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RECENT DEVELOPMENTS IN DIFFRACTIVE STUDIES*

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1. Introduction

In the following lectures I plan to review recent developments in diffractive studies. However before discussing the latest data it will be useful to introduce the various pictures of diffraction and their respective language, and to summarize the characteristics of diffractive reactions.[1]

Diffraction scattering is described from two quite different points of view: the t-channel picture in which the process is mediated by the exchange in the t-channel of a Regge pole - a special singularity called the Pomeron, and the s-channel picture in which diffraction is seen in terms of the shadow due to the absorption of the incoming wave by all of the open inelastic final states.

Study of the energy dependence and angular dependence of the scattering process allows an understanding of the details of the dynamics - either by providing information on the properties of the Pomeron trajectory or on the size and blackness (opacity) of the scatterer, depending which point of view is being used in the analysis.

Both pictures have proved useful in describing two-body scattering processes. In general the exchange picture has been best in dealing with energy dependences and the optical picture most successful in describing the angular dependences in scattering. We shall refer to both points of view in describing the data.

The characteristics of diffraction scattering have been reviewed extensively,[1] but we list them below again for convenience:

- energy independent cross section (to factors of ln s)
- sharp forward peak in $d\sigma/dt$
- mainly imaginary amplitude
- particle cross section equal to antiparticle cross sections

- exchange processes characterized by the quantum numbers of the vacuum, in the t-channel. The spin structure of elastic processes being mainly s-channel helicity conserving.

These properties may be studied in the three "laboratories" we have for observing the diffractive process:

- elastic scattering, and through the optical theorem the total cross section
- exclusive diffraction, or diffraction dissociation
- inclusive diffraction scattering, in which the leading particle effects are observed.

New data on total cross sections are available from NAL, and on elastic scattering from SLAC, NAL, and ISR. These are discussed in Sections 2 and 3 below. Some interesting new data on diffraction dissociation from NAL and ISR are presented in Section 4, together with leading particle inclusive data from SLAC.

2. Total Cross Sections and Real Part of Forward Scattering Amplitude

Cross Sections

Study of the total cross section is the classical method of learning about the elastic scattering amplitude. The optical theorem relates the imaginary part of the forward scattering amplitude to the total cross section at the same energy

$$\sigma_T(s) \propto \mathrm{Im} \int_{el} (s, t = 0).$$

This linear relationship allows a study of the different contributions to the elastic amplitude and their separate energy dependence. Prior to 1972 the total cross-section data were explained[2] within the framework of Regge theory by two components:

- an energy independent term due to the exchange of the Pomeron, with trajectory $\alpha(t) = 1$,
- an energy dependent term due to the exchange of the ρ, ω, f, and A_2 trajectories, which are assumed to have the same form - $\alpha(t) = \frac{1}{2} + t$.

This formalism leads to the simple parametrization:

$$\sigma_T(AB) = a(AB) + b(AB) \cdot p^{-\frac{1}{2}}$$

$$\sigma_T(\bar{A}B) = a(\bar{A}B) + b(\bar{A}B) \cdot p^{-\frac{1}{2}}$$

The terms a(AB), b(AB) represent the two contributions described above, and p is the laboratory momentum of the particles. The Pomeranchuk theorem, which states that at infinite energy particle cross sections will be equal to anti-particle cross sections, is satisfied by $a(AB) = a(\bar{A}B)$. Such a description of the total cross sections is shown in Fig. 1, together with the data available in 1972. The rising trend for K^+p was accommodated with this model by assuming a negative contribution from Pomeron-cuts which died out as the energy increased. Clearly this picture gives a good representation of the data. However, new data on high energy pp scattering became available from the CERN ISR[3] and showed the total cross section increasing by 5-6 mb through the range (200-2000) GeV/c. (See Fig. 2.) At this point it became clear that the region which gave credibility to the notion of flat asymptotic total cross sections (see 10-40 GeV data in Fig. 1) was only a local minimum. In fact, the asymptotic behavior of the total cross sections may, or may not be, to become energy-independent - see Fig. 3. It is for experiment to determine whether σ_T continues to rise indefinitely, or whether it approaches a constant value from below.

The latest data on total cross sections from NAL[4] are shown in Fig. 4. The NAL experiment has extended their old measurements to lower and higher energies and joins smoothly on to the existing data. All the processes except $\bar{p}p$ show the total cross section increasing as the energy increases - the K^+ and K^- cross sections clearly grow, the π^+ and π^- cross sections increase by $\sim 1\frac{1}{2}$ mb over the NAL range while the pp cross section increases by $\sim$ 6 mb from $\sim$ 60 GeV/c through 2000 GeV/c. The $\bar{p}p$ data show no energy dependence from 100 to 280 GeV/c and one may expect the cross section to begin to rise to meet the pp total cross section and satisfy the Pomeranchuk's theorem.

The energy dependence of the increase in the total cross section is of considerable interest, but at present there is no unique description - the data may be fit with ln s or $\ln^2 s$ or even more complicated functions of logarithms of the energy. Basically there is not a long enough lever arm in ln s to determine unambiguously the energy dependence. The NAL experimenters' favorite current description[5] is in terms of

$$\sigma_T = C_0 + C_1 \ln^{C_2}(s/s_0) ,$$

which fits the data from the minimum value of cross section C_0 at energy $\sqrt{s_0}$ (input by hand), and describes the increase in terms of the amplitude of the rising term, C_1, and the power of ln s by which the cross section is growing, C_2. This form fits the data well, but I don't want to emphasize the form of the fit but rather the comparison of the coefficients using the same form to describe the K^+p and pp measurements.

For K^+p they input $C_0 = 17.25$ mb, and $s_0 = 20$ GeV^2 and find

$$C_1 = 0.367 \pm 0.030$$

$$C_2 = 1.82 \pm 0.08$$

For pp they input $C_0 = 38.24$ mb, and $s_0 = 93.7$ GeV^2 and find

$$C_1 = 0.395 \pm 0.02$$

$$C_2 = 1.92 \pm 0.11$$

If the CERN ISR data on pp total cross sections are included with their own pp data, then they find

$$C_1 = 0.376 \pm 0.015$$

$$C_2 = 2.10 \pm 0.07$$

in good agreement with their fit to NAL energies alone.

It is interesting to note that both cross sections are found to rise at the same rate (i.e., $\ln^2 s$), and be driven by the same amplitude (i.e., $C_1 \sim 0.37$).

Real Parts

Dispersion relations provide a connection between the total cross section and the real part of the elastic scattering amplitude in the forward direction. Given the energy dependences of the total cross sections discussed above, it is of interest to study the energy dependence of the real part of the scattering amplitude. Fig. 5 shows the expected behavior for the ratio of the real to imaginary parts of the elastic scattering amplitude at $t = 0$, for different behavior of the total cross section; for σ_T approaching a constant from above, the ratio (α) is expected to approach zero from below, while for the case of a rising cross section the ratio (α) should go through zero, becoming positive, and then flatten out at a positive value of α for a continuously rising cross section, or turn over and approach zero from above (like ln s) for a σ_T which becomes asymptotically constant.

The real part of the scattering amplitude may be experimentally identified through its interference with the Coulomb amplitude near the very forward direction. The scattering angular distribution may be written

$$\frac{d\sigma}{dt} = \frac{F_c^2}{t^2} + \frac{\sigma_T^2(1 + \alpha^2)}{16\pi} e^{-bt} - 2Q\alpha F_c \frac{\sigma_T}{\sqrt{16\pi}} \cdot \frac{e^{-bt/2}}{t} \qquad (2.1)$$

where F_c is the Coulomb amplitude

Q is the charge of the scattered particle

b is the slope of the nuclear scattering cross section

σ_T is the total cross section

α is the ratio of the real to imaginary parts of the nuclear scattering.

The first term describes the pure Coulomb contribution and falls of like t^{-2}, the second term is the pure nuclear contribution with an exponential slope and the third term describes the interference between the Coulomb amplitude and the real part of the nuclear amplitude. This interference term is strongest when the value of both the Coulomb and real nuclear amplitudes are comparable, which occurs for t-values around 0.005 GeV^2. Therefore experiments measuring this quantity have to be designed to study elastic scattering at such small values of momentum transfer.

There are good experiments for pp scattering from a few GeV/c up through 500 GeV/c. Examples of the data from the ISR experiments[6] are shown in Fig. 6, where the contributions from the three terms in Eq. (2.1) are separately shown. The interference effect is clearly observed. The energy dependence of the ratio of real to imaginary parts of the scattering amplitude is shown in Fig. 7, where α is seen to cross zero around 250 GeV/c and become positive as expected for a reaction in which the total cross section increases with energy. In fact there is good quantitative agreement between the value of α calculated from the total cross-section data via dispersion relations and the measured values of α, as shown in Fig. 7.

It is of interest to look at α for other processes, especially since we know that all total cross sections (except $\bar{p}p$) have been observed to rise in the NAL energy range (Fig. 4). Of special interest is the K^+p system for which the σ_T continues to rise from 10 GeV. The existing data on α for K^+p are shown in Fig. 8, together with the dispersion relation calculation[7] for α using the measured total cross sections from Serpukov and NAL. The agreement is not good.

New measurements of forward K^+p elastic scattering have been performed at SLAC using a wire spark chamber spectrometer.[8] They are part of a systematic study of particle and antiparticle scattering in the (6-14) GeV/c range. The measured cross sections for 10 and 14 GeV/c scattering are shown in Fig. 9. The very forward region only is shown again in Fig. 10, where the nuclear, Coulomb, and interference contributions are separately indicated. The real part in K^+p scattering at these energies is well measured, and is found to be in agreement with the predictions of dispersion relations. (See Fig. 11.)

We can also turn the dispersion relation calculations around, and see if real part measurements in a given energy range can be used as a probe of the total cross section behavior at higher energies. Such calculations have been performed[9] for the pp system, and are shown in Fig. 12. Here the energy dependence of the real to imaginary ratio is shown for different assumptions on the behavior of the high energy total cross section; for σ_T becoming constant at 38 mb at an energy of 120 GeV the real part smoothly approaches zero from below; for σ_T rising to a constant value of 47 mb at 10,000 GeV the ratio crosses zero and becomes positive in the (200-300) GeV region and then reaches a maximum value for energies of ~ 5000 GeV, beyond which it turns over and goes to zero; for σ_T continuing to rise indefinitely the ratio again crosses zero around (200-300) GeV and becomes positive, rising to a plateau for energies around 10,000 GeV at a value of ~ 0.10. From inspection of these curves it appears that careful measurements of the real part through the energy range of the ISR could indicate the s-dependence of the total cross section up to a few 10^4 GeV - or, more specifically, if the cross section becomes constant at energies around 10^4 GeV, one should be able to see a flattening (or even a turn-over) in the s-dependence of α through the ISR region.

Such studies seem of great interest for K^+p at NAL. The K^+p system is the most precocious, having shown signs of rising total cross section from 10 GeV and, as such, is the prime candidate for revealing the asymptotic behavior of the total cross section. Therefore, we should watch with interest the NAL experiment measuring K^+p real parts, as it may prove the most useful probe of the high energy s-dependence of total cross sections.

As a final comment on total cross sections, we show in Fig. 13 the breakdown of σ_T into the elastic and inelastic parts, for pp scattering.[10] The elastic cross section is known to rise by the same fraction as the total in the (100-2000) GeV/c range, and the inelastic cross section to smoothly increase at all energies. The inelastic cross sections for all processes - $p^{\pm}p$, $K^{\pm}p$, $\pi^{\pm}p$ - are shown in Fig. 14.[10] It appears that it is the increase in the inelastic cross section which is driving all the total cross sections to increase at high energies.

3. Elastic Differential Cross Sections

General

The angular distribution for elastic scattering, at energies above 1 GeV, is observed to be sharply forward peaked as expected for a diffractive process. The shape of this forward peak is well represented by an exponential form -

$$\frac{d\sigma}{dt} = \left.\frac{d\sigma}{dt}\right)_0 e^{bt} \quad ,$$

where the slope parameter, b, characterizes the sharpness of the scattering peak. Most processes show a trend for the slope to increase as the energy increases - a phenomenon which may be interpreted as an increase in the size of the scatterer. Fig. 15 summarizes the energy dependence of the slope parameter, b, for the momentum transfer region about $t \sim 0.2$ GeV^2, for $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ scattering. The general trend is for π^+, K^+, and p reactions to shrink strongly as the energy increases, for π^- and K^- to show little energy dependence, and for $\bar{p}$ to display strong antishrinkage. Further, the particle and antiparticle slopes appear to converge at high energies, as expected for a purely diffractive process.

High Energy pp Scattering

In 1972, high statistics studies of pp elastic scattering uncovered interesting structure in this forward scattering peak. They found the slope of the differential cross section changed by ~ 2 GeV^{-2} for momentum transfers ~ 0.15 GeV^2. Examples of the shape of the cross section are shown in Fig. 16 for momentum of 1400 GeV/c and 130 GeV/c, as measured at ISR[11] and NAL[12] respectively. These experiments also observe quite different energy dependence in the shape of the scattering distribution for $t < 0.15$ GeV^2, and for $t > 0.2$ GeV^2. The slope of the cross section has been evaluated for both momentum transfer regions and its energy dependence is displayed in Fig. 17. The small t region exhibits quite rapid shrinkage as energy increases, the increase being linear in ln s. The change in slope is often parametrized in the form

$$b = b_0 + 2\alpha' \ln s$$

and for $t < 0.15\ \mathrm{GeV}^2$, α' is found to be $0.27 \pm .02\ \mathrm{GeV}^{-2}$. For the larger t region, ($0.2 \leq t \leq 0.5\ \mathrm{GeV}^2$), the slope is almost constant with an α' lying somewhere between 0 and $0.1\ \mathrm{GeV}^{-2}$.

Extending these studies to larger t at the ISR revealed more interesting structure in the elastic scattering peak.[13] Fig. 18 shows large t np scattering cross sections in the (13-21) GeV range. The np data show the same behavior as the pp data, with the cross section falling steeply out to $t \sim 1\ \mathrm{GeV}^2$, then a broad shoulder around $t \sim 1.5\ \mathrm{GeV}^2$ and the cross section again falling off quite steeply as t increases further. At the ISR, this shoulder region develops into a deep hole, very reminiscent of a diffraction pattern minimum, as shown in Fig. 19. The energy dependence of this phenomenon, as seen in the original experiment, is shown in Fig. 20. New data on large t pp scattering have been obtained on the Split-Field Magnet Facility[14] at equivalent energies of 260 and 2000 GeV/c, shown in Fig. 21, and from NAL[15] at 100 and 200 GeV/c, shown in Fig. 22. The NAL data show the dramatic onset of the diffraction minimum for energies around 200 GeV/c; presumably this is related to the fact that the real part of the scattering amplitude is contributing most of the cross section at the dip t-values and is going very rapidly to zero in the 200 GeV region, as discussed above and shown in Fig. 7. The energy dependence of the position of the dip and of the height of the second maximum are shown in Fig. 23.

Geometric Scaling

All of the data we have discussed to date are neatly summarized in a geometrical picture. The features of the data were:

- σ_T^{pp} increasing by $(10 \pm 2)\%$ in NAL-ISR energy range - (200-2000) GeV/c; σ_{el}^{pp} increasing by 10% through the same energy range; σ_{inel}^{pp} is responsible for most of the rise in σ_T, growing by (3-4) mb in the above energy range.
- all σ_T (except $\bar{p}p$) show increase as energy increases in NAL energy range; the increase is either ln s or $\ln^2 s$.
- the real part of the pp and K^+p scattering amplitude at $t = 0$ are seen to be in agreement with the dispersion relation prediction using the measured total cross sections as input.
- the small t slope of $d\sigma/dt$ for $t < 0.15$ GeV^2 is steep ($b \sim 12$ GeV^{-2}), and grows like ln s; parametrizing this shrinkage in terms of $b = b_0 + 2\alpha' \ln s$ yields $\alpha' = 0.27 \pm 0.02$ GeV^{-2} for pp scattering.
- the slope of the cross section changes rapidly by ~ 2 GeV^{-2} for t-values around 0.15 GeV^2; the cross sections for larger t-values show weak energy dependence.
- the break in the cross sections, $d\sigma/dt$, observed in (10-30) GeV/c pp and np scattering for t-values around 1.5 GeV^2, develops into a beautiful diffraction minimum at high energies; the position of the minimum moves to smaller t-values as the energy increases - by $\sim 10\%$ in the (200 - 2000) GeV/c range.

These data are all consistent with an optical picture of a gray absorbing disc of constant opacity, and with the radius increasing with energy - or, more generally, with the idea of geometric sealing. In such a picture, we expect

$$\sigma_{el} \propto R^2$$

$$\sigma_T \propto R^2$$

$$b \propto R^2$$

$$\sigma_{el}/\sigma_T \propto \text{constant}$$

$$b/\sigma_T \propto \text{constant}$$

$$t_{dip} \propto R^{-2} \quad ,$$

where R is the radius of interaction.

All of the above relationships are observed in the data, but at present it is not clear whether R^2 is growing like ln s or $\ln^2 s$ or some more complicated function of ln s (i.e., the incremental change in each of the above quantities is measured to scale by the same amount, but it is difficult to determine the functional form of the energy dependence, due to the small lever arm in (lns) for the existing experiments).

New Data from SLAC and NAL

Several new experiments have been performed studying elastic scattering, and it is interesting to examine how the new results fit into our conventional picture of diffractive processes.

The SLAC experiment[16] has measured K^+p and $K\bar{p}$ scattering at 6.4, 10.4, and 14 GeV/c and $\pi^{\mp}p$ and $p^{\mp}p$ scattering at 10.4 GeV/c. This experiment emphasized the systematic study of particle and antiparticle scattering, with high statistics and good relative normalization - each scattering distribution contains approximately 200,000 events, and the relative normalization of the cross sections is better than 0.5%. The differential cross sections from this experiment are shown in Figs. 24 through 33. The experiment for K^+p scattering at

14 GeV/c was extended to 650,000 events in order to look for possible small t structure comparable to that observed in elastic pp scattering, as discussed above. The cross section for the full statistics at 14 GeV/c is shown in the upper part of Fig. 34. To demonstrate the presence of curvature, the data are divided by an exponential function, e^{Bt}, which is the best fit to the cross section in the small t interval between 0.02 and 0.20 GeV^2. The resulting distribution is shown in the bottom part of Fig. 34, where the cross section is clearly seen to deviate from a single exponential form. There is no evidence for a sharp break in the distribution, but rather the data imply a continuous curvature of the cross section. The 10 GeV/c K^+p cross section is repeated in Fig. 35, together with the distribution when divided by the best fit to an exponential form in the small t interval. The deviations from a single exponential are not very pronounced at 10 GeV/c; the lower energy K^+p scattering data (from CERN[17] and Argonne[18]) are well explained by a single exponential form. Therefore we see evidence in the exotic K^+p scattering for a curvature effect at small t values growing with energy above 10 GeV/c. Such a curvature is unlikely to be generated by peripheral nondiffractive exchanges since the K^+p system is exotic and receives little contribution from such exchanges. Furthermore, the energy dependence expected from exchange processes is opposite to that exhibited by the data. It is natural to ascribe this behavior to diffraction scattering, and one would then expect the phenomenon to become more pronounced at higher energies.

To investigate the structure of the forward elastic cross sections in more detail, we examine the scattering distributions for $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ differentially over the measured t-range, and fit locally for the logarithmic slope,

$$B(t) = \frac{d}{dt}\left(\ln\frac{d\sigma}{dt}\right).$$

These fits were performed over small t intervals - typically (0.1) GeV^2 near to the forward direction, growing to (0.3) GeV^2 for t-values near 1 GeV^2. Each fit contained around 50 - 80,000 events. In addition to the statistical error, there are systematic effects due to the spatial variation of the detection efficiencies. These systematic effects contribute an uncertainty of $\sim 0.1\ \mathrm{GeV}^{-2}$ to the determination of B(t) in this experiment. The results of these fits for each of the processes measured are shown in Fig. 36.

It is interesting to note that such an analysis could be performed not only because of the large statistical power of this experiment, but also due to the very good understanding that was achieved on the details of the local and global angular acceptance of the apparatus. These effects were carefully studied using fully constrained elastic events in which the recoil proton was measured, and a very large sample of $K \to \mu\nu$ decays. To set a scale for these effects, an uncertainty of 1% in the acceptance over one if the t-bins used in the fits would introduce an error in the slope of $\sim 0.1\ \mathrm{GeV}^{-2}$.

Rich structure is observed in all of the scattering distributions. The $\pi^{\pm}p$, $K^{-}p$ cross sections all display similar behavior as a function of t; they have a steep slope in the very forward direction which smoothly decreases as t increases. By contrast the slope in $\bar{p}p$ scattering increases as t changes from zero to about 0.3 GeV^2, drops sharply around $t \sim 0.5\ \mathrm{GeV}^2$, and then remains constant as t increases further. These scattering processes are all nonexotic in the s-channel and have corresponding strong imaginary (Regge) exchange amplitudes. Such scattering processes have been successfully explained within the framework of the Dual Absorptive Model.[19] In this model there are two main amplitudes - a diffractive amplitude which is central and represents the absorption of the incoming particle via all the open inelastic channels, and an exchange amplitude which is strongly absorbed for small impact parameters and peaks at the edge of the interaction volume. These two amplitudes would then have contributions to the differential cross section as shown in Fig. 37 - an exponential and a modified Bessel function, $J_0\left(R\sqrt{-t}\right)$, respectively. The peripheral amplitude causes the curvature at small t, and when it becomes very large, as in the case for $\bar{p}p$ scattering, even causes the flattening in $\ln\frac{d\sigma}{dt}$ at very small values of t. So this picture can qualitatively account for the structures observed in the nonexotic channels.

The K^+p and pp cross sections both exhibit structure in the forward direction (see Fig. 36). The K^+p data, as discussed above, indicate a steepening of the cross section for small t values, and the effect seems to grow with increasing energy. For pp scattering a sharp change of slope is observed for $t < 0.2$ GeV^2 with the slope increasing by $\sim 1\frac{1}{2}$ GeV^{-2} from its value at larger t. This is very similar to the behavior observed in pp scattering at the ISR, and is the first time the phenomenon has been clearly observed at lower energies.[20] The K^+p and pp reactions are exotic in the s-channel and are expected[19] to be accounted for by just one contribution - the central Gaussian diffractive amplitude, leading to purely exponential behavior in t. The data for both of these reactions clearly do <u>not</u> support such a simple description of the exotic elastic scattering reactions.

New data on elastic scattering are also available from two experiments at NAL. Preliminary cross sections for 200 GeV/c π^-p scattering and 100 GeV/c K^+p scattering are shown in Fig. 38 and 39 respectively. These data are from the Michigan-Argonne-NAL experiment.[15] Figs. 40 and 41 show the preliminary differential cross sections for π^+p and K^+p scattering at 100 GeV/c from the NAL Single Arm Spectrometer Collaboration.[21] All of these scattering distributions show considerable curvature and would not be well represented by a single exponential form.

Two interesting comments on forward structure emerge from these experiments. In Fig. 42 we plot the slope parameter for K^+p scattering as a function of energy for $t < 0.2\ \mathrm{GeV}^2$ and for $t > 0.3\ \mathrm{GeV}^2$. The low energy ANL are well fit by a single exponential and so the slope value from fitting the whole scattering region is plotted. The 6, 10, and 14 GeV SLAC points indicate that the slope is strongly increasing as energy increases for the small t region, and almost independent of energy for the larger t region. The dots at high energy in Fig. 42 are the equivalent slope parameters derived from the preliminary SASG data.[21] Good agreement is observed with the s-dependence indicated by the lower energy experiment. The shrinkage of the slope, when analyzed in terms of $b = b_0 + 2\alpha' \ln s$, yields $\alpha' \sim 0.5\ \mathrm{GeV}^{-2}$ for the forward region, and $\alpha' \sim 0.1\ \mathrm{GeV}^{-2}$ for the larger t region. It is interesting to remember that similar behavior is observed in pp scattering, but there the shrinkage is found to be $\alpha' \sim 0.27$ and $\sim 0.1\ \mathrm{GeV}^{-2}$ for the two t regions respectively. We see, then, that the K^+p scattering displays similar structure in t and s to the pp scattering reaction.

The SAS group have made a detailed comparison of π^+p and pp scattering in the NAL energy range. They study the ratio of π^+p and pp cross sections as a function of t, and in this way eliminate many of the systematic errors of their measurement. They find that the ratio of the cross sections can be well represented by a single exponential over the t region $(0.015 - 0.4)\ \mathrm{GeV}^2$ for all energies studied between 50 and 170 GeV/c. An example of such a distribution is shown in Fig. 43. Now we know that the pp scattering has a change of slope for t-values around $0.15\ \mathrm{GeV}^2$ and that the slope shrinks strongly for t less than this value, and has little s-dependence for larger t values. Therefore, the new NAL data strongly suggest that the π^+p scattering must show the same structure, both in t and for the s-dependence of the separate t regions. So once again we conclude that high energy elastic scattering is not described by a single exponential, but that the diffractive amplitude has some interesting structure in t and in s, and moreover that this structure is seen in πp and Kp scattering as well as in pp scattering.

4. Inelastic Diffraction Scattering

Having reviewed the experimental data on the elastic scattering amplitude and the insights it provides on diffractive processes, it is of interest to turn now to inelastic diffraction scattering. These processes may be discussed in two broad categories - inclusive scattering, where the leading particle effects are dominated by diffraction, and exclusive scattering, where the classic Good-Walker[22] excitation of the target or projectile particles - into low mass (3π), (5π), ($K\pi\pi$), ($N\pi$), or ($N\pi\pi$) final states is also dominated by diffractive amplitudes.

Inclusive Scattering

The inclusive scattering process is usually discussed in terms of small momentum transfer collisions in which the target is excited into a state X, as shown diagrammatically in Fig. 44a. Such processes are usually analyzed in terms of the Feynman variable x, which is the fractional longitudinal momentum carried by the leading particle, and is also equal to $(1 - M^2/s)$, where M_X is the mass of the excited system and s is the square of the center-of-mass energy. The cross section in this variable x is shown schematically in Fig. 44b, and is characterized by a broad peak about $x = 0.5$, which represents general particle production by exchange processes, then a valley around $x = 0.9$ leading into a sharp peak near $x = 1.0$. It is this peak for $0.9 < x < 1.0$ which is dominated by the diffraction process, and whose properties we will now try to summarize. These processes are characterized by

- the production of a low mass peak, which favors low multiplicity and falls off like M^{-2}.

  In Fig. 45 the missing mass distributions from the 200 GeV pp bubble chamber experiment[23] at NAL are shown. A low mass peak is clearly visible in the 2 prong and 4 prong distributions with only a hint left in the 6 prong spectrum. More complete analyses of the multiplicity distributions for πp and pp interactions around 200 GeV show that the multiplicity associated with diffractive reactions is about half the multiplicity for all processes.[24] The shape of the mass distribution is shown in Fig. 46, where the solid line is a M^{-2} shape. The data are compatible with such a falloff. A stronger statement on the M^2 dependence is obtained from the NAL experiment using the internal deuterium gas jet target and detecting the recoil deuteron in a solid state counter array.[25] Observation of the reaction

$$pd \rightarrow dX$$

assures that the exchange is isoscalar. The M^2 dependence of the cross section multiplied by M^2 from this experiment is shown in Fig. 47. For each energy studied the distribution is a flat spectrum for $M^2 > 5\ \mathrm{GeV}^2$, implying a M^{-2} behavior of the low mass diffractive peak.

- the low mass peak is produced in a sharply forward peaked angular distribution, as expected for a diffractive process. The sharpness of the peak depends strongly on the mass, M_x, of the excited system.

The differential cross section for pp $\rightarrow$ pX from an NAL bubble chamber[23] experiment is shown in Fig. 48 for several regions of missing mass M_x. The slope of the scattering distribution is seen to become flatter as the mass increases. New data from the pd $\rightarrow$ dX experiment[25] gives more detail on this dependence of the sharpness of the scattering distribution on the mass of the recoil system, as shown in Fig. 49. The slope is about twice the elastic slope for masses near threshold, falling rapidly to a slope about half the elastic slope.

- the cross section for production of the $x = 1$ peak is almost s-independent, and may grow at high energies like $\ln s$.

The energy dependence of the diffraction peak has been measured at NAL[26] and ISR[27] for pp scattering, and the results are shown in Fig. 50 and 51, respectively. The NAL data only measure to x-values of ~ 0.93, but survey an energy region from $s = 100$ to $s = 750$ GeV^2 for momentum transfers in the range $0.14 < t < 0.38$ GeV^2. The cross section is seen to fall $\sim 20\%$ through this energy range. The ISR data are compatible with a less than 10% variation of the cross section through the ISR energy range. However, once more the $pd \rightarrow dX$ experiment from NAL[25] has the most interesting new information on this question - they showed that the mass distribution falls off as M^{-2}, as discussed above. More explicitly they found that

$$\frac{d\sigma}{dM^2} = \frac{0.7\ \text{mb}}{M^2} \quad .$$

If we integrate this distribution to find the total diffractive cross section, we find

$$\sigma \propto \ln s \ .$$

This can be seen clearly in Fig. 52, where the cross section $\frac{d\sigma}{dx}$ is plotted against $(1-x)$. The diffractive cross section is the integration for all $x > 0.9$. As the energy of the scattering process increases, the region kinematically accessible grows, and so the total diffractive cross section also grows.

There is some new information on inclusive leading particle scattering from the SLAC wire chamber experiment that we discussed extensively in the elastic scattering section above. High statistics data on K^+p and K^-p scattering at 6, 10, and 14 GeV and for $\pi^{\pm}p$ and $p^{\pm}p$ at 10 GeV have been obtained that allow detailed study of the cross section as a function of M_x and t and also a good comparison of particle and antiparticle scattering.[28]

The differential cross sections for three regions of missing mass are shown in Fig. 53. For $M_x < 1.2$ GeV the distribution falls very steeply to $t \sim 0.3$ GeV^2, then flattens out (or even dips) before falling off at larger t; in the intermediate range with $1.4 < M_x < 1.5$ GeV the cross section is well represented by two exponentials, while for large masses, $M_x > 1.8$ GeV, it is well described by a single exponential.

To investigate more fully the structure of the scattering distribution the slope of the cross section has been evaluated for $t < 0.25$ GeV^2 and $t > 0.4$ GeV^2 as a function of M_x. The results of these studies are shown in Fig. 54. The slope for the small t region is very steep for masses near threshold - approximately twice the elastic slope - and falls very sharply for masses between 1.3 and 1.5 GeV, until around $M_x \sim 1.8$ GeV it settles down to a constant value of about half the elastic slope. The K^+ and K^- distributions are essentially the same. In the large t region the slope is almost independent of M_x, showing no strong s-dependence and very little dependence on the particle.

The $\pi^{\pm}p$ and $p^{\pm}p$ data show very similar structure.

In summary, the inclusive scattering produces a low mass, low multiplicity enhancement with a slowly varying cross section which may even rise like ln s. The enhancement is produced with a sharp forward peak whose shape depends strongly on the mass being produced. For small M_x and small t there is strong structure, very reminiscent of a $J_0(R\sqrt{-t})$ behavior for all scattering processes; for small t and M_x increasing beyond threshold the cross section is represented first by the sum of two exponentials, and then beyond $M_x \sim 1.8$ GeV by just a single exponential (this is true for π, K, and p scattering); for the larger t region, $t > 0.4$ GeV^2, the scattering distribution shows no strong M_x or s-dependence.

Exclusive Scattering

The exclusive diffraction scattering process is observed when pions are excited into (3π), (5π) systems, kaons are excited into a $(K\pi\pi)$ system, and nucleons into $(N\pi)$ or $(N\pi\pi)$ systems. These excited systems are characterized by

- the production of a low mass enhancement, with the same isospin as the incident particle and characterized by zero isospin exchanged in the t-channel.

  Fig. 55 shows the various contributions to the diffractive excitation $p \rightarrow (N\pi)$ in 16 GeV/c πp scattering.[29] The isoscalar exchange amplitude leading to an $I = 1/2$ $N\pi$ final state shows a smooth, featureless bump rising quickly from threshold. Of the other contributions, only the isovector production of the $\Delta(1238)$ resonance is substantial. Fig. 56 shows the $n\pi^+$ mass spectrum from $pp \rightarrow pn\pi^+$ at 1500 GeV/c.[30] A very similar shape is seen.

- the cross section for the production of the low mass enhancement is only weakly s-dependent.

  Fig. 57 shows the cross section for the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^+ \pi^- p$ from a few GeV/c up to 200 GeV/c. At high energy this reaction is dominated by the diffraction dissociation channels $p \rightarrow (p\pi^+\pi^-)$ and $\pi^- \rightarrow (\pi^-\pi^+\pi^-)$. Above 20 GeV/c there is very little energy dependence observed. In Fig. 58 the cross section for $pp \rightarrow pp\pi^+\pi^-$ is summarized. The cross section is seen to fall slowly even from low energy, and be consistent with no s-dependence at high energy. Finally, Figs. 59 and 60 show the energy dependence for $p \rightarrow (n\pi^{+})$ and $n \rightarrow (p\pi^{-})$, respectively. The proton dissociation data show a point at 1500 GeV/c from the ISR,[30] confirming the slow falloff of the diffractive cross section, while the neutron dissociation data from NAL[31] exhibit essentially flat cross-section behavior in the (100-300) GeV/c range

- can be produced coherently on nuclear targets
- produced with a sharp forward peak in the differential cross section, $\frac{d\sigma}{dt}$, with the slope of the cross section showing a strong dependence on the mass of the excited system.

Fig. 61 shows the differential cross sections for the dissociation of a neutron into a low mass $(p\pi^{-})$ system as measured by the Rochester - Northwestern collaboration at NAL.[31] Similar behavior is seen in 1500 GeV/c $p \rightarrow n\pi^{+}$ from ISR[30] in Fig. 62.

- the production of the low mass enhancement is thought to contain two contributions - (1) a kinematic enhanced process like the Deck effect, and (2) dissociation of the particle into resonant states. It is difficult to isolate these two contributions.

The $pp \rightarrow pn\pi^{+}$ experiment at the ISR[30] shows indications that these two amplitudes may be separable. In Fig. 63 the scattering process is shown diagrammatically. For a pure resonance formation and subsequent decay into $n\pi^{+}$, the scattering angular distribution should be symmetric in $\cos\theta$ and isotropic in ϕ (unless the resonance were aligned, in which case it should be symmetric about $\phi = 90^{0}$). For a Deck type background in which the phase and amplitude of the process did not change much from one partial wave to the next, one would expect reinforcement in the forward direction and cancellation backwards. This would result in a sharp forward peak in cos θ for the background. Fig. 64 shows the $p \rightarrow (n\pi^+)$ mass distribution cut on this scattering angle. For backward produced neutrons the resonances stand out clearly, and for forward neutrons the smooth Deck-like spectrum is observed. These cuts work just as one would expect if the two contributions - Deck and resonance formation - were both present. Fig. 65 shows the ϕ distribution also cut on cos θ.

The "resonance cut" shows a distribution symmetric in ϕ about 90°, while the "background" is strongly peaked around $\phi \sim 0^\circ$. Such distributions would result (a) if the resonances were produced aligned and (b) if the high energy neutron from the background process "bremsstrahlunged" pions.

It is interesting to see signs of these two processes in exclusive diffraction, and one hopes that shortly some clues as to the presence of both resonance and kinematic amplitudes in π- and K-meson processes will emerge.

Factorization

If we really believed that diffraction reactions are dominated by the exchange of a simple Pomeron, we should be able to factorize, or separate, the different vertices appearing in these processes. It is interesting to see how well the data support such a hypothesis.

An interesting test is found in the three sets of reactions, shown in Fig. 66, involving the excitation of the proton to N*(1688) in π^-, K^-, or p^- interactions. These cross sections should have the same ratio with respect to elastic scattering, independent of the nature of the incident particle. The results of the test are shown in Fig. 67, where the ratio $\left[\frac{Ap \rightarrow AN^*(1688)}{Ap \rightarrow Ap}\right]$ is plotted against momentum transfer for two energies - 8 and 16 GeV/c. Factorization is observed to hold within 20% and even works well as a function of momentum transfer, at least out to $t \sim 0.2\ \mathrm{GeV}^2$.[32]

Next consider the processes illustrated in Fig. 68, with elastic pion and proton scattering at the upper vertex, and proton diffraction into proton plus zero, one, two, or three pions at the bottom vertex. The ratio between cross sections for reactions involving the upper two vertex processes should be the same, independent of which of the four bottom vertices they interact. That is, $R_1 = \sigma(\pi p \rightarrow \pi p)/\sigma(pp \rightarrow pp)$ should equal $R_2 = \sigma(\pi p \rightarrow \pi(p\pi^o))/\sigma(pp \rightarrow p(p\pi^o))$, etc.

The cross section for each of the bottom vertices was isolated in 16 GeV/c $\pi^- p$ and 19 GeV/c pp bubble chamber experiments,[33] using the Van Hove Longitudinal Phase Space analysis to isolate the diffractive components. The results are given in Fig. 68. Good agreement is observed.

Another interesting test of factorization in diffractive processes is shown schematically in Fig. 69.[33] If the pomeron contribution were well-behaved and factorizable, we would expect the ratio of cross sections for each of the upper vertex processes, $\gamma \rightarrow \rho^o$, $\pi \rightarrow \pi$, $p \rightarrow p$, being joined in turn to both of the bottom vertex processes, $p \rightarrow p$, $p \rightarrow (p\pi^+\pi^-)$, to be equal. That is, we would expect to find

$$R_1 = \frac{\sigma(\gamma p \rightarrow pp)}{\sigma(\gamma p \rightarrow \rho p\pi\pi)},\ \text{equal to}\ R_2 = \frac{\sigma(pp \rightarrow pp)}{\sigma(pp \rightarrow pp\pi\pi)},\ \text{equal to}\ R_3 = \frac{\sigma(\pi p \rightarrow \pi p)}{\sigma(\pi p \rightarrow \pi p\pi\pi)}\ .$$

The diffractive component for these reactions was again isolated using the LPS analysis.

The experimental values for R_1, R_2, and R_3 are given in Fig. 69 for three different energy regions. The agreement is surprisingly good.

Yet another interesting factorization test has been made possible by the study of the four body exclusive reaction in pp and $\pi^- p$ collisions at 205 GeV/c.[34] The diffraction of the target proton into a $(p\pi^+\pi^-)$ system has been isolated in each experiment - see Fig. 70 -

the cross section for $\pi p \to \pi(p\pi\pi)$, $\sigma_1 = (180 \pm 36)\,\mu b$

the cross section for $pp \to p(p\pi\pi)$, $\sigma_2 = (370 \pm^{40}_{140})\,\mu b$

the cross section for $\pi^- p$ reaction is proportional to $g^2_{\pi\pi\mathscr{P}} \cdot g^2_{pN^*\mathscr{P}}$

the cross section for the pp reaction is proportional to $g^2_{pp\mathscr{P}} \cdot g^2_{pN^*\mathscr{P}}$

Now

$$\frac{\sigma_1}{\sigma_2} = g^2_{\pi\pi\mathscr{P}} \cdot g^2_{pN^*\mathscr{P}} / g^2_{pp\mathscr{P}} \cdot g^2_{pN^*\mathscr{P}}$$

$$= g^2_{\pi\pi\mathscr{P}} / g^2_{pp\mathscr{P}}$$

$$= \frac{\sigma(\pi p \to \pi p)}{\sigma(pp \to pp)}, \text{ or } \frac{\sigma_{el}(\pi p)}{\sigma_{el}(pp)}$$

$$= \frac{3.0 \pm 0.3}{6.8 \pm 0.2}$$

$$= 0.44 \pm 0.05$$

while σ_1/σ_2 is measured to be ~ 0.5. Again, surprisingly good agreement.

The NAL gas target experiment[25] allows a comparison of the processes $pp \to pX$ and $pd \to Xd$ for small missing mass and small t. These are just the data one would expect to be diffraction dominated and hope to exhibit factorization. The data are observed to agree to slightly better than 20%.

Finally, if one believes in factorization one must take the double dissociation process seriously, and believe that its properties may be explained in terms of the single dissociation data. The various related diffractive reactions are shown schematically in Fig. 71, where Kp scattering was taken as our example.

Writing down the separate vertex couplings, we find

$$\left.\frac{d\sigma}{dt}\right)_{DD} \propto \left| g_{K\mathscr{P}Q}(t) \cdot g_{p\mathscr{P}N'}(t) \right|^2 \sim \left| \frac{A(Q) \circ A(N')}{A_{el}} \right|^2$$

If $A = Be^{\lambda t}$, then

$$\left.\frac{d\sigma}{dt}\right)_{DD} \sim \left(\frac{B_Q \cdot B_{N'}}{B_{el}}\right) e^{-(\lambda_Q + \lambda_{N'} - \lambda_{el})t} \sim B_{DD} \circ e^{-\lambda DDt}$$

Thus we have a relation between the slopes of the various diffractive processes, and also between the cross sections, viz.

$$\sigma_{DD} \sim \frac{\sigma_{SD} \circ \sigma_{SD'}}{\sigma_{el}}$$

Further, since the single diffractive cross section depends strongly on the mass of the excited state, similar correlations are expected for double diffraction.

There have been several experiments identifying such a reaction in 14 GeV/c K^-p[35] and 16 GeV/c $\pi^\pm p$ reactions, but at such low energies the t_{min} corrections make it very difficult to be other than qualitative in the analysis. New data from NAL and ISR also identify the process and demonstrate that the factorization hypothesis works to (10-20)%.[37] One experiment has even studied the ratio of elastic single and double diffraction as a function of t at the ISR,[38] and shown that factorization works to about 15%. The results are summarized in Table 1. Even the expected slope correlations are being observed[39] - see Fig. 72. There the differential cross section for two ($p\pi\pi$) systems recoiling against each other is shown. When the total low mass enhancement is used, the cross section has a flat slope of $\sim 3.6\ \mathrm{GeV}^{-2}$, but if the cross section is plotted for only low mass systems ($\sim$ 1400 MeV) recoiling from one another, the slope increases to $\sim 12\ \mathrm{GeV}^{-2}$. This is again just what you would expect for double dissociation in a factorization picture.

5. Impact Parameter Picture of Diffractive Processes

Reminder of the Data

- The total cross sections for all processes, except $\bar{p}p$, grow with increasing energy. The growth is consistent with a $\ln s$ or $\ln^2 s$ dependence. It seems that the growth of the inelastic cross section is driving the increase in the total cross section.
- The small t slope for the pp differential cross section, $\frac{d\sigma}{dt}$, is $\sim 10.5\ \mathrm{GeV}^{-2}$ at $s \sim 100\ \mathrm{GeV}^2$, and grows like $\ln s$ (or $\ln^2 s$). If the slope is parameterized as $b = b_0 + 2\alpha' \ln s$, then for this small t region ($t < .15\ \mathrm{GeV}^2$), α' is found to be $\sim 0.27\ \mathrm{GeV}^{-2}$. The slope changes rapidly by about $2\ \mathrm{GeV}^{-2}$ around $t \sim .15\ \mathrm{GeV}^2$, and the slope for the larger t region, ($.2 < t < .5\ \mathrm{GeV}^2$), shows very little s-dependence ($\alpha' \sim 0.1\ \mathrm{GeV}^{-2}$).

Similar conclusions are obtained for πp, Kp scattering—viz., there is a sharp change in slope for t values in the neighborhood of $0.02\ \mathrm{GeV}^2$, and the small t region shrinks quite strongly as s increases ($\alpha' \sim .5$ for K^+p), while the larger t region is essentially flat in s.

- For inelastic diffraction, the slope of the differential cross section is seen to be a strong function of the mass of the diffracted system, falling from a value about twice the elastic slope at threshold to about half the elastic slope about 1 GeV above threshold.

For masses near threshold, there is a striking structure in $d\sigma/dt$, with a very steep exponential forward region followed by a dip (or a shoulder) for $t \sim 0.3\ \mathrm{GeV}^2$, and then falling off slowly for larger t values. As the mass of the excited system increases, this structure is lost, and the distribution is well described by the sum of two exponentials with the size of the second, flatter component increasing as the mass increases. Finally, for masses greater than 2 GeV, the data are well represented by a single exponential.

s-Channel Unitarity and the Overlap Functions

We have described diffraction scattering above in simple geometrical terms, as the shadow due to the absorption of the initial wave caused by the many open inelastic final states. This approach was extended and made quantitative by Van Hove in 1963[40] when he introduced the elastic and inelastic overlap functions. The unitarity relation for the transition amplitude, T_{fi}, is

$$\text{Im } T_{fi} = \sum T^*_{el,f} \cdot T_{el,i} + \sum T^*_{in,f} \cdot T_{in,i}$$

where the two terms on the right-hand side are the elastic and inelastic overlap functions. This equation is graphically illustrated in Fig. 73.

It is interesting to transform this relation into impact parameter space. This may be performed using the Fourier-Bessel transform

$$A(s,\ t) \propto \int B(b,s) \quad J_0\,(b\,\sqrt{-t})\; b\; db,$$

and results in the relation

$$2\ \text{Im}\ a\,(s,\ b) \ = \ |a\,(s,b)\,|^2 + a_{in}\,(s,b)$$

Notice that the imaginary part of the elastic amplitude is built of two parts: the shadow of the inelastic channels plus the actual elastic contribution. This is shown in Fig. 74 where it may be observed that for small inelasticity, the elastic amplitude is very small indeed. As the inelasticity grows, the elastic amplitude also grows until for complete absorption, the two contributions are equal. It is also important to notice that this equation relates the total, the elastic and the inelastic overlap functions at the same impact parameter.

Before going on to look at the overlap functions derived from the elastic and inelastic scattering data, it is interesting to remind ourselves how the details of an amplitude in four momentum transfer space reflect themselves into impact parameter space, and vice versa. In Fig. 75, examples are shown of how two peripheral and two central amplitudes would transform. Peripheral production at a radius, R, would result in a Bessel function shape, $J_0(R\sqrt{-t})$, in t-space; for $R \sim 1$ fermi, the zero in F(t) should occur around $t \sim 0.2\ \mathrm{GeV}^2$. For a central Gaussian amplitude in b-space, we would expect an exponential shape in t-space, where the coefficient of the exponential form is a measure of the mean radius of the Gaussian profile. If we were to add on some extra-high partial waves to a Gaussian distribution in b-space (see Fig. 76), we would observe an additional contribution at small t in F(t), inducing an upward curvature in the cross section at small t. On the other hand, if we were to remove some low partial waves from our Gaussian profile in b-space, the effect on F(t) would be to induce some large t structure (see, again, Fig. 76).

We have observed the presence of both these small and large t phenomena in the high-energy pp elastic scattering data. We now follow a more quantitative analysis of these effects on the amplitude profile in b-space.

High Energy pp Scattering Analysis

The s-channel unitarity relation discussed above may be used to derive the inelastic overlap given a description of the total elastic amplitude and its imaginary part. Since the measured data provide the necessary information for the elastic amplitude, we remain with the problem of isolating the imaginary and|or real parts. Most simply, we might assume that at high energies, the elastic scattering amplitude is purely imaginary—this is not too bad an approach. The next stage of sophistication is to attempt to identify the real part and correct the measured cross section to determine the imaginary part of the scattering amplitude. The real part is known only at $t = 0$, through coulomb interference experiments (as discussed in Section 2 above); however, a reasonable estimate of the phase is obtained by assuming that all of the cross section at the $t = 1.3\ \mathrm{GeV}^2$ dip is due to the real part of the amplitude (i.e., the imaginary part of the amplitude has a zero at this point). Smoothly connecting the known $t = 0$ real part to the derived $t \sim 1.3\ \mathrm{GeV}^2$ real part allows an estimate of the real part of the scattering amplitude at all t. (The analysis is found not to be sensitive to the real part assumed.)

Figure 77 shows the total,inelastic and elastic overlap functions for pp scattering at $\sqrt{s} = 53$ GeV, using such a prescription. These results are from the calculations of Pirila and Miettinen,[41] but all of the analyses are in fairly good agreement. The "blackness" of the proton is observed to be ~74% of the unitary black disc limit, and the inelastic overlap function looks like a Gaussian with average radius just a little under 1 fermi. On closer inspection, $G_{inel}(b)$ flattens out near $b = 0$ and has a longer tail than you would expect for a purely Gaussian profile.

It is interesting to study the energy dependence of the inelastic profile. The above analysis was performed on the high energy pp elastic data for $\sqrt{s} = 21$, 30, 44, and 53 GeV, and the results are shown in Fig. 78. The immediate conclusion of this analysis is that the proton gets <u>bigger</u> and <u>not blacker</u> as the energy increases. The value of $G_{inel}(b)$ at $b = 0$ does not change as the momentum increases from 200 GeV/c to 1400 GeV/c but the mean radius of the Gaussian-like profile does increase. If we look in more detail at how the overlap function changes, we find the increase coming from a ring around impact parameters near 1 fermi. The lower plot in Fig. 78 is the difference between the $\sqrt{s} = 53$ and 30 GeV profiles, implying that the extra contribution to the inelastic profile is rather peripheral.

The measurements of the inclusive proton spectra at NAL and ISR indicate that at high energies, inelastic diffractive scattering and non-diffractive scattering populate different regions of phase space. This suggests that it may be useful to consider their contributions to the elastic amplitude separately. To do this, we split the inelastic overlap into two parts:

$$G_{inel}(t) = G_{prod}(t) + G_D(t)$$

where $G_{prod}(t)$ is the overlap function due to the absorption from all the non-diffractive inelastic processes, and $G_D(t)$ is the inelastic overlap function for inelastic diffractive processes. The s-channel unitarity relation now reads

$$I_m T_{fi}(t) = G_{el}(t) + G_D(t) + G_{prod}(t) \quad .$$

The analysis closely parallels the elastic study described above except that now we have to take into account the spins and helicity structure of the inelastic scattering. For the elastic scattering case, we had to deal with spin 1/2 for all particles, and could safely assume one amplitude with no spin flip. However, in the inelastic scattering case, the spin of the excited state grows quickly as the mass of the excited system increases, and the inelastic scattering processes do not seem to conserve s-channel helicity—so where to now?

Sakai and White[42] have done some interesting calculations abstracting the inelastic diffraction overlap, inputting the spin dependence on the mass of the recoil system, and assuming t-channel helicity conservation. It turns out that the details of their analysis are not strongly dependent on the assumption of the helicity structure as long as there is an appreciable s-channel helicity flip component, which the data strongly support. The profile functions obtained from their analysis are shown in Fig. 79. The inelastic diffractive profile function is found to be peripheral, implying that these diffractive processes take place in a ring around the edge of the interaction volume, with a characteristic impact parameter a little under 1 fermi.

These observations leave us with an intriguing situation:

- the total cross section, σ_T, is seen to rise
- the rise in σ_T is mainly in σ_{inel}
- the increase in σ_{inel} is mainly from large impact parameters, with $b \sim 1$ fermi
- inelastic diffraction is observed to take place largely from large impact parameters, in fact from a ring with $b \lesssim 1$ fermi
- the cross section for inelastic diffraction may be rising like $\ln s$.

All of these features are very suggestive that diffraction is, at the very least, a major component of the rise in the total cross section in hadron-hadron collisions.

Another interesting "fall-out" of this picture of diffraction scattering is that it allows a very natural explanation of the observed strong correlation between the slope of the differential cross section and the mass of the excited system (see Section 4 above).

If the inelastic diffraction scattering is peripheral, it will contribute a term, $e^{at} J_{\Delta\lambda}(R\sqrt{-t})$, to the differential cross section, $d^2\sigma/dt\,M^2$. The exponential factor e^{at} determines how thick the peripheral ring is for any given process and $\Delta\lambda$ is the helicity flip involved in the scattering process. For scattering close to threshold, the spin of the recoil system is small and the contribution of helicity flip amplitudes is small. In this case, the cross section is dominated by the term $J_0(R\sqrt{-t})$ and for $R \sim 1$ F should have a dip near $t \sim 0.3$ GeV^2. Such a situation is shown in Fig. 80a. As the mass of the excited system increases, the spins involved increase quite rapidly and the helicity flip contributions grow, leading to the situation shown in Fig. 80b. These are just the features we have observed in the data; see Fig. 81 and 82.

Elastic Scattering in the (5 - 15) GeV Region—or "Fun and Games"

To finish this section on interpretation of the data, we take at face value the picture of diffraction developed above and attempt to reconstruct the elastic scattering amplitude in the energy range of the SLAC experiment, viz. (5 - 15) GeV/c. The differential cross section may then be written[43]

$$\frac{d\sigma}{dt} \propto \left| P_1 + P_2 + R \right|^2$$

where we neglect the effect of small real parts and where P_1 is the classic diffraction amplitude - central and Gaussian in impact parameter space and essentially constant with energy; P_2 is the peripheral diffractive contribution discussed above, which comes from a ring with $b \sim 1$ fermi and grows with increasing energy like ln s (or $\ln^2 s$); R represents the non-diffractive (Regge) exchange contribution. The t-dependence of the separate contributions is defined by

$$P_1 = A_p e^{B_p t}$$

$$P_2 = A'_p e^{B'_p t} J_0(R_p\sqrt{-t})$$

$$R = A_R e^{B_R t} J_0(R_R\sqrt{-t})$$

All of the parameters in the above amplitudes are determined from our knowledge of the total cross sections and exchange amplitudes, except for B'_p which was set by inspection of the data. The strength of each contribution A_p, A'_p, A_R is determined for the $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ scattering by the three different energy dependences observed in the total cross sections: constant, logarithmic, and $s^{-1/2}$. (See Fig. 4.) The parameters defining the impact profile of the exchange contribution, B_R and R_R, are obtained from the study of the $C = -1$ exchange amplitudes from particle and anti-particle elastic scattering. See Fig. 83. B_p is set "geometrically" from A_p (i.e., $B_p \propto A_p$) and R_p was set equal to 1 fermi.

The t-dependence of the slope of the forward scattering cross section <u>predicted</u> by this model for the six reactions studied is shown in Fig. 84, together with the measured slopes. The agreement is good.

The simple model appears to account for all of the features observed in the measured logarithmic slope. It demonstrates clearly that the diffractive amplitude is not a simple exponential and that a second component is necessary, even at energies as low as 10 GeV; the data on both K^+p and pp reactions require such a term. Extrapolating the observed energy dependence of the two-component amplitude provides a good description of the curvature measured in high energy K^+p and pp scattering. The strong structure observed in the non-exotic scattering where the slope grows steeper as t increases for processes with large exchange amplitudes (like $\bar{p}p$ and K^-p) and the general feature of concave cross sections for $t \gtrsim .2\ \mathrm{GeV}^2$, are well accounted for both in magnitude and in slope by the peripheral (Regge) exchange contribution. As the energy increases, this exchange term dies away, but the peripheral diffractive term grows so that even at high energy, the π^+p, π^-p, K^-p, and $\bar{p}p$ will exhibit curvature of the differential cross section.

6. Summary and Conclusions

Rather than give a tidy summary of how well we understand diffraction scattering (which I cannot do), I will close by listing some of the interesting questions which have been raised.

- What is the asymptotic behavior of the total cross sections?

  We may get a hint of this from careful study of the energy dependence of the total cross sections through NAL and ISR, but other clues could come from study of the s-dependence of the ratio of elastic to total cross sections, the real part of the forward scattering amplitude, and the slope of the forward elastic scattering distribution. The K^+p system is a prime candidate for such studies since it was so precocious in displaying the rise in the total section.

- Are there really two components to the diffractive contribution to $\mathrm{Im}\, A_{el}$?

  If there is a peripheral piece of the Pomeron, then interesting structure should be observed in the differential cross section for reactions like $\bar{p}p$ and K^-p through the energy range of NAL. Both of these processes have strong exchange amplitudes which will die rapidly as the energy increases, and both have a growing contribution from the peripheral diffractive amplitude. Despite both being peripheral, these two contributions (the exchange and the diffractive contributions) have quite different detailed profiles in impact parameter space. Very good elastic experiments in the (50 - 400) GeV/c region should uncover interesting structure.

The reaction $\gamma p \rightarrow \phi p$ is also a very interesting test of these ideas, since the diffractive amplitude dominates the cross section. However, we badly need new experiments here.

- What is causing the low mass enhancement and the "slope-mass correlation" in inelastic diffraction?
- Did the ISR pp $\rightarrow p n \pi^+$ experiment really separate out the kinematic and resonance production amplitudes in the inelastic diffraction process?
    - Can it be done at other energies?
    - Can it help to study the helicity structure of the inelastic diffraction process? Is the helicity structure of the resonance production and the Kinematic (Deck) amplitude different?
    - Where are the resonant components in the meson diffraction process (i.e., the A_1 and Q)?
- Why is the "blackness" of the proton so constant as the energy changes, and why is it only 92%?
- If the proton gets "bigger and not blacker" as the energy increases, how does it grow?
    - Does it all grow, or is it only the ring getting thicker or blacker or bigger?

There are many interesting questions, and some hope that there will be more experimental information to help throw some light (we hope!) on many of them in the near future. It is very much as if the old Chinese curse has come true (to our good/bad fortune?) —

"We live in interesting times."

* * *

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Table 1

t (GeV^2)	$R = \dfrac{\left.\frac{d\sigma}{dt}\right)_{DD} \cdot \left.\frac{d\sigma}{dt}\right)_{el}}{\left.\frac{d\sigma}{dt}\right)_{SDI} \cdot \left.\frac{d\sigma}{dt}\right)_{SD2}}$
(.15 - .28)	1.18 ± .14
(.28 - .40)	1.21 ± .18
(.40 - .53)	1.02 ± .15

Figure Captions

1. Energy dependence of the total cross sections for $\bar{p}p$, pp, π^+p, π^-p, K^-p, and K^+p up through 60 GeV/c.
2. The energy dependence of the total pp cross section up through the CERN ISR energy range.
3. Possible asymptotic energy dependence of the total cross section: (a) continuing growth with increasing energy; (b) asymptotically constant cross section being approached from below.
4. Measured energy dependence of the total cross sections for $\bar{p}p$, pp, π^+p, π^-p, K^-p, and K^+p up through 280 GeV/c, showing the latest results from NAL.
5. The relationship between the energy dependence of the total cross section and the corresponding behavior expected for the ratio of the real and imaginary parts of the forward elastic scattering amplitudes.
6. Differential cross sections for pp elastic scattering at two energies of the CERN ISR in the very forward direction. The separate contributions of the coulomb scattering, nuclear scattering, and coulomb-nuclear interference are indicated.
7. The energy dependence of the measurements of α, the ratio of the real to imaginary parts of the forward scattering amplitude for p-p scattering. The shaded region is the prediction for α using dispersion relations and the data on the total pp cross section.
8. The energy dependence of the measurements of α, the ratio of the real to imaginary parts of the forward scattering amplitude for K^+p scattering. The shaded region is the prediction for α using dispersion relations and the data on the total K^+p cross section.

9. The elastic scattering differential cross section for K^+p at 10 and 14 GeV/c: There are two separate measurements at each energy differing in the small t cut-off—the elastic geometry data extend down to 0.01 GeV^2 while the coulomb geometry data extend down to $t \sim 0.003$ GeV^2.
10. The differential cross section for K^+p elastic scattering at 10.4 and 14 GeV/c; the data are shown for the t interval below 0.03 GeV^2 and the separate contributions of the coulomb, nuclear and interference amplitudes are shown.
11. The energy dependence of the ratio of the real and imaginary parts of the forward elastic K^+p scattering amplitude. The shaded region is the ratio expected from dispersion relation calculations using the measured K^+p total cross section.
12. Calculated energy dependence of the ratio of the real to imaginary parts of the forward scattering amplitude in pp elastic scattering, assuming that the total cross section stops rising at 10^2, 10^3, 10^4, and 10^5 GeV, respectively.
13. The energy dependence of σ_T, σ_{inel}, and σ_{el} for pp scattering.
14. The energy dependence of the inelastic cross section for $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ interactions.
15. The energy dependence of the slope of the elastic differential cross section (evaluated at $t = 0.2$ GeV^2), for $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ scattering.
16. Differential cross sections for elastic pp scattering at: (a) the ISR by the ACGHT Group, and (b) at NAL by the US-USSR collaboration.
17. The energy dependence of the slope, (b), of the elastic differential cross section evaluated for $t < 0.15$ GeV^2, and for the interval $0.2 < t < 0.5$ GeV^2.

18. Differential cross section for np elastic scattering from (13 - 21) GeV/c.

19. The differential cross section for pp elastic scattering in the large t region, as measured at the ISR.

20. The differential cross section for pp elastic scattering in the large t region measured at four energies at the CERN ISR.

21. New measurements of the differential cross section for pp elastic scattering at $\sqrt{s}$ = 23 and 62 GeV, from the Split-Field Magnet at the CERN ISR.

22. Measurement of pp elastic scattering at 100 and 200 GeV/c at NAL by the Michigan-Argonne-FNAL collaboration.

23. Systematics of the large t dip region in pp elastic scattering: (a) the s-dependence of the position of the dip, and (b) the s-dependence of the cross section for the second maximum.

24. Differential cross section for 14 GeV/c K^+p elastic scattering, measured at SLAC.

25. Differential cross section for 14 GeV/c K^-p elastic scattering, measured at SLAC.

26. Differential cross section for 10.4 GeV/c K^+p elastic scattering, measured at SLAC.

27. Differential cross section for 10.4 GeV/c K^-p elastic scattering, measured at SLAC.

28. Differential cross section for K^+p elastic scattering at 6.4 GeV/c, measured at SLAC.

29. Differential cross section for 6.4 GeV/c K^-p elastic scattering, measured at SLAC.

30. Differential cross section for 10.4 GeV/c $\bar{p}p$ elastic scattering, measured at SLAC.

31. Differential cross section for 10.4 GeV/c pp elastic scattering, measured at SLAC.

32. Differential cross section for 10.4 GeV/c $\pi^+ p$ elastic scattering, measured at SLAC.

33. Differential cross section for 10.4 GeV/c $\pi^- p$ elastic scattering, measured at SLAC.

34. Differential cross section for 14 GeV/c $K^+ p$ elastic scattering containing ~ 650,000 events. Below, the cross section is divided by an exponential, Ae^{Bt}, which was fit to the data for $t < 0.2\ \mathrm{GeV}^2$.

35. The differential cross section for 10.4 GeV/c $K^+ p$ elastic scattering. Below, the cross section is divided by an exponential, Ae^{Bt}, which was fit to the data for $t < 0.2\ \mathrm{GeV}^2$.

36. The logarithmic slope of the elastic differential cross section as a function of momentum transfer, t, for $\pi^\pm p$, $K^\pm p$, $p^\pm p$ at 10.4 GeV/c and $K^\pm p$ at 14 GeV/c.

37. The two components of the forward elastic cross section, in the Dual Absorptive Model—an exponential part usually associated with the Pomeron contribution, and a "J_0" Bessel function piece, which may be associated with the peripheral Regge contribution, or with an additional peripheral part of the Pomeron. This second contribution causes an upward curvature in the cross section at small t.

38. Differential cross section for elastic $\pi^- p$ scattering at 200 GeV/c, measured at NAL by the Michigan-Argonne-FNAL collaboration.

39. Differential cross section for elastic $K^+ p$ scattering at 100 GeV/c, measured at NAL by the Michigan-Argonne-FNAL collaboration.

40. The differential cross section for 100 GeV/c π^+p elastic scattering, measured at NAL by SASG.

41. The differential cross section for 100 GeV/c K^+p elastic scattering, measured at NAL by SASG.

42. The energy dependence of the slope of the differential cross section for K^+p elastic scattering, evaluated separately in the small t and intermediate t intervals.

43. The ratio of the differential cross section for π^+p and pp elastic scattering as a function of t, as measured by the SASG at NAL.

44. Schematic of inclusive leading particle pp scattering, and general features of the invariant cross section.

45. The missing mass distribution for 200 GeV/c pp inclusive scattering, broken down topology by topology.

46. The missing mass distribution in $pp \rightarrow pX$ at 200 GeV/c, as measured in the NAL HBC experiment. The solid line shows an M^{-2} fall-off in the mass spectrum.

47. The cross section multiplied by M^2, $\left(\text{i.e.}, M^2 \frac{d^2\sigma}{dM^2\,dt}\right)$ for $pd \rightarrow dX$, as a function of the missing mass squared.

48. The differential cross section, $\frac{d\sigma}{dt}$, for $pp \rightarrow pX$, as a function of the missing mass, as measured in the 200 GeV/c NAL HBC experiment.

49. The slope of the differential cross section for $pd \rightarrow Xd$, as a function of the missing mass squared, as measured by the US-USSR collaboration at FNAL.

50. The invariant cross section for $pp \rightarrow pX$ as a function of x for $100 < s < 750$ GeV^2, as measured by the Rutgers-Imperial College group at NAL. The data are presented in four t intervals.

51. The invariant cross section for $pp \to pX$ as a function of x, for a fixed $p_T = 0.8$ GeV. Data comes from the CHLM group at the ISR, for $\sqrt{s}$ = 22, 31, and 45 GeV.

52. The invariant cross section for $pd \to dX$ as a function of x (and 1 - x), for small t values. The data comes from the US-USSR collaboration at NAL, in the energy range (50 - 400) GeV/c.

53. The differential cross section, $\frac{d\sigma}{dt}$, for $K^+p \to K^+X$ at 14 GeV/c for the missing mass region: (a) $M_x < 1.2$ GeV, (b) $1.4 \leq M_x \leq 1.5$ GeV, (c) $2.0 \leq M_x \leq 2.2$ GeV. The solid lines in (a) and (b) represent the result of the fit with an exponential e^{-Bt} for $t < 0.25$ GeV^2. In (c), the exponential fit was done for the whole t range.

54. The dependence of the slope on the missing mass, M_x, for $K^+p \to K^+X$ and $K^-p \to K^-X$ at 10.4 and 14 GeV/c.

55. The mass spectra for the various production and decay isospin amplitudes for $\pi^{\pm}p \to (\pi N)\pi$ at 16 GeV/c.

56. The $(n\pi^+)$ mass spectrum from the reaction $pp \to pn\pi^+$ as studied by the Split-Field Magnet group at the ISR.

57. The energy dependence of the cross section for the reaction $\pi^-p \to \pi^-\pi^+\pi^-p$.

58. The energy dependence of the cross section for the reaction $pp \to p\pi^+\pi^-p$.

59. The energy dependence of the cross-section for the reaction $pp \to N(N\pi)_{I=1/2}$.

60. The energy dependence of the cross section for the reaction $np \to p\pi^-p$, as measured at NAL by the Rochester-Northwestern collaboration.

61. The differential cross section for the process $pn \to pp\pi^-$ for different regions of $p\pi^-$ mass, as measured at NAL by the Rochester-Northwestern collaboration.

62. The differential cross section for the process $pp \rightarrow pn\pi^+$ for different regions of $n\pi^+$ mass, as measured by the Split-Field Magnet group at ISR.

63. Schematic of the diffraction dissociation process $pp \rightarrow (n\pi^+)p$.

64. The mass spectrum $(n\pi^+)$ from the process $pp \rightarrow pn\pi^+$ as measured at the ISR: (a) the neutron comes backwards in the $n\pi^+$ center-of-mass system, while (b) the neutron goes forward in the $n\pi^+$ center-of-mass.

65. The azimuthal distribution of the π^+ from the breakup of the $n\pi^+$ system.

66. Schematic diagrams for elastic π^-, K^-, $\bar{p}$ scattering and for diffractive production of N*(1690).

67. The ratio of the elastic to N* production cross section for incident π^-, K^-, and $\bar{p}$ at 8, 16 GeV/c, as a function of momentum transfer.

68. A schematic of diffractive reactions studied in a test of factorization. The ratios R_1, - R_4 refer to the ratio of the cross section for the reaction when the top two vertices (pion and proton elastic vertices) are joined successively to the bottom four vertices, representing proton diffraction into a proton plus zero, one, two, or three pions, respectively, e.g.,

$$R_1 = \frac{\sigma(\pi p \rightarrow \pi p)}{\sigma(pp \rightarrow pp)}\ , \qquad R_2 = \frac{\sigma(\pi p \rightarrow \pi p \pi)}{\sigma(pp \rightarrow pp\pi)}\ , \quad \ldots$$

69. A schematic of diffractive reactions studied in a test of factorization. The ratios R_1, R_2, R_3 refer to the ratio of the cross sections when each of the upper vertices ($\gamma \rightarrow \rho$, $p \rightarrow p$, $\pi \rightarrow \pi$) is connected with the two lower vertices representing proton scattering into a proton or a $(p\pi\pi)$system, respectively.

70. Schematic diagrams for the diffractive production of $(p\pi^+\pi^-)$ systems in the reactions $\pi p \rightarrow \pi(p\pi\pi)$ and $pp \rightarrow p(p\pi\pi)$.

71. Schematic diagrams for elastic scattering, single diffraction dissociation, and double diffraction dissociation.

72. The differential cross section for $pp \rightarrow (p\pi\pi) \cdot (p\pi\pi)$ at $\sqrt{s} = 22$ GeV from the ISR.

73. Schematic diagram for the s-channel unitarity relation.

74. Relationship between the total, elastic and inelastic overlap functions.

75. Schematic of several peripheral and central amplitudes and how they transform from impact parameter space to momentum transfer space.

76. Schematic of the effect on the scattering amplitude in momentum transfer space, of adding extra partial waves.

77. The impact structure of proton-proton scattering at $\sqrt{s} = 53$ GeV. Shown are $\mathrm{Im}\, h_{el}(s,b)$ and the inelastic and elastic overlap functions extracted from the experimental data. The "black disc limit" indicates the maximum value of the inelastic overlap function allowed by unitarity (i.e., 100% absorption).

78. (a) Inelastic overlap functions calculated from the $\sqrt{s} = 21$, 31, 44, and 53 GeV ISR data. (b) The difference of the $\sqrt{s} = 53$ and 31 GeV inelastic overlap functions, $\Delta G_{incl}(s,b)$, showing that the cross-section increase comes from a rather narrow region around 1 fermi.

79. Inelastic overlap function separated into a diffractive and non-diffractive piece.

80. Schematic illustration of the origin of the mass-slope correlation in the peripheral model of inelastic diffraction. (a) Small M^2; the non-flip amplitude dominates, faking a steep exponential at small t and inducing structure around $t \sim 0.3$ GeV^2. (b) Large M^2; several helicity amplitudes contribute appreciably. The differential cross section is much flatter, and the structure around $t \sim .3$ GeV^2 gets filled in.

81. The differential cross section for $pp \rightarrow pn\pi^+$ for different intervals of $n\pi^+$ mass.

82. The differential cross section, $\frac{d^2\sigma}{dMdt}$, for the reaction $K^+p \rightarrow K^+X$ at 14 GeV/c, for intervals of the missing mass, M_x.

83. The measured difference in particle and anti-particle cross sections,

$$A(s,t) = \frac{\frac{d\sigma}{dt}(X^-p) - \frac{d\sigma}{dt}(X^+p)}{\sqrt{8\left[\frac{d\sigma}{dt}(X^-p) + \frac{d\sigma}{dt}(X^+p)\right]}} ,$$

for the scattering of $K^{\pm}p$ at 6.4, 10.4, and 14 GeV/c and $\pi^{\pm}p$ and $p^{\pm}p$ at 10.4 GeV/c. The lines represent the fit to the data using $\Delta(s,t) = Ae^{Bt}J_0(R\sqrt{-t})$.

84. The momentum transfer dependence of the slope of the forward scattering cross section, as predicted from a simple model incorporating a two-component diffractive amplitude and a peripheral exchange amplitude. The t range of the $\pi^{\pm}p$ curve extends only to $t \sim 0.75$ GeV^2 as limited by the data, while that for $p^{\pm}p$ was limited to the t interval in which the exchange amplitude is well understood.