SLAC-PUB-1677
November 1975
(T/E)

DYNAMICAL CALCULATION OF THE A_1*

D. D. Brayshaw†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A relativistic three-body theory previously used to generate the ω has been applied to the 1^+ state of three pions. An A_1 resonance pole is produced with $M_{A_1} = 1160$ MeV, $\Gamma/2 = 90$ MeV but there is no associated phase variation of the amplitude.

(A shorter version of this paper has been submitted to Phys. Rev. Letters.)

*Work supported in part by the U.S. Energy Research and Development Administration.
†Alfred P. Sloan Foundation Fellow.

There has been growing concern over the embarassments suffered by the quark model in predicting the meson spectroscopy, and in particular the experimental absence of 1^+ states such as the Q and A_1. Thus, the diffractively produced enhancement observed in $\pi p \to (3\pi)p$ at 1.1 GeV has recently been subjected to a number of independent and rather sophisticated analyses, and all agree on the absence of a resonant A_1 signal.[1] As developed by Ascoli and collaborators, the procedure is a variant of the isobar model in which one writes the amplitude for a three-body "decay" in the form $T = \Sigma_\alpha f_\alpha t_\alpha$, where t_α is the two-body scattering amplitude for the β-γ pair ($\alpha \neq \beta \neq \gamma$) and depends explicitly on the pair subenergy s_α. The f_α are treated as complex fitting parameters, and one subsequently studies their phase variation with respect to some (nonresonant) reference amplitude. This works quite nicely for the A_2, which exhibits Breit-Wigner behavior with the phase varying through 90°. In contrast, the A_1 phase variation is quite flat, and one is apparently forced to a nonresonant interpretation such as the Reggeized Deck effect.[2] A recent attempt by Ascoli and Wyld to answer previous criticism of the methodology[3] has led to the same negative conclusion.[4]

In this Letter we show that an absence of distinctive phase variation is not only compatible with an A_1 resonance pole, but (1) is an automatic consequence of a simple dynamical model. Moreover, we argue that (2) such behavior is a general feature of a properly unitarized amplitude, <u>providing the effect is dynamical</u>, and (3) has a simple physical interpretation. Specifically, we observe that a resonant amplitude need not exhibit a large change of phase, as assumed in these analyses. In fact, this is a familiar phenomenon associated with very <u>inelastic</u> resonances. For example, consider a system in which two orthogonal channels are dynamically coupled (e.g., $\pi\pi$ and $K\bar{K}$); in such a

system the elastic amplitudes have the form $\tau_\alpha = (\eta\, e^{2i\delta_\alpha} - 1)/2i$. If the amplitudes are resonant and $\eta < 1/2$, the phase shifts δ_α are roughly sinusoidal, passing through zero (rather than $\pi/2$) when Re $(\tau_\alpha) = 0$. Typically, the magnitude of this oscillation is quite small ($< 30^\circ$), in which case one may easily show that the phase of the amplitude itself exhibits a similar oscillation about $(-)\pi/2$. The net phase variation may thus be quite small, and one would not expect to detect it in an Ascoli-type analysis, particularly in view of simplifying approximations (e.g., neglect of the subenergy dependence of the f_α parameters). In fact, phase variations in the production amplitudes alone could easily mask the effect. We then observe that in the language of the isobar model, the A_1 state of three pions is precisely such a system, containing two strong competing channels ($\rho\pi$ and $\epsilon\pi$). Therefore, it would not be at all surprising if the relevant phase behavior were of the second, weaker type, in which case one must clearly employ alternative techniques.

This conjecture is supported by explicit calculations based on the author's covariant boundary condition formalism (BCF), which has recently been applied to a number of relativistic three-particle systems.[5,6] In the present context, we have used it to study the amplitude $T_3 = \Sigma_\alpha \tau_\alpha$ describing 3π scattering in a 1^+ (I=1) state. We thus consider $T(N\pi \to N3\pi)$ to be of the form $T = T_p * T_3 + \ldots$, where T_p is an appropriate production amplitude. Providing that the A_1 is indeed a dynamical effect, we would expect T_3 to contain the corresponding resonance pole (in alternative mechanisms such as the Deck effect, the enhancement would originate in T_p). For this purpose the BCF may be employed in two complementary ways. In its most general form, it provides a general solution of the three-particle unitarity relations, and hence any physical amplitude can be constructed given suitable input. Conversely, the class of allowable input

exhausts the possible physical amplitudes, and thus one can determine whether any input which produces an A_1 peak in T_3 will also produce a large phase variation. The answer turns out to be "no", which is a model-independent result. Furthermore, the formalism permits an explicit analytic continuation onto the second sheet of the total (3π) energy; in this way it has been verified that each such peak corresponds to an associated pole.

Secondly, input to the BCF has a straightforward dynamical interpretation, and can be estimated for a simple model which has previously been applied to calculate the π and ω as dynamical 3π effects[5] (these turn out to be the only "particles" generated in the 0^-, 1^- states for any isospin). It is therefore interesting that the same model "predicts" an A_1 state of approximately the right mass and width (and with negligible phase variation). Together, these results indicate that the 0^-, 1^-, 1^+ states can be understood in terms of potential-like forces generated by particle exchange. In contrast, a similar calculation in the 2^+ state shows that the A_2 can only be produced via explicit coupling to associated inelastic channels ($K\bar{K}, \eta\pi$). This is in fact desirable, since the I=1, 2 2^+ states are otherwise degenerate in the model.

As noted by Amado,[7] a minimal scheme for unitarizing three-particle amplitudes must take the form of a one-dimensional integral equation. The key ingredients of such an equation are well known; in order to produce the primary (model-independent) singularity structure, the kernel must contain a pole corresponding to the free propagation of all three particles, and two-particle propagators characterized by elastic phase shifts. The minimal form of the BCF corresponds to a trivial dynamical model possessing all of these features; additional parameters in the general form correct the dynamical details via the addition of nonsingular terms. Although the BCF is particularly efficient for

this type of analysis, it should be emphasized that our more general results ((2) and (3) above) will be a feature of any properly unitarized amplitude.

The BCF builds on a trivial model in which the pairwise interaction of β-γ is compressed to the surface of an impenetrable boundary at $|\vec{r}_\beta - \vec{r}_\gamma| = a_\alpha$. The scattering is described by an energy-dependent logarithmic derivative of the wave function at that radius, $\lambda_{\alpha\ell}(\kappa_\alpha^2)$. The two-body amplitudes are then $t_{\alpha\ell}(\kappa_\alpha) = N_{\alpha\ell}/D_{\alpha\ell}$, with

$$\begin{aligned} N_{\alpha\ell}(\kappa_\alpha) &= \left[a_\alpha \lambda_{\alpha\ell}(\kappa_\alpha^2) - \ell\right] j_\ell(a_\alpha \kappa_\alpha) + a_\alpha \kappa_\alpha j_{\ell+1}(a_\alpha \kappa_\alpha) \quad , \\ D_{\alpha\ell}(\kappa_\alpha) &= i\kappa_\alpha \left\{\left[a_\alpha \lambda_{\alpha\ell}(\kappa_\alpha^2) - \ell\right] h_\ell(a_\alpha \kappa_\alpha) + a_\alpha \kappa_\alpha h_{\ell+1}(a_\alpha \kappa_\alpha)\right\} \quad . \end{aligned} \tag{1}$$

Here κ_α is the c.m. momentum of the β-γ pair; $\lambda_{\alpha\ell}(\kappa_\alpha^2)$ and a_α are fitted to scattering data in the physical region $\kappa_\alpha^2 \geq 0$. Since $\lambda_{\alpha\ell}$ must be <u>meromorphic</u> in κ_α^2 in order to produce unitary (elastic) amplitudes, this fit permits analytic continuation of $N_{\alpha\ell}$, $D_{\alpha\ell}$ for $\kappa_\alpha^2 < 0$. Below we use the notation $N^c_{\alpha\ell}$, $D^c_{\alpha\ell}$ to denote $N_{\alpha\ell}$, $D_{\alpha\ell}$ evaluated with $\lambda_{\alpha\ell}(\kappa_\alpha^2)$ replaced by the constant value $\lambda_{\alpha\ell}(-\mu^2)$. If one considers the three-particle system with β-γ separated by a_α in their c.m. frame, it is obvious that there is a characteristic distance b_α of particle α from the β-γ c.m. such that $|\vec{r}_\alpha - \vec{r}_\beta| \leq a_\gamma$ and or $|\vec{r}_\alpha - \vec{r}_\gamma| \leq a_\beta$; b_α characterizes the interior region in which the cores may overlap. The BCF describes the system <u>outside</u> of this region ($b_\alpha = 3/2\ a_\alpha$ for identical particles).

The channel amplitudes $\tau^\alpha_{J\ell\lambda}(s, s_\alpha)$ required to form T_3 may then be computed from the solution of a one-variable integral equation

$$X^\alpha_{J\ell\lambda}(s, s_\alpha) = \Omega^\alpha_{J\ell\lambda}(s, s_\alpha) + \sum_{\ell', \lambda', \beta \neq \alpha} \int_0^\infty dq'_\beta q'^2_\beta K^J_{\alpha\ell\lambda;\beta\ell'\lambda'}(s_\alpha, s'_\beta; s) * X^\beta_{J\ell'\lambda'}(s, s'_\beta) \quad , \tag{2}$$

written in terms of the variable q_α (3-momentum of particle α in the β-γ c.m. frame, equivalent to s_α for fixed s). Here J, λ, ℓ are angular momentum labels corresponding to the total system, the motion of α in the β-γ c.m. frame, and the spin of the β-γ subsystem, respectively. Apart from purely kinematic factors, the relation of τ to X is such that $X^\alpha_{J\ell\lambda}(s, s_\alpha)/N^c_{\alpha\ell}(\kappa_\alpha)$ is to be compared with the isobar amplitude f_α. Similarly, $\Omega^\alpha_{J\ell\lambda}(s, s_\alpha)/N^c_{\alpha\ell}(\kappa_\alpha)$ is essentially an angular momentum projection onto an initial plane-wave state (for precise definitions see BCR). However, for our present purposes these details do not turn out to be relevant. A pole in τ_α (and hence T_3) can arise only via a pole in the operator $(1-K)^{-1}$; this corresponds to a complex zero of the determinant $D(s) \equiv |1-K|$. One can thus study the resonant properties by constructing D(s); the rapidly varying factor of an appropriate cross section is proportional to $|D|^{-2}$.

Consequently, we need only consider the kernel of Eq. (2). In the present application, we take the 1^+ state to be composed of two components corresponding to ℓ=1, λ=0 and ℓ=0, λ=1. For convenience we label these by ρ and ϵ, respectively, although π-π phases were used directly as input and an ϵ pole was not assumed explicitly. Taking into account Bose symmetrization and isospin, the kernel of the minimal equation can be expressed as

$$K^S_{ij}(q_i', q_j) = \Lambda_{ij} \frac{N^S_{ij}(q_i', q_j)}{D_j(\kappa_j)} \frac{N_j(\kappa_j)}{N^c_j(\kappa_j)} ,$$

$$N^S_{ij}(q_i', q_j) = -\frac{2\kappa_j}{\pi} \int_{-1}^{1} dz\, G_{ij}(z, \hat{K}_{ij}\cdot\hat{q}_j, \hat{Q}_{ij}\cdot\hat{q}_j) \frac{g_i(b_i q_i', bQ_{ij})}{q_i'^2 - Q^2_{ij} - i\epsilon} \frac{N^c_i(K_{ij})}{Q_{ij}} , \tag{3}$$

where i, j take on the values ρ, ϵ and hence index the appropriate combinations of ℓ, λ and isospin. Here Λ_{ij} is an isospin recoupling coefficient ($\Lambda_{\rho\rho} = 1/2$,

$\Lambda_{\epsilon\rho} = -\Lambda_{\rho\epsilon} = 1/\sqrt{3}$, $\Lambda_{\epsilon\epsilon} = 1/3$), and G_{ij} is a geometrical recoupling coefficient which would be unity if all particles were in relative s-waves. The three-vectors $\vec{K}_{ij}$, $\vec{Q}_{ij}$ are the values of $\vec{\kappa}_i, \vec{q}_i$ in the i c.m. corresponding to $\vec{\kappa}_j, \vec{q}_j$ in the j frame, and $z = \hat{k}_j \cdot \hat{q}_j$. The function g_i arises from excluding the inner region;

$$g_i(x,y) = iy\left[yj_\lambda(x)\, h_{\lambda+1}(y) - xj_{\lambda+1}(x)\, h_\lambda(y)\right] \quad . \tag{4}$$

We note that $g_i(x,x) = 1$, and hence the residue of the integrand at the Green's function pole $q_i' = Q_{ij}$ is given by the two-body amplitude $t_j(\kappa_j)$.

The general form of the BCF is obtained by replacing $N^s_{ij} \to N^s_{ij} + A_{ij}$, where $A_{ij}(q_i', q_j)$ is an <u>arbitrary</u> L_2 function which must be real-valued to describe elastic three-body scattering. As noted in BCR, a rough estimate of A_{ij} can be derived if one assumes it to be dominated by off-shell corrections to the $\rho\pi\pi$ vertex. This leads to the specific model

$$\begin{aligned} A_{\rho\rho}(q',q) &= \frac{\gamma_{\rho\rho}}{\kappa_\rho}\,\frac{\kappa_\rho^2 + 4\mu^2}{3\mu^2}\, g_\rho(q')\, g_\rho(q) \quad , \\ g_\rho(q) &= (q^2 + 4\mu^2)^{-1} \quad , \end{aligned} \tag{5}$$

with $\gamma_{\rho\rho} \simeq 1/2$ (similar estimates give $A_{\epsilon\epsilon}$, $A_{\rho\epsilon}$, $A_{\epsilon\rho}$, which turn out to be numerically unimportant). More generally, if potential-like mechanisms dominate one expects A_{ij} to have a relatively weak dependence on s, and to be a smooth function of q', q.[8] It can then be expanded in a complete set $A = \Sigma_\lambda c_\lambda |\lambda\rangle\langle\lambda|$, and the c_λ treated as real fitting parameters. A number of such forms were employed to test the model-dependence of our result.

The required numerical procedures are straightforward: one distorts the q_j-integration contour to avoid the singularity at $q_i' = Q_{ij}$, and employs Gaussian quadrature to reduce the equation to finite matrix form in order to calculate

D(s). To simplify the numerics a cut-off was employed at $q_j^{max} = 30\ fm^{-1}$; the calculation was quite insensitive to this choice (a 1% effect for $25\ fm^{-1} < q_j^{max} < 35\ fm^{-1}$). Several choices of s- and p-wave π-π phases were employed corresponding to the range of models reported by Basdevant et al.,[9] as well as a simple s-wave which does not exhibit the rapid change of phase at the $K\bar{K}$ threshold (no S*). In practice, the S* region turns out to be relatively unimportant since it requires a very small spectator momentum ($q_j \simeq 0$), and this is suppressed both by the λ=1 character and the $q_j^2 dq_j$ integration weight.

Given this input and the simple model of Eq. (5), a 1^+ resonance is indeed generated in the vicinity of 1100 MeV for $\gamma_{\rho\rho}$ in the estimated range. A typical example is illustrated in Fig. 1 (solid curve), corresponding to $\gamma_{\rho\rho} = .57$. Writing $T = N/D(s)$, this result would imply a width of 220 MeV if the s-dependence of N were negligible. It is clear that the phase $\phi(D)$, which would normally signal the presence of simple Breit-Wigner behavior, exhibits no noticeable variation associated with the enhancement. One of the unique advantages of this approach is that one can unambiguously determine whether or not such an effect corresponds to a true resonance pole. Since the fitting parameters ($\gamma_{\rho\rho}$, or more generally the c_λ) carry only weak s-dependence (in the absence of inelastic thresholds), one can hold them fixed and perform an explicit analytic continuation. Thus, the dashed and dashed-dot curves illustrate the effect of taking $\sqrt{s} = M_{3\pi} - i\ (30\ MeV)$, and $\sqrt{s} = M_{3\pi} - i\ (60\ MeV)$, respectively. In this way we confirm the existence of a pole 90 MeV below the real axis on the second sheet, with a mass of 1160 MeV. Although it was possible to vary A in such a way as to produce no peak, it was found that peak, pole and minor phase variation were always correlated.

With regard to the proposed "inelastic" mechanism, it is very suggestive that as we take $\sqrt{s}$ deeper onto the second sheet, $\phi(D)$ increasingly takes on the characteristic appearance of such a resonance. We note that by doing so we also approach closer to the ρ and ϵ poles which occur in the factor $D_j^{-1}(\kappa_j)$, and hence more closely approach a coupled channel problem involving stable "particles". Numerical studies confirm that the interplay between the ρ and ϵ channels is vital in producing the effect (whereas an S* pole is <u>not</u> required). On the other hand, the limitations of the isobar model are apparent in the damping of the effect for real $\sqrt{s}$. Thus, one cannot escape the fact that we are dealing with <u>three</u> particles, with an associated three-particle cut as well as ρ and ϵ thresholds. The net s-dependence is an integrated product of these factors, and is necessarily quite complicated; this shows up both in the nondescript phase behavior and the shape of the bump in $|D|^{-2}$ (solid curves). We conclude that such an A_1 cannot be established by an Ascoli-type analysis, and may well be present in the data.

REFERENCES AND FOOTNOTES

1. Yu. M. Antipov et al., Nucl. Phys. B63, 153 (1973); G. Ascoli et al., Phys. Rev. D 7, 669 (1973); M. Tabak et al., talk reported in AIP Proceedings No. 21 (1974).
2. G. Ascoli et al., Phys. Rev. D 8, 3849 (1973).
3. R. Aaron and R. D. Amado, Phys. Rev. Letters 31, 1157 (1973).
4. G. Ascoli and H. W. Wyld, Phys. Rev. D 12, 43 (1975).
5. D. D. Brayshaw, Phys. Rev. D 11, 2583 (1975); hereafter designated by BCR (Boundary Condition Relativistic).
6. D. D. Brayshaw, Phys. Rev. C 11, 1196 (1975); D. D. Brayshaw and H. Pierre Noyes, Phys. Rev. Letters 34, 1582 (1975).
7. R. D. Amado, Phys. Rev. Letters 33, 333 (1974).
8. Explicit connections with potential theory are derived in D. D. Brayshaw, SLAC-PUB-1648, Stanford Linear Accelerator Center (September 1975).
9. J. L. Basdevant, C. D. Froggatt and J. L. Petersen, Phys. Letters 41B, 178 (1972).

FIGURE CAPTION

1. Dependence of $|D|^{-2}$ (upper figure) and $\phi(D)$ (lower figure) on the three-pion mass. The curves correspond to complex energies $\sqrt{s} = M_{3\pi} - i\Delta$ with Δ=0, 30, 60 MeV for the solid, dashed, and dashed-dot curves, respectively.

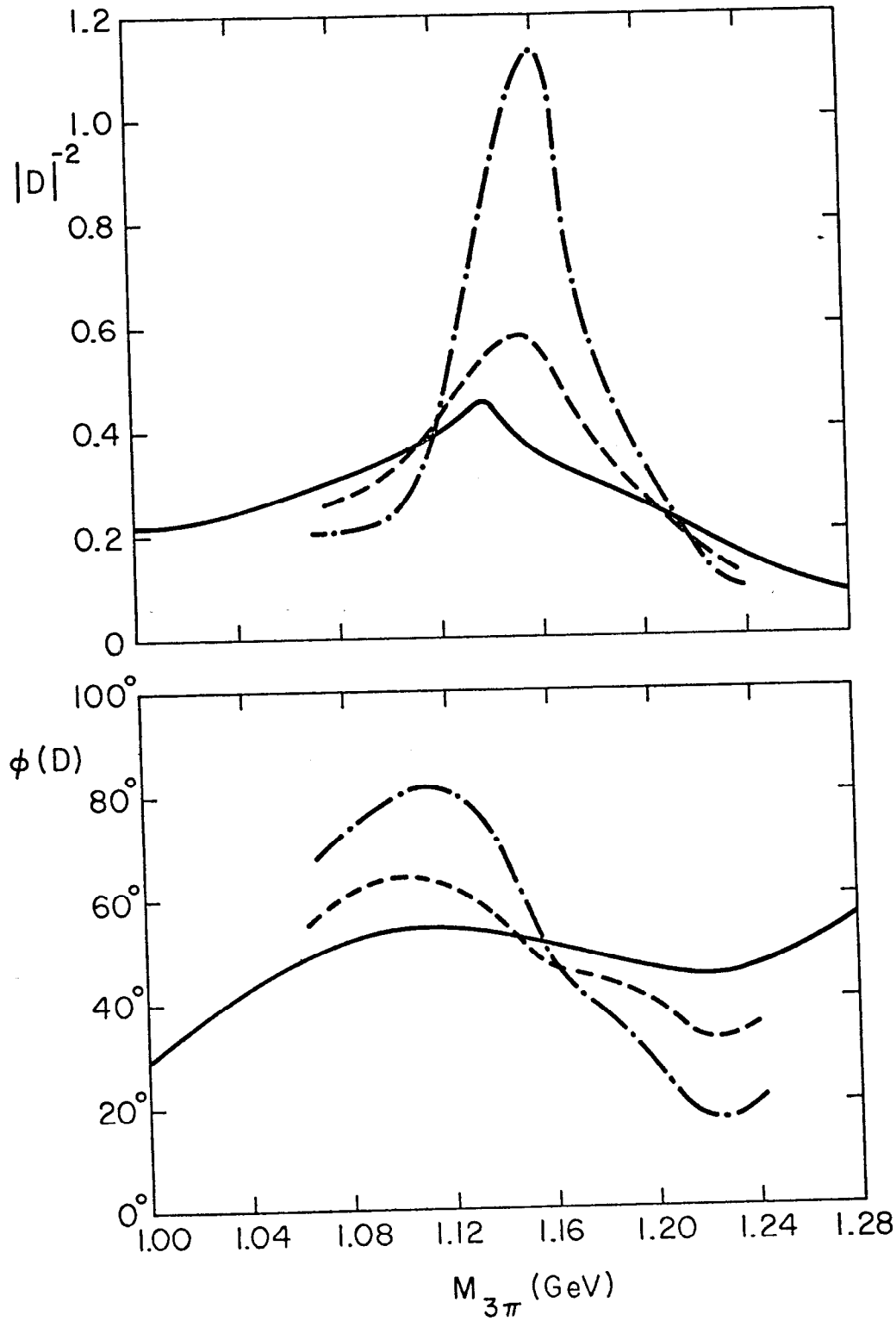
1.2
1.0
$|D|^{-2}$
0.8
0.6
0.4
0.2
0
100°
$\phi(D)$
80°
60°
40°
20°
0°
1.00
1.04
1.08
1.12
1.16
1.20
1.24
1.28
$M_{3\pi}$ (GeV)

Fig. 1