence implies a finite asymptotic trajectory at $t \rightarrow-\infty$. Following T. Uematsu [4], we write that $A$ satisfies the Mandelstam Representation. Because of eq. (5), we can write it without any subtraction. In the spirit of the Constituent Interchange Model [3] the single dispersion terms, which correspond to quark-quark scattering, are not dominant so that we have

$$
\begin{gather*}
A(s, t, u)=\frac{1}{\pi^{2}} \iint d s^{\prime} d t^{\prime} \frac{\rho_{s t^{\prime}\left(s^{\prime}, t^{\prime}\right)}^{\left(s^{\prime}-s\right)\left(t^{\prime}-t\right)}+\frac{1}{\pi^{2}} \iint d t^{\prime} d u^{\prime} \frac{\rho_{t u}\left(t^{\prime}, u^{\prime}\right)}{\left(t^{\prime}-t\right)\left(u^{\prime}-u\right)}+}{}+ \\
+\frac{1}{\pi^{2}} \iint d u^{\prime} d s^{\prime} \frac{\rho_{u s}\left(u^{\prime}, s^{\prime}\right)}{\left(u^{\prime}-u\right)\left(s^{\prime}-s\right)} \tag{6}
\end{gather*}
$$

Assuming that we can expand eq. (6) in the powers of $1 / \mathrm{s}$ and pick out the leading terms compatible with eq. (5), leads to the determination of the angular dependence of the amplitude as [4]

$$
\begin{equation*}
s^{n} A=f_{s t}^{n}(z)+f_{t u}^{n}(z)+f_{u s}^{n}(z) \tag{7}
\end{equation*}
$$

with

$$
\begin{align*}
& \mathrm{f}_{\mathrm{st}}^{\mathrm{n}}(\mathrm{z})=\sum_{\mathrm{m}=0}^{\mathrm{n}-2}(-1)^{m+1} \mathrm{C}_{\mathrm{n}-2-\mathrm{m}, \mathrm{~m}}^{\mathrm{st}}(2 / 1-\mathrm{z})^{\mathrm{m}+1} \\
& \mathrm{f}_{\mathrm{tu}}^{\mathrm{n}}(\mathrm{z})=(-1)^{\mathrm{n}} \sum_{m=0}^{\mathrm{n}-2} \mathrm{C}_{\mathrm{n}-2-m, m}^{\mathrm{tu}}(2 / 1-\mathrm{z})^{\mathrm{n}-\mathrm{m}-1}(2 / 1+\mathrm{z})^{m+1}  \tag{8}\\
& \mathrm{f}_{\mathrm{us}}^{\mathrm{n}}(\mathrm{z})=\sum_{\mathrm{m}=0}^{\mathrm{n}-2}(-1)^{\mathrm{m}+1} C_{m, n-2-m}^{\mathrm{us}} \quad(2 / 1+\mathrm{z})^{m+1}
\end{align*}
$$

where we have noted

$$
\begin{align*}
& C_{\mathrm{nm}}^{\mathrm{st}}=\frac{1}{\pi^{2}} \iint \mathrm{dsdt} \mathrm{~s}^{\mathrm{n}} \mathrm{~m}^{\mathrm{m}} \rho_{\mathrm{st}}(\mathrm{~s}, \mathrm{t}) \\
& \mathrm{C}_{\mathrm{nm}}^{\mathrm{tu}}=\frac{1}{\pi^{2}} \iint \mathrm{dt} d \mathrm{du} \mathrm{t}^{\mathrm{n}} \mathrm{u}^{\mathrm{m}} \rho_{\mathrm{tu}}(\mathrm{t}, \mathrm{u})  \tag{9}\\
& \mathrm{C}_{\mathrm{nm}}^{\mathrm{us}}=\frac{1}{\pi^{2}} \iint \mathrm{duds} \mathrm{u}_{\mathrm{s}}^{\mathrm{m}} \rho_{\mathrm{us}}(\mathrm{u}, \mathrm{~s})
\end{align*}
$$

This result in itself leads to few predictions unless we make some other assumptions. They will be of two kinds, dealing with a requirement of
nonexoticity and with the connection to Regge regime: a) In the spirit of dual models, we will assume that the discontinuity of the amplitude in an exotic channel gives a nonleading contribution to the amplitude via the dispersion relation. This is a straightforward application of the concept of exoticity that has proven to be very efficient in the phenomenology of hadronic processes. b) We will assume that the effective asymptotic trajectories in the two regions of Regge behavior

$$
\begin{align*}
& s=-u \gg-t \gg m^{2}  \tag{10}\\
& s=-t \gg-u \gg m^{2} \tag{11}
\end{align*}
$$

obey in the case of the exchange of a bosonic system the relation obtained in the Constituent Interchange Model [5].

$$
\begin{equation*}
\alpha_{\mathrm{ac}}(-\infty)=\left(4-\mathrm{n}_{\mathrm{a}}-\mathrm{n}_{\mathrm{c}}-\mathrm{n}_{\mathrm{int}}\right) / 2 \tag{12}
\end{equation*}
$$

where $n_{a}$ and $n_{c}$ are the number of valence quarks in particles a and $c$ and $n_{\text {int }}$ is the minimum number of exchanged quarks compatible with the internal states. This relation is obtained assuming a smooth connection between fixed (but not small) $t$ and fixed angle regimes. However in the case of the exchange of a fermionic system this result is likely to be modified. We will only need to assume that this modification is not more than one unit. This can easily be shown to be equivalent, in the physical processes studied below, to the requirement that there is no fixed pole at $J=-1 / 2, i_{0} e_{0}$, that $\alpha_{a c}(-\infty) \neq-1 / 2$ 。 A modification of eq. (12) by more than one unit would be difficult to justify in the framework of the model. We will now illustrate this method in the simple case of the elastic scattering of two bosons. For definiteness we will study $\mathrm{K}^{+} \pi^{-}$ scattering。
2. Determination of the Differential Cross Section of $\mathrm{K}^{+} \pi^{-}$Elastic Scattering

Dimensional scaling tells us that in that case the amplitude will be of the form

$$
\begin{equation*}
A=s^{-2} f(z) \tag{13}
\end{equation*}
$$

Because in this reaction the $u$ channel process is exotic, we write eq. (7) and (8) as

$$
\begin{equation*}
A=s^{-2} f_{s t}^{2}(z)=\alpha s^{-2} /(1-z) \tag{14}
\end{equation*}
$$

In other words, the amplitude is of the form $\alpha /$ st. The differential cross section can then be written as

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{+} \pi^{-} \rightarrow \mathrm{K}^{+} \pi^{-}\right)=\frac{\beta}{\mathrm{s}^{4} \mathrm{t}^{2}}=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{+} \pi^{-} \rightarrow \mathrm{K}^{+} \pi^{-}\right)\right|_{90^{\circ}} \frac{1}{(1-\mathrm{z})^{2}} \tag{15}
\end{equation*}
$$

The asymptotic trajectory is seen to be equal to -1 for the region given by eq. (10) and -2 for the region given by eq. (11) in accordance with eq. (12). In this example our assumptions seem to be redundant. However, that will not be the case in the more interesting processes studied thereafter.
3. Study of Meson-Baryon Scattering

The complete amplitude of the scattering process

$$
\mathrm{M}_{1}+\mathrm{B}_{1} \rightarrow \mathrm{M}_{2}+\mathrm{B}_{2}
$$

can be written as [6]

$$
\begin{equation*}
\mathscr{M}=\bar{u}\left(\mathrm{~B}_{2}\right)(\mathrm{A}+\mathrm{B} \gamma \mathrm{Q}) \mathrm{u}\left(\mathrm{~B}_{1}\right) \tag{16}
\end{equation*}
$$

where $Q$ is the sum of the 4 -momenta of $M_{1}$ and $M_{2}$. It can be assumed, if we have in mind an underlying field theory with $\gamma_{5}$ in variance, that in the limit
of asymptotic energies the helicity of the baryon is conserved so that A can be neglected. The differential spin-averaged cross section is then proportional to

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}} \propto-\frac{\mathrm{us}}{\mathrm{~s}^{2}}|\mathrm{~B}|^{2} \tag{17}
\end{equation*}
$$

We will now apply our method to the reduced amplitude $B$, the energy dependence of which is, following eq. (1),

$$
B=s^{-4} f(z)
$$

a. $\mathrm{K}^{+} \mathrm{p}$ and $\mathrm{K}^{-} \mathrm{p}$ elastic scattering

The process $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \mathrm{p}$ is s - channel exotic so that eq. (7) and eq. (8) simplify to

$$
\begin{aligned}
& B=s^{-4} f_{t u}(z) \\
& f_{t u}(z)=\frac{16 \alpha}{(1+z)^{3}(1-z)}+\frac{16 \beta}{(1+z)^{2}(1-z)^{2}}+\frac{16 \gamma}{(1+z)(1-z)^{3}}
\end{aligned}
$$

In other words, $B$ can be written as

$$
\begin{equation*}
B=\frac{\alpha}{u^{3} t}+\frac{\beta}{u^{2} t^{2}}+\frac{\gamma}{u t^{3}} \tag{18}
\end{equation*}
$$

We now use our argument about the asymptotic trajectory in the Regge regime. Using that, in the region $-t \ll s, \alpha{ }_{\mathrm{K}^{+} \mathrm{K}^{+}}{ }^{(-\infty)}=-1$, as given by eq. (11), and that in the crossed region $\alpha_{K^{+}}{ }^{\prime}(-\infty) \neq-1 / 2$, we require that $\alpha$ and $\gamma$ vanish so that the angular dependence of the differential cross section is completely determined as

$$
\begin{equation*}
\frac{d \sigma}{d t}\left(K^{+} p \rightarrow K^{+} p\right)=-\frac{\sigma_{0} u}{s t^{4} u^{4}}=\left.\frac{d \sigma}{d t}\left(K^{+} p \rightarrow K^{+} p\right)\right|_{90^{\circ}} \frac{1+z}{\left(1-z^{2}\right)^{4}} \tag{19}
\end{equation*}
$$

$$
\frac{d \sigma}{d t}\left(K^{+} p \rightarrow K^{+} p\right)=-\frac{\sigma_{0}{ }^{\mathrm{u}}}{s t^{4} u^{4}}=\left.\frac{d \sigma}{d t}\left(K^{+} p \rightarrow K^{+} p\right)\right|_{90^{o}} \frac{1+z}{\left(1-z^{2}\right)^{4}}
$$

The process $\mathrm{K}^{-} \mathrm{p} \rightarrow \mathrm{K}^{-} \mathrm{p}$ can be easily obtained from $\mathrm{s}-\mathrm{u}$ crossing and one gets:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{-} \mathrm{p} \rightarrow \mathrm{~K}^{-} \mathrm{p}\right)=\frac{-\sigma_{0}^{\mathrm{u}}}{\mathrm{~s}^{5} \mathrm{t}^{4}}=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{-} \mathrm{p} \rightarrow \mathrm{~K}^{-} \mathrm{p}\right)\right|_{90^{\circ}} \frac{1+\mathrm{z}}{(1-\mathrm{z})^{4}} \tag{20}
\end{equation*}
$$

Moreover one obtains the relation

$$
\begin{equation*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{+} \mathrm{p} \rightarrow \mathrm{~K}^{+} \mathrm{p}\right)\right|_{90^{\circ}}=\left.16 \frac{\mathrm{~d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{-} \mathrm{p} \rightarrow \mathrm{~K}^{-} \mathrm{p}\right)\right|_{90^{\circ}} \tag{21}
\end{equation*}
$$

One can deduce of these results the form of the differential cross section for the annihilation process $p \bar{p} \rightarrow K \bar{K}$ by an additional $s-t$ crossing. One obtains

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{~K} \overline{\mathrm{~K}})=\frac{\sigma_{0} \mathrm{u}}{2 \mathrm{~s}^{6} \mathrm{t}^{3}}=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{~K} \overline{\mathrm{~K}})\right|_{90^{\circ}} \frac{1+\mathrm{z}}{(1-\mathrm{z})^{3}} \tag{22}
\end{equation*}
$$

together with the relation

$$
\begin{equation*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{-} \mathrm{p} \rightarrow \mathrm{~K}^{-} \mathrm{p}\right)\right|_{90^{\circ}}=\left.4 \frac{\mathrm{~d} \sigma}{\mathrm{dt}}\left(\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}\right)\right|_{90^{\circ}} \tag{23}
\end{equation*}
$$

b. $\quad \pi-\mathrm{p}$ scattering

We will study here the elastic scattering of $\pi^{+}$and $\pi^{-}$on a proton together with the charge exchange reaction $\pi^{-} p \rightarrow \pi^{\circ} n$. Because of isospin invariance the latter can be deduced from the two first reactions by the following relation between the amplitudes

We will use in this section one more assumption related to the impulse approximation, namely that the contribution of $f_{u s}$ as defined in eq. (7) and eq. (8) can be neglected. The argument is as follows: Neglecting spin effects, the process $a+b \rightarrow c+d$ can be visualized as in Fig. 2. In the impulse approximation, only one constituent can be interchanged between the meson and the baryon. However the presence of the two spectators in the baryon is reflected in the physical amplitude by a form factor $\mathrm{F}_{\mathrm{bd}}{ }^{(t)}$ such that:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{ab} \rightarrow \mathrm{~cd})=\frac{\mathrm{d} \boldsymbol{\sigma}}{\mathrm{dt}}(\mathrm{aq} \rightarrow \mathrm{cq}) \quad\left|\mathrm{F}_{\mathrm{bd}}(\mathrm{t})\right|^{2} \tag{25}
\end{equation*}
$$

The dimensional counting technique tells us that that the spin averaged form factor $F_{b d}(t)$ behaves as $t^{-2}$. In simple models $\frac{d \sigma}{d t}(a q \rightarrow c q)$ does not depend on $t$ and we can thus reasonably assume that this quantity does not vanish faster than $t$ as $t \rightarrow 0$, so that $f_{u s}(z)$ which is the only non-singular part with respect to $t$ in the amplitude can be neglected. The resulting expression for the elastic scattering of $\pi^{+} p$ is then

$$
\begin{equation*}
\mathrm{B}^{+}=\frac{\alpha_{1}}{\mathrm{st}^{3}}+\frac{\alpha_{2}}{\mathrm{~s}^{2} \mathrm{t}^{2}}+\frac{\alpha_{3}}{\mathrm{~s}^{3} \mathrm{t}}+\frac{\beta_{1}}{\mathrm{ut}^{3}}+\frac{\beta_{2}}{\mathrm{u}^{2} \mathrm{t}^{2}}+\frac{\beta_{3}}{\mathrm{u}^{3} \mathrm{t}} \tag{26}
\end{equation*}
$$

and for $\pi^{-} p$ elastic scattering, the $s-u$ crossed expression

$$
\begin{equation*}
\mathrm{B}^{-}=\frac{\beta_{1}}{\mathrm{st}^{3}}+\frac{\beta_{2}}{\mathrm{~s}^{2} \mathrm{t}^{2}}+\frac{\beta_{3}}{\mathrm{~s}^{3} \mathrm{t}}+\frac{\alpha_{1}}{\mathrm{ut}^{3}}+\frac{\alpha_{2}}{\mathrm{u}^{2} \mathrm{t}^{2}}+\frac{\alpha_{3}}{\mathrm{u}^{3} \mathrm{t}} \tag{27}
\end{equation*}
$$

Imposing the asymptotic trajectory to be $\alpha_{\pi}(-\infty)=-1$ and $\alpha_{\pi p}(-\infty) \neq-\frac{1}{2}$, one obtains $\alpha_{1}-\beta_{1}=0$ and $\alpha_{3}=\beta_{3}=0$ so that the following expressions for the differential cross sections hold

$$
\begin{align*}
& \frac{d \sigma}{d t}\left(\pi^{+} \mathrm{p} \rightarrow \pi^{+} \mathrm{p}\right)=-\frac{\sigma_{0}^{\mathrm{u}}}{\mathrm{st}^{4}}\left|\frac{\alpha_{1}}{\mathrm{st}}+\frac{\alpha_{2}}{\mathrm{~s}^{2}}+\frac{\alpha_{1}}{\mathrm{ut}}+\frac{\beta_{2}}{\mathrm{u}^{2}}\right|^{2}  \tag{28}\\
& \frac{d \sigma}{d \mathrm{t}}\left(\pi^{-} \mathrm{p} \rightarrow \pi^{-} \mathrm{p}\right)=-\frac{\sigma_{0}{ }^{\mathrm{u}}}{\mathrm{st}^{4}}\left|\frac{\alpha_{1}}{\mathrm{st}}+\frac{\beta_{2}}{\mathrm{~s}^{2}}+\frac{\alpha_{1}}{\mathrm{ut}}+\frac{\alpha_{2}}{\mathrm{u}^{2}}\right|^{2} \tag{29}
\end{align*}
$$

These results are not very predictive but,using eq. (24), one deduces the following form for the charge exchange case

$$
\begin{align*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\pi^{-} \mathrm{p} \rightarrow \pi^{\mathrm{o}} \mathrm{n}\right) & =-\frac{\sigma_{1} \mathrm{u}}{\mathrm{st}^{4}}\left(\frac{1}{\mathrm{~s}^{2}}-\frac{1}{\mathrm{u}^{2}}\right)^{2} \\
& =\left.\frac{1}{9} \frac{\mathrm{~d} \sigma}{\mathrm{dt}}\left(\pi^{-} \mathrm{p} \rightarrow \pi^{\circ} \mathrm{n}\right)\right|_{90^{\circ}} \frac{1+\mathrm{z}}{(1-\mathrm{z})^{4}}\left(1-\frac{4}{(1+\mathrm{z})^{2}}\right)^{2} \tag{30}
\end{align*}
$$

Using $\operatorname{SU}(3)$ symmetry, we can simplify eqs. (28) and (29), because the amplitude is then a linear combination of the amplitudes for $\mathrm{K}^{+} \mathrm{p}$ and $\mathrm{K}^{-} \mathrm{p}$ elastic scattering so that $\alpha_{1}$ vanishes.

## 4. Baryon - Baryon Scattering

We now turn to the case of baryon-baryon scattering. In that case the matrix element can be written as the sum of five independent amplitudes. However, because of helicity conservation at asymptotic energies, we can retain only two of these amplitudes, namely the vector - vector and axial vector-axial vector ones. Dealing with the process

$$
\mathrm{B}_{1}\left(\mathrm{p}_{1}\right)+\mathrm{B}_{2}\left(\mathrm{p}_{2}\right) \longrightarrow \mathrm{B}_{3}\left(\mathrm{p}_{3}\right)+\mathrm{B}_{4}\left(\mathrm{p}_{4}\right)
$$

one thus writes the matrix element as
$\mathscr{M}=\mathrm{V} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{\mu} \mathrm{u}\left(\mathrm{p}_{1}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right)+\mathrm{A} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{5} \gamma_{\mu} \mathrm{u}\left(\mathrm{p}_{1}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma_{5} \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right)$
in the case where the two baryons are distinct, and as the antisymmetrized form.
$\mathscr{M}=\mathrm{V} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{\mu} \mathrm{u}\left(\mathrm{p}_{1}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right)+\mathrm{A} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{5} \gamma_{\mu}^{\gamma} \mathrm{u}\left(\mathrm{p}_{1}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma_{5} \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right)$
$-\widetilde{\mathrm{V}} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{1}\right)-\tilde{\mathrm{A}} \overline{\mathrm{u}}\left(\mathrm{p}_{3}\right) \gamma_{5} \gamma_{\mu} \mathrm{u}\left(\mathrm{p}_{2}\right) \overline{\mathrm{u}}\left(\mathrm{p}_{4}\right) \gamma_{5} \gamma^{\mu} \mathrm{u}\left(\mathrm{p}_{1}\right)$
in the case where the two initial (and final) baryons are identical, where we have noted

$$
\begin{equation*}
\tilde{\mathrm{V}}(\mathrm{~s}, \mathrm{t}, \mathrm{u})=\mathrm{V}(\mathrm{~s}, \mathrm{u}, \mathrm{t}) \quad \widetilde{\mathrm{A}}(\mathrm{~s}, \mathrm{t}, \mathrm{u})=\mathrm{A}(\mathrm{~s}, \mathrm{u}, \mathrm{t}) \tag{33}
\end{equation*}
$$

a. $\mathrm{pp} \rightarrow \mathrm{pp}$

We take the matrix element given by eq. (32) and square it. We obtain that the differential cross section can be written as

$$
\begin{equation*}
\frac{d \sigma}{d t}(p p \rightarrow p p)=\frac{\alpha}{s^{2}}\left[s^{2}|V+A+\widetilde{V}+\widetilde{A}|^{2}+u^{2}|v-A|^{2}+t^{2}|\tilde{V}-\widetilde{A}|^{2}\right] \tag{34}
\end{equation*}
$$

The dimensional counting rules lead us to the power dependence upon the energy of $V$ and $A$ as

$$
V=s^{-5} f_{V}(z) \quad A=s^{-5} f_{A}(z)
$$

The s-channel exchange being exotic eqs. (7) and (8) simplify as

$$
\begin{equation*}
\mathrm{V}=\mathrm{s}^{-5} \mathrm{f}_{\mathrm{tu}}^{\mathrm{V}}(\mathrm{z}) \quad \mathrm{A}=\mathrm{s}^{-5} \mathrm{f}_{\mathrm{tu}}^{\mathrm{A}}(\mathrm{z}) \tag{35}
\end{equation*}
$$

$$
\begin{align*}
& V=\frac{\alpha_{0}}{t u^{4}}+\frac{\alpha_{1}}{t^{2} u^{3}}+\frac{\alpha_{2}}{t^{3} u^{2}}+\frac{\alpha_{3}}{t^{4} u} \\
& A=\frac{\beta_{0}}{t u^{4}}+\frac{\beta_{1}}{t^{2} u^{3}}+\frac{\beta_{2}}{t^{3} u^{2}}+\frac{\beta_{3}}{t^{4} u} \tag{36}
\end{align*}
$$

The asymptotic trajectory, as given by eq。(12), being $\alpha(-\infty)=-2$ for the two Regge regions (10) and (11) leads to great simplification, finally giving

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{pp} \rightarrow \mathrm{pp})=\frac{\sigma_{0}}{\mathrm{~s}^{2} \mathrm{t}^{4} \mathrm{u}^{4}}=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{pp} \rightarrow \mathrm{pp})\right|_{90^{\circ}} \frac{1}{\left(1-\mathrm{z}^{2}\right)^{4}} \tag{37}
\end{equation*}
$$

## b. $p \bar{p} \rightarrow p \bar{p}$

That case is easily deduced from the preceding calculation by s-u crossing. We obtain

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{p} \overline{\mathrm{p}})=\frac{\sigma_{0}}{\mathrm{~s}^{6} \mathrm{t}^{4}}=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\overline{\mathrm{p}} \rightarrow \mathrm{p} \overrightarrow{\mathrm{p}})\right|_{90^{\circ}} \frac{1}{(1-\mathrm{z})^{4}} \tag{38}
\end{equation*}
$$

together with the relation

$$
\begin{equation*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{pp} \rightarrow \mathrm{pp})\right|_{90^{\circ}}=\left.16 \quad \frac{\mathrm{~d} \sigma}{\mathrm{dt}}(\mathrm{pp} \rightarrow \mathrm{p} \overline{\mathrm{p}})\right|_{90^{\circ}} \tag{39}
\end{equation*}
$$

c. $n p \rightarrow n p$

We have to use the unsymmetrized form given by eq. (31). The differential cross section is then given by

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{np} \rightarrow \mathrm{np})=\frac{\alpha}{\mathrm{s}^{2}}\left(\mathrm{~s}^{2}|\mathrm{~V}+\mathrm{A}|^{2}+\mathrm{u}^{2}|\mathrm{~V}-\mathrm{A}|^{2}\right) \tag{40}
\end{equation*}
$$

Expanding V and A in the form given by eq。(36) and using the result given by eq. (12) for the two Regge regions (10) and (11), namely that $\alpha(-\infty)$ is equal
to -2 ，the expression simplifies so that the differential cross section has the same angular dependence as in the pp case

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{np} \rightarrow \mathrm{np})=\left.\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{np} \rightarrow \mathrm{np})\right|_{90^{\circ}} \quad \frac{1}{\left(1-\mathrm{z}^{2}\right)^{4}} \tag{41}
\end{equation*}
$$

## 5．Summary and Conclusions

We summarize in tables 1 and 2 our main results together with earlier pre－ dictions obtained in that same framework of the Constituent Interchange Model． Greek letters represent adjustable parameters．In ref．［3］Gunion et al。do not assume any analytic properties for the amplitude．They neglect exotic channel contributions as we do．In ref．［7］Freund and Nandi take advantage of the compatibility of scaling and Regge behavior in a similar manner as in our method．They however do not include spin considerations in their analysis， which renders it doubtful in the case of baryon－baryon scattering．In ref．［8］ Matveev，Maradyan，and Tavkhelidze make extensive use of naive quark count－ ing in the $\operatorname{SU}(3)$ limit．We show in figs。 3 and 4 comparisons between theoret－ ical predictions and experimental measurements of the angular dependence of the differential cross section in the case of elastic $\mathrm{K}^{+} \mathrm{p}$ and pp scattering。 The agreement is quite good for $\mathrm{K}^{+} \mathrm{p}$ but in the latter case a distribution of the form $1 /\left(1-z^{2}\right)^{6}$ seems to be favored．Higher energy data are clearly needed for these reactions，together with $\pi p$ charge exchange and $n p$ elastic scattering， before any conclusive distinction can be drawn．As a final remark let us note that our results show that in the case where a fermion is exchanged the values obtained using eq．（12）in ref．［5］for the asymptotic trajectories must be cor－ rected by an amount of half a unit giving in the same notation

$$
\begin{gather*}
\alpha_{\pi \mathrm{p}}(-\infty)=\alpha_{K_{p}}^{+}(-\infty)=-3 / 2 \\
\alpha_{K-p}(-\infty)=-7 / 2 \tag{42}
\end{gather*}
$$

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## References

［1］S．J．Brodsky and G．R．Farrar，Phys．Rev．Letters 31 （1973）1153； Phys．Rev．D 11 （1975）1309。
［2］V．Matveev，R．Muradyan，and A．Tavkhelidge，Nuovo Cimento Letters 7 （1973） 779.
［3］J．F．Gunion，S．J．Brodsky，and R．Blankenbecler，Phys．Rev．D 8 （1973） 287.
［4］T．Uematsu，Kyoto University Preprint（1975），to be published in Progr． Theor．Phys．
［5］R．Blankenbecler and S．J．Brodsky，Phys．Rev．D 10 （1974）2973；D．D． Coon，J．F．Gunion，J．Tran Thanh Van，and R．Blankenbecler，SLAC－PUB－ 1483.
［6］G．F．Chew，M．L．Goldberger，F．E．Low，and Y．Nambu，Phys．Rev． 106 （1957）1337。
［7］P．G．O．Freund and S．Nandi，Nuovo Cimento 25A（1975） 395.
［8］V．Matveev，R．Muradyan，and A．Tavkhelidge，JINR Preprint E2－8048 （1974）。
［9］D．Sivers，S．J．Brodsky，and R．Blankenbecler，Phys．Reports 23C （1976）1．This paper contains other references to related publications．
［10］The $\operatorname{SU}(3)$ factors as calculated by R．Pearson（cited in ref．［9］）lead， however，to different predictions for $\pi p$ scattering，namely，that the form of $\mathrm{d} \sigma / \mathrm{dt}\left(\pi^{+} \mathrm{p} \rightarrow \pi^{+} \mathrm{p}\right.$ ）depends on an unknown parameter and that the charge exchange process shows the same angular distribution as derived by our method．
［11］A．Eide et al．，Nucl．Phys．B60（1973）173．
［12］J．V．Allaby et al．，Phys．Letters 25B（1967） 156.
［13］$I(z)$ is a function whose precise form is not predicted．

## Figure Captions

1. Examples of quark interchange (a) and quark scattering (b) processes for the description of elastic scattering at large momentum transfer.
2. The factorization of the form factor of the target as seen in the Constituent Interchange Model.
3. Angular distribution of $\mathrm{K}^{+} \mathrm{p}$ elastic scattering. Experimental data are from ref.[11] ( $s=10.55 \mathrm{GeV}^{2}$ ). The curve is our prediction (eq. (19)).
4. Angular distribution of pp elastic scattering. Experimental data are from ref. [12] ( $s=33.5 \mathrm{GeV}^{2}$ and $38 \mathrm{GeV}^{2}$ ). The full line represents our prediction (eq. (37)), the broken line the prediction of ref. [7].
Table 1
Angular Dependence of $\mathrm{d} \sigma / \mathrm{dt}$

| Process | Our alalalation | Ref. 3 | Ref. 7 | Reef. 8 (10) |
| :---: | :---: | :---: | :---: | :---: |
| $K^{\prime} \mathrm{p}-\mathrm{K}_{\mathrm{p}}^{\prime}$ | $\frac{1+2}{(1-2)^{4}}$ | $\frac{1+2{ }^{(1-2)^{4}}}{}$ | $\frac{1+2}{(1-2)^{4}}$ | $\frac{1+2}{(1-2)^{4}}$ |
| $\pi^{+} p \rightarrow \pi_{p}^{+}$ | $\frac{1+2,}{(1-2)^{4}}{ }^{\left[\frac{a}{1+2}+\frac{2 \alpha_{2}}{1-2^{2}}\left(\frac{\alpha_{3}}{(1+2)^{2}}\right]\right.}$ | $\frac{1+2}{\left.(1-2)^{2}\right)^{[ }\left[\frac{\left[a+\left(\frac{\beta}{2}\right.\right.}{(1+2)^{2}}\right]^{2}}$ |  | $\frac{1+2}{(1-2)}\left(1+\frac{8}{(1+2)^{2}}\right)^{2}$ |
| $\pi^{-p}-z^{\circ} \mathrm{n}$ | $\frac{1+z}{\left.\left.(1-z)^{4}\right)^{( }-\frac{4}{(1+2)^{2}}\right)^{2}}$ | $\frac{1+2,}{\left.(1-2)^{2}\right)^{(1+}}\left(\frac{4}{(1+2)^{2}}\right)^{2}$ |  | $\frac{1+2 x^{2}}{\left(1-22^{2}\right.}\left(1+\frac{4}{(1+2)^{2}}\right)^{2}$ |
| ${ }^{p q-p p}$ | $\frac{1}{\left(1-22^{2}\right)^{4}}$ | $\frac{1}{\left.(1-2)^{2}\right)^{1}}{ }^{1(2)[13]}$ | $\frac{1}{\left(1-z^{2}\right)^{6}}$ | $\frac{1}{\left(1-z^{2}\right)}\left[\text { [ }\left[(x+2)^{2}+8(1-2)\right)^{2}\right]^{2}+(z-2)$ |
|  |  |  |  |  |
| $\mathrm{mp}_{\mathrm{n}} \mathrm{np}$ | $\frac{1}{\left(1-22^{2}\right)^{4}}$ |  |  |  |

Table 2
Ratios at $90^{\circ}$ of the Differential Cross Section

| Ratios | This <br> Calculation | Ref。3 | Ref。7 | Ref。8 | Exp。 <br> Value |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\frac{\mathrm{pp} \rightarrow \mathrm{pp}}{\mathrm{pp} \rightarrow \mathrm{pp}}$ | 16 | $*$ | 1024 | 16 | $\sim 100$ |
| $\frac{\mathrm{~K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{+} \mathrm{p}}{\mathrm{K}_{\mathrm{p}}^{+} \rightarrow \mathrm{K}^{-} \mathrm{p}}$ | 16 | 16 | 16 | 16 | $\sim 3$ |


(a)

(b)


Fig. 1


Fig. 2


Fig. 3


Fig. 4


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