# COMMENT ON THE ASSOCIATED MULTIPLICITY IN SEMI-INCLUSIVE REACTIONS* 

A. C. D. Wright $\dagger$<br>Stanford Linear Accelerator Center<br>Stanford University, Stanford, California 94305

J. L. Alonso $\dagger \dagger$<br>Departamento de Fisica Teorica Universidad de Zaragoza, Spain<br>and<br>Stanford Linear Accelerator Center Stanford University, Stanford, California 94305


#### Abstract

The average charged multiplicity $\overline{\mathrm{n}}_{\mathrm{c}}$ for the semi-inclusive reaction $p_{1} p_{2} \rightarrow p_{3} p_{4} X$ is studied in the context of a previously proposed twocomponent model. Recent data for $\bar{n}_{c}$ at $28.5 \mathrm{GeV} / \mathrm{c}$ for $\mathrm{p}_{3 \mathrm{~T}} \leq 1 \mathrm{GeV} / \mathrm{c}$ are shown to be satisfactorily described by the soft component, without requiring any contribution from hard scattering. Reasonable assumptions for the hard component are made in order to estimate $\bar{n}_{c}$ as a function of $p_{3 T}$.


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[^0]In a previous paper ${ }^{1}$ we proposed a two-component model for the average charged multiplicity $\overline{\mathrm{n}}_{\mathrm{c}}$ and single particle distributions in inclusive reactions. Our work was motivated by the need to understand the behavior of $\bar{n}_{c}$ for the reactions $\mathrm{pp} \rightarrow \mathrm{pX}$ and $\mathrm{pp} \rightarrow \pi^{+} \mathrm{X}$ observed ${ }^{2}$ at $\mathrm{p}_{\text {lab }}-28.5 \mathrm{GeV} / \mathrm{c}$. The steplike dependence on the transverse momentum $\mathrm{p}_{\mathrm{T}}$ of the trigger for fixed missing mass in these data does not seem to be of kinematic origin, ${ }^{3}$ but rather suggests a transition in the dynamics of particle production at $p_{T} \approx 1 \mathrm{GeV} / \mathrm{c}$. In our model, the steplike behavior of $\overline{\mathrm{n}}_{\mathrm{c}}$ is attributed to the transition from the "soft" coherent-scattering regime at $\mathrm{p}_{\mathrm{T}} \lesssim 1 \mathrm{GeV} / \mathrm{c}$ to the region where "hard", incoherent scattering from the hadronic constituents dominates at $p_{T} \gtrsim 1 \mathrm{GeV} / \mathrm{c}$.

The advent of recent data ${ }^{4}$ on the associated multiplicity in the semi-inclusive reaction

$$
\begin{equation*}
\mathrm{p}\left(\mathrm{p}_{1}\right)+\mathrm{p}\left(\mathrm{p}_{2}\right) \rightarrow \mathrm{p}\left(\mathrm{p}_{3}\right)+\mathrm{p}\left(\mathrm{p}_{4}\right)+\mathrm{X} \tag{1}
\end{equation*}
$$

poses a further challenge for our model. Consequently, in the present note we analyze the data of Ref. 4 in our picture.

For completeness, we first recall the relevant results of paper I. Particle production in an inclusive reaction is assumed to proceed through either of two mechanisms:
(i) incoherent, hard scattering from the hadronic constituents,
(ii) coherent, soft scattering from the constituents.

Process (ii) dominates at small $\mathrm{p}_{\mathrm{T}}$, and corresponds to a Mueller-Regge description of particle production, whereas process (i) takes over at large $\mathrm{p}_{\mathrm{T}}$ because of its power-law dependence on $\mathrm{p}_{\mathrm{T}}$. The invariant single particle distribution for $p\left(p_{1}\right)+p\left(p_{2}\right) \rightarrow p\left(p_{3}\right)+X$ is given by

$$
\begin{equation*}
\mathrm{f}\left(\mathrm{p}_{3}\right) \equiv \mathrm{E}_{3} \frac{\mathrm{~d} \sigma}{\mathrm{~d}^{3} \mathrm{p}_{3}}=\mathrm{f}_{\mathrm{S}}\left(\mathrm{p}_{3}\right)+\mathrm{f}_{\mathrm{h}}\left(\mathrm{p}_{3}\right) \tag{2}
\end{equation*}
$$

where $f_{s}\left(p_{3}\right)$ and $f_{h}\left(p_{3}\right)$ are the soft and hard components, respectively. The associated mean total charged multiplicity is obtained by averaging the contributions from soft and hard components with appropriate weights,

$$
\begin{equation*}
\bar{n}_{c}\left(p_{3}\right) f\left(p_{3}\right)=\bar{n}_{s}\left(p_{3}\right) f_{s}\left(p_{3}\right)+\bar{n}_{h}\left(p_{3}\right) f_{h}\left(p_{3}\right) \tag{3}
\end{equation*}
$$

The multiplicity associated with the soft component is assumed to have a simple logarithmic dependence on the missing mass M ,

$$
\begin{equation*}
\overline{\mathrm{n}}_{\mathrm{s}}\left(\mathrm{p}_{3}\right)=\mathrm{a}+\mathrm{b} \ln \mathrm{M}^{2} \tag{4}
\end{equation*}
$$

where $\mathrm{a}=\mathrm{b}=1$ was found in paper I to give a good fit to data ${ }^{5}$ at $205 \mathrm{GeV} / \mathrm{c}$ (we do not include the trigger proton in this expression). We note that (4) can be obtained in multiperipheral models ${ }^{6}$ and corresponds to the idea that the asymptotic multiplicity in a system of particles is determined principally by the invariant mass of the system. ${ }^{7}$ We mean this statement to apply asymptotically to soft processes only. For instance, at low invariant mass the multiplicity for a $B$ (baryon number) $=2$ system is necessarily lower than that for a $B=0$ system.

For the hard component, the multiplicity is given by

$$
\begin{equation*}
\overline{\mathrm{n}}_{\mathrm{h}}\left(\mathrm{p}_{3}\right)=\mathrm{a}+\ln \left(\sqrt{\mathrm{s}}-\sqrt{\mathrm{s}}^{\prime}\right)^{2}+\mathrm{a}_{1}+\mathrm{b}_{1}\left|\overrightarrow{\mathrm{p}}_{3}\right| \tag{5}
\end{equation*}
$$

where $a$ is the same as in (4), and $a_{1}$ andb $b_{1}$ are given by ${ }^{1} a_{1}=0.7, b_{1}=0.5(\mathrm{GeV} / \mathrm{c})^{-1}$. The variable $s^{\prime}$ is the invariant mass squared of the irreducible subprocess. For a full discussion of (5) including the determination of the parameter values we refer the reader to paper I. ${ }^{8}$

We now turn to the application of these ideas to the semi-inclusive reaction (1). Analogously to (2) and (3), we have for the invariant distribution and charged multiplicity

$$
\begin{equation*}
\mathrm{g}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \equiv \mathrm{E}_{3} \mathrm{E}_{4} \frac{\mathrm{~d} \sigma}{\mathrm{~d}^{3} \mathrm{p}_{3} \mathrm{~d}^{3} \mathrm{p}_{4}}=\mathrm{g}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)+\mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \tag{2a}
\end{equation*}
$$

and

$$
\begin{equation*}
\overline{\mathrm{n}}_{\mathrm{c}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \mathrm{g}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)=\overline{\mathrm{n}}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \mathrm{g}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)+\overline{\mathrm{n}}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \tag{3a}
\end{equation*}
$$

respectively. For $\overline{\mathrm{n}}_{\mathrm{S}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ we shall make the reasonable assumption, discussed above, that the multiplicity depends on the missing mass in the usual way,

$$
\begin{equation*}
\overline{\mathrm{n}}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)=\mathrm{a}^{\prime}+\ln \mathrm{M}_{34}^{2} \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{M}_{34}^{2}=\left(\mathrm{p}_{1}+\mathrm{p}_{2}-\mathrm{p}_{3}-\mathrm{p}_{4}\right)^{2} \tag{7}
\end{equation*}
$$

The parameter $a^{\prime}$ will be determined below. To calculate $\overline{\mathrm{n}}_{\mathrm{c}}$ in (3a) we need to know $\overline{\mathrm{n}}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ and the ratio $\mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) / \mathrm{g}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$. In the absence of theoretical information on this ratio we make the following assumption in the phase space region of Ref. 4,

$$
\begin{equation*}
\frac{\mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)}{\mathrm{g}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)} \ll 1 \tag{8}
\end{equation*}
$$

This is based on the following arguments:
(i) Disregarding $\overrightarrow{\mathrm{p}}_{3}, \mathrm{p}_{4 \mathrm{~T}}$ is too small in the kinematic range of Ref. 4 for dominance of the hard component (see Fig. 3 of Ref. 4).
(ii) In Fig. 1 we plot, using the results of paper $I$, the ratio $f_{h}\left(p_{3}\right) / f\left(p_{3}\right)$ for $\mathrm{M}_{3}^{2} \equiv\left(\mathrm{p}_{1}+\mathrm{p}_{2}-\mathrm{p}_{3}\right)^{2}=3.5^{2} \mathrm{GeV}^{2}$, and $\mathrm{p}_{3 \mathrm{~T}} \leq 1 \mathrm{GeV} / \mathrm{c}$. These are the values of $\mathrm{M}_{3}$ and $\mathrm{p}_{3 \mathrm{~T}}$ used in Ref. 4. We see from Fig. 1 that for these values of $\mathrm{p}_{3 \mathrm{~T}}$ the hard component gives a minor contribution when $\vec{p}_{4}$ is averaged over.
(iii) In Fig. 2, along with the data of Ref. 4, we plot the empirical mean charged multiplicities for soft and hard components (dashed-dotted lines) for the process $\mathrm{pp} \rightarrow \mathrm{pX}$ at $\mathrm{M}_{3}=3.57 \mathrm{GeV}$. From this figure it is clear that the data of Ref. 4 do not correspond to typical hard or soft events because of their, in general, very different multiplicities.

Argument (ii) shows that the hard component is almost zero for $\mathrm{p}_{3 \mathrm{~T}}<1$ $\mathrm{GeV} / \mathrm{c}$, and argument (iii) suggests that the constraint on $\overrightarrow{\mathrm{p}}_{4}$ made in Ref. 4 is insufficient to select the rare hard events because of their ambiguous characterization as hard or soft. Consequently, we assume (8) in what follows. To make (8) more precise, we would require knowledge of the semi-inclusive distributions $\mathrm{g}_{\mathrm{S}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ and $\mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$; ${ }^{9}$ however, we show below that (8) gives a satisfactory description of the data of Ref. 4. Therefore, we have

$$
\begin{equation*}
\overline{\mathrm{n}}_{\mathrm{c}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)=\overline{\mathrm{n}}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right), \tag{9}
\end{equation*}
$$

with $\overline{\mathrm{n}}_{5}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ given by (6).
To apply this picture to the $28.5 \mathrm{GeV} / \mathrm{c}$ data of Ref. 4 we take $\mathrm{p}_{3 \mathrm{~T}}=0.25$ $\mathrm{GeV} / \mathrm{c}$, and find that $\mathrm{a}^{\prime}=1.5$ gives the solid line in Fig. 2. ${ }^{10}$ Choosing different values of $p_{3 T}$ in the range $0 \leq p_{3 T} \leq 1 \mathrm{GeV} / \mathrm{c}$ (the region covered by the experiment), results only in small changes in the normalization constant a', ${ }^{11}$ while the essential $\theta_{4}$ behavior is always the same. In Fig. 2 we also show as the dashed line the parametrization $\overline{\mathrm{n}}_{\mathrm{c}}=-0.2+\ln \mathrm{M}_{4}^{2}$, where ${ }^{4} \mathrm{M}_{4}^{2} \equiv\left(\mathrm{p}_{1}+\mathrm{p}_{2}-\mathrm{p}_{4}\right)^{2}$. The two parametrizations are similar, although (6) is in slightly better agreement with the data. The fact that (6) is the natural prediction of our model, together with its reasonable success in fitting the data in Fig. 2 (taking into account the fact that $M_{34}^{2}$ is always too small $\left(\leq 7 \mathrm{GeV}^{2}\right)$ to expect the asymptotic expression (6) to provide detailed agreement with the data) encourages us to believe that $\mathrm{M}_{34}$ is the important variable here.

As further support for the identification of $\bar{n}_{c}$ with a general soft-component multiplicity we show in Fig. 3 the data of Ref. 4 (open circles) along with data on $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons (closed circles, Ref. 12), $\pi^{-} \mathrm{p} \rightarrow \mathrm{pX}$ (closed squares, Ref. 6), and $\bar{p} p \rightarrow$ hadrons (closed triangles, Ref. 13). For comparison, we also include data for $\mathrm{pp} \rightarrow \mathrm{pX}$ (open squares) from Ref. 5. The abscissa in Fig. 3 is the invariant
mass squared of the unobserved system in each case. The data for $p p \rightarrow p p X$ are seen to follow the general trend of the $\mathrm{e}^{+} \mathrm{e}^{-}, \overline{\mathrm{p}} \mathrm{p}$ and $\pi^{-} \mathrm{p}$ data, although the $p p \rightarrow p X$ data (with $B=1$ ) lie consistently lower. All of these data correspond to a general soft component because either they are averaged over all final states $\left(e^{+} e^{-} \rightarrow X, \bar{p} p \rightarrow X\right)$ and the soft contribution is therefore dominant, or the $p_{T}$ value of the observed particle is very small and the same conclusion holds $\left(\pi^{-} \mathrm{p} \rightarrow \mathrm{pX}, \mathrm{pp} \rightarrow \mathrm{pX}\right)$. We also show the parametrization $\overline{\mathrm{n}}_{\mathrm{c}}=1.5+\ln \mathrm{M}_{\mathrm{X}}^{2}$ as the solid line in Fig. 3.

The experimentalists ${ }^{4}$ have also measured $\bar{n}_{c}$ for reaction (1) at fixed $M_{3}^{2}$ for $\theta_{4}<15^{\circ}\left(\left|\overrightarrow{\mathrm{p}}_{4}\right| \leq 1 \mathrm{GeV} / \mathrm{c}\right)$ and $0 \leq \mathrm{p}_{3 \mathrm{~T}} \leq 2 \mathrm{GeV} / \mathrm{c}$. They find that $\overline{\mathrm{n}}_{\mathrm{c}}$ is independent of $p_{3 T}$, in contrast to the result when $p_{4}$ is not constrained. In our picture, for $p_{3 T}$ smaller than the value at which the break in $\bar{n}_{c}$ occurs in the inclusive data and for $a \vec{p}_{4}$ value in the phase space region of Ref. 4 we have that $\overline{\mathrm{n}}_{\mathrm{c}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)=\overline{\mathrm{n}}_{\mathrm{s}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$. In the region where both contributions are comparable, even if we are able to give an expression for $\bar{n}_{h}\left(p_{3}, p_{4}\right),{ }^{15}$ we do not know $\mathrm{g}_{\mathrm{S}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ and $\mathrm{g}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ and therefore it is impossible to predict $\overline{\mathrm{n}}_{\mathrm{c}}$. When the hard contribution is dominant $\bar{n}_{c}\left(p_{3}, p_{4}\right)=\bar{n}_{h}\left(p_{3}, p_{4}\right)$. In this case we shall assume that for $\left(\mathrm{p}_{4 \mathrm{~L}}, \mathrm{p}_{4 \mathrm{~T}}\right) \approx(-1.0,0.2) \mathrm{GeV} / \mathrm{c}$, corresponding to $\theta_{4} \approx 8^{\circ}, \overline{\mathrm{n}}_{\mathrm{h}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right) \approx$ $\bar{n}_{h}\left(p_{3}\right)$, becausc this value of $\vec{p}_{4}$ is probably not very different from that of the "average" hard event. ${ }^{16}$

Using this method we show our estimate for $M_{3}=3.57$, 4.56 and 5.47 GeV as the solid lines in Fig. 4. The dashed lines represent smooth interpolations joining the estimated multiplicities in the transition region. We see that the $p_{3 T}$ dependence is slight for the largest values of $M_{3}$, the numerical value of $\overline{\mathrm{n}}_{\mathrm{c}}\left(\mathrm{p}_{3}, \mathrm{p}_{4}\right)$ coinciding with $\overline{\mathrm{n}}_{\mathrm{c}}\left(\mathrm{p}_{3}\right)$ for the reaction $\mathrm{pp} \rightarrow \mathrm{pX}$ after the rise in agreement with the behavior reported in Ref. 4. The absence of a
step in the semi-inclusive data results, in our model, from the selection of soft events with $\left|\vec{p}_{4}\right|$ small, and consequently, $\mathrm{M}_{34}^{2}$ large. It is perhaps reasonable that events with low $\left|\overrightarrow{\mathrm{p}}_{4}\right|$ should have a higher multiplicity than the "average" soft event, because in the former case more energy is available for pion production.

In conclusion, we have applied our two-component model for inclusive reactions to a limited phase space region of the semi-inclusive case by means of reasonable, though perhaps simplistic, assumptions. In this picture, our essential results for the data ${ }^{4}$ at $28.5 \mathrm{GeV} / \mathrm{c}$ are;
(i) The low $p_{3 T}$ data are reasonably well described by pure soft scattering with a multiplicity depending only on the overall missing mass. The rise in multiplicity at small $\vec{p}_{4}$ corresponds to the general idea that $\bar{n}_{c}$ increases when going from diffractive to nondiffractive production.
(ii) Our picture is consistent with the almost complete absence of a step in $\bar{n}_{c}$ as a function of $p_{3 T}$ at fixed $M_{3}$ and $\vec{p}_{4}$ (for the largest values of $M_{3}$ ) and its value is roughly equal to $\bar{n}_{c}$ in the reaction $\mathrm{pp} \rightarrow \mathrm{pX}$ above the rise for the same value of $\mathrm{M}_{3}$.

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8. In paper I we assumed, for simplicity, that the c.m. system for the irreducible collision coincides with the c.m. system for the pp collision. This assumption and the reasonable supposition that the multiplicity associated with the irreducible collision depends asymptotically only on $s^{\prime}$ (in our picture, on $\left|\overrightarrow{\mathrm{p}}_{3}\right|$ in the c.m. system for the irreducible collision) are essentially the reasons why (5) depends only on $M^{2}$. For fixed $\vec{p}_{3}$, the c.m. system for the irreducible process which minimizes s' (and which therefore favors the cross section) is that system for which $s^{\prime} \approx\left(2 p_{3 T}\right)^{2}$, and is not, in general, the overall c.m. system. Thus, independent of the dynamics of the irreducible process, expression (5) oversimplifies the true expression for $\overline{\mathrm{n}}_{\mathrm{h}}\left(\mathrm{p}_{3}\right)$. More experimental and theoretical work is needed to clarify this point, and a possible $p_{3 T}$ dependence beyond that coming from the transition from one mechanism to the other should be kept in mind. In this connection, see R. O. Raitio and G. A. Ringland, SLAC Report (1975) unpublished.

Further details are discussed in D. Sivers, S. J. Brodsky, and R. Blankenbecler, SLAC Report No. SLAC-PUB-1595 (1975) unpublished.
9. For an example of the difficulties encountered in obtaining a simple parametrization of the semi-inclusive distribution, see the study of the correlation coefficients in $\mathrm{pp} \rightarrow \pi^{\circ} \pi^{\circ} \mathrm{X}$ performed by D. Schiff, A. P. Contogouris, and J. L. Alonso, Phys. Letters 55B, 87 (1975).
10. Here we have assumed that the protons $p_{3}$ and $p_{4}$ are observed on opposite sides of the beam (i.e., $\Delta \phi=180^{\circ}$ ). This corresponds to the experimental setup (see, for example J. R. Ficenec et al., in Experimental Meson Spectroscopy, edited by C. Baltay and A. Rosenfeld (Columbia University Press, New York, 1970), p. 581).
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15. This is perhaps, not too difficult. This case is different from that studied in paper $I$ in that now, in $\overline{\mathrm{n}}_{\mathrm{R}}\left(\mathrm{p}_{3}\right), \overrightarrow{\mathrm{p}}_{4}$ is not averaged over, and therefore one must "separate" this $\overrightarrow{\mathrm{p}}_{4}$ from the remaining energy in a similar way to the separation made for the soft component.
16. Here, we take $\left|\vec{p}_{4}\right|$ small because we have assumed in our model that only one proton is involved in the irreducible subprocess, and that the remaining energy is too small to favor $\left|\vec{p}_{4}\right|$ large. In particular, we take $\theta_{4} \approx 8^{\circ}$ because this corresponds to the central value of the experimental range $\left(\theta_{4}<15^{\circ}\right)$. Our results are not significantly altered by using other values of $\theta_{4}<15^{\circ}$.

## FIGURE CAPTIONS

1. 'The ratio of hard to total inclusive cross section at $28.5 \mathrm{GeV} / \mathrm{c}$ plotted versus $p_{T}$ for $M_{3}=3.5 \mathrm{GeV}$ in the reaction $\mathrm{pp} \rightarrow \mathrm{pX}$.
2. Data from Ref. 4 for $\bar{n}_{c}$ (trigger protons not included) plotted versus $\theta_{4}$, the average laboratory scattering angle of $p_{4}$ in the reaction $p_{1} p_{2} \rightarrow p_{3} p_{4} X$ at $\overline{\mathrm{M}}_{3}=3.5 \mathrm{GeV}$. The inset shows the corresponding c.m. momentum of $p_{4}$, and the solid line is our fit as explained in the text. The dashed line is the parametrization $\overline{\mathrm{n}}_{\mathrm{c}}=-0.2+\ln \mathrm{M}_{4}^{2}$. We also show the empirical charged multiplicities for soft and hard components when $\mathrm{M}_{3}=3.57 \mathrm{GeV}$ in the reaction $\mathrm{pp} \rightarrow \mathrm{pX}$ (dashed-dotted lines).
3. Data for $\bar{n}_{c}$ plotted versus the invariant mass squared of the unobserved system $M_{X}^{2} ; \quad p p \rightarrow p p X$ (open circles, Ref. 4), $e^{+} e^{-} \rightarrow X$ (closed circles, Ref. 12), $\pi^{-} \mathrm{p} \rightarrow \mathrm{pX}$ (closed squares, Ref. 6), $\overline{\mathrm{p}} \mathrm{p} \rightarrow \mathrm{X}$ (closed triangles, Ref. 13), and $p p \rightarrow p X$ (open squares, Ref. 5). The solid line is the parametrization $\bar{n}_{c}=1.5+\ln M_{X}^{2}$
4. Estimates for $\bar{n}_{c}$ at $28.5 \mathrm{GeV} / \mathrm{c}$ plotted versus $\mathrm{p}_{3 \mathrm{~T}}$ for various fixed values of $M_{3}$ and for $\left(p_{4 L}, p_{4 T}\right)=(-1.0,0.2) \mathrm{GeV} / \mathrm{c}$. We show, for comparison, data for $\overline{\mathbf{n}}_{c}$ in the reaction $p p \rightarrow \mathrm{pX}$ from Ref. 2 .


Fig. 1


Fig. 2


Fig. 3


Fig. 4


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