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## A NEW DYNAMICS OF NN SCATTERING\*

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The meson theory of nuclear forces provides a good qualitative, and often quantitative, description of NN scattering, but nevertheless suffers from some major defects. Thus, by adjusting certain  $\pi N$  and  $\pi \pi$  amplitudes (including a fictitious " $\sigma$ ") one can construct a  $2\pi$  exchange potential which, together with OBE terms corresponding to  $\pi$ ,  $\rho$ , and  $\omega$  exchange, can produce a reasonably good fit to the phase shifts (with certain exceptions such as the  ${}^{3}D_{1}$ ). See, for example, A. D. Jackson [1]. However, after 25 years of effort the form of the (crucial)  $2\pi$  contribution is still somewhat ambiguous, and one cannot with any certainty predict the effect of still higher order diagrams. Furthermore, effective "potentials" derived from this theory come equipped with a sizeable number of adjustable "coupling constants" and "regularization parameters", which are not (for the most part) independently measurable. This introduces a latitude in the description which is highly unsatisfactory for a basic theory.

Moreover, this approach obscures our understanding of related phenomena, such as  $\pi$  production and absorption, and mesonic corrections to electromagnetic (EM) form factors. The reason is simply that the mesonic degrees of freedom are lost in constructing the effective "potential". The resulting predicament is well illustrated by attempts to marry field theory and nonrelativistic wave functions in calculating exchange corrections to the deuteron form factor, and the rather embarassing comparison to recent data at large q<sup>2</sup> reported by R. G. Arnold [2]. A similar situation could well arise in the near future when accurate data on  $\pi$ -N and  $\pi$ -nucleus scattering become available. My objective in this talk is to discuss an alternative approach suggested by recent developments in hadron scattering at high energies.

Historically, strong interaction field theory was constructed in imitation of the EM interaction, going back to the Yukawa postulate in 1934 that the force is mediated via the exchange of a massive analogue of the photon. The discovery of the  $\pi$  in 1947 was of course a major triumph, and the subsequent success of QED suggested an appropriate formalism. Physically, the concept of a quantized field is strongly related to the idea of point particles, in that the Wick argument  $R < \hbar c/\Delta E$ , relating the range R to the energy fluctuation  $\Delta E$ , permits the exchange of an arbitrarily large number n of  $\pi$ 's ( $\Delta E = n\mu c^2$ ) providing that R can be arbitrarily small. This implies that the NN system is intrinsically a many-body system at small spacial separations, and a second-quantized field is a natural way of building in the essentially infinite degrees of freedom. On the other hand, if the nucleons had some finite intrinsic size (not due to the pion field), the number of  $\pi$ 's would be finite and the field concept inappropriate.

A sizeable body of evidence has accumulated in the last several years which indicates that this is in fact the case. Thus, a great variety of direct experiments suggest that hadrons are in fact composite (e.g., made up of quarks), in addition to the indirect evidence of unitary symmetry. In fact, both EM and weak probes (deep inelastic electron scattering at SLAC and neutrino experiments at CERN) have given considerable support to the Gell-Mann-Zweig quark model. The relevance to nuclear physics has been noted by Neudatchin and coworkers [3], who observe that the concept of composite nucleons provides a simple explanation of the repulsive core seen empirically in NN (and other dihadron) scattering. The physics is quite simple; given that the constituents obey some exclusion principle, two clusters such as NN will resist interpenetration. The effect is thus to keep hadrons apart (they have a characteristic "size"). As I noted at Laval, this interpretation also provides a ready explanation of the approximate constancy of the logarithmic derivative (LD) of the NN wave function at  $r_b = .7$  fm, and the relation of  $r_b$  to the hard core radius  $r_c$ . This empirical fact has led to a very successful phenomenology as developed by Feshbach, Lomon and collaborators [4].

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(Presented at the 7th Internat'l. Conference on Few Body Problems in Nuclear and Particle Physics, Delhi, India, 29 December 1975 - 4 January 1976) This suggests a rather different picture of dihadron scattering, in which the primary effects arise from the core properties, but are obscured at low energies by the exchange of (at most a few) light mesons. Phenomenologically, the core-core behavior can be well represented by an effective radius and a constant LD, which can be determined empirically from NN data at  $T_L \gtrsim 200$  MeV. Thus, to the lowest level of approximation, one takes

$$\psi_{\ell}(\mathbf{r}) = 0 , \qquad \mathbf{r} < \mathbf{r}_{b}$$
$$= j_{\ell}(\kappa \mathbf{r}) + i e^{i\delta_{\ell}} \sin \delta_{\ell} h_{\ell}(\kappa \mathbf{r}) , \qquad \mathbf{r} > \mathbf{r}_{b} ; \qquad (1)$$

with  $\delta_{\ell}$  determined by the boundary condition (BC)  $(\psi_{\ell}^*/\psi_{\ell})_{r_b} = \lambda_{\ell}^C$ . By suitably adjusting  $r_b$  and  $\lambda_{\ell}^C$  one can fit  $\delta_{\ell}$  to the data at energies approaching the  $\pi$ -production threshold. However, the low energy behavior will be incorrect, and one will in general need to introduce an energy-dependent LD  $\lambda_{\ell}(\kappa^2)$  of the form

$$\lambda_{\ell}(\kappa^2) = \lambda_{\ell}^{c} + \sum_{i} \frac{r_{\ell,i}}{\kappa^2 - \beta_{\ell,i}^2}$$
(2)

in order to obtain realistic phases. Even if this is done, of course,  $\psi_{\ell}(\mathbf{r})$  will not be realistic except asymptotically.

It is clear that what this description lacks is the effect of meson exchanges at distances  $r > r_b$ . The approach of Feshbach and Lomon is to add meson theoretic potentials to represent 1- and 2- $\pi$  exchange (since  $r_b \simeq (2\mu)^{-1}$ , this is presumably sufficient). This leads to many of the same problems noted above, since the exchange potentials cannot be unambiguously constructed and the pionic coordinates are lost. However, there is an alternate way to proceed. One may instead treat this as a coupled channel problem, in which the (virtual) pionic channels are taken explicitly into account. This means that at the next level of approximation one regards NN scattering as a special case of NN $\pi$  scattering in which the pion is only present in intermediate states. In practice, this requires that one employ a fully covariant description of NN $\pi$  scattering as a three-body problem, extract the part corresponding to NN initial and final states, and analytically continue this amplitude to energies below the threshold for actual pion production. Technically, one just identifies  $t_{NN}$  as the residue of the double pole arising in  $T_{NN\pi}$  from a  $P_{11} \pi$ N combination at the N mass in both the initial and final states. This is formally equivalent to regarding the nucleon as an N $\pi$  bound state, and the prescription is identical with that used in extracting Nd scattering from 3N  $\rightarrow$  3N. Actually, the same prescription arises in field theory, or S-matrix theory; the difference here is that  $T_{NN\pi}$  is to be calculated on the basis of a three-body scattering theory, and not according to some set of field-theoretic diagrams.

In order to play this game one clearly requires a fully covariant three-body formalism capable of dealing with two-particle "interactions" characterized by boundary conditions. Fortunately, such a theory may be derived as an <u>unambiguous</u> generalization of the corresponding nonrelativistic formalism, as I have recently shown [5]. Despite the fact that the "interaction" is nonseparable, the corresponding equations reduce to one-dimensional form in a partial-wave decomposition, and hence are readily amenable to numerical solution. Preliminary applications to  $\pi d$  scattering and the  $\omega \ 3\pi$  system were reported at Laval; more recent results include an analysis of the A<sub>1</sub> state of three pions [6].

The obvious first approximation is to use the nucleon as an s-wave spectator of the  $\pi N$  state which contains the nucleon pole (P<sub>11</sub>), and the pion as a p-wave spectator of the appropriate NN s-wave ( ${}^{1}S_{0}$  to drive the  ${}^{3}S_{1}$  calculation, and  ${}^{3}S_{1}$  to drive the  ${}^{1}S_{0}$  calculation). After antisymmetrization in the nucleon variables, the equation takes the form

$$X_{i}(q_{i}^{\prime}) = \overline{K}_{i2}(q_{i}^{\prime}) + \sum_{j=1}^{2} \int_{0}^{Q_{j}} dq_{j} q_{j}^{2} K_{ij}(q_{i}^{\prime}, q_{j}) X_{j}(q_{j})$$
(3)

Here  $X_1$  and  $X_2$  represent series of pairwise rescatterings initiated by a  $\pi N$  pair at the nucleon pole;  $X_1(X_2)$  corresponds to a final NN ( $\pi N$ ) scattering. The variable  $q_j$  is the three-momentum of the spectator particle in the c.m. frame of the pair (j=1 corresponds to a spectator pion, j=2 to a nucleon). The Lorentz frame used to describe each pair configuration is uniquely specified by requiring that the pair remain in its own c.m. system when the spectator recedes to infinite distance. This reduces to the usual definition nonrelativistically, but introduces important kinematic effects in eq. (3). For three particles of mass  $m_{\alpha}$ ,  $m_{\beta}$ , and  $m_{\gamma}$  treated as free outside the region excluded by the cores, and using a real spectator momentum  $q \ge 0$ , the c.m. energy for the  $\beta\gamma$  pair is

$$\left(m_{\beta}^{2}+\kappa^{2}\right)^{1/2}+\left(m_{\gamma}^{2}+\kappa^{2}\right)^{1/2}=\left(s+m_{\alpha}^{2}q^{2}/M_{\alpha}^{2}\right)^{1/2}-\left(m_{\alpha}^{2}+m_{\alpha}^{2}q^{2}/M_{\alpha}^{2}\right)^{1/2},$$
(4)

with  $M_{\alpha}^{-1} = m_{\alpha}^{-1} + (m_{\beta} + m_{\gamma})^{-1}$ , and  $s = P^2$ , the invariant four-momentum squared of the threeparticle system. The upper limit on this energy, and hence on the energy where we need the two-body input for our equation, is achieved at  $q^{2}=0$ , while the lower limit, implied by the fact that eq. (4) can be satisfied only for  $\kappa^2 \ge -\min\left(m_{\beta}^2, m_{\gamma}^2\right)$ , fixes an upper limit  $q=Q_{\alpha}$ (infinite only if  $m_{\beta}=m_{\gamma}$ ).

Since the c.m. energy of the  $\beta\gamma$  pair is bounded by  $\sqrt{s} - m_{Q}$ , any three-body treatment of the NN system requires two-body input always a pion mass below the two-body output to be computed. Thus, in order to calculate NN scattering near elastic threshold ( $\sqrt{s} \simeq 2M$ ), we require only NN input for  $-M^2 \le \kappa_{NN}^2 \le -M_{\mu} (1-\mu/4M)$ , and  $\pi N$  input in the narrow range  $-\mu^2 \le \kappa_{\pi N}^2 \le -\mu^2 (1-\mu^2/4M^2)$ ; the immediate vicinity of the nucleon pole. As noted above, at any level of approximation one can partially account for neglected channels by employing energy-dependent LD's obtained from NN and  $\pi N$  scattering data and analytically continued to the required region (since  $\lambda_{\ell}$  must be meromorphic, the extrapolation is essentially unique). Having obtained such fits for the NN system, it turns out that  $\lambda_{\ell}$  has essentially achieved its asymptotic value ( $\lambda_{\ell}^{C}$ ) in the kinematic region required for a threshold calculation ( $\kappa_{NN}^2 \le -M_{\mu}$ ). The NN input thus consists of the constant LD parameters taken to represent the core-core (quark) structure, and obtained empirically from the high energy NN phase shifts. Using  $r_b = .7$  fm, we take  $\lambda_0^c = 0.30$  for the  ${}^{1}S_0$ , and  $\lambda_0^c = 1.8$  for the  ${}^{3}S_1$  (from the  ${}^{3}S_1 - {}^{3}D_1$  coupled channel fit of Feshbach and Lomon [7]).

The input for the  $P_{11}$  amplitude presents more of a problem, since the nucleon pole is only a pion mass below  $\pi N$  threshold and, in contrast to the NN situation, we are most sensitive to data up to about a pion mass above threshold, where they are poorly known. We know the position of the pole, and its residue can be inferred from the requirement that our three-body formulation yields the correct OPE singularity. The simplest fit (one pole term in eq. (2)) thus requires only a single parameter (in addition to the core radius  $r_{\pi N}$ ), and it is possible to obtain quite reasonable fits to the  $P_{11}$  phase shift obtained by J. R. Carter [8] for  $r_{\pi N} \simeq .2$  fm (approximately  $\hbar c/M$ ). However, in view of the uncertainties in this phase we simultaneously consider several alternative fits, and also compare values based on  $G^2 = 14.6$  and 15.3.

At this level of approximation all parameters are thus determined, and we may apply eq. (3) to calculate the low energy properties of the NN s-waves, and in particular the existence of bound states. The results of the calculation are given in Table I. We see that

r <sub>πN</sub> (fm)	$G^2$	<sup>€</sup> d (MeV)	€ <sub>0</sub> (MeV)
0.180 0.186 0.196 0.198 0.220	$14.6 \\ 14.6 \\ 15.3 \\ $	3.26 3.14 2.96 3.02 2.59	$1.41 \\ 1.34 \\ 1.10 \\ 1.17 \\ 0.73$

Table I

in spite of uncertainties engendered by the P<sub>11</sub> amplitude, the most significant features of the NN s-waves—namely, two bound states close to zero in units of the  $\pi$  mass and split by approximately 2 MeV —are stably reproduced. Considering the simplicity of the model at this level and its close connection to empirical results found in quite different experiments, this close agreement with experiment ( $\epsilon_d$ =2.2,  $\epsilon_0$ =-.07) is quite remarkable [9]. In order to go beyond these results one must work a bit harder. Clearly, there is no point in computing phase shifts until the scattering lengths are brought into agreement with experiment. By introducing a phenomenological term of range  $(2\mu)^{-1}$  and adjusting its size to produce a singlet scattering length  $a_s = -24.4$  fm, it is possible to produce an excellent prediction for the  ${}^{1}S_0$  phase up to  $T_L = 50$  MeV (i.e., the effective range and shape parameter are generated automatically). However, if a similar adjustment is made to produce a correct value for  $\epsilon_d$  in the  ${}^{3}S_1$  state the fit is not nearly as good ( $a_t \simeq 4.6$  fm instead of 5.4 fm). The reason is that while coupling to neglected channels may indeed be represented by such a term, it is virtually impossible to guess energy-dependence to sufficient accuracy. This problem is much more acute in the triplet channel, which properly must be treated as a coupled  ${}^{3}S_1 - {}^{3}D_1$  system by including the nucleon as a d-wave spectator of the  $P_{11} \pi N$  state.

Unfortunately, the code used to produce these results was not sufficiently general to permit an investigation of additional three-body channels. Quite recently, a new covariant threebody code was completed which is suitable for this purpose. In order to complete a calculation in time for this conference, I have concentrated on the simpler  ${}^{1}S_{0}$  state. Although a number of possible contributions were assayed (S<sub>11</sub>, S<sub>31</sub>, P<sub>31</sub>), the only significant channel turned out to be P<sub>33</sub> coupled to a d-wave nucleon spectator; this is suppressed by the d-wave character at low energy, but becomes important as one nears the  $\pi$ -production threshold. This is shown in fig. 1, in which the solid curve corresponds to the two-channel



Fig. 1. Calculated  ${}^{1}S_{0}$  phase (solid curve) for two channel model; experimental points from ref. [10]. Effects due to the P<sub>33</sub> coupling (dashed curve) and energy-dependence in the  ${}^{3}S_{1}$  (dashed-dot) are also shown.

model discussed above, and the dashed shows the effect of including the  $P_{33}$  (all are adjusted to produce  $a_s = -24.4$ ). It is clear that this channel contributes a significant repulsion which brings the computed curve into good agreement with the experimental points of M. MacGregor [10] for  $T_L < 200$  MeV (this role for the  $\Delta(1236)$  was also noted in a simpler calculation by P. Haapakoski [11]). It should be emphasized that the curves shown are not a "fit", but an unadjusted theoretical prediction based on data from other experiments. It would almost certainly be possible to produce a high precision fit with minor adjustments of the parameters, but our purpose at this stage is merely to explore the general consequences of our dynamical picture. Fine details will require more effort in pinning down accurate input parameters, as well as including more channels. This becomes apparent in

the figure as  $T_L$  approaches the  $\pi$  threshold. For example, the energy-dependence of the  ${}^{3}S_1$  LD parameter begins to show up, as shown by comparing the dashed to the dashed-dot curve (which includes this effect). Nevertheless, it seems fair to conclude that such an embodiment of quark dynamics may provide an attractive alternative to field theoretic models.

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