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A SUPERSYMMETRIC GAUGE MODEL OF THE ELECTRON AND ITS NEUTRINO*

G. B. Mainland
Department of Physics, The Ohio State University
Columbus, Ohio 43210

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305**

and

K. Tanaka
Department of Physics, The Ohio State University
Columbus, Ohio 43210

and

Department of Theoretical Physics, The University of Helsinki
Helsinki 17, Finland**

ABSTRACT

A supersymmetric $SU(2) \times U(1)$ gauge model for the electromagnetic, weak, and neutral weak interactions of the electron are constructed. After masses are created via spontaneous symmetry breaking, the electron and its neutrino remain massless to lowest order while a heavy electron and an accompanying neutral lepton acquire masses comparable respectively to those of the intermediate vector bosons $W^{\mu}_{\pm}$ and Z^{μ}. Lepton number is conserved in a natural way without requiring bosons to carry lepton number.

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Recently there has been interest in constructing supersymmetric unified gauge theories for leptons.[1] The motivation for wedding supersymmetry to gauge theories is the following: When supersymmetry is broken spontaneously, a (massless) Goldstone fermion appears.[2] Thus by incorporating supersymmetry, a possible explanation is provided for the existence of a massless fermion in nature, the neutrino. By requiring the model to be supersymmetric, the number of arbitrary parameters is severely limited, thereby increasing its predictive value. In the present model there are three parameters: the $SU(2)$ and $U(1)$ coupling constants and the coefficient ξ of the symmetry breaking term.

The most significant new features of the model are (1) the natural manner in which lepton number is conserved without requiring bosons to carry lepton number[3] and (2) the smaller number of unobserved fields as compared to previous models.[1] The restrictions imposed by gauge invariance and supersymmetry require the unbroken Lagrangian to be massless with the result that the model automatically possesses a conserved lepton number. After spontaneously breaking the Lagrangian using the technique of Fayet and Iliopoulos,[4] the electron and its neutrino remain massless to lowest order while a heavy electron E_- and a heavy Dirac neutrino E_0 acquire masses $m(E_-) = \sqrt{2}\, m(W_\pm)$, $m(E_0) = m(Z)$ where $W_\pm^\mu$ and Z^μ are the charged and neutral vector bosons respectively.

We expect that the masslessness of the electron in lowest order will be rectified in higher order. The electron interacts with the photon A^{μ} and the intermediate weak vector boson $W^{\mu}_{\pm}$ in the usual manner.

Our model has a $SU(2) \times U(1)$ group structure and is constructed from the following four superfields:

$V(W,\lambda,D, \ldots)$: $SU(2)$ triplet vector superfield

$V'(W',\lambda',D', \ldots)$: $U(1)$ vector superfield

$S(A,B,\psi,F,G)$: $SU(2)$ doublet left-handed scalar superfield

$S'(A',B',\psi',F',G')$: $U(1)$ right-handed scalar superfield.

Under a generalized gauge transformation,[5] the four superfields transform respectively as

$$e^{2gV} \to e^{-2ig\Lambda^{\dagger}} e^{2gV} e^{2ig\Lambda} \tag{1a}$$

$$e^{2fV'} \to e^{-2if\Lambda'^{\dagger}} e^{2fV'} e^{2if\Lambda'} \tag{1b}$$

$$S \to e^{-2ig\Lambda} e^{-2if\Lambda'} S \tag{1c}$$

$$S' \to e^{-4if\Lambda'^{\dagger}} S' \; . \tag{1d}$$

In (1) Λ is a $SU(2)$ triplet left-handed scalar superfield, Λ' is a left-handed $U(1)$ scalar superfield, and f and g are coupling constants.

The Lagrangian, when written in terms of superfields, is given by the expression

$$\begin{aligned} \mathscr{L} = & -\frac{1}{2}(V,V)_{F\text{ term}} - \frac{1}{2}(V',V')_{F\text{ term}} + [S^* e^{2gV} e^{2fV'} S]_{D\text{ term}} \\ & + [S'^* e^{-4fV'} S']_{D\text{ term}} \; . \end{aligned} \tag{2}$$

The terms $[S'^*e^{-4fV'}S']_D$ and $[S^*e^{2gV}e^{2fV'}S]_D$ are the simple U(1) model previously discussed by the authors[6] and its $SU(2)\times U(1)$ generalization and does not conserve parity. Equation (2) is manifestly invariant under the transformation (1). Because we have taken the "F term" for scalar multiplets and the "D term" for vector multiplets, the Lagrangian transforms as a four divergence under a supersymmetry transformation thus yielding an invariant action. By going to the special gauge[5] in which only the fields W, λ, D and W', λ', D' survive respectively in the vector superfields V and V', the Lagrangian (2) in the notation of Ref. 7 takes the form

$$
\begin{aligned}
\mathscr{L} = {} & \frac{1}{2}\,\mathrm{Tr}\Big(-\frac{1}{4}W_{\mu\nu}^2 - \frac{i}{2}\bar{\lambda}\not{\mathscr{D}}\lambda + \frac{1}{2}D^2\Big) - \frac{1}{4}W'^2_{\mu\nu} - \frac{i}{2}\bar{\lambda}'\not{\mathscr{D}}\lambda' + \frac{1}{2}D'^2 \\
& - \frac{i}{2}\bar{\Psi}_L\not{\mathscr{D}}\Psi_L - \frac{i}{2}\bar{\Psi}_R\not{\mathscr{D}}\Psi_R - \frac{1}{2}|\mathscr{D}_\mu\varphi_+|^2 - \frac{1}{2}|\mathscr{D}_\mu\varphi_-|^2 + \frac{1}{2}F^\dagger F + \frac{1}{2}G^\dagger G \\
& + \frac{f}{2}(\varphi_-^\dagger\varphi_- + \varphi_+^\dagger\varphi_+)D' + \frac{g}{2}(\varphi_-^\dagger D\varphi_- + \varphi_+^\dagger D^T\varphi_+) + \sqrt{2}f\bar{\lambda}'(\varphi_+^\dagger R - \varphi_-^\dagger L)\Psi \\
& + \sqrt{2}g(\varphi_+^\dagger\bar{\lambda}^T R - \varphi_-^\dagger\bar{\lambda}L)\Psi - \frac{i}{2}\bar{\Psi}'_L\not{\mathscr{D}}\Psi'_L - \frac{i}{2}\bar{\Psi}'_R\not{\mathscr{D}}\Psi'_R - \frac{1}{2}|\mathscr{D}_\mu\varphi'_+|^2 - \frac{1}{2}|\mathscr{D}_\mu\varphi'_-|^2 \\
& + \frac{1}{2}F'^2 + \frac{1}{2}G'^2 - f(\varphi'^\dagger_-\varphi'_- + \varphi'^\dagger_+\varphi'_+)D' + 2\sqrt{2}f\bar{\lambda}'(\varphi'_-L - \varphi'_+R)\Psi' ,
\end{aligned}
\tag{3}
$$

where

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \qquad \mathscr{D}_\mu\lambda = \partial_\mu\lambda + ig[W_\mu,\lambda] , \tag{4a}$$

$$\mathscr{D}_\mu\Psi_L = (\partial_\mu + ifW'_\mu + igW_\mu)\Psi_L, \qquad \mathscr{D}_\mu\Psi_R = (\partial_\mu - ifW'_\mu - igW^T_\mu)\Psi_R , \tag{4b}$$

$$\mathscr{D}_\mu\varphi_- = (\partial_\mu + ifW'_\mu + igW_\mu)\varphi_-, \qquad \mathscr{D}_\mu\varphi_+ = (\partial_\mu - ifW'_\mu - igW^T_\mu)\varphi_+ , \tag{4c}$$

$$\mathscr{D}_\mu\Psi'_L = (\partial_\mu - 2ifW'_\mu)\Psi'_L, \qquad \mathscr{D}_\mu\Psi'_R = (\partial_\mu + 2ifW'_\mu)\Psi'_R , \tag{4d}$$

$$\mathscr{D}_\mu\varphi'_+ = (\partial_\mu - 2ifW'_\mu)\varphi'_+, \qquad \mathscr{D}_\mu\varphi'_- = (\partial_\mu + 2ifW'_\mu)\varphi'_- . \tag{4e}$$

The helicity projection operators L and R are given respectively by $\frac{1}{2}(1 - i\gamma_5)$ and $\frac{1}{2}(1 + i\gamma_5)$. An SU(2) triplet is written as

$$W \equiv \begin{pmatrix} W_3 & W_1 - iW_2 \\ W_1 + iW_2 & -W_3 \end{pmatrix} \equiv \begin{pmatrix} W_3 & \sqrt{2}W_- \\ \sqrt{2}W_+ & -W_3 \end{pmatrix} \equiv W_i \tau_i \ , \tag{5}$$

and the scalar fields $\varphi'_{\pm}$ and $\varphi_{\pm}$ are equal to the expressions

$$\varphi'_{\pm} = (A' \mp iB')/\sqrt{2} \ , \tag{6a}$$

$$\varphi_{\pm} = (A \pm iB)/\sqrt{2} \ . \tag{6b}$$

We have chosen the above definitions for $\varphi'_{\pm}$ and $\varphi_{\pm}$ so that after the electromagnetic field has been identified, $\varphi'_{\pm}$ and the upper component of $\varphi_{\pm}$ will have a charge indicated by the subscript while the lower component of $\varphi_{\pm}$ will be neutral.

Because the restrictions of gauge invariance and supersymmetry prohibit mass terms in the unbroken Lagrangian, the Lagrangian is invariant under the transformation

$$\begin{aligned} \lambda &\to e^{\gamma_5 \alpha}\lambda \ , & \lambda' &\to e^{\gamma_5 \alpha}\lambda' \ , \\ \psi &\to e^{-\gamma_5 \alpha}\psi \ , & \psi' &\to e^{-\gamma_5 \alpha}\psi' \ . \end{aligned} \tag{7}$$

After the Lagrangian is broken spontaneously and the physical fields are identified, the invariance of the Lagrangian under (7) will be associated with conservation of lepton number. As can be seen from (7), no bosons participate.

An important feature of the Lagrangian is that the left- and right-hand components of ψ' and ψ are associated with opposite charges as can be seen in Eqs. (4). As a result, neither ψ' nor ψ represents a physical field because neither is an eigenstate of charge. Instead, for example, ψ'_R must be the right-hand component of one physical field and ψ'_L must be the left-hand component of the charge conjugate physical field (see for example Eq. (14)). Our method for constructing the charged physical fields represents a departure from the technique used here-to-fore. In the past, charged fields were constructed using the "complexification method."[5] The complexified fields are eigenstates of charge and can be taken to be the physical fields. The helicity components of the complexified fields can also be used as building blocks for constructing physical fields by combining the left-hand component of one complexified field with the right-hand component of a second identically charged complexified field, a technique used extensively in Ref. 1.

To spontaneously break the supersymmetry of the Lagrangian, we use the technique of Fayet and Iliopoulos[4] and add the gauge invariant term

$$- \xi f D' \, , \qquad \xi = \text{constant} \, . \tag{8}$$

Under a supersymmetry transformation D' transforms as a four divergence. After eliminating the non-dynamical fields D, D', F, G, F', and G', the Lagrangian (3) takes the form

$$
\begin{aligned}
\mathscr{L} = {} & \frac{1}{2}\,\mathrm{Tr}\left(-\frac{1}{4}W_{\mu\nu}^{2} - \frac{i}{2}\,\bar{\lambda}\not{\mathscr{D}}\lambda\right) - \frac{1}{4}W_{\mu\nu}'^{2} - \frac{i}{2}\,\bar{\lambda}'\not{\partial}\lambda' \\
& - \frac{i}{2}\,\bar{\psi}_L\not{\mathscr{D}}\psi_L - \frac{i}{2}\,\bar{\psi}_R\not{\mathscr{D}}\psi_R - \frac{1}{2}|\mathscr{D}_\mu\varphi_+|^2 - \frac{1}{2}|\mathscr{D}_\mu\varphi_-|^2 \\
& - \frac{i}{2}\,\bar{\psi}_L'\not{\mathscr{D}}\psi_L' - \frac{i}{2}\,\bar{\psi}_R'\not{\mathscr{D}}\psi_R' - \frac{1}{2}|\mathscr{D}_\mu\varphi_+'|^2 - \frac{1}{2}|\mathscr{D}_\mu\varphi_-'|^2 \qquad (9) \\
& + \sqrt{2}f\bar{\lambda}'(\varphi_+^\dagger R - \varphi_-^\dagger L)\psi + \sqrt{2}g(\varphi_+^\dagger\bar{\lambda}^T R - \varphi_-^\dagger\bar{\lambda}L)\psi + 2\sqrt{2}f\bar{\lambda}'(\varphi_-' L - \varphi_+' R)\psi' \\
& - \frac{f^2}{8}(2\varphi_-'^\dagger\varphi_-' + 2\varphi_+'^\dagger\varphi_+' - \varphi_-^\dagger\varphi_- - \varphi_+^\dagger\varphi_+ + 2\xi)^2 - \frac{g^2}{8}(\varphi_-^\dagger\varphi_- + \varphi_+^\dagger\varphi_+)^2 \,.
\end{aligned}
$$

To spontaneously break the gauge symmetry while maintaining electromagnetic charge conservation, we allow only the (neutral) lower components of the doublet

$$\varphi_{\pm} \equiv \begin{pmatrix} U_{\pm} \\ D_{\pm} \end{pmatrix}, \qquad (10)$$

to have a non-zero vacuum expectation value. The condition that the potential V

$$V = \frac{f^2}{8}(2\varphi_-'^\dagger\varphi_-' + 2\varphi_+'^\dagger\varphi_+' - \varphi_-^\dagger\varphi_- - \varphi_+^\dagger\varphi_+ + 2\xi)^2 + \frac{g^2}{8}(\varphi_-^\dagger\varphi_- + \varphi_+^\dagger\varphi_+)^2 \,, \qquad (11)$$

be at a minimum is $\langle D_-^\dagger D_- \rangle = \xi f^2/(f^2 + g^2)$, so we break the gauge symmetry by making the substitution

$$D_{\pm} \rightarrow D_{\pm} \pm ia \,, \qquad a = a^* = [\xi f^2/(f^2 + g^2)]^{1/2} \,. \qquad (12)$$

The minimum of the potential is lowered from $\frac{1}{2}(\xi f)^2$ to $\frac{1}{2}(\xi f)^2 g^2/(f^2 + g^2)$, thereby implying global stability.

We are now interested in diagonalizing the fermion mass matrix in order to identify the physical fermions. Defining the photon field A_μ and the massive neutral vector meson field Z_μ by the relations

$$A_\mu = W_{3\mu} \sin\theta + W'_\mu \cos\theta \,, \tag{13a}$$

$$Z_\mu = W_{3\mu} \cos\theta + W'_\mu \sin\theta \,, \tag{13b}$$

allows us to identify λ_{-L}, λ_{-R}, ψ'_R and ψ_{uL} as components of fields with the same charge. The requirement that the electron be the lighter of the two charged leptons and that its neutrino be left-handed leads to the identification

$$\begin{aligned} e_- &= \lambda_{-L} + \psi'_R \,, \qquad & \nu_e &= \lambda_{3L} \sin\theta + \lambda'_L \cos\theta \,, \\ E_- &= \lambda_{-R} + \psi_{uL} \,, \qquad & E_0 &= \psi_{dL} - \lambda_{3R} \cos\theta + \lambda'_R \sin\theta \,, \end{aligned} \tag{14}$$

where

$$\tan\theta = f/g \,, \qquad \lambda_{-L} \equiv L\lambda_- \quad \text{etc.,} \tag{15}$$

and

$$m(e_-) = m(\nu) = 0, \quad m(E_0) = [2\xi f^2]^{1/2}, \quad m(E_-) = [4\xi f^2 g^2/(f^2+g^2)]^{1/2} \,. \tag{16}$$

In the tree approximation, the electron remains massless.

After defining the photon A_μ and the massive neutral vector meson Z_μ by the relations (13), the vector meson masses are found to be

$$m(A) = 0, \quad m(Z) = [2\xi f^2]^{1/2}, \quad m(W_-) = [2\xi f^2 g^2/(f^2 + g^2)]^{1/2} \,. \tag{17}$$

The scalar bosons $U_\pm$ and the lower component of A, A_d, are eliminated by the Higgs mechanism while the remaining scalars $\varphi'_\pm$ and the lower component of B, B_d, acquire the masses

$$m(\varphi'_-) = [2\xi f^2 g^2/(f^2 + g^2)]^{1/2}, \quad m(B_d) = [2\xi f^2]^{1/2} . \tag{18}$$

Under the transformation (7), the physical fields transform as

$$\begin{aligned} e_- &\to e^{i\alpha} e_- , \qquad & \nu_e &\to e^{i\alpha} \nu_e , \\ E_- &\to e^{-i\alpha} E_- , \qquad & E_0 &\to e^{-i\alpha} E_0 . \end{aligned} \tag{19}$$

By assigning a lepton number +1 to e_- and ν_e and a lepton number -1 to E_- and E_0, lepton number is conserved without assigning lepton number to bosons.

The fermions interact with the photon according to the term

$$g \sin \theta (\bar{e}_- \gamma^\mu e_- + \bar{E}_- \gamma^\mu E_-) A_\mu . \tag{20}$$

Omitting unobserved heavy leptons, the weak and neutral vector-fermion interactions are respectively

$$- 2g \sin \theta (\bar{e}_{-L} \gamma_\mu \nu_e) W^\mu_- , \tag{21}$$

and

$$g \sin \theta \, \bar{e}_- \gamma_\mu (\cot \theta \, L - \tan \theta \, R) e_- Z^\mu . \tag{22}$$

We note that the electron neutrino does not couple to the massive neutral vector meson.[1]

Again omitting unobserved heavy leptons, the scalar-fermion interaction is

$$2\sqrt{2} g \sin \theta (\bar{e}_{-R} \nu_e \phi'_-) . \tag{23}$$

Because the coupling constants in (21) and (23) are comparable and the masses of W_-^μ and φ_- are identical, scalar interaction terms of the type (23) could cause problems in a model which included the muon sector.[1]

Three problems are immediately encountered when attempting to extend the model to include the muon. First, the electron neutrino is massless because it is a Goldstone fermion. If the muon neutrino is also to be massless, some other mechanism must be responsible. Second, it is difficult to have independently conserved electron number and muon number since the fermions from various scalar multiplets all couple to fermions from the vector multiplets. Finally, if scalar-fermion interaction terms (of the type in (23)) occur, they will be incompatible with experiment if they contribute significantly to muon decay.

This $SU(2) \times U(1)$ model represents another step toward obtaining a realistic supersymmetric extension of a spontaneously broken $SU(2) \times U(1)$ Yang Mills theory. It has the desirable features of electromagnetic, weak, and neutral weak interactions which conserve lepton number and involve a relatively small number of unobserved fields. The major defect of the model is its inability to accommodate the muon. It remains to be seen how this problem can be solved within the framework of supersymmetry.

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REFERENCES

1. P. Fayet, Nucl. Phys. B90, 104 (1975).
2. J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974).
   A. Salam and J. Strathdee, Phys. Letters 49B, 465 (1974).
3. A. Salam and J. Strathdee, Nucl. Phys. B87, 85 (1975).
3. G. B. Mainland and K. Tanaka, "Fermion Number Conservation in Supersymmetric Lagrangians," Proceedings of the Fifth International Conference on Neutrino Science, Balaton, Hungary (1975).
4. P. Fayet and J. Iliopoulos, Phys. Letters 51B, 461 (1974).
5. J. Wess and B. Zumino, Nucl. Phys. B78, 413 (1974).
6. G. B. Mainland and K. Tanaka, Ohio State University report COO-1545-152, 1975 (to be published in Phys. Rev. D).
7. J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974).