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WEAK NEUTRAL CURRENTS IN $e^+ - e^-$ ANNIHILATION*

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ABSTRACT

The differential cross section of $e^+e^- \to F^+F^-$ ($F^+ = \mu^+, p, \pi^+$, etc.) is calculated for arbitrarily polarized incident and outgoing beams by exchanging one photon and one neutral Z-meson coupled to neutral weak currents. The interference effects manifest themselves in significant parity violating asymmetries for longitudinal polarizations which are about 4 (4.5)% at q = 28 GeV, 9 (12.5)% at q = 40 GeV for $p\bar{p}$ ($\pi^+\pi^-$) in the Weinberg-Salam model at $\sin^2\Theta_W = 0.38$. The relative changes of rates are about 7-15% at these energies.

It is shown that the Z-meson causes only unessential effects in the two-photon processes at high energies and arbitrary polarizations; furthermore, the pure two-photon contributions are negligibly influenced by including transverse polarizations.

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I. INTRODUCTION

By now significant azimuthal asymmetries have been observed at the center-of-mass energy 7.4 GeV of SPEAR II both[1] in $e^+e^- \to e^+e^-$ and $\mu^+\mu^-$ and inclusive hadron production[2] indicating strong transverse beam polarizations.[3] Probably, for the next generation of storage rings, the problems of creating longitudinally polarized circulating beams will be solved.[4] Longitudinally polarized beams are very useful for finding the parity violating effects of neutral weak currents.

Although the existence of neutral weak currents is well established, many of their properties are unknown, leading also to several predictions in e^+-e^- collisions.[5,6] In the present note the differential cross section of $e^+e^- \to F^+F^-$ is calculated for arbitrarily polarized incoming and outgoing beams by exchanging one γ and one Z, providing a unified treatment for $F^+ = \mu^+$, baryons, and spin-0 mesons. Mass terms are neglected in the cross section. In particular, P, C, T violating asymmetries are discussed, which are quite meaningful at higher energies as is shown in the Weinberg-Salam model; their measurements would provide valuable information on the relevant couplings and form factors of neutral weak currents, respectively[7] (Section II).

In Section III the influence of Z is discussed in the two-photon process $e^+e^- \to e^+e^-F^+F^-$ with polarized incident beams. It turns out that the shift of the $\gamma\gamma$-contribution due to transverse polarizations and the contributions of Z are both negligible.

II. WEAK EFFECTS IN $e^+e^- \to F^+F^-$

We denote the four-momenta of e^-, e^+, F^-, F^+ by $p_{i\mu}$, $i=1,\ldots,4$ and the corresponding three-vectors representing the rest-frame spin directions by $\vec{n}_i$, $i=1,\ldots,4$. The magnitudes of the polarizations for $e^\pm$ are included into $\vec{n}_{1,2}$. For the sake of simplicity we neglect the masses of $e^\pm$, $F^\pm$ in the cross section.

The differential cross section of $e^+e^- \to F^+F^-$ can be written as $\alpha^2 |M_\gamma + M_Z|^2 (4e^4 q^2)^{-1}$ where $q_\mu = p_{3\mu} + p_{4\mu}$ and the amplitudes M_γ, M_Z are due to one photon and one neutral Z-meson exchanges, respectively. In the Bjorken-Drell conventions[8] M_γ and M_Z are given by

$$M_\gamma = -\frac{ie^2}{q^2} \langle F^+F^- | J^\mu_\gamma(0) | 0 \rangle \, \bar{v}(p_2)\, \gamma_\mu (p_1) \quad , \tag{1}$$

$$M_Z = -ig_Z^2 \, \frac{g_{\mu\nu} - q_\mu q_\nu / m_Z^2}{q^2 - m_Z^2} \langle F^+F^- | J^\mu_Z(0) | 0 \rangle \, \bar{v}(p_2)\, \gamma^\nu \left(g_V + \gamma_5 g_A\right) u(p_1) \quad . \tag{2}$$

Here g_Z means the coupling constant of the (hermitian) neutral weak hadronic current J^μ_Z to Z, while $g_Z g_V$ and $g_Z g_A$ are the corresponding real leptonic coupling constants. The remaining matrix elements have the following general covariant decompositions[9]

$$\langle F^+F^- | J^\mu_\gamma(0) | 0 \rangle = \bar{u}(p_3) \left(\gamma^\mu G + (p_3 - p_4)^\mu F\right) v(p_4) \quad , \tag{3}$$

$$\langle F^+F^- | J^\mu_Z(0) | 0 \rangle = \bar{u}(p_3) \Big[\gamma^\mu G_V + (p_3 - p_4)^\mu F_V + q^\mu H_V + \\ + \left(\gamma^\mu G_A + (p_3 - p_4)^\mu F_A + q^\mu H_A\right) \gamma_5 \Big] v(p_4) \quad . \tag{4}$$

All the form factors are complex functions of q^2, G and F can be expressed by the usual magnetic and electric form factors G_M, G_E

$$G = G_M \quad , \qquad F = 2m_F (G_M - G_E) \left(q^2 - 4m_F^2\right)^{-1} \quad . \tag{5}$$

G_A, H_V, H_A measure the nonconservation of J^μ_Z; H_V, H_A, G_A, F_A characterize the nonelectromagnetic part of J^μ_Z. Assuming J^μ_Z is a first class current, T-invariance would require $F_A = H_V = 0$.

If the e^--beam is incident in the positive z-direction, $\vec{p}_1 = (0, 0, q/2)$ and in the natural center-of-mass system of the storage ring $\vec{p}_2 = -\vec{p}_1$, as well as

$\vec{p}_4 = -\vec{p}_3$. The polar and azimuthal angles of $\vec{p}_3$ with respect to $\vec{p}_1$ are denoted by θ and ϕ, respectively.

In order to present a convenient structure in the differential cross section, we introduce the following quantities

$$\vec{p}_\pm = O_3\left[(n_3 n_4 \pm n_4 n_3)\,\vec{p}_3\right] \quad ,$$

$$\vec{n}_i' = \vec{n}_i - \frac{4}{q^2}\,(\vec{p}_i \vec{n}_i)\,\vec{p}_i \equiv O_i \vec{n}_i \quad , \qquad i = 1, \ldots, 4$$

$$\tau_i = \begin{cases} 1 + \dfrac{4}{q^2}\,(\vec{p}_i \vec{n}_i)(\vec{p}_i \vec{n}_{i+1}) \quad , & i = 1, 3 \\[2ex] \dfrac{2}{q}\left((\vec{n}_i + \vec{n}_{i-1})\,\vec{p}_{i-1}\right) \quad , & i = 2, 4 \end{cases} \tag{6}$$

$$A_i = (\vec{n}_i' \vec{n}_{i+1}') - (n_i' n_{i+1}' + n_{i+1}' n_i') \quad , \qquad i = 1, 3 \quad ,$$

the second term of the matrix A_i abbreviates the symmetric matrix product of $\vec{n}_i'$ and $\vec{n}_{i+1}'$. We need also the three-vectors

$$\vec{d}_{1,2} = \left(a\,|F|^2,\ 2f\,\mathrm{Re}(F^* F_V),\ t\left(|F_V|^2 \pm |F_A|^2\right)\right) ,$$

$$\vec{d}_3 = 2\mathrm{Im}(0, f F^* F_A,\ t F_V^* F_A) \quad ,$$

$$\vec{d}_4 = 2\mathrm{Re}\left(aFG^*,\ f(FG_V^* + F_V G^*),\ t F_V G_V^*\right) \quad ,$$

$$\vec{d}_5 = 2\mathrm{Im}(0, 0, t F_A G_A^*) \quad ,$$

$$\vec{d}_6 = 2\mathrm{Re}(0, f F G_A^*,\ t F_V G_A^*) \quad ,$$

$$\vec{d}_7 = 2\mathrm{Im}(0, f F_A G^*,\ t F_A G_V^*) \quad ,$$

$$\vec{g}_1 = \left(1, g_V, g_V^2 - g_A^2\right) ,$$

$$\vec{g}_2 = \left(\tau_1, \tau_1 g_V + \tau_2 g_A,\ \tau_1\left(g_V^2 + g_A^2\right) + 2\tau_2 g_V g_A\right) \quad ,$$

$$\vec{g}_3 = \left(\tau_2, \tau_1 g_A + \tau_2 g_V,\ 2\tau_1 g_V g_A + \tau_2\left(g_V^2 + g_A^2\right)\right) \quad ,$$

$$\vec{g}_4 = \left(a\,|G|^2,\ 2f\,\mathrm{Re}(G^* G_V),\ t\left(|G_V|^2 - |G_A|^2\right)\right) \quad ,$$

$$\vec{g}_5 = \left(a\tau_3\,|G|^2,\ 2f\,\mathrm{Re}\left[G^*(\tau_3 G_V - \tau_4 G_A)\right],\ t\left[\tau_3\left(|G_V|^2 + |G_A|^2\right) - 2\tau_4 \mathrm{Re}(G_V G_A^*)\right]\right),$$

$$\vec{g}_6 = \left(a\tau_4\,|G|^2,\ 2f\,\mathrm{Re}\left[G^*(\tau_3 G_A + \tau_4 G_V)\right],\ t\left[2\tau_3 \mathrm{Re}(G_V G_A^*) + \tau_4\left(|G_V|^2 + |G_A|^2\right)\right]\right) \tag{7}$$

with

$$a = \frac{e^4}{q^4} \quad , \qquad f = \frac{e^2 g_Z^2}{q^2\left(q^2 - m_Z^2\right)} \quad , \qquad t = \left(\frac{g_Z^2}{q^2 - m_Z^2}\right)^2 \quad . \tag{8}$$

Making use of T-invariance for the electromagnetic interaction, the differential cross section is calculated from Eqs. (1) - (4); in the notations of Eqs. (6) - (8) its TCP-invariant form is given by[10]

$$\frac{4e^4}{\alpha^2}\frac{d\sigma}{d\Omega} = \frac{1}{q^2}\,|M_\gamma + M_Z|^2 = -(\vec{g}_1\vec{g}_4)\left[\frac{q^2}{2}(\vec{n}_3' A_1 \vec{n}_4') + (\vec{p}_3 A_1 \vec{p}_3)(\vec{n}_3'\vec{n}_4')\right] -$$

$$- (\vec{g}_2\vec{g}_4)(\vec{p}_1 A_3 \vec{p}_1) - (\vec{g}_1\vec{g}_5)(\vec{p}_3 A_1 \vec{p}_3) + \frac{q^2}{4}(\vec{g}_2\vec{g}_5)(1 + \cos^2\theta) -$$

$$- \frac{q^2}{2}\cos\theta\left[(\vec{g}_3\vec{g}_6) + 2(\vec{n}_1'\vec{n}_2')(\vec{n}_3'\vec{n}_4')\ f g_A \mathrm{Re}(G G_A^*)\right] +$$

$$+ 4\left(\vec{p}_1(\vec{n}_1 \times \vec{n}_2)\right)\mathrm{Re}\left\{\vec{p}_3\left[4t g_V g_A G_V G_A^* + \left(1 + \frac{1}{2}\cos\theta - \frac{q^2}{8m_Z^2}(1 + \cos\theta)\right) 2f g_A G G_A^*\right] +\right.$$

$$\left. + \frac{q^2}{4} O_3 \vec{p}_1 \left(1 - \frac{q^2}{m_Z^2}\right) f g_A G H_A^*\right\}(\vec{n}_3 \times \vec{n}_4) + 2(\vec{p}_1 \times \vec{p}_3)\left\{\vec{p}_-\ (\vec{g}_3\vec{d}_5) +\right.$$

$$\left. + \frac{q}{2}(\vec{n}_3' - \vec{n}_4')(\vec{g}_3\vec{d}_7) + f g_A A_1 \mathrm{Im}\left[4\vec{p}_3\left(\vec{p}_3(\vec{n}_4' - \vec{n}_3')\right) F^* F_A + q(\vec{n}_3' - \vec{n}_4') F_A G^*\right]\right\} +$$

$$+ q\vec{p}_1\left\{2f g_A(\vec{n}_1'\vec{n}_2')\,\mathrm{Re}\left[(\vec{n}_3' + \vec{n}_4')\frac{q}{2}(G^* F_V - F G_V^*) + \vec{p}_+ F G_A^*\right] + (\vec{n}_3' + \vec{n}_4')\frac{q}{2}(\vec{g}_3\vec{d}_4) - \vec{p}_+(\vec{g}_3\vec{d}_6)\right\} +$$

$$+ O_1\vec{p}_3\Big\{A_1\Big[2(\vec{p}_-\times\vec{p}_3)(\vec{g}_1\vec{d}_7) - q\Big((\vec{n}_3'-\vec{n}_4')\times\vec{p}_3\Big)(\vec{g}_1\vec{d}_5) + q\vec{p}_+(\vec{g}_1\vec{d}_4) +$$

$$+ \frac{q^2}{2}(\vec{n}_3'+\vec{n}_4')(\vec{g}_1\vec{d}_6) + q\vec{p}_3\Big[q\Big(1-\frac{4}{q^2}(\vec{p}_3\vec{n}_3)(\vec{p}_3\vec{n}_4)\Big)(\vec{g}_1\vec{d}_1) +$$

$$+ q(\vec{n}_3'\vec{n}_4')(\vec{g}_1\vec{d}_2) + 2\Big(\vec{p}_3(\vec{n}_4\times\vec{n}_3)\Big)(\vec{g}_1\vec{d}_3)\Big]\Big]\Big\} + O_1\vec{p}_3\Big\{A_1\to 1, \vec{g}_1\to\vec{g}_2\Big\} \quad . \tag{9}$$

Because of TCP-invariance the cross section of the reversed process, $F^+F^- \to e^+e^-$, is the same as above, with reversed spins.

The cross section for a spin-0 meson pair can be obtained by multiplying the right hand side of Eq. (9) with[11] $\left[\frac{q^2}{2}\left(1-\frac{4}{q^2}(\vec{p}_3\vec{n}_3)(\vec{p}_3\vec{n}_4)+(\vec{n}_3'\vec{n}_4')\right)\right]^{-1}$ and omitting G, G_A, G_V, F_A, H_A, then only the terms $\vec{d}_1, \vec{d}_2$ survive and

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 q^2}{8e^4}\sin^2\theta\Big\{\Big[\cos 2\phi(n_{1y}n_{2y}-n_{1x}n_{2x}) - \sin 2\phi(n_{1x}n_{2y}+n_{1y}n_{2x}) +$$

$$+ 1+n_{1z}n_{2z}\Big]\Big(a|F|^2 + 2fg_V\mathrm{Re}(F^*F_V) + t\Big(g_V^2-g_A^2\Big)|F_V|^2\Big) +$$

$$+ 2tg_A^2|F_V|^2(1+n_{1z}n_{2z}) + (n_{1z}+n_{2z})\Big(2fg_A\mathrm{Re}(F^*F_V) + 2tg_Vg_A|F_V|^2\Big)\Big\} . \tag{10}$$

This is the TCP-invariant version of the cross section in Ref. 12.

The case of the final state $\mu^+\mu^-$ with arbitrary polarizations follows from Eq. (9) provided[13] G=1, F=0, $F_{V,A}=H_{V,A}=0$, $G_{V,A}=g_{V,A}$. For a hadronic final state F^+F^-, however, all the form factors contribute to Eq. (9), the largest contributions come from G, $G_{V,A}$. The pure photon piece of $e^+e^- \to F^+F^-$ is described by the first components of $\vec{d}_1$, $\vec{d}_2$, $\vec{d}_4$, $\vec{g}_1$-$\vec{g}_6$ in Eq. (9) at arbitrary polarizations.

Another important case arises when F^+, F^- are in helicity states, $\vec{n}_3 \to h_3\vec{p}_3\frac{2}{q}$, $\vec{n}_4 \to -h_4\vec{p}_3\frac{2}{q}$, then

$$\frac{1}{q^2}|M_\gamma + M_Z|^2 = \frac{q^2}{4}(1+\cos^2\theta)(\vec{g}_2\vec{g}_5) - (\vec{p}_3 A_1\vec{p}_3)(\vec{g}_1\vec{g}_5) -$$

$$-\frac{q^2}{2}\cos\theta\,(\vec{g}_3\vec{g}_6) + q^2(1+h_3h_4)\,O_1\vec{p}_3\left[A_1(\vec{g}_1\vec{d}_1) + (\vec{g}_2\vec{d}_1)\right]\vec{p}_3 -$$

$$-4\left((\vec{p}_1\times\vec{p}_3)\,A_1\vec{p}_3\right)q(h_3+h_4)\,fg_A\,\mathrm{Im}(F^*G_A) \quad . \tag{11}$$

If the electrons and positrons are polarized transversely to their momenta as in the storage rings, Eq. (11) reduces to the cross section discussed in Ref. 9.[14]

Let us consider the changes caused by the transformations P, C, T in the cross section (9), where

$$\begin{aligned}
&P: \quad \theta\to\theta\,, \qquad \phi\to\pi-\phi\,, \qquad \vec{n}_i \to (n_{ix}, -n_{iy}, -n_{iz})\\
&C: \quad \theta\to\theta\,, \qquad \phi\to-\phi\,, \qquad \vec{n}_i \to (-n_{i+1x}, n_{i+1y}, -n_{i+1z})\,, \quad i=1,3 \qquad (12)\\
&T: \quad \theta\to\theta\,, \qquad \phi\to-\phi\,, \qquad \vec{n}_i \to (n_{ix}, -n_{iy}, n_{iz}) \quad .
\end{aligned}$$

Under P, C the following asymmetry emerges in Eq. (9)

$$\sum_{n_3,n_4}(1-P)\,|M_\gamma + M_Z|^2 = \sum_{n_3,n_4}(1-C)\,|M_\gamma + M_Z|^2 =$$

$$= 4(n_{1z}+n_{2z})\,q^4\Big\{(1+\cos^2\theta)\left[f\mathrm{Re}(G^*G_V) + tg_V\left(|G_V|^2 + |G_A|^2\right)\right]g_A +$$

$$+\sin^2\theta\,q^2\left[f\mathrm{Re}(F^*F_V) + tg_V\left(|F_V|^2 + |F_A|^2\right)\right]g_A -$$

$$-2\cos\theta\left[fg_V\mathrm{Re}(GG_A^*) + t\left(g_V^2+g_A^2\right)\mathrm{Re}(G_VG_A^*)\right]\Big\} \quad , \tag{13}$$

similarly, for $\vec{n}_3' = -\vec{n}_4'$, $\vec{n}_{1,2} = (0, 0, n_{1,2z})$, $(\vec{p}_1 \vec{n}_3') = 0$, keeping only $O\left(q^2/m_Z^2\right)$ terms and neglecting F's except for the T-violating F_A we get

$$(1-T)\,|M_\gamma + M_Z|^2 = 2q^5 \sin\theta\, |\vec{n}_3'|\, f\mathrm{Im}(F_A G^*) \Big[\Big(g_A(1+n_{1z}n_{2z}) + g_V(n_{1z}+n_{2z})\Big) - \cos\theta(h_4-h_3)\Big(g_V(1+n_{1z}n_{2z}) + g_A(n_{1z}+n_{2z})\Big)\Big] \,. \tag{14}$$

All the other asymmetries of the cross section (9) can be found by looking at the transformations leaving the pure photon piece invariant, but this will not be pursued here.

For fermions, F, $F_{V,A}$, $H_{V,A}$ can be neglected beside G, $G_{V,A}$, then the relative P-, C-asymmetries are

$$\eta = \frac{(1-P)\frac{d\sigma}{d\Omega}}{(1+P)\frac{d\sigma}{d\Omega}} = \frac{(1-C)\frac{d\sigma}{d\Omega}}{(1+C)\frac{d\sigma}{d\Omega}} = 2\,\frac{n_{1z}+n_{2z}}{1+n_{1z}n_{2z}} \Big\{(1+\cos^2\theta)\, g_A \cdot \Big[f\,\mathrm{Re}(G^*G_V) + t g_V\Big(|G_V|^2 + |G_A|^2\Big)\Big] - 2\cos\theta\,\mathrm{Re}\Big[fG^*G_A g_V + t G_V^* G_A\Big(g_V^2 + g_A^2\Big)\Big]\Big\}\Big\{(1+\cos^2\theta)\Big[a\,|G|^2 + 2f g_V \mathrm{Re}(G^*G_V)\Big] - 4\cos\theta\, f g_A \mathrm{Re}(G^*G_A)\Big\}^{-1} \tag{15}$$

($\vec{n}_3, \vec{n}_4$ are summed). This asymmetry is realized for longitudinally polarized beams ($\sigma_{\rightarrow\rightarrow} - \sigma_{\leftarrow\leftarrow}$) and it picks up the parity violating combinations of $g_i G_j$.

For longitudinal incident polarizations, from Eqs. (9), (14) the relative T-asymmetry is

$$\frac{(1-T)\frac{d\sigma}{d\Omega}}{(1+T)\frac{d\sigma}{d\Omega}} = \frac{\mathrm{Im}(F_A^* G) q \sin\theta}{a|G|^2(1+n_{1z}n_{2z})} \Big\{-2f\Big[g_A(1+n_{1z}n_{2z}) + g_V(n_{1z}+n_{2z})\Big]\Big\} \equiv \frac{\mathrm{Im}(F_A^* G) q \sin\theta}{|G|^2} A(T) \tag{16}$$

provided $h_3 = h_4 = 0$, and $|\vec{n}_3'| = 1$.

In order to have some insight into the numerical details of the effects caused by the weak neutral currents, we consider the Weinberg-Salam model[15] where the relevant parameters have the forms

$$g_Z = e \sin \Theta_W , \quad m_Z = \frac{73 \text{ GeV}}{\sin 2\Theta_W} , \quad g_V = \frac{1}{2}\left(4 \sin^2 \Theta_W - 1\right) , \quad g_A = \frac{1}{2} . \quad (17)$$

In the case of the final state $p\bar{p}$ one can write[16]

$$G_V = \frac{1}{2}\left(G_V^W - 4 \sin^2 \Theta_W G\right) , \quad G_A = \frac{1}{2} G_A^W ; \quad (18)$$

G_V^W, G_A^W mean the corresponding form factors of the charged weak hadronic current. Furthermore, it is usually assumed[16] that $G_V^W(q^2)$, $G_A^W(q^2)$ are proportional to $G(q^2)$; from experiments at $q^2 \approx 0$, $G(0) \approx 2.79$, $G_V^W(0) \approx 4.70$, $G_A^W \approx 1.20$.

For a spin-0 meson pair,[12] clearly

$$F_V = \cos 2\Theta_W F \quad . \quad (19)$$

The longitudinal asymmetry (15) for $e^+e^- \to p\bar{p}$ is plotted in Fig. 1 with $n_{1z} + n_{2z} = 1 + n_{1z} n_{2z}$ (e.g., maximum polarizations). For instance, at $\theta = \pi$, $\sin^2 \Theta_W = 0.38$ (Ref. 17) $|\eta|$ is about 4% (8.7%) if $q = 28$ GeV (40 GeV). The corrections of the order f^2/a^2 are about 0.1% (0.4%) at these energies. The $\cos \theta$-dependence of the asymmetry η makes it possible to separate $g_A G_V$ and $g_V G_A$, however, around $\sin^2 \Theta_W \approx 0.3$ the term $g_A G_V$ becomes dominant even if $\theta \approx \pi$. The longitudinal asymmetry calculated by Eqs. (10), (17), (19) for a spin-0 meson pair[12] turns out to be independent of θ, for a comparison it is drawn in Fig. 1. For instance, at $\sin^2 \Theta_W = 0.38$, $q = 28$ GeV (40 GeV) $-\eta(\text{meson}) \approx 4.5\%$ (12.5%) provided $n_{1z} + n_{2z} = 1 + n_{1z} n_{2z}$. From these examples it follows that the weak neutral hadronic currents give rise to remarkable longitudinal asymmetries for $q \gtrsim 28$ GeV in $e^+e^- \to F^+F^-$.[18]

The T-violation due to F_A is characterized by Eq. (16), it is largest at $|\sin\theta| = 1$, increasing strongly with the center-of-mass energy as in shown in Fig. 2 for $n_{1z}+n_{2z} = 1+n_{1z}n_{2z}$. Apparently, the T-violation in Eq. (16) comes from an amplitude F_A not proportional to G.

It is easy to see that for $e^+e^- \leftrightarrow p\bar{p}$ the relative change in the total cross section coming from Eq. (9) due to the weak effects is the same as in Ref. 9, that is it grows from 0 to 10% as Θ_W goes from 40° to 60°. For $e^+e^- \to \pi^+\pi^-$ we get from (10)

$$\frac{\sigma^{\gamma}_{tot} - \sigma_{tot}}{\sigma^{\gamma}_{tot}} = \begin{cases} -\dfrac{2fg_V}{a}\cos 2\Theta_W \ , & \text{transverse polarizations} \\ -\dfrac{2f(g_V+g_A)}{a}\cos 2\Theta_W \ , & \begin{cases}\text{longitudinal polarizations} \\ \text{with } n_{1z}+n_{2z} = 1+n_{1z}n_{2z}\end{cases} \end{cases} , \qquad (20)$$

In the Weinberg-Salam model the rate correction (20) is plotted in Fig. 3. The longitudinal configuration is again more favorable, for example, at $\sin^2\Theta_W = 0.38$ and 28 GeV (40 GeV) the change is about 7% (15%).

III. WEAK EFFECTS IN TWO-PHOTON PROCESSES

In what follows we show that the Z-meson causes negligible effects in two-photon processes[19] at least for $q \lesssim 40$ GeV. The relevant contributions to the amplitude squared are

$$|M_{\gamma\gamma}|^2 + 2\mathrm{Re}(M_{\gamma\gamma}M^*_{\gamma Z}) + 2\mathrm{Re}(M_{\gamma\gamma}M^*_{Z\gamma}) \ , \qquad (21)$$

the graphs describing these amplitudes M_{ij} are drawn in Fig. 4a-c and the usual $k_1^2 \approx 0$, $k_2^2 \approx 0$ are assumed leading to the largest contributions (forward scattering). The circles attached to the produced pair F^+F^- give rise to the matrix elements $\langle F^+F^- | T^*\left(J_i^{\mu} J_j^{\nu}\right) | 0\rangle$ with $i, j = \gamma, \gamma; \gamma, Z; Z, \gamma$ whose Fourier transforms $M_{ij}^{\mu\nu}$ describe the processes $i+j \to F^+ + F^-$. For instance, if $F^{\pm} = \pi^{\pm}$, in Born

approximation, for quasi-real photons

$$M^{\mu\nu}_{\gamma\gamma} = F\left(k_1^2\right)F\left(k_2^2\right)\left[\frac{(2q_1-k_1)^{\mu}(2q_2-k_2)^{\nu}}{2q_1k_1} + (q_1 \longleftrightarrow q_2) - 2g^{\mu\nu}\right] \quad , \tag{22}$$

including also the terms due to the crossed graph and the seagull. Then, only the vector part of J^{ν}_{Z} contributes to $M^{\mu\nu}_{\gamma Z}$, $M^{\mu\nu}_{\gamma Z} = M^{\mu\nu}_{Z\gamma} = c\, M^{\mu\nu}_{\gamma\gamma}$. (In the Weinberg-Salam model $c = \cos 2\Theta_W$.)

In general, one can write

$$|M_{\gamma\gamma}|^2 = \left(\frac{e^4}{k_1^2 k_2^2}\right)^2 E_{\mu\nu}(1,0)\, P_{\rho\lambda}(1,0)\, M^{\mu\rho}_{\gamma\gamma} M^{\nu\lambda+}_{\gamma\gamma} \quad , \tag{23}$$

$$\mathrm{Re}(M_{\gamma\gamma}M^{*}_{\gamma Z}) = \frac{e^4}{k_1^2k_2^2}\,\frac{e^2g_Z^2}{k_1^2\left(-m_Z^2\right)}\,\mathrm{Re}\left(E_{\mu\nu}(1,0)\, P_{\rho\lambda}(g_V, g_A)\, M^{\mu\rho}_{\gamma\gamma} M^{\nu\lambda+}_{\gamma Z}\right) \quad , \tag{24}$$

$$\mathrm{Re}(M_{\gamma\gamma}M^{*}_{Z\gamma}) = \frac{e^4}{k_1^2k_2^2}\,\frac{e^2g_Z^2}{k_2^2\left(-m_Z^2\right)}\,\mathrm{Re}\left(E_{\mu\nu}(g_V, g_A)\, P_{\rho\lambda}(1,0)\, M^{\mu\rho}_{\gamma\gamma} M^{\nu\lambda+}_{Z\gamma}\right) \quad , \tag{25}$$

where we have neglected higher order effects coming from $k_\mu k_\nu/m_Z^2$ in the propagator of Z and omitted k_i^2 beside m_Z^2. The influence of the spinors is contained in $E_{\mu\nu}(P_{\mu\nu})$ for e^- (e^+)

$$E_{\mu\nu}(a,b) = \sum_{s_1'} \bar{u}(p_1's_1')\,\gamma_\mu\, u(p_1s_1)\left[\bar{u}(p_1's_1')\,\gamma_\nu(a+b\gamma_5)\, u(p_1s_1)\right]^{+} = \quad ,$$

$$= \frac{1}{4m_e^2}\Big\{a\left[(p_1+p_1')_\mu\,(p_1+p_1')_\nu - k_{1\mu}k_{1\nu} + g_{\mu\nu}k_1^2\right] + ib\,\epsilon_{\mu\alpha\nu\beta}(p_1+p_1')^{\alpha}\,k_1^{\beta} -$$

$$- 2iam_e\,\epsilon_{\mu\sigma\nu\alpha}\,s_e^{\sigma}k_1^{\alpha} + 2m_e b(p_1+p_1')_\mu\, s_{e\nu} - 2bm_e\left(g_{\mu\nu}k_{1\rho}s_e^{\rho} - s_{e\mu}k_{1\nu}\right)\Big\} \quad , \tag{26}$$

$$P_{\lambda\rho}(c,d) = \sum_{s_2'} \bar{v}(p_2 s_2)\gamma_\lambda v(p_2' s_2')\left[\bar{v}(p_2 s_2)\gamma_\rho(c+d\gamma_5)v(p_2' s_2')\right]^+ =$$

$$= \frac{1}{4m_e^2}\left\{c\left[(p_2+p_2')_\lambda (p_2+p_2')_\rho - k_{2\lambda}k_{2\rho} + g_{\lambda\rho}k_2^2\right] - id\,\epsilon_{\lambda\alpha\rho\beta}(p_2+p_2')^\alpha k_2^\beta -\right.$$

$$\left. - 2icm_e\,\epsilon_{\lambda\rho\sigma\alpha}s^\sigma k_2^\alpha - 2m_e d(p_2+p_2')_\lambda s_\rho - 2m_e d\left(s_\lambda k_{2\rho} - g_{\lambda\rho}k_{2\nu}s^\nu\right)\right\} \qquad (27)$$

with s_e^ν, s^ν the polarization vectors for e^-, e^+.

Looking at the two-photon amplitude (23), we see that for real $M_{\gamma\gamma}^{\mu\rho}$ (e.g., Eq. (22)) the terms linear in s_e^ν or s^ν are absent, otherwise, from Eq. (26) the polarization dependence appears in the form $m_e^2 s_e s$. Consequently, the influence of the transverse polarization can be completely neglected in pure two-photon processes; however, for longitudinally polarized beams there remains a contribution due to polarization. The terms linear in the polarizations are present in the electromagnetic and weak interference under Eqs. (24), (25), and they are proportional to the parity violating coupling constant g_A. As before, the largest contributions are given by the longitudinal polarizations, while negligible for transverse ones.

Turning to the magnitude of the interference, we first remark that Eqs. (23)-(25) present high energy behaviors opposite to those coming from the one γ and one Z exchanges yielding a relative interference of the order q^2/m_Z^2 at large q and $q^2 \ll m_Z^2$. In case of Eqs. (23) - (25) both the $\gamma\gamma$ and the interference terms are growing strongly with the energy ($k_i^2 \to 0$), in such a way that the integrated relative interference decreases at large q and m_Z^2 remains uncompensated. To see this phenomenon more closely, we consider the ratio of the amplitude squares

in (23) - (25) and write

$$2\,\frac{(24)+(25)}{(23)} \approx -2\,\frac{g_Z^2 c}{e^2 m_Z^2}\left(k_1^2+k_2^2\right) \quad , \qquad k_1^2, k_2^2 \approx 0 \tag{28}$$

keeping the most relevant pieces in $E_{\mu\nu}$, $P_{\mu\nu}$ $\left(\rightarrow E_{\mu\nu}(1,0),\ P_{\mu\nu}(1,0)\right)$. Even if one chooses the relatively large value $k_1^2 \approx q(m_e q/2)^{1/2}$,[19,20] Eq. (28) provides a small ratio, for instance, in the Weinberg-Salam model at $q = 40$ GeV, $\sin^2\Theta_W = 0.4$ we get -0.06%. For the total cross section, the ratio (28) is expected to be around[19]

$$-4\,\frac{m_\pi^2}{m_Z^2}\,\frac{g_Z^2 c}{e^2}\left[\ln\left(\frac{q}{2m_e}\right)\right]^{-1} \tag{29}$$

(pion pair production) leading to very small interferences even at lower energies.[21]

IV. CONCLUSIONS

In the present paper the differential cross section of $e^+e^- \rightarrow F^+F^-$ is calculated by exchanging one γ and one neutral meson Z coupled to neutral weak currents. Polarizations of incoming and outgoing particles are arbitrary, mass terms are neglected in the cross section. The treatment is valid both for $F^+ = \mu^+$, baryons, and spin-0 mesons, as well as for baryon-antibaryon annihilation into electron-positron pair at vanishing masses. In the cross section the dependence of the neutral weak current emerges in the weak leptonic couplings $g_Z g_V$, $g_Z g_A$ and through the form factors of the matrix element $\langle F^+F^- | J_Z^\mu | 0 \rangle$. At $q^2 \ll m_Z^2$ the interference term behaves as $e^2 g_Z^2/q^2 m_Z^2$ and the relative interference grows with q^2 as $q^2 g_Z^2/e^2 m_Z^2$.

Several weak asymmetries appear in the cross section, in particular, for longitudinally polarized incoming beams a significant parity violating asymmetry is shown depending on the relevant form factors G, G_V, G_A, the magnitudes of

the polarizations through $(n_{1z}+n_{2z})(1+n_{1z}n_{2z})^{-1}$ and the function $\cos\theta(1+\cos^2\theta)^{-1}$ of the scattering angle θ. In the Weinberg-Salam model the region $\theta \approx \pi$ is most favorable for measuring η, numerical examples in Figs. 1, 3 present weak effects measurable in the energy range of the next generation of storage rings.[18]

A small T-asymmetry is also found (Eq. (16)) related to the hadronic form factor F_A.

In Section III we have discussed the influence of Z in two-photon processes in the most favorable kinematic configuration (forward lepton scattering). It turns out that the weak interferences due to Z are completely negligible at high energies. From the kinematics it follows that in contrast to the one photon exchange the contributions from pure two photon diagrams are negligibly shifted when transverse polarizations are included for the incident beams.

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20. This corresponds to $\theta^2 \approx (2m_e/q)^{1/2}$ giving the three quarters of the cross section for two-photon processes.

21. For q = 2 GeV, $\sin^2 \Theta_W = 0.4$ it is about $10^{-4}\%$.

FIGURE CAPTIONS

1. Longitudinal asymmetries in the Weinberg-Salam model with $n_{1z} + n_{2z} = 1 + n_{1z} n_{2z}$.

2. The coefficient of the T-violating asymmetry (16) in the Weinberg-Salam model with $n_{1z} + n_{2z} = 1 + n_{1z} n_{2z}$.

3. Relative changes in the total cross section of $e^+e^- \to \pi^+\pi^-$ in the Weinberg-Salam model.

4. Relevant graphs for $e^+e^- \to e^+e^- F^+F^-$, (a) two-photon exchange, (b) γ-Z exchange, (c) Z-γ exchange.

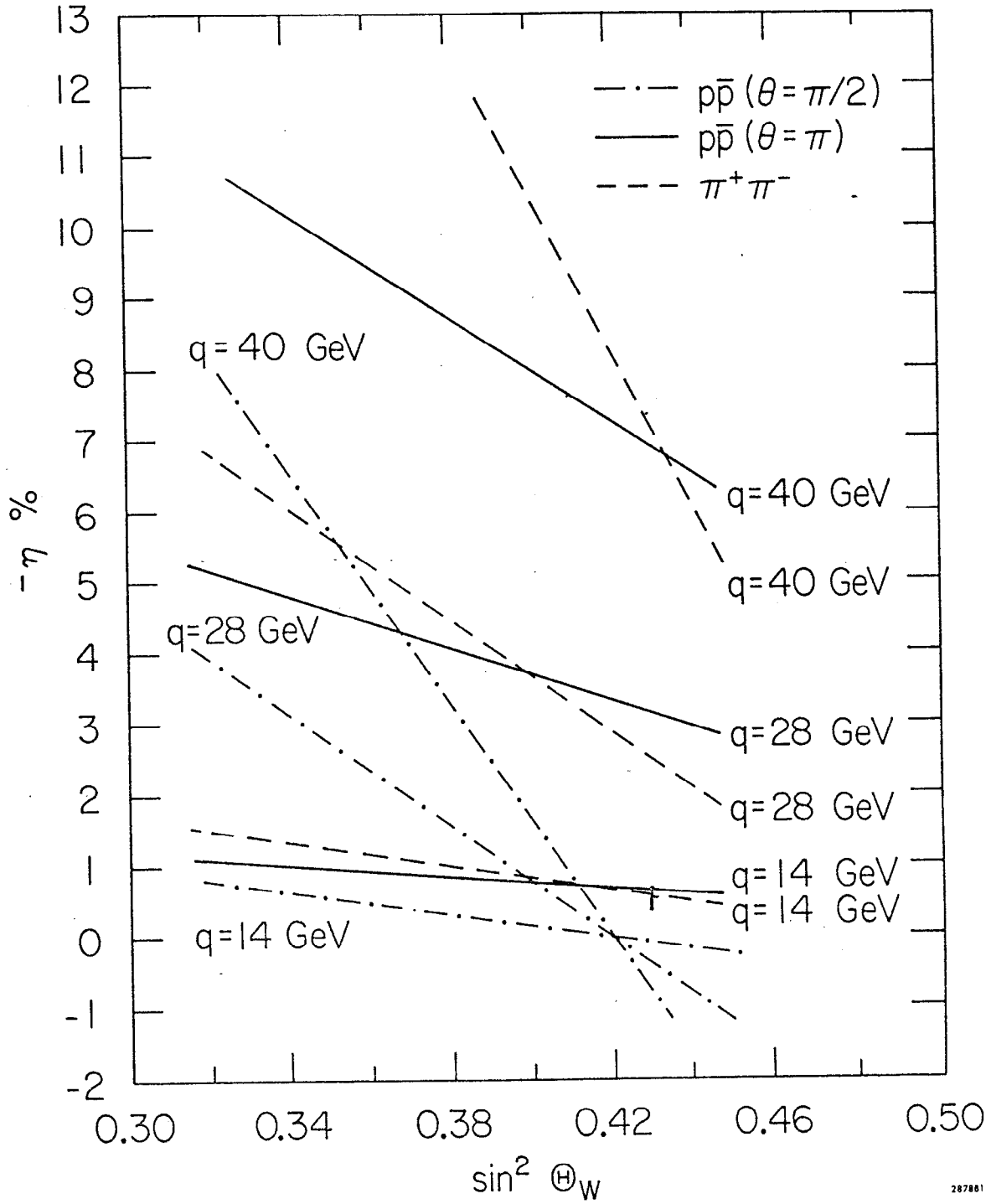
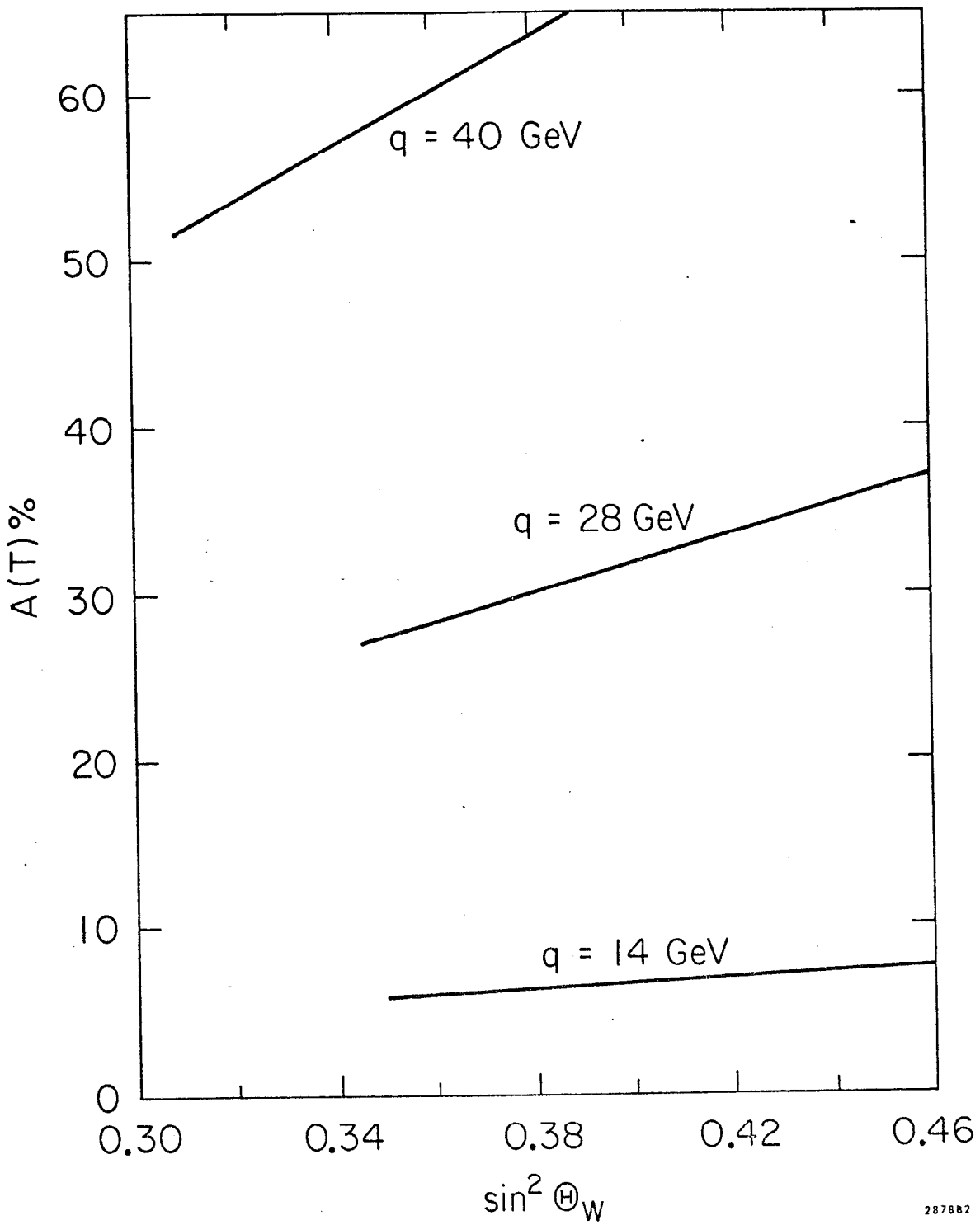

13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
$-\eta$ %
0.30
0.34
0.38
0.42
0.46
0.50
$\sin^2 \Theta_W$
$p\bar{p}$ ($\theta = \pi/2$)
$p\bar{p}$ ($\theta = \pi$)
$\pi^+\pi^-$
q = 40 GeV
q = 28 GeV
q = 14 GeV
q=40 GeV
q=40 GeV
q=28 GeV
q=28 GeV
q=14 GeV
q=14 GeV
287881

Fig. 1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50
40
30
20
10
0
A(T) %
q = 40 GeV
q = 28 GeV
q = 14 GeV
0.30
0.34
0.38
0.42
0.46
sin² Θ_W
287882

Fig. 2

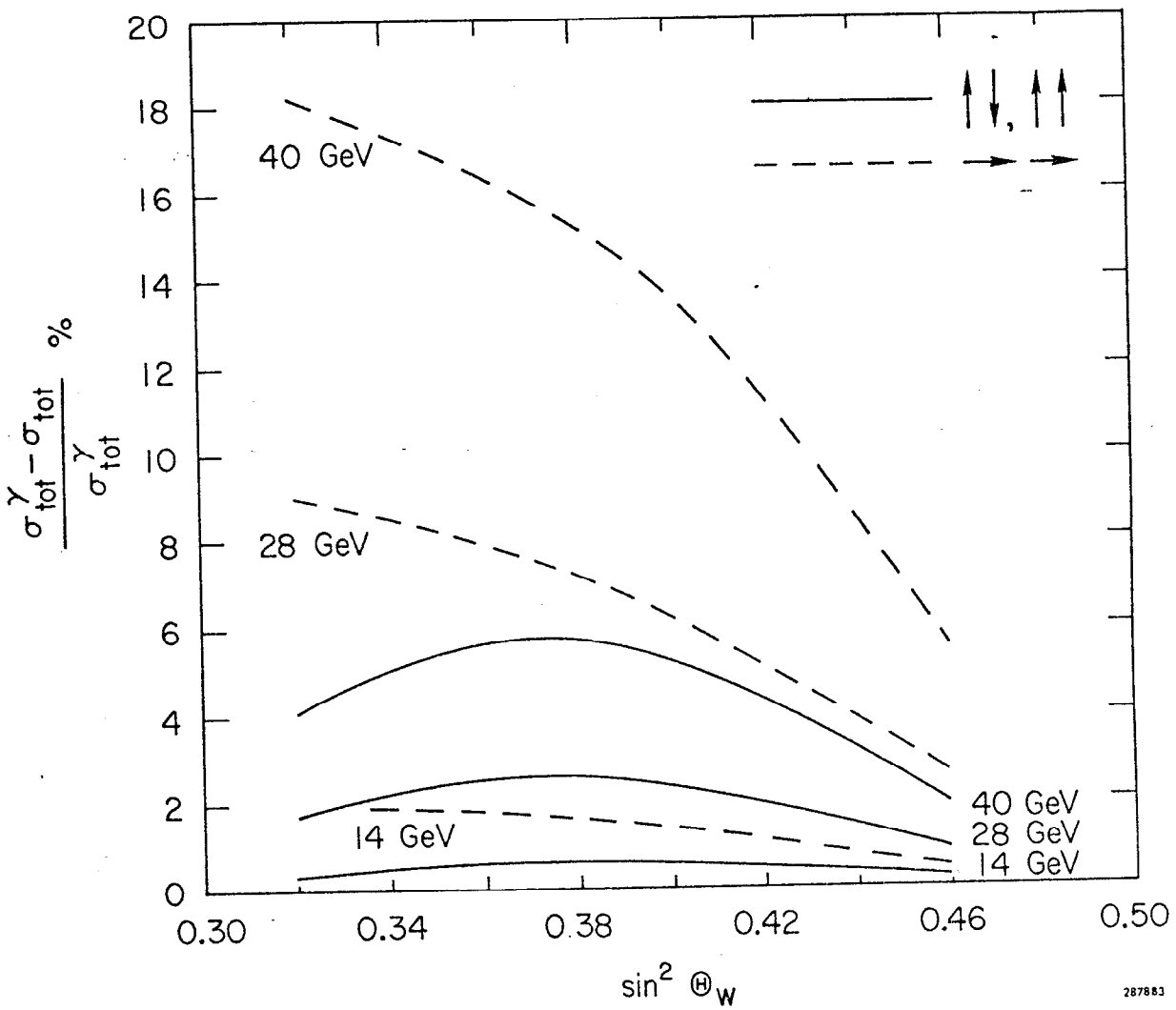
20
18
16
14
12
10
8
6
4
2
0
0.30
0.34
0.38
0.42
0.46
0.50
$\sin^2 \Theta_W$
$\frac{\sigma^{\gamma}_{tot} - \sigma_{tot}}{\sigma^{\gamma}_{tot}}$ %
40 GeV
28 GeV
14 GeV
40 GeV
28 GeV
14 GeV
287883

Fig. 3

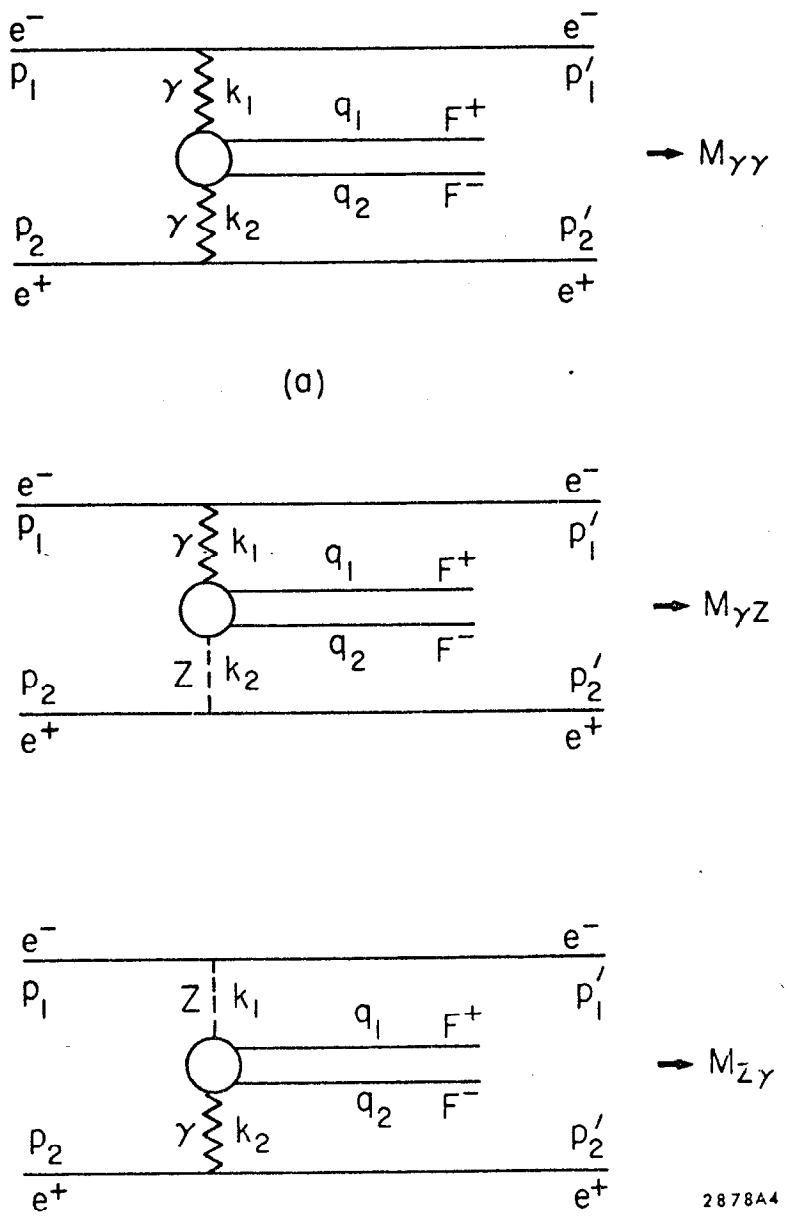
e⁻
e⁻
p₁
p′₁
γ
k₁
q₁
F⁺
→ Mγγ
q₂
F⁻
γ
k₂
p₂
p′₂
e⁺
e⁺
(a)
e⁻
e⁻
p₁
p′₁
γ
k₁
q₁
F⁺
→ MγZ
q₂
F⁻
Z
k₂
p₂
p′₂
e⁺
e⁺
e⁻
e⁻
p₁
p′₁
Z
k₁
q₁
F⁺
→ MZγ
q₂
F⁻
γ
k₂
p₂
p′₂
e⁺
e⁺
2878A4

Fig. 4