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THE IMPACT OF QUANTUM ELECTRODYNAMICS*

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I. INTRODUCTION

The remarkable and continued success of quantum electrodynamics in precisely predicting the interactions of leptons and the properties of atoms stands as a testament to the applicability of mathematics and physical theory to the physical world. Despite, what at times seems to be a confrontation with the inevitable, it is essential that experimental tests of QED be pursued as far as possible. Among the immediate reasons:

(1) It is not known whether the expansion in α is reliable; for example, the asymptotic behavior of vacuum polarization may be different at large momentum transfers ($|t| >> m^2$) then what is predicted by perturbation theory.[1]

(2) We need to know the limits on possible structure of the leptons, and possible new interactions which distinguish the muon and electron.

(3) The precision tests determine the value of the fundamental dimensionless constants α and m_e/m_μ.

(4) The effects of the weak and strong interactions are inevitably intertwined in the precision tests; thus QED cannot be studied in isolation. The weak interactions will play a major role in the tests of QED at the new storage rings at $E_{cm} \gtrsim 30$ GeV.

(5) The pursuit of the heavy leptons—and the unexpected.

The purpose of this review is two-fold: First I will review recent developments in QED, particularly tests which involve the dynamics of the muon and tests which bear on the above five topics.[2] A new limit on the possible composite structure of the muon will also be given. In the second part of the talk, I will focus on the impact on QED and its generalizations—the gauge theories—to other areas of physics, including the weak and strong interactions and the atomic spectrum of the new particles. This discussion, which is intended for non-specialists, also briefly reviews the consequences of scale invariance in hadron, atomic, and nuclear physics.

II. QED AT SHORT DISTANCES

The most direct experimental test of the structure of QED in the high momentum transfer regime is the measurements of the cross sections for $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\gamma\gamma$ performed at the storage rings at Novosibirsk, Stanford, Orsay, Frascati, and SPEAR. At the present precision only the Born approximation amplitudes are tested (modified by the standard, classical, radiative corrections), but the momentum transfers are so large ($\sqrt{s} = E_{cm}$ up to 8 GeV at SPEAR) that modifications of QED—parametrized in terms of an intrinsic scale or size of the leptons—can all be ruled out at the 15 GeV (or 10^{-15} cm) level.[3]

The most sensitive test,[4] performed at SPEAR using $e^+e^- \to e^+e^-$ and $\mu^+\mu^-$ gives lower limits on heavy photon masses: $\Lambda_+ \geq 35$ GeV, and $\Lambda_- \geq 47$ GeV (95% confidence level) assuming a photon propagator of the form $1/k^2 \to 1/k^2 \pm 1/\left(k^2 - \Lambda_\pm^2\right)$.

Alternatively, the cross section can be parametrized in the form $\sigma(e^+e^- \to \mu^+\mu^-) = A\, s^{-n}$. A Frascati measurement[5] at $\sqrt{s} < 3$ GeV gives $n = 0.99 \pm 0.02$ and $A/(4\pi\alpha^2/3) = 1.00 \pm .02$. The absence of an intrinsic length scale in the perturbation theory calculation and the elementarity of the leptons are directly tested by the scaling $s\sigma(s) = \text{const}$. Further implications of scale-invariance in QED and gauge theories will be discussed in Section V.

The anomalous $e\mu$ events[6] seen recently at SPEAR in e^+e^- annihilation are consistent with the hypothesis of a new heavy lepton $e^+e^- \to \ell^+\ell^-$ (decaying as

$\ell^{\pm} \to \mu^{\pm}\nu\bar{\nu}$ and $\ell^{\mp} \to e^{\mp}\nu\bar{\nu}$) with a mass of order 1.8 GeV. If heavy leptons are indeed being observed, then this may provide a further new testing ground of QED (heavy leptonic atoms!) at a mass scale which seems more appropriate to heavy hadrons.

III. PRECISION TESTS OF QED

The historic testing grounds of quantum electrodynamics, which are sensitive to the renormalization procedure, the perturbation expansion, and the nature of vacuum polarization and self-energy effects, are the precision g-2 and atomic physics tests. As we shall emphasize, the anomalous magnetic moments of the leptons provide an extraordinary test of lepton elementarity. This past year could well be called the "year of the muon" since several, new, very sensitive muon tests have been reported. The $(g-2)_\mu$ and muonic atom measurements are sensitive to effects at a distance scale two orders of magnitude smaller than the corresponding electron measurements. It seems certain that continued progress in this important area will come from the increased flux of muons to be available soon here at LAMPF.

A. The anomalous moment of the Muon

The results of the new elegant measurement at CERN of the muon's precession frequency by J. Bailey et al.[7] have recently been reported. A central innovation in this experiment was the use of a storage ring where muons are trapped by a uniform magnetic field, with vertical confinement being effected by a transverse electric quadrupole field. At the "magic" momentum $p_\mu = 3.094$ GeV, the electric field does not affect the precession frequency. Defining $\vec{\mu} = (e\hbar/2mc)g\vec{S}$, and $a = (g-2)/2$,

$$a_\mu^{\mathrm{expt}} = \begin{cases} 1165895(27)\ 10^{-9} \\ 1165901(27)\ 10^{-9} \end{cases} \quad (23\ \mathrm{ppm})$$

where the first entry uses the ratio $\lambda = (\omega_L)^\mu/(\omega_L)^p$ of muon/proton Larmor frequencies measured by Crowe et al.[8] [direct measurement in a chemical environment] and the second entry uses the value λ deduced from the recent muonium hyperfine splitting measurements of the Yale group[9] as discussed in Section B.

The theoretical value can be predicted reliably at the 10 ppb level. The present breakdown is (see Fig. 1)

$$
\begin{array}{lll}
 & & 10^{-9} \\
a_\mu^{QED} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) & & 1161409.8 \\
\quad + .76578\left(\frac{\alpha^2}{\pi^2}\right) & & 4131.8 \\
\quad + \left[1.195^a(\pm.026) + 1.944^b + 21.32^c(\pm.05)\right]\left(\frac{\alpha}{\pi}\right)^3 & & 306.6 \pm 0.7 \\
\quad + \left[150^d(\pm 70)\right]\left(\frac{\alpha}{\pi}\right)^4 & & \underline{4.4 \pm 2.0} \\
 & & 1165853. \quad \pm 3. \\
a_\mu^{had} = [\underset{(\rho)}{55} + \underset{(\omega)}{6} + \underset{(\phi)}{5} + \underset{(2/5\ \mathrm{GeV})}{4} + \underset{(>5\ \mathrm{GeV})}{3\,(\pm 3)}] & & \underline{73 \quad \pm 10} \\
 & & 1165926 \quad \pm 10
\end{array}
$$

The experimental value of $\alpha^{-1} = 137.035987(29)$ [0.21 ppm] is taken from the e/h Josephson measurements and the recent determination of $\gamma_{p'}$. [See Ref. 15.] The sixth order contributions are: (a) the sixth order graphs (common to a_e) without vacuum polarization insertions [due to Cvitanovic and Kinoshita,[10] Levin and Wright,[11] and Levine and Roskies[12] (analytic results)]; (b) the electron pair vacuum polarization interactions [evaluated numerically in Ref. 13 and analytically in Ref. 14]. Contribution (c) is the electron-loop "light by light" scattering insertion diagram which turns out to extraordinarily large. The calculations of Aldins, Brodsky, Dufner and Kinoshita[16] have been confirmed by Peterman and Calmet,[17] and by Levine and Chang.[18] The numerical value quoted here has recently been given by Samuel,[19] based on a new numerical analysis of the required seven dimensional integration. [The result is slightly higher than the value 19.79 ± 0.16 reported by Calmet and Peterman.[17]] The combination of light-by-light graphs plus vacuum polarization insertion leads to the estimate (d) of the eight order graphs as reported by Lautrup[20] and Samuel.[21]

The hadronic vacuum polarization contribution to the muon moment is directly computable as an integral over the measured $e^+e^- \to "\gamma" \to$ hadron cross section (as long as $\sigma(s) \to 0$ as $s \to \infty$). The estimate for 5 GeV < $\sqrt{s} < \infty$ covers the whole range of possibilities consistent with unitarity.[22] The major uncertainty ($\pm 9 \times 10^{-9}$) comes from the unmeasured threshold region $\sqrt{s} > 2m_\pi$ up to the ρ, ω, ϕ region. The weak interaction contribution (discussed in Ref. 23 and references therein) used to be ambiguous—dependent on the magnetic moment of the intermediate vector boson and a possible logarithmic cutoff. Now, if we accept the renormalizable gauge theory models of the weak interaction, the result is finite (since the W has zero anomalous moment as defined by the Drell-Hearn[24]-Gerasimov[25] sum rule), and a_μ^{weak} is not large (of order of 2×10^{-9}). The gauge theories are discussed further in Section III.

Comparing theory and experiment, we have

$$a_\mu^{expt} - a_\mu^{theory} = \begin{cases} 31(29) \times 10^{-9} \\ 26(29) \times 10^{-9} \end{cases}$$

(for the chemical and muonium determinations of λ, respectively). Note that ±30 ppb checks the light-by-light contribution to ±10%, the hadronic contribution to ±40%, and the electron loop vacuum polarization insertion (the α^2/π^2 term) to ±1.5%.

What is truly extraordinary is that the experimental determination of the g factor

$$g_\mu = 2[1.001165895\,(27)]$$

ninth significant figure

is predicted by QED correctly through nine significant figures! Note that there is no a priori reason for a spin 1/2 particle to have g near 2 (as witnessed by the nucleon). The Dirac value g=2 holds only if the fermion is elementary. Thus, suppose the muon were actually a composite structure—a bound state of say two or three more "fundamental" sub units. Then the coupling to an internal charged current leads to the general contribution

$$a_\mu \sim 0\left(\frac{m_\mu}{m^*}\right)$$

where m* is a characteristic internal mass, the mass of the first excited state, or continuum threshold. Alternately, we can use the Drell-Hearn[24]-Gerasimov[25] sum rule

$$a_\mu^2 = \frac{m_\mu^2}{8\pi\alpha}\int_{s_{th}}^{\infty}\left[\sigma^P_{\gamma\mu} - \sigma^A_{\gamma\mu}\right]\frac{ds}{s}$$

where $\sigma^{P[A]}_{\gamma\mu}$ is the photoabsorption cross section with parallel [antiparallel] photon and target spins. In general $<\sigma> \sim 0\left[\alpha/m^{*2}\right]$ for the contribution of the excited states of continuum. This contribution (together with the modification of the near-threshold region of $\sigma_{\gamma\mu}$) again gives $a_\mu \sim 0(m_\mu/m^*)$. Taking this effect to be less than 10^{-7} in a_μ gives the bound $m^* \gtrsim 0\left[10^5 \text{ GeV}\right]$! Thus the precision measurement on g-2 leads to an important limit on possible lepton substructure.[26]

The electron g-2 is also in agreement with the QED prediction. Reviews are given in Ref. 2.

B. The Muonium Hyperfine Splitting

The measurements of the ground state splitting of the fundamental muonium atom (μ^+e^-) performed in recent years by the Chicago[27] and Yale[28] groups have now reached the extraordinary precision of less than 1/2 ppm. The latest result, measured here at LAMPF by the Yale group[28] is

$$\Delta\nu_{expt} = 4463302.2\,(1.4)\ \mathrm{kHz}\ [0.3\ \mathrm{ppm}] \quad ,$$

The measurements are performed at low pressure in a weak magnetic field using both the "old muonium" and "separated oscillating fields" methods to obtain narrow line widths.[29]

The theory of the ground state splitting[30] begins with the Fermi formula E_F (for the spin coupling of the exchanged transverse photon) modified by corrections from the Dirac wavefunctions of order $0(Z\alpha)^2 E_F$ for the electron in a Coulomb field. Higher transverse photon exchange relativistic recoil corrections as computed from the Bethe-Salpeter equation are of order $Z\alpha\ m_e/m_\mu$, $(Z\alpha)^2\ m_e/m_\mu \log Z\alpha$. [For convenience in identifying terms the muon charge is taken as Ze .] QED corrections to the muon are incorporated in the muon moment. Self-energy corrections to the electron and vacuum polarization lead to terms of order $\left[\alpha + \alpha(Z\alpha) + \alpha^2 + \alpha(Z\alpha)^2 \log^2 Z\alpha,\ \alpha(Z\alpha)^2 \log Z\alpha\right] E_F$. Also some terms of order $\alpha(Z\alpha)^2 E_F$ have been evaluated, but only the ones associated with the $\log^2 Z\alpha$, $\log Z\alpha$ terms.[31] Further progress in the theory of muonium thus requires the computation of all the terms of two-order plus the evaluation of the relativistic recoil terms of order $(Z\alpha)^2\ m_e/m_\mu\ E_F$.[32] [An effort to compute the latter terms is now in progress using an effective potential method.[33]] Taking a reasonable estimate of the uncalculated terms, one obtains

$$\Delta\nu_{th} = \alpha^2 \mu_\mu/\mu_p \left[2.632\ 957\ 87 \pm 0.6\ \text{ppm}\right] \times 10^7\ \text{MHz}$$

which together with $\Delta\nu_{exp}$ implies[28]

$$\lambda = \mu_\mu/\mu_p = 3.183\ 329\quad 9\,[25] \qquad 0.8\ \text{ppm}$$

$$m_\mu/m_e = 206.769\quad 27\,[17] \qquad 0.8\ \text{ppm}$$

The value for λ disagrees by 2 σ with value obtained for μ's in liquids by the Berkeley group.[8]

Casperson et al.[28] also note that the latest comparison of theory and experiment for the hydrogen hfs yields a nucleon polarization contribution

$$\delta_{p'} = \left[0.5 \pm 1.2\right]\ \text{ppm}$$

This is in agreement with the preliminary values deduced from the spin-dependent structure function measurements of the Yale-SLAC group currently in progress at SLAC.[34]

It should be emphasized that the computation of the higher order terms in the muonium hfs is probably the most critical calculation required for the progress of precision QED and the utilization of the high accuracy of the experiments. A similarly critical situation also holds for the positronium hfs, although the theoretical work is even more difficult in the case of equal constituent masses.

C. The Helium Fine Structure $[2^3P]$

The theory of the fine structure of the helium atom has now reached the stage where comparisons between theory and experiment can be compared at below the 1 ppm level. There are a great number of theoretical heroes: The calculation of the spin-dependent Breit operator matrix elements for the two electron atom were performed by Schwartz,[35] and Schiff, Pekeris, and Lifson.[36]

The systematic reduction[37] of the Bethe-Salpeter equation to an effective Hamiltonian was done by M. Douglas, N. Kroll, and J. Daley. The calculation[38] of the required terms to second order in perturbation theory was done by A. Delgano, J. Lewis, and L. Hambro, with the final variational calculations performed numerically (up to 455 terms!) by M. L. Lewis and P. M. Serafino.[39] The comparison with the measurement of the Yale group of A. Kiponou, V. W. Hughes, C. F. Johnson, S. A. Lewis, and F.M.J. Pichanick[40] is

$$\underset{[\mathrm{MHz}]}{\Delta\nu\left(2^3P_0 - 2^3P_1\right)} = \begin{cases} 29\ 616\ .864\,(36) & \text{expt} \quad [1.2\ \text{ppm}] \\ 29\ 616\ .883\,(43) & \text{th} \quad [1.4\ \text{ppm}] \end{cases}$$

$$\Delta\nu[\text{th-exp}] = 0.019\ \text{MHz} \qquad\qquad [0.64\ \text{ppm}]$$

This can be used also to derive the most precise value of α to be obtained from a QED measurement:

$$\alpha^{-1} = 137.03608\,(13) \qquad 0.94\ \text{ppm}$$

which can be compared with the latest non-QED determination [using e/h, $\gamma_{p'}$ (gyromagnetic ratio of proton in H_2O)][15]

$$\alpha^{-1} = 137.035987\,(29) \qquad 0.21\ \text{ppm}$$

A graph of the various determinations of the fine structure constant is shown in Fig. 2.

D. High Z Muonic Atoms

Over the past decade the precision of theory and experiment for the spectra of muonic atoms μ^-Z has steadily increased, giving important information and checks on nucleon size and polarization parameters, as well as checks on the electrodynamics of the muon. The large n transitions in high Z muonic atoms are relatively insensitive to nuclear and electron screening effects and lead to an important check on vacuum polarization potential (due to virtual electron pairs) at average momentum $<p> \sim Z\alpha m_\mu /n \sim 50\ m_e$. The breakdown of the various theoretical contributions for the μPb$\left[5g_{9/2} - 4f_{7/2}\right]$ transition is shown in Fig. 3. (Only order of magnitudes are given.) Contribution (a) is the Dirac transition energy for a spin 1/2 particle (with the muon magnetic moment) in a point Coulomb field. The Serber-Uehling vacuum polarization potential (b) is tested at about 1% accuracy. The next largest term (c) is the famous Wichman-Kroll[41] $\alpha(Z\alpha)^3$ contribution (actually evaluated to all orders in the Coulomb potential). The calculation of these terms with finite nuclear size is given by Rinker and Wilets.[42] The contribution shown in (2) starting with order $\alpha^2(Z\alpha)^2$ has been the subject of a great deal of theoretical work. The original claim by Chen[43] that the term is negative and anomalously large has been contradicted by a number of different calculations, including those by Sundaresan and Watson,[44] Wilets and Rinker,[42] Cahn, Brown, and McKerran,[45] Fujimoto,[46] and Borie.[42] Kroll[47] has given an argument showing that the sign of (d) should be opposite to that of (c). The last contribution (e) is due to the radiative corrections to the muon line, i.e.: the muon Lamb shift, as has been calculated by Barrett, Brodsky, Goldhaber, and Erickson,[48] and by Bethe and Negele.[49] Recent reviews of the calculations are given in Refs. 44 and 50.

The comparison for μPb$\left[5g_{9/2} - 4f_{7/2}\right]$ between theory and the most recent results of two experimental groups is

Tausher *et al.*[51]	431.353 (14) keV
Dixit *et al.*[52]	431.341 (11) keV
Theory[44]	431.338 (7) keV

As noted in Ref. 52, previous experiments were in disagreement with these results, presumably due to unknown systematic errors. The present agreement with QED places severe limits on new scalar particles of low mass (< 20 MeV).[44] Also it had been conjectured that Adler's ansatz for a finite QED theory might lead to an observable modification (in fact a flattening) of the vacuum polarization potential at large momentum transfer due to nonperturbative effects.[1,53] The present comparison with theory places a new limit on where such modifications can occur.[54]

E. Muonic Helium

A long sought goal of muon physics is the formation and measurement of the properties of the fundamental hydrogenic atom $\mu^- p$. The Lamb shift (which is inverted because of the strong effect of electron pair vacuum polarization) and hyperfine splitting are particularly interesting as relatively short distance probes of the size and polarization of the proton. One is of course also anxious to check whether a specific short range muon-proton interaction is present. An even more exciting system is the $\mu\pi$ atom which is now being sought[55] by M. Schwartz's group at BNL via a rare decay of $K_L \to (\pi\mu)\nu$. A more complete review of exotic muon atoms can be found in V. Hughes recent contribution to the Santa Fe conference.[2]

Although measurements of $\mu^- p$ have not yet been reported, Zavattini's group at CERN has reported[57] the first laser excitation measurement of the $2S_{1/2}$-$2P_{3/2}$ transition of the muonic helium ion $\left[\mu\,{}^4\mathrm{He}\right]^4$. The predicted spectrum is shown in Fig. 4. The comparison of theory and experiment can be expressed in the form[58]

$$\Delta E_{exp}\left(2P_{3/2}-2S_{1/2}\right) = 1527.4 \pm .9 \ \text{MeV}$$

$$\Delta E_{th}\left(2P_{3/2}-2S_{1/2}\right) = 1811.4 - 103.1 \langle r^2\rangle + (\Delta E)_{pol} \ \text{MeV}$$

where $\langle r^2\rangle$ is taken in fm^2. If the nuclear size $\langle r^2\rangle^{1/2}$ and polarization were known to the accuracy of ΔE_{exp}, one could test the e^+e^- vacuum polarization ($\sim$ 1680 MeV) to 0.05% and the Lamb shift ($\sim$ -14 MeV) to 6% at momentum transfer $\sim 0(\alpha m_\mu)$. Taking the present measurement from e - ^{4}He scattering: $\langle r^2\rangle^{1/2} = 1.650 \pm 0.025$ fm, equality between theory and experiment requires

$$(\Delta E)_{pol} = -3.3 \pm 9 \ \text{MeV} .$$

However, according to Bernabéu and Jarlskog,[58] models for the nuclear polarization give (within ±20%)

$$(\Delta E)_{pol} = 2.8 + 40\ (\alpha - 0.07) \ \ \text{MeV}$$

where α (in fm^3) is electric polarization of ^{4}He (see Ref. 28):

$$\alpha = \frac{1}{2\pi^2} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2}\, \sigma(\nu) = 0.073 \pm .004 \ \text{fm}^3 \quad ,$$

and $\sigma(\nu)$ is the total photoabsorption cross section. The number for α presently has to be based on partial cross sections, but complete measurements will be available. Data on low q^2 inelastic scattering will also be valuable here. Clearly, as the experiments progress to other hydrogenic muonic atoms, including $\mu\,^3$He, μD, μp, and $\mu\pi$, there will be a wealth of information and checks on the basic nuclear and hadronic structure. It will be crucial, however, to have the most accurate information possible on $\langle r^2\rangle$ from electron scattering experiments.

F. Positronium/Charmonium

The positronium atom (like muonium) is a pure quantum electrodynamic system, but its theoretical structure (see Fig. 5) is potentially even more interesting, because of its sensitivity to the annihilation channels $e^+e^- \rightarrow \gamma \rightarrow e^+e^-$, $e^+e^- \rightarrow \gamma + \gamma \rightarrow e^+e^-$, and because the full complexity of the Bethe-Salpeter equation comes into play. Unfortunately this complexity also makes the calculations extraordinarily difficult, and it has only recently been that all the terms through order $\alpha^2 (\log \alpha) E_F$ in the ground state splitting have been identified and computed.[59] However, since a large number of terms of order $\alpha^2 E_F$ have not yet been computed, a decisive test at the 6 ppm precision of the latest experiments cannot be made. The present comparison is

$$\Delta E_{exp}\left(1^3S_1 - 1^1S_0\right) = \begin{cases} 203.3870\,(16)\ \text{GHz (Mills and Berman}^{60}) \\ 203.3849\,(12)\ \text{GHz (Egan, Frieze, Hughes, and Yam}^{61}) \end{cases}$$

$$\Delta E_{th} = 203.4013\ (?)\ \text{GHz (T. Fulton } \underline{\text{et al}}.^{59})$$

The computation of the order α^2 terms is the most challenging calculation of the problems remaining to be done in QED.[62]

The first measurement of the fine structure of the first excited state of positronium was reported by a Brandeis University group[63] during this past year. The comparison with the theoretical value of Fulton and Martin[64] is

$$\Delta E_{exp}\left(2^3S_1 - 2^3P_2\right) = 8628.4\,(2.8)\ \text{MHz}$$

$$\Delta E_{th}\left(2^3S_1 - 2^3P_2\right) = 8625.14\ (?)\ \text{MHz}$$

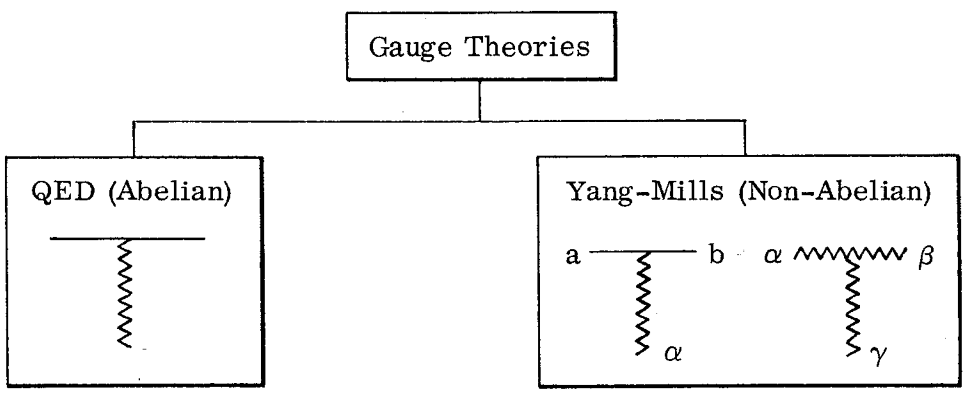
It is quite striking that the same graph which labels the positronium spectrum seems to fit (at least tentatively) the spectrum of new particles discovered this past year at SPEAR (SLAC),[65] BNL,[66] and DORIS (DESY).[67] (See Fig. 6.) The indicated photon transitions indicate radiative decays that have been observed in the storage ring experiments. In addition to these, the other decays which have been identified and which restrict the quantum numbers are: $\psi \to e^+e^-$, $\mu^+\mu^-$, plus many hadronic modes; $\psi' \to \psi\pi\pi$, $\psi\eta$, e^+e^-, $\mu^+\mu^-$; $\chi(3530) \to 4\pi$, 6π, $\pi\pi KK$; $\chi(3410) \to 4\pi$, 6π, $\pi\pi$ or KK; and $\eta_c(2750) \to p\bar{p}$. The P_c state is observed in the decay $\psi' \to P_c + \gamma_1$ but its mass is ambiguous since the prompt photon γ_1 has not been $\hookrightarrow \psi + \gamma_2$ identified. If the P_c mass turns out to be 3500 MeV, then the state is more likely to be P-state, instead of the indicated ${}^1S_0$ assignment.

The rate of decay of the ψ and ψ' into hadrons is three orders of magnitude suppressed compared to the width expected for hadrons of such mass; this together with the similarity of the spectra of Figs. 5 and 6 suggests a new "atomic" system with the degrees of freedom of two charged spin 1/2 fermions—which must have a distinct quantum number to distinguish them from the quark constituents of the usual hadrons. At least one additional quark is in fact needed to understand strangeness-changing weak interactions: hence the charm model and charmonium.[68] The identification may be premature, however, in view of the complicated structure of $e^+e^- \to$ hadrons in the $E_{cm} = 4$ GeV region, and difficulties in understanding the radiative transition rates. A more complicated system involving several quarks with new quantum numbers may be required.[68]

The present challenge for theorists is to fit the charmonium spectrum and the known decay rates to a reasonable effective potential for binding spin 1/2 constituents. Theoretical prejudice and the quark-gluon models suggest effective potentials which mimic the α/r Coulomb potential at short distance and which lead to confinement at large distances. The problems of calculation of the new particle spectrum parallel those of positronium and many QED results can be immediately rescaled to the ψ domain. Among the difficult questions are how to include relativistic corrections, annihilation channels, and the relation of the effective potential of a reduced-mass Dirac equation back to the interaction Lagrangian of a basic field theory. The new particle spectrum has thus renewed the search for efficient calculational techniques applicable to the two body relativistic atom. It is interesting to note that the first observation[63] of the excited positronium state was made in the same year as the discovery of the corresponding spectrum of ψ states.

IV. GENERALIZATIONS OF QED

The success of QED in describing the electrodynamics of the leptons has continually led theorists to a search for other local field theories which could possibly be applicable to the weak or even strong interactions. In fact QED is a subcategory of a more general set of renormalizable theories of spin 1/2 plus spin one fields called gauge theories.[69]



QED is an "Abelian" theory since a single fermion line carries a conserved current (more formally, this corresponds to invariance of the Lagrangian under a local phase change $e^{i\Lambda(x)}$ of the fermion field). In the non-Abelian Yang-Mills theories, the conserved current couples together different fermion lines and must also be carried by some of the spin one "gluon" fields. Thus the gluon lines are themselves "charged". (Such a conserved current arises due to an invariance of the Lagrangian under a local unitary transformation of an irreducible representation of fermion fields of some Lie group.) Both fermions and gluons can be given non-zero masses in these theories if one introduces additional spin-zero fields with non-zero vacuum expectation values. Symmetries are then broken and currents become non-conserved, but renormalizability is maintained.

The application of the non-Abelian gauge theories to the weak interactions, as suggested by Weinberg[70] and Salam,[71] has led to some impressive successes, including the prediction of neutral currents. Assuming the Lie group symmetry is $SU(2)\times U(1)$, the weak interaction is mediated by an "isovector" triplet of intermediate vector (and axial vector) bosons (W^+, W^o, W^-) which couple to doublets of leptons $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$, $\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$, etc. or quarks $\begin{pmatrix} d \\ u \end{pmatrix}$, $\begin{pmatrix} s \\ c \end{pmatrix}$, etc. The W^o vector boson mixes with the photon. This scheme, with V-A charged currents, appears to be consistent with present weak interaction experiments,[72] and implies a unification of electromagnetic and weak interactions: Only one coupling constant, e, enters. Because of renormalizability, consistent calculations of higher order weak interaction contributions can now be carried out, such as the weak interaction perturbation to the muon moment as we discussed earlier, and electromagnetic interactions of the neutrino.[23] In some models, with six quarks, the neutral current need only have a vector interaction, so parity—violating effects in atoms and nuclei are not expected from lowest order weak interactions.[73] Such models also can incorporate the superweak model of CP violation.[74] In addition, heavy leptons and then neutral counterparts may also be included in the gauge theory models.[75]

In the case of the strong interactions, there is considerable optimism that a non-Abelian gauge theory of quarks and spin-one gluons may provide a viable model of the hadronic world.[76] The most promising candidate theory, called "Quantum Chromodynamics" (QCD) by Gell-Mann, is based on $SU(3)_{color}$ where the strong interaction is mediated by an octet of vector gluons which couple together triplets of quarks

$$\begin{pmatrix} q_R \\ q_Y \\ q_B \end{pmatrix}$$

(with "color" labels red, yellow, and blue), as well as to themselves. The gluon coupling is independent of the quark type (or "flavor") u, d, s, etc. The meson and baryon are then identified as the color singlet (i.e.: white or neutral) composites of quark fields, $(q\bar{q})$ and (qqq) respectively. Among the features of this model are (1) the Fermi statistics problem is solved for the baryons since the color singlet (qqq) state is antisymmetric in the color label. (2) The $q\bar{q}$ and qqq forces are attractive. (3) Except for the pion, the known hadron spectrum can be reasonably fit in a phenomological "bag" model based on QCD.[76] The spin-dependent $q\bar{q}$ interactions play an essential role here. (4) There are indications, but no proof, that QCD may only possess solutions for the color singlet states which would imply absolute quark (and gluon) confinement within hadrons. The infrared divergence due to the emission of massless soft gluons from an interacting quark is evidently more severe in QCD than QED since the soft gluons themselves radiate ad infinitum.[77] However, as in QED, the infrared divergences due to soft bremsstrahlung is not present if the external particles are all neutral.[78, 79] (5) Because the sign of the vacuum polarization (due to virtual gluon pairs) is opposite to that of QED, the effective strength of the gluon interactions in QED becomes weaker at short distances ("asymptotic freedom"),[80] leading to Bjorken scaling for deep inelastic scattering and the general features of the parton model.

V. SCALING LAWS AND FORM FACTORS

A common link between all the gauge theories is the fact that the coupling constant is dimensionless. This fact, together with the applicability of perturbation theory (due to the small value of α in QED, and asymptotic freedom at short distances in QCD), implies that there is no intrinsic mass scale at large momentum transfers in these theories (modulo calculable logarithmic corrections). Thus the scaling laws of QED must also be the scaling laws of the weak and strong interactions if the gauge theories are applicable.

A general prediction based on scale invariance at short distances is the dimensional counting rule for fixed angle scattering ($t/s = -1/2\,(1-\cos\theta_{cm})$, $s = E_{cm}$)[78,81]

$$\frac{d\sigma}{dt}(A+B \to C+D) \Rightarrow \frac{1}{s^{n-2}} f(t/s)$$

where $n = n_A + n_B + n_C + n_D$ is the total number of elementary fields (e, μ, γ, q, etc.) in A, B, C and D. Thus the scaling behavior of elastic positronium-positronium scattering in QED is predicted to be the same as that for $\pi + \pi \to \pi + \pi$! ($d\sigma/dt \sim s^{-6} f(\theta_{cm})$). The predictions for hadron scattering and photoproduction are consistent with experiment and are reviewed in Ref. 82. The prediction for $pp \to pp$ is $d\sigma/dt = s^{-10} f(\theta_{cm})$. Figure 7 shows a comparison with the scaling law; the best fit giving[83] $s^{-9.7\pm0.5}$. The scaling law can be heuristically derived by neglecting binding of the constituents and calculating the scaling law in perturbation theory for the n-particle amplitude obtained if each composite particle's momentum is partitioned among its constituents in proportion to their mass.[78]

The counting rule is also applicable to electron-hadron (elastic or inelastic) scattering, thus determining the power-law falloff of the form factors at large t. For the spin averaged form factor, scale-invariance predicts[78,81]

$$t^{n_A-1} F_A(t) \rightarrow \text{const}$$

where n_A is the number of constituent fields in A. Thus the form factor falloff is controlled by the number of constituents which must change momentum from along p_i to p_f. This implies $F(t) \sim t^{-1}$ for mesons or positronium (inelastic transition), $F_1(t) \sim t^{-2}$ for baryons or three-lepton atoms $(e^- \mu^+ e^-)$, and $F(t) \sim t^{-5}$ for the deuteron (six quarks). The prediction $t^{n_A-1} F_A(t) \rightarrow$ const is compared with experiments in Figs. 8 and 9 for π,[84] N,[85] and D.[86]

The structure of the deuteron form factor in a relativistic theory can be understood in some detail from Fig. 10. If we neglect the binding of the deuteron, then the calculation of the form factor requires the change of momentum from $1/2\,p$ to $1/2\,(p+q)$ for each nucleon. Thus the natural scaling form for the deuteron at large t is[87]

$$F_D(t) = F_N^2(t/4) \frac{1}{1 - t/m_0^2}$$

where F_N is the nucleon form factor and the last factor accounts for the additional power in the counting rule. [Note that the scaling of $F_D(t)$ is identical to that of a system with two elementary fields after the nucleon form factors are removed.] This can obviously be generalized to He, etc. A comparison of the prediction $\left[1-t/m_0^2\right] F_D(t)/F_N^2(t/4) \rightarrow$ const with the recent data[86] of Chertok's group is given in Fig. 11.

When the momentum of the deuteron is finally partitioned out to the quarks then perturbation theory diagrams of the type shown at the bottom of Fig. 10

have to be considered. The first graph, with the exchange of a colored (octet) gluon between (singlet) nucleons, is forbidden in the QED model. The quark interchange graph shown at the bottom right is allowed and has the structure of the form given for $F_D(t)$. Furthermore, the quark interchange diagram is clearly the lowest order perturbation theory contribution to what can be identified as an exchange current contribution to the deuteron form factor.

The application of the quark model and gauge theories to the structure of the deuteron provides important constraints on the nuclear potential at momentum transfers beyond ~1 GeV. The dimensional counting predictions imply that the effective nucleon potential is dominated at short distances by the interchange of quarks, and after the nucleon structure is accounted for, has remarkably simple scaling behavior.

Further applications of the counting rules to hadron physics are given in Refs. 78, 82, and 88. A more comprehensive discussion of the applications to the form factors of hadrons and nucleii will be presented in Ref. 89.

Acknowledgement

I would like to thank Professors H. Anderson and V. Hughes for helpful conversations and the hospitality of the LAMPF Users group.

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90. I wish to thank B. Chertok and R. Arnold for providing Figs. 9 and 11.

FIGURE CAPTIONS

1. Representative contributions to the magnetic moment of the muon: (a) electron-pair vacuum polarization; (b) the photon-photon scattering contribution calculated in Refs. 16-19; (c) hadronic vacuum polarization; (d) the weak interaction contribution calculable in gauge theories.

2. Representative contributions to the muonic x-ray transition $5\,g_{9/2} - 4f_{7/2}$ in μPb: (a) the level spacing for a Dirac muon in the Coulomb field of the nucleus; (b) the electron-pair vacuum polarization contribution; (c) the lowest order Wichman-Kroll[41] contribution from photon-photon scattering; (d) the order $\alpha^2(Z\alpha)^2$ contribution calculated in Refs. 44-46; (e) the muon self-energy contribution[48]. Only orders of magnitude are indicated in this figure.

3. Determinations of the fine structure constant. See text and Ref. 39.

4. The n=1 and n=2 levels of muonic helium ion. The measurement of the $2P_{3/2} - 2S_{1/2}$ transition is reported in Ref. 57.

5. The spectrum of positronium. The n=2 transition $^3S_1 - {}^3P_2$ is measured in Ref. 63.

6. A possible placement of the spectrum of new particles in the Charmonium scheme, and the observed radiative transitions. [See text and Refs. 65-68.]

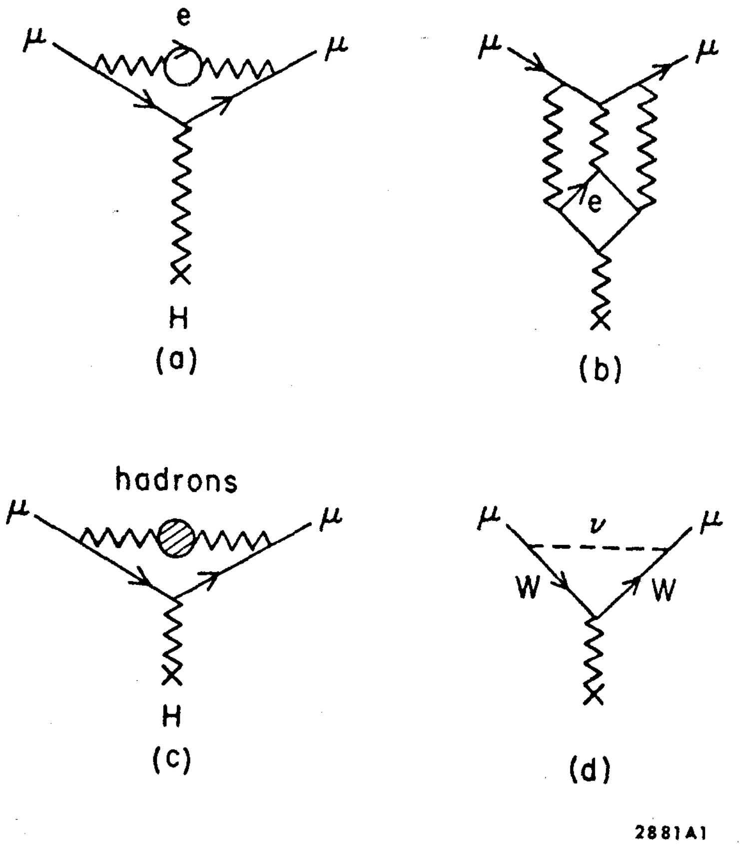
7. Comparison of data for the elastic pp cross section $d\sigma/dt$ [in $\mathrm{cm}^2/\mathrm{GeV}^2$] with the fixed angle scaling law $d\sigma/dt = s^{-N} f(\theta_{cm})$ (solid line). The best fit is $N = 9.7 \pm 0.5$. [From Ref. 83.]

8. Comparison of the dimensional counting prediction $t^2 G_M^p(t) \rightarrow \text{const}$ with the SLAC experimental data.[85]

9. Tests of the dimensional counting prediction $t^{n_A - 1} F_A(t) \to \text{const}$ with the experimental data for the pion (electroproduction data), proton, neutron, and elastic deuteron form factors.[90]

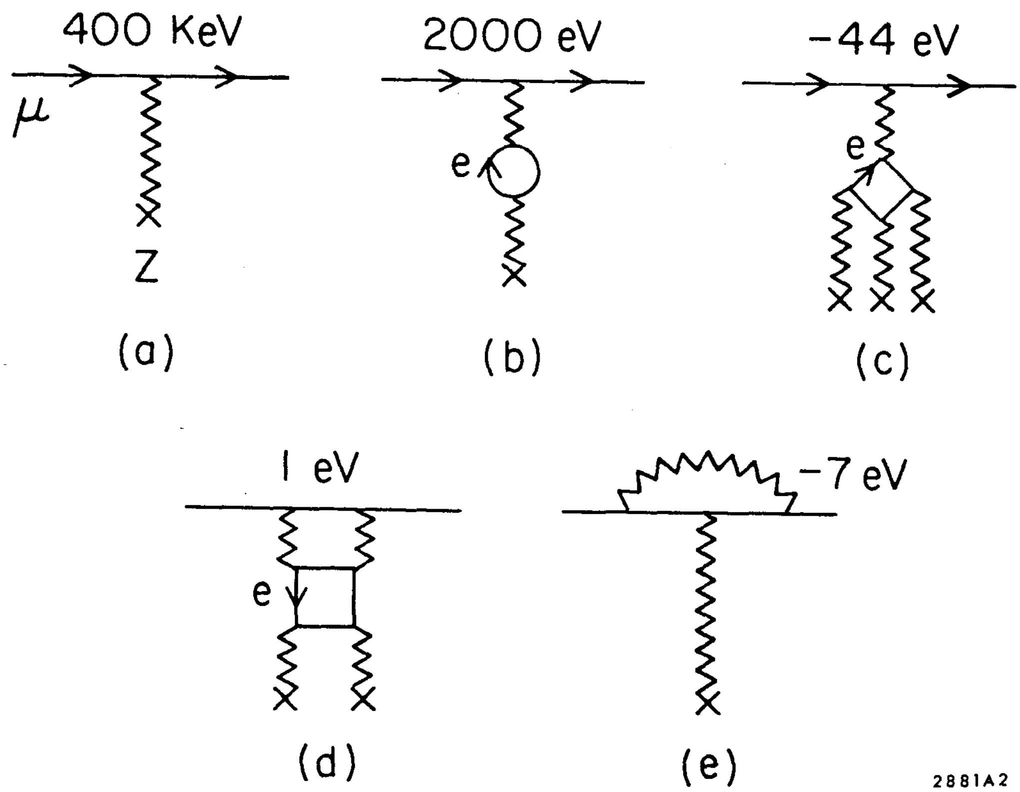
10. Structure of the deuteron form factor using the partition procedure.

11. Test of the quark model prediction $F_D(t)/\left[F_N^2(t/4) \dfrac{1}{1-t/m^2}\right] \to \text{const}$ with the experimental data of Ref. 86.[90]



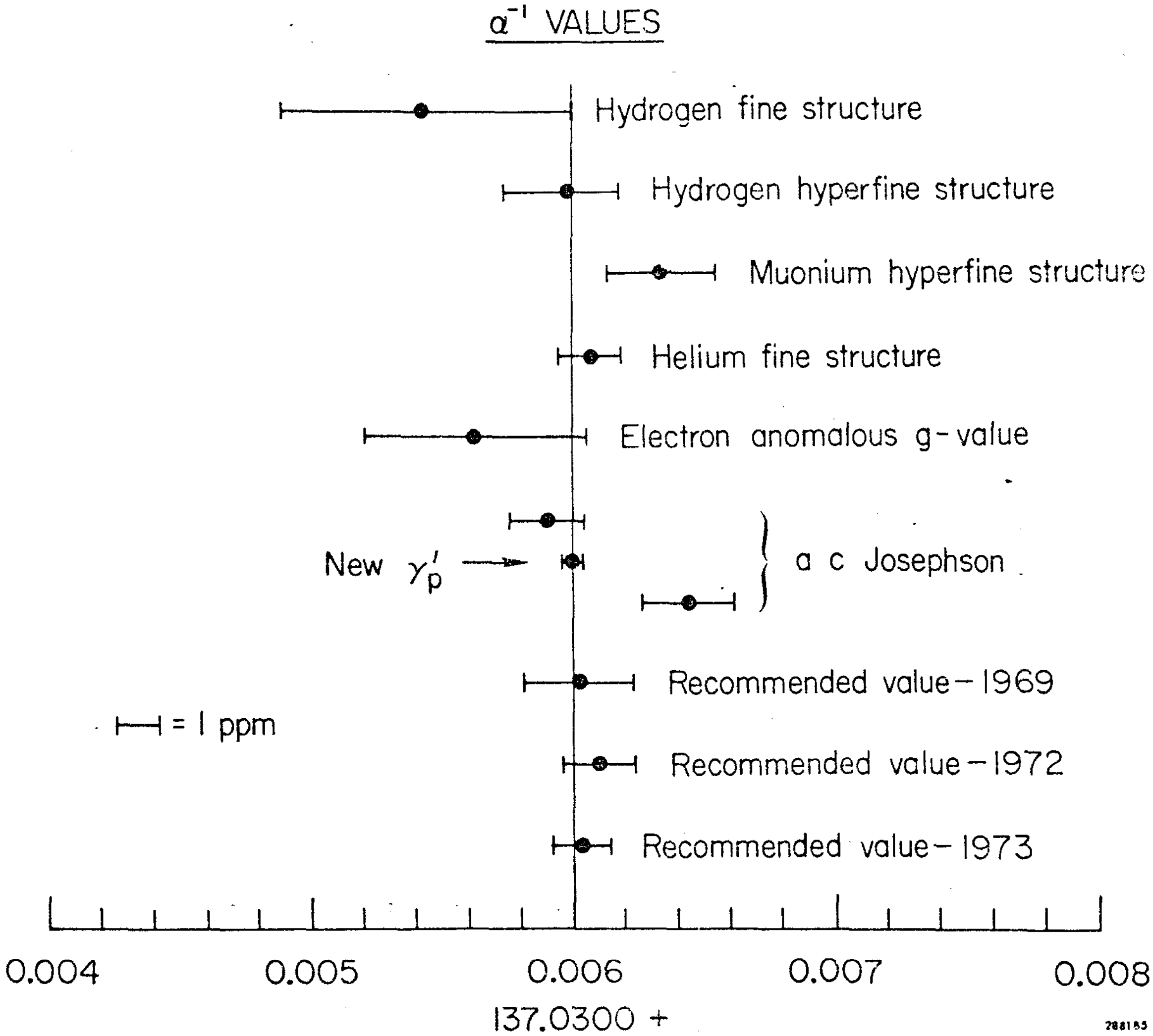
e
μ
μ
H
(a)
μ
μ
e
(b)
hadrons
μ
μ
H
(c)
μ
ν
μ
W
W
(d)
2881A1

Fig. 1



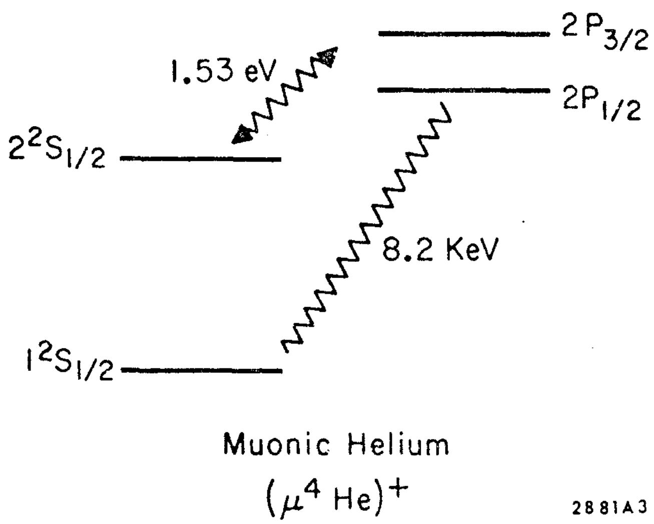
400 KeV
μ
Z
(a)
2000 eV
e
(b)
−44 eV
e
(c)
1 eV
e
(d)
−7 eV
(e)
2881A2

Fig. 2



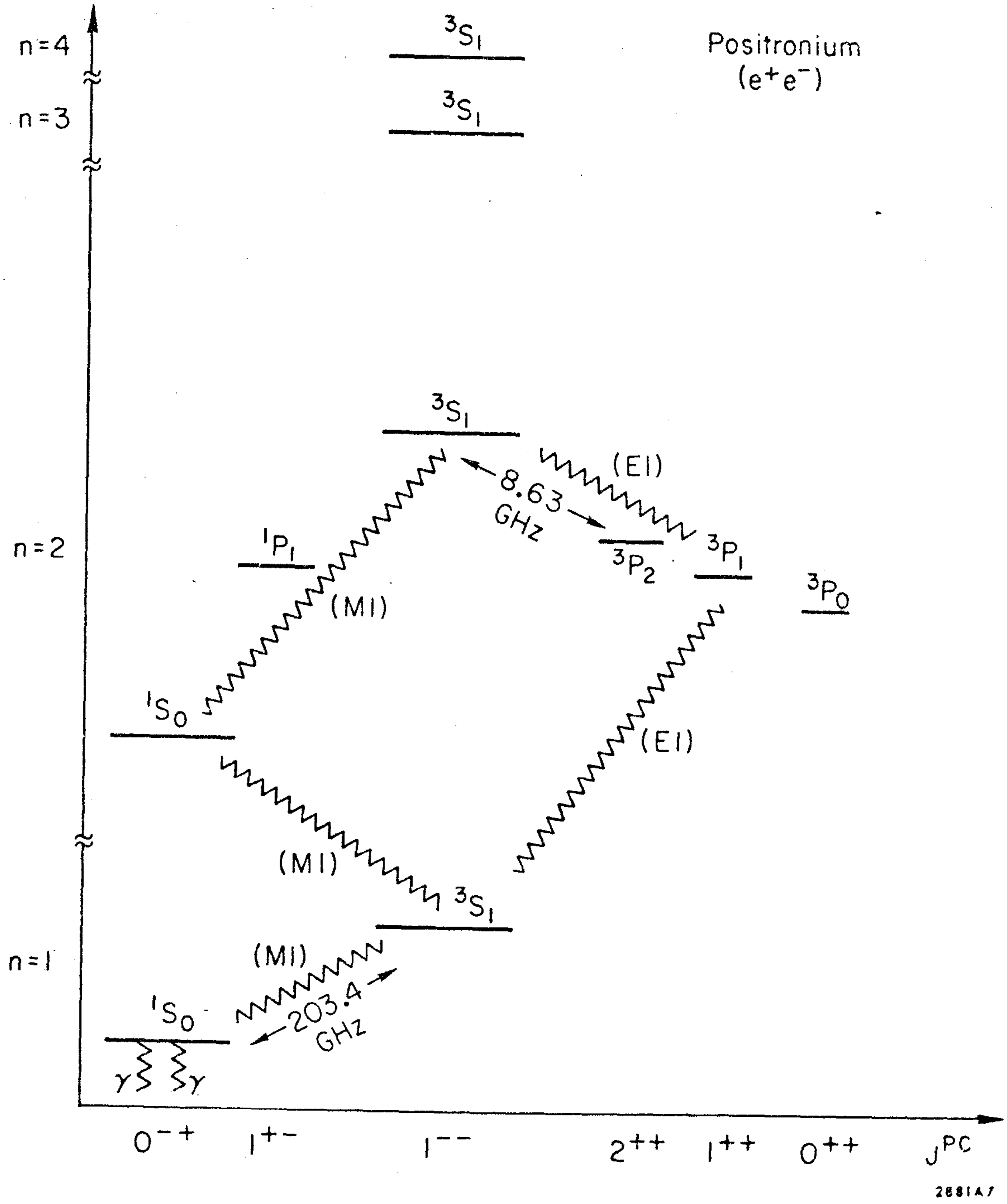
α⁻¹ VALUES
Hydrogen fine structure
Hydrogen hyperfine structure
Muonium hyperfine structure
Helium fine structure
Electron anomalous g-value
New γ_p'
a c Josephson
Recommended value-1969
Recommended value-1972
Recommended value-1973
= 1 ppm
0.004
0.005
0.006
0.007
0.008
137.0300 +
288153

Fig. 3



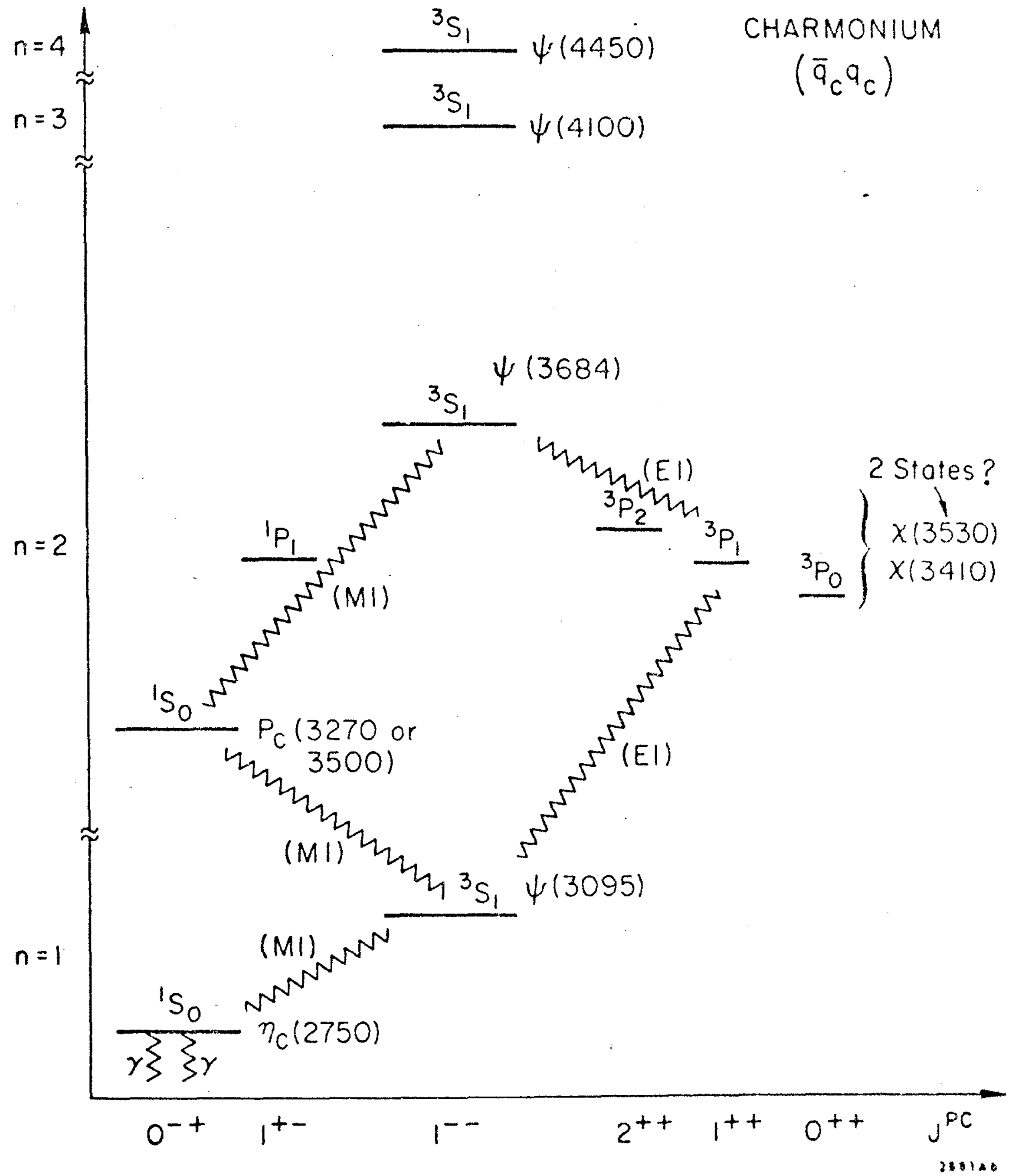
$2P_{3/2}$
1.53 eV
$2P_{1/2}$
$2^2S_{1/2}$
8.2 KeV
$1^2S_{1/2}$
Muonic Helium
$(\mu^4\,He)^+$
2881A3

Fig. 4



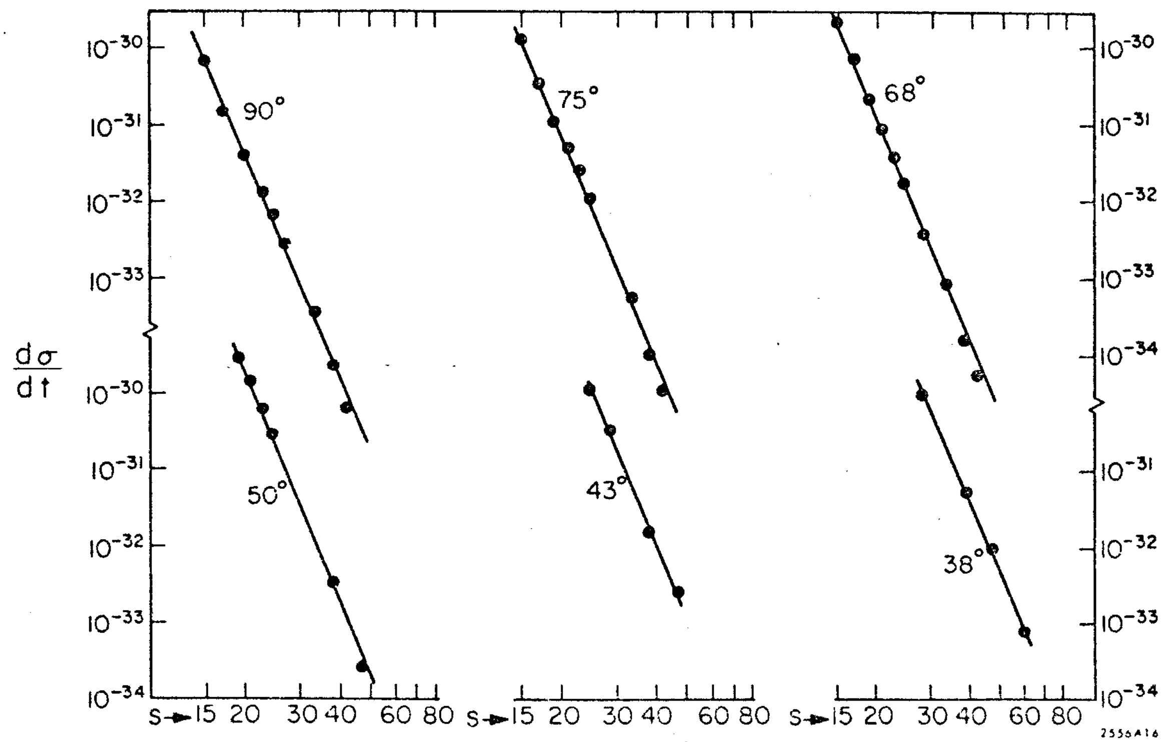
n=4
n=3
n=2
n=1
$^3S_1$
$^3S_1$
Positronium
(e^+e^-)
$^3S_1$
(E1)
8.63 GHz
$^1P_1$
$^3P_2$
$^3P_1$
$^3P_0$
(M1)
$^1S_0$
(E1)
(M1)
$^3S_1$
(M1)
203.4 GHz
$^1S_0$
γ
γ
0^{-+}
1^{+-}
1^{--}
2^{++}
1^{++}
0^{++}
J^{PC}
2881A7

Fig. 5



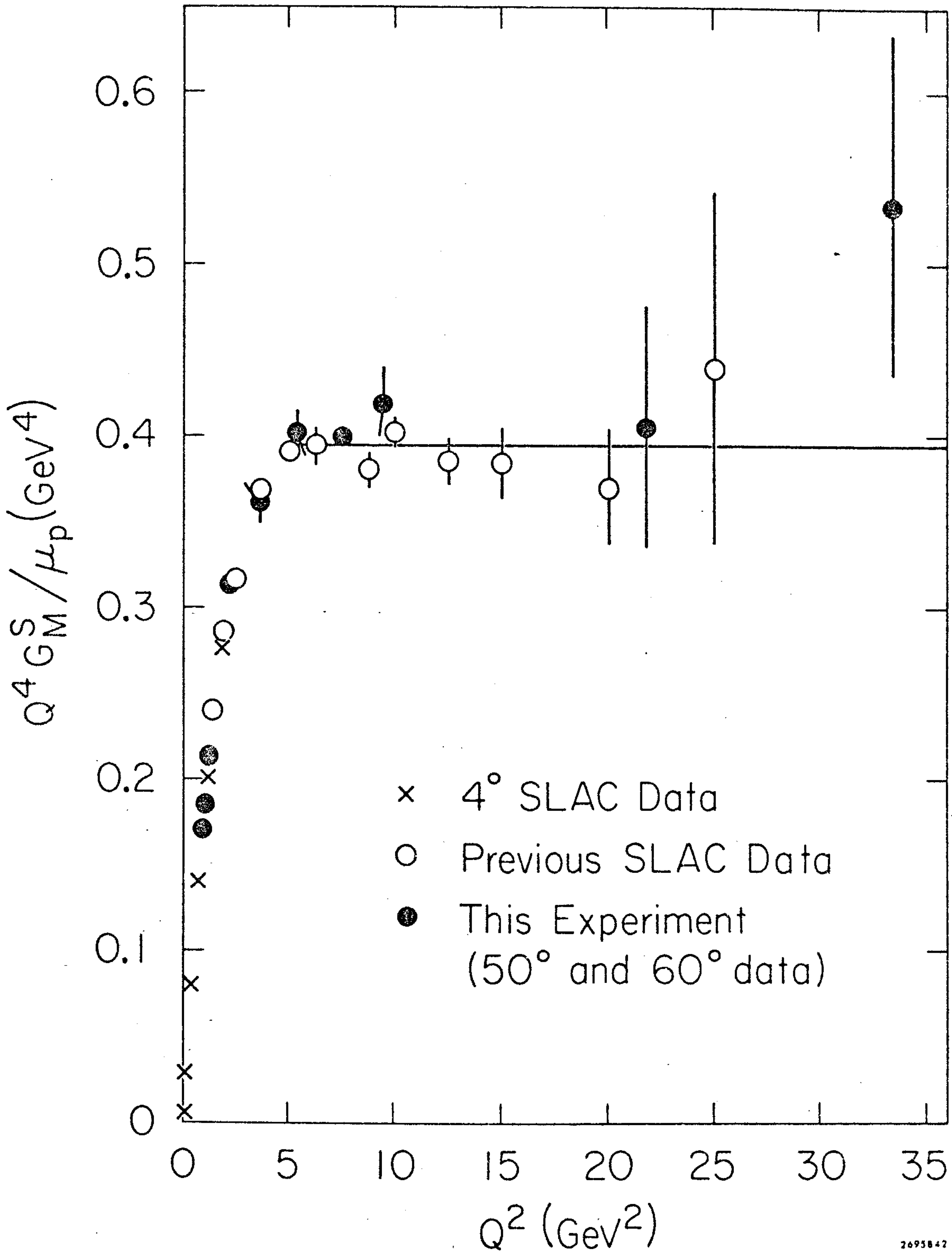
CHARMONIUM
$(\bar{q}_c q_c)$
n = 4
n = 3
n = 2
n = 1
$^3S_1$ $\psi(4450)$
$^3S_1$ $\psi(4100)$
$\psi(3684)$
$^3S_1$
(E1)
2 States?
$^3P_2$
$^3P_1$
$\chi(3530)$
$^1P_1$
$^3P_0$
$\chi(3410)$
(M1)
$^1S_0$
P_c (3270 or 3500)
(E1)
(M1)
$^3S_1$ $\psi(3095)$
(M1)
$^1S_0$
$\eta_c(2750)$
γ γ
0^{-+} 1^{+-} 1^{--} 2^{++} 1^{++} 0^{++} J^{PC}

Fig. 6



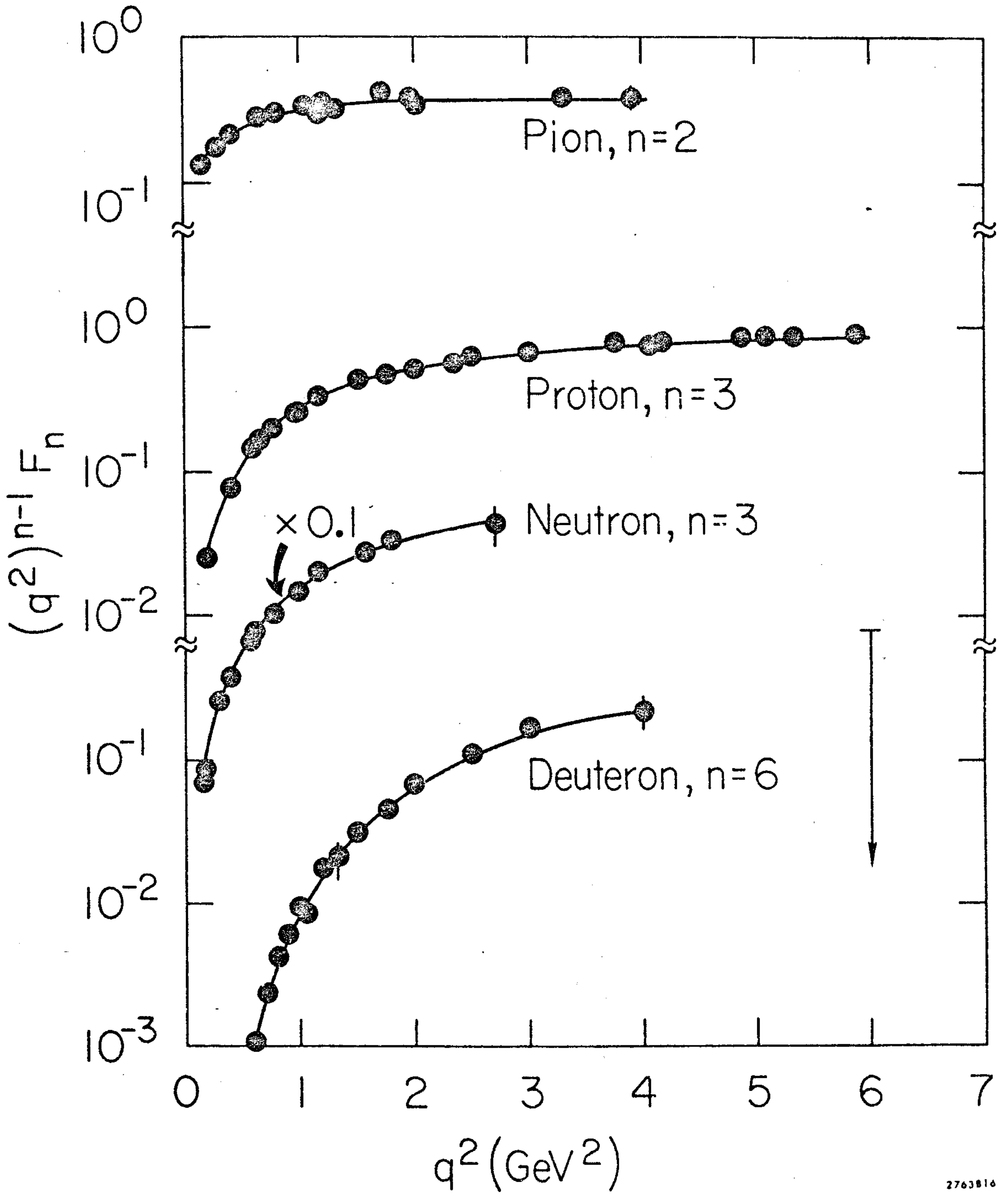
$\frac{d\sigma}{dt}$
10^{-30}
10^{-31}
10^{-32}
10^{-33}
10^{-34}
90°
75°
68°
50°
43°
38°
S→15 20 30 40 60 80
2556A16

Fig. 7



0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
$Q^4 G_M^S / \mu_p (\mathrm{GeV}^4)$
$Q^2 (\mathrm{GeV}^2)$
× 4° SLAC Data
○ Previous SLAC Data
● This Experiment (50° and 60° data)
2695B42

Fig. 8



10^0
10^{-1}
10^0
10^{-1}
10^{-2}
10^{-1}
10^{-2}
10^{-3}
Pion, n=2
Proton, n=3
Neutron, n=3
×0.1
Deuteron, n=6
$(q^2)^{n-1} F_n$
0
1
2
3
4
5
6
7
q^2 (GeV^2)
2763B16

Fig. 9

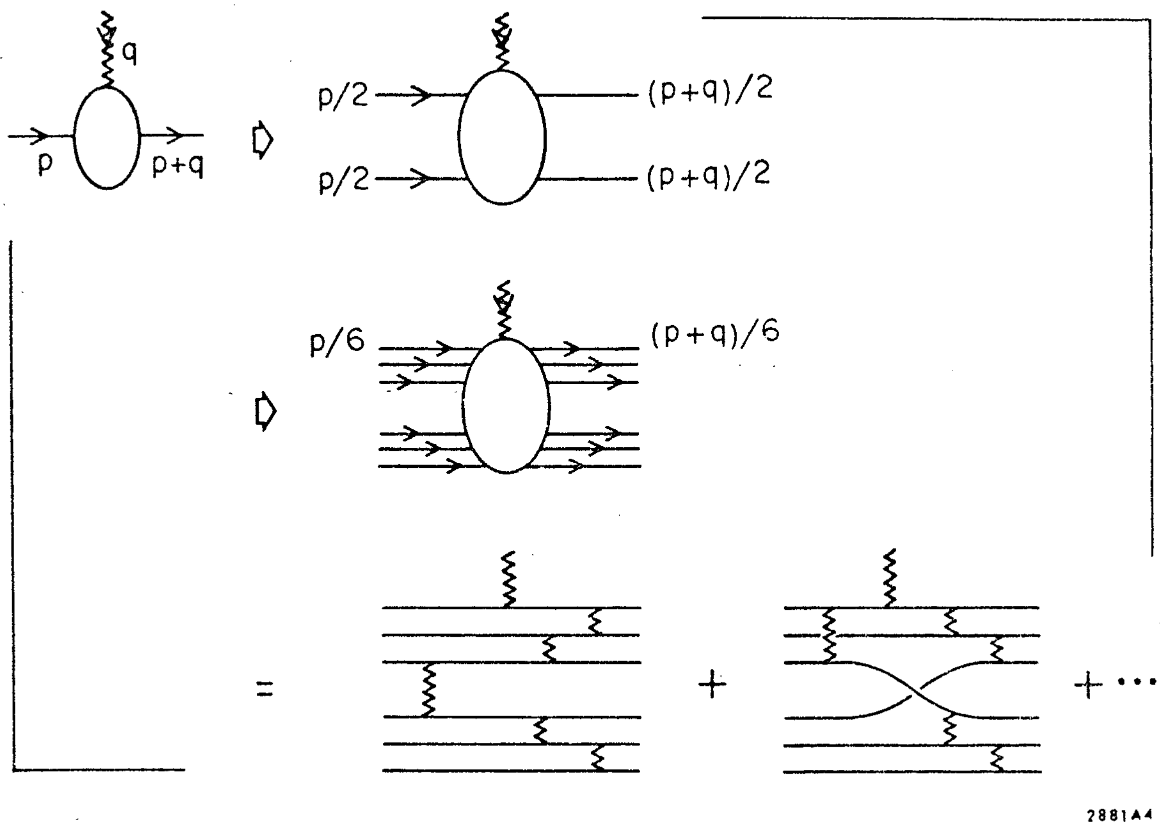
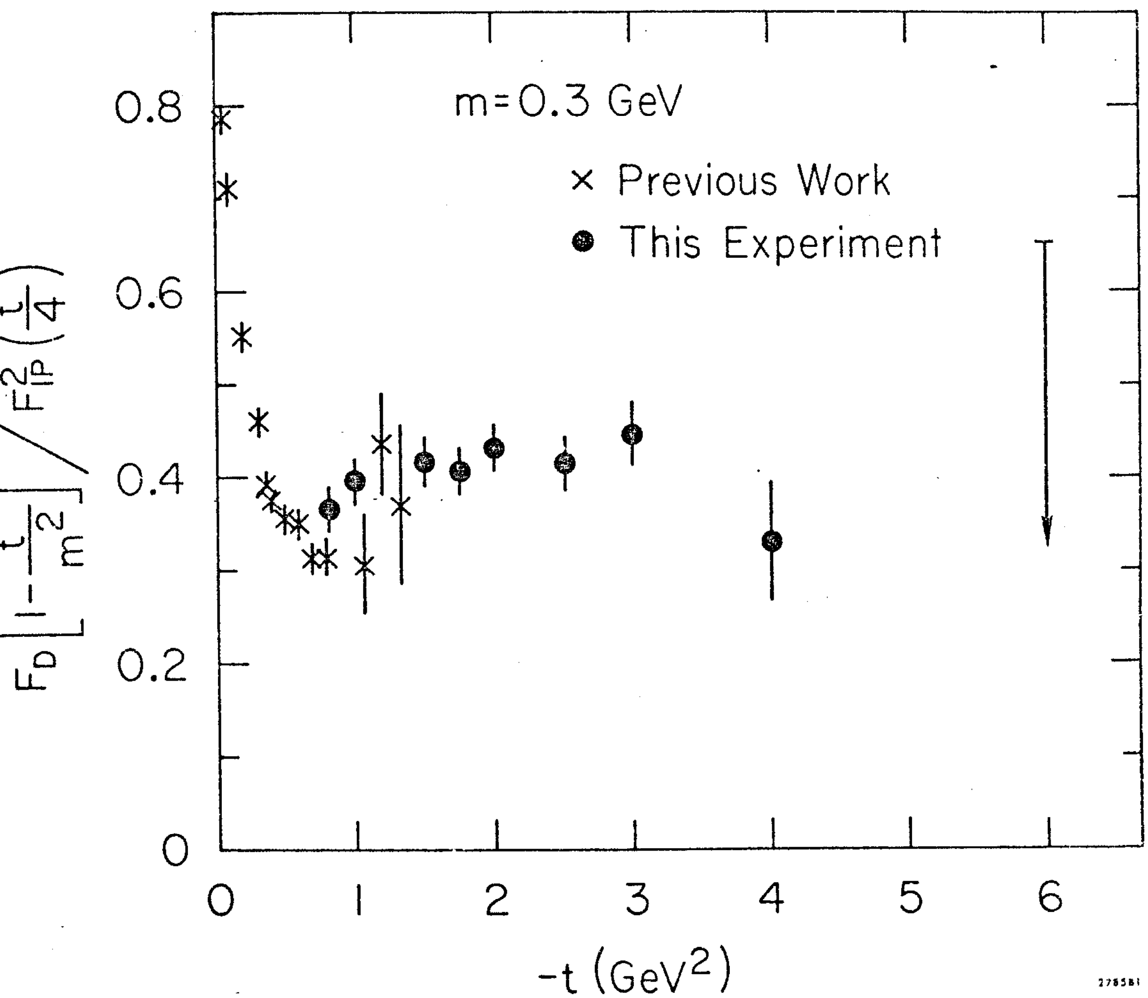
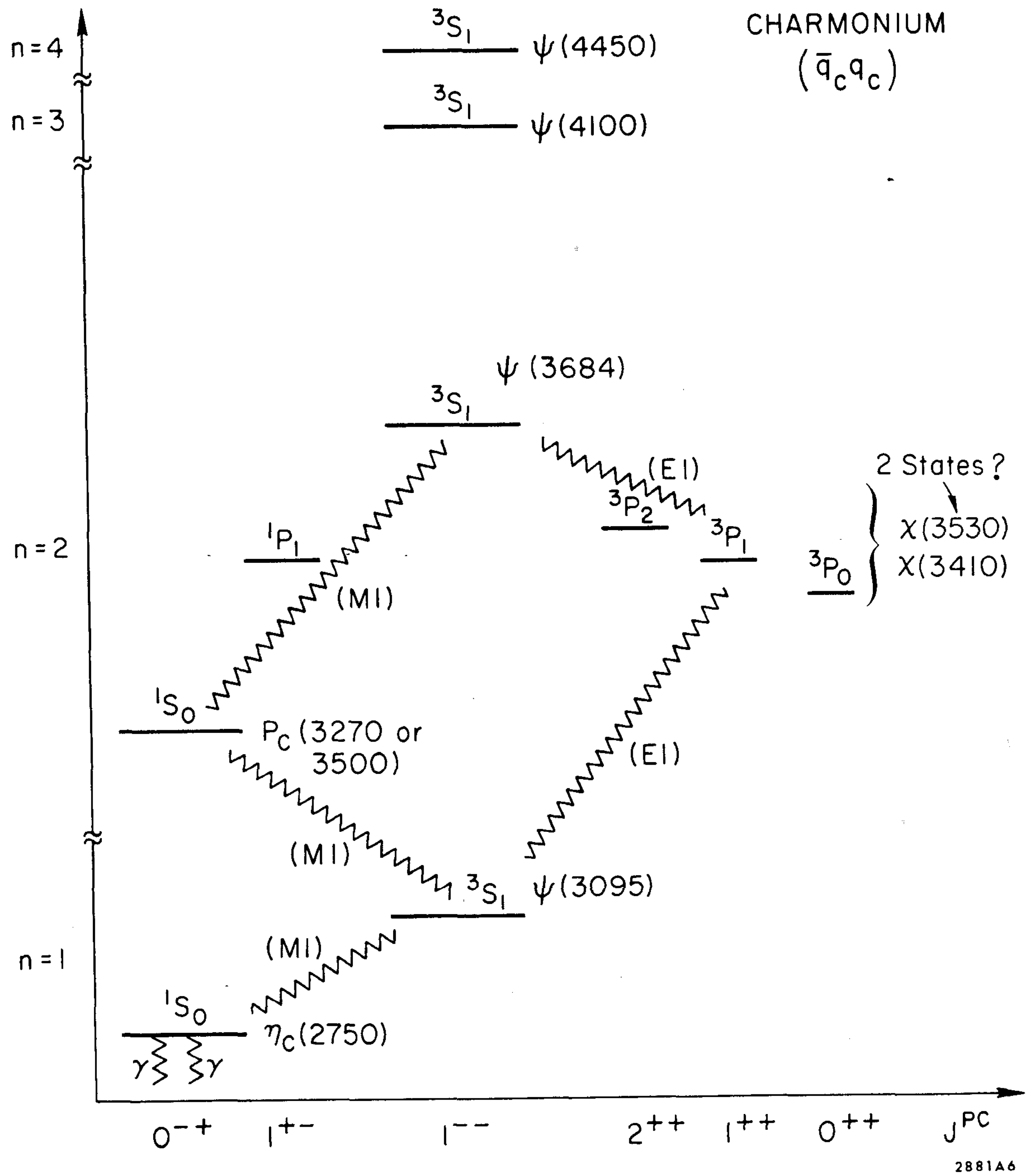


Fig. 10



m=0.3 GeV
× Previous Work
● This Experiment
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
$F_D\left[1-\frac{t}{m^2}\right] \Big/ F_{1P}^2\left(\frac{t}{4}\right)$
$-t\ (\mathrm{GeV}^2)$

Fig. 11



CHARMONIUM
($\bar{q}_c q_c$)
n = 4
n = 3
n = 2
n = 1
$^3S_1$ $\psi(4450)$
$^3S_1$ $\psi(4100)$
$\psi(3684)$
$^3S_1$
(E1)
2 States ?
$^3P_2$
$^3P_1$
$^3P_0$
$\chi(3530)$
$\chi(3410)$
$^1P_1$
(M1)
$^1S_0$
P_c (3270 or 3500)
(E1)
(M1)
$^3S_1$ $\psi(3095)$
(M1)
$^1S_0$
$\eta_c(2750)$
γ γ
0^{-+} 1^{+-} 1^{--} 2^{++} 1^{++} 0^{++} J^{PC}
2881A6

Fig. 6