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NUCLEON NUMBER DEPENDENCE OF LARGE TRANSVERSE MOMENTUM REACTIONS AND MULTIPLE SCATTERING*

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Abstract

We examine the effect of multiple scattering on large transverse momentum reactions in a simple hard scattering model. It is shown that the measured A dependence can be explained under assumptions on the hard scattering component, which are consistent with experiment. We comment on the effect of Fermi motion.

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I. GENERAL EXPERIMENTAL SITUATION

Hadronic reactions on nuclear targets might offer the unique chance for direct studies of the space-time development of hadronic processes. Although one could expect increasing complications with the number of nucleons involved in those reactions, some experiments show that these processes have sometimes surprisingly simple properties.

The total cross section for proton nucleus reactions is proportional A^{α} with $\alpha \approx .7$,[1] as expected from the Glauber theory. From recent NAL experiments, we learn that the increase of multiplicity with the nucleon number A comes mainly from the target fragmentation region.[2] If we generalize this result to the proton nucleus case and take a slight increase of multiplicity in the central region into account, we expect that also the inclusive cross section for pion production behaves proportional A^{α} with $\alpha \approx .85$, and, in fact, the low p_T data of the Chicago-Princeton collaboration[3] show clearly this behavior. For large transverse momenta, the situation changes drastically. Let us briefly recall some essential aspects of the CP experiment: It measures the secondaries at an angle of .077 radian, which corresponds to 90° in the nucleon-nucleon center-of-mass frame at the relevant laboratory energy of 300 GeV. The inclusive cross section

$$E \frac{d\sigma}{d^3p} (p + A \rightarrow \pi^- + X) \propto A^{n(p_T)} \tag{1}$$

where n changes from .85 at $p_T = .7$ GeV to 1.1 for p_T between 4 and 6 GeV. (Fig. 1.) The energy dependence of n is unknown. Corresponding results hold for kaons with $n \approx 1.15$ and protons with $n \approx 1.3$ for the high p_T values.

The assumption of a power law is in good agreement with the data, although also $A + c(p_T)\, A^{4/3}$ fits reasonably well. However, any functional form

$\alpha(A + c_2(p_T) A^2 + c_3(p_T) A^3 + \ldots)$ with c_i positive—as proposed by some authors[4,5]—is in complete disagreement with the data.

We stress that the particular choice of the angle is only related to 90^o in the center-of-mass frame if most of the events originate in collisions between individual nucleons, and if neither the projectile gets slowed down in the nucleus before the collision nor the secondaries are strongly disturbed in the nucleus after the collision.

In the following, we shall show that by double scattering—under assumptions on the hard scattering cross section which are consistent with experiment—a simple parton model can in fact account for the observed A dependence. We shall comment on the effect of Fermi motion on the observed cross section.

II. MULTIPLE SCATTERING MODEL

(a) General Description

It is now well known that low and high p_T cross sections differ in many respects. Low p_T events account for most of the cross section and falloff $\propto e^{-2 p_T / \langle p_T \rangle}$. They can be described, for example, by independent cluster production or bremsstrahlung-type models.[6,7] Large p_T events show weaker decrease with p_T and clear s dependence, as predicted by the scaling laws of parton models.[8,9] It is, therefore, tempting to relate the different A-dependence to specific properties of the different models for large and small p_T.

We want to describe proton nucleus scattering as follows: Most of the secondaries are produced by conventional scattering mechanisms—for example, by independent cluster production, hadronic bremsstrahlung,[6,7] or the energy flux cascade.[11] This component is $\propto A^{\alpha}$ with $\alpha \approx .85$.

"Hard scattering events" are responsible for the excess over the exponential p_T distribution. We interpret these as scattering of the high momentum fragments of the original hadrons, for example, by the mechanism of the constituent interchange model (CIM),[9] but our treatment relies only on the short-distance nature of the interaction, which leads to large transverse momentum fragments. These processes occur independently of the first ones. Neither the large x components of the incoming proton nor the large p_T secondaries undergo absorption, which may be made plausible by the short-distance nature of the interaction (one might even say by the smallness of the partons). This assumption is common to all parton models, which explain the rise of n from .8 to 1.[4,5,14]

Scattered fragments may then undergo subsequent hard scatterings. (Fig. 2.) These will, in general, be different from the first one. As we shall see, double scattering gives a contribution $A^{4/3}$. We should point out that for both large and small p_T the objects which emerge immediately after a collision can not be identified with ordinary hadrons. Since they had not had time to restore their self-field, they might resemble bare objects. For ordinary hadrons, one would expect a cascading process which would lead to far higher multiplicities and only low-momentum secondaries. Furthermore, it is hard to see how a high p_T particle (which, in the experiment, has very high $p_{\parallel}$, $p_{\parallel} = 1/.077\ p_T$) could leave the nucleus without losing all its energy by cascading.

Let us summarize our input:

1. Soft and hard components of the cross section can be added incoherently.
2. The soft part is proportional to $\exp(-2\ p_T/\langle p_T\rangle)\ A^{\alpha}$, with $\alpha = .85$, and is dominant for small p_T. ($2/\langle p_T\rangle = 6\ \mathrm{GeV}^{-1}$ for pions.)
3. The hard part is responsible for the deviation from the exponential behavior. There is no shadowing of the large x components of the wave function of the incident proton and of the fragments.

4. The form of the cross section for the second scattering is as suggested by parton models and consistent with experiment (cf Eq. (13)).

(b) Detailed Calculation

To be more specific, we shall concentrate on the following in the case of large p_T pions. The ratio

$$r(A, p_T) = \frac{E \frac{d\sigma}{d^3p} (p + A \to \pi^- + X)}{E \frac{d\sigma}{d^3p} (p + Be \to \pi^- + X)} \frac{A_{Be}}{A} , \tag{2}$$

as given by the experiment, is shown in Fig. 1.

If we ignore for the moment the double scattering contribution, then

$$E \frac{d\sigma}{d^3p} (A) = c\, e^{-6p_T} A^{\alpha} + E \frac{d\sigma_H}{d^3p} \cdot A \tag{3}$$

where c is fixed by the low p_T data to 132 mb/GeV^2, in good agreement with ISR results,[10] and $E\frac{d\sigma_H}{d^3p}$ describes the "hard part" of $E(d\sigma/d^3p)$, which can be deduced from the CP data (reduced to A = 1 by Eq. (1)), also in good agreement with ISR results.[10] (Fig. 3.) We, therefore, can express r(A) parameter-free by the ratio of soft contribution to measured cross section for Be.

$$r_1(A, p_T) = 1 - \frac{c e^{-6p_T} A_{Be}^{\alpha}}{E \frac{d\sigma}{d^3p}(Be)} \left(1 - \left(\frac{A_{Be}}{A}\right)^{1-\alpha}\right) . \tag{4}$$

The data for $p_T = .76$ fix α to .85 (cf Fig. 2 of Ref. 3).

Equation (4) gives already a good description for the lower p_T values. The value of α can be easily understood if we take $\sigma_{tot} \sim A^{.7}$ (cf Ref. 1) and take a slight increase of multiplicity $\sim A^{.15}$ at $\theta_{cm} = 90^{\circ}$ into account, which is, in fact, observed in various experiments (Ref. 2 and Fig. 3 of Ref 11).

If we now include the multiple scattering contribution, we have to be more specific about the second scattering. Although we formulate things again in the quark-parton picture, our treatment is more general. Assume the object, which emerges after the first scattering, is a $q\bar{q}$ state with high $p_{\parallel}$ and p_T, which scatters again inside the nucleus (Fig. 2). We do not have to consider the effect of the remaining particles X, as long as they are not able to produce again a high p_T particle.

We identify the first cross section with $E\,\frac{d\sigma_H}{d^3p}$. If we assume that the second and all successive cross sections have the same form

$$E\,\frac{d\sigma'}{d^3p} = \sigma'\rho'; \quad \left(\int \frac{d^3p}{E}\,\rho' = 1\right),$$

the multiple scattering cross section is given by*

$$E\,\frac{d\sigma}{d^3p} = \int_0^R 2\pi r\,dr \sum_{n\geq 1} P_n(r)\,\rho_n + c\,e^{-6p_T}\,A^{\alpha} \tag{5}$$

R = nuclear radius = $R_0A^{1/3}$

P_n = probability for n times scattering, is the generalization of the Poisson distribution to our case

ρ_n = momentum distribution of the $q\bar{q}$ state after n scatterings

$$\underset{n\geq 1}{P_n} = \frac{\lambda\,\lambda'^{\,n-1}}{(\lambda'-\lambda)^n}\,e^{-\lambda'}\left[e^{\lambda'-\lambda} - \sum_{i=0}^{n-1} (\lambda'-\lambda)^i/i!\right] \tag{6}$$

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*For our actual calculation, we only need the first two terms in the sum which give contributions $\sim$ A and $A^{4/3}$.

with

$$\lambda = \sigma n d(r), \qquad \lambda' = \sigma' \, n \, d\,(r)$$

$$n = \text{nuclear density} = 1/\frac{4\pi}{3} R^3 \; ; \qquad d = 2 \sqrt{R^2 - r^2}$$

In two limiting cases, P_n exhibits a particular simple behavior. As expected, for $\lambda' \to \lambda$, P_n reproduces the familiar Poisson distribution

$$P_n \underset{\lambda' \to \lambda}{\longrightarrow} e^{-\lambda} \lambda^n / n! \tag{7}$$

In the case $\lambda \to \infty$, P_n reproduces the Poisson distribution for n - 1 events. This shows that it is important to treat hard and soft scattering independently to get λ small. Else the single scattering contribution would be $\sim A^{2/3}$ and the double scattering contribution $\sim A$.

The invariant momentum distribution after n scattering processes ρ_n is given by

$$\rho_n(p, s) = \int \frac{d^3 p_1}{E_1} \cdots\cdots \frac{d^3 p_n}{E_n} \circ \rho_H(p_1, s_1) \cdot \rho'(p_2, s_2) \cdots\cdots \rho'(p_n, s_n)$$

$$\cdot\, \delta^{(2)}\left(\left(\vec{p}_{1T} / |p_1| + \cdots\cdots + \vec{p}_{nT} / |\vec{p}_n| \right) |\vec{p}_n| - \vec{p}_T \right) \cdot E_n \delta(p_\parallel - p_{n\parallel}) \tag{8}$$

$$s_i = 2 E_{i-1} m \, ,$$

where we assumed the scattering angles in the lab frame to be small.

By the first (two-dimensional) delta function, the successive scattering angles $\theta_i = \vec{p}_{iT} / |\vec{p}_i|$ add up to the final angle $\vec{p}_T / |\vec{p}|$. The second delta function requires that after the n-th scattering, the parallel momentum has to be $p_\parallel$. This should be contrasted to the case where the second (and all successive) scatterings are elastic:

$$\rho_n(p,s) = \int d^2p_{1T} \cdots d^2p_{nT}\, \rho_H(p_1,s)\, \frac{1}{\pi}\rho'_{e\ell}\left(t=\vec{p}^{\,2}_{2T}\right) \cdots \frac{1}{\pi}\, \rho'_{e\ell}\left(t=\vec{p}^{\,2}_{nT}\right)$$

$$\delta^{(2)}\left(\sum \vec{p}_{iT} - \vec{p}_T\right) \tag{8a}$$

Assuming λ and $\lambda' \ll 1$, we get in second order*

$$E\frac{d\sigma}{d^3p} = c\, e^{-6p_T} A^{\alpha} + E\frac{d\sigma_H}{d^3p} A + \frac{9}{16}\frac{\sigma_H \sigma'}{\pi R_0^2}\, \rho_2\, A^{4/3}$$

$$- \frac{9}{16}\, \frac{\sigma_H + \sigma'}{\pi R_0^2}\; E\,\frac{d\sigma_H}{d^3p}\; A^{4/3} \tag{9}$$

The last term corresponds to absorption, is small, and will be ignored in the following.** The third term is due to double scattering and will now be examined. We define

$$c_2(p_T) \equiv \frac{9}{16}\,\frac{\sigma_H \sigma'}{\pi R_0^2}\; \rho_2 / E\,\frac{d\sigma_H}{d^3p} =$$

$$= \frac{9}{16}\,\frac{1}{\pi R_0^2}\,\frac{1}{E\frac{d\sigma_H}{d^3p}} \int \frac{d^3p_1}{E_1}\, E_1\frac{d\sigma_H}{d^3p}(p_1,s)\; E\frac{d\sigma'}{d^3p}\left(\vec{p}_{2T} = \vec{p}_T - \left|\frac{\vec{p}_1}{\vec{p}}\right| \vec{p}_{1T},\, p_{2\parallel} = p_{\parallel},\, s_2\right) \tag{11}$$

c_2 should be around .1 - .2 to fit the data.

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*Actually we should write $\approx (A-1)^{4/3}$ instead of $A^{4/3}$ in order to obtain a reasonable behavior (no multiple scattering) for $A \to 1$. For the following, this makes no difference, however.

**This was the main reason why we decomposed the cross section into a soft and a hard component (which has small total cross section), and treat both processes independently.

Explicit calculations show that this can be fulfilled if the p_T distribution of ρ_H and ρ' is roughly a power law. For r, we obtain

$$r(A, p_T) = \frac{c_0(p_T)\, A^{\alpha - 1} + 1 + c_2(p_T)\, A^{1/3}}{c_0(p_T)\, A_{Be}^{\alpha - 1} + 1 + c_2(p_T)\, A_{Be}^{1/3}} \tag{11}$$

where $c_0(p_T) = ce^{-6p_T} / E \frac{d\sigma_H}{d^3p}$.

The results of a simple model calculation are shown in Fig. 1. For the form of $E \frac{d\sigma_H}{d^3p}$ we have chosen*

$$E \frac{d\sigma_H}{d^3p} = .65 \cdot 10^6 (1 - x_T)^5 x_T^2 / \left(p_T^2 + 2.8\right)^8 + 5(1 - x_T)^{11} / \left(p_T^2 + 1.3\right)^4 \left[\frac{\text{mb}}{\text{GeV}^2}\right] \tag{12}$$

in agreement with the cross section which we get by extrapolating the nuclear cross sections by Eq. (1) to A = 1 and which is also in agreement with ISR data[10] of the corresponding energy (Fig. 3), imposing a suitable cutoff in $x_{\parallel}$, as suggested by the measured angular dependence of the cross section at ISR.[12]

If we take as intermediate fragment a $q\bar{q}$ state, the CIM would suggest

$$\frac{d\sigma'}{d^3p} \sim p^{2^{-4}} \epsilon^3$$

(if we restrict ourselves to the most elementary subprocess). In the relevant energy range ($\sqrt{s} \sim 15$ - 20 GeV), the measured cross section[12] for

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*We do not mean to give by Eq. (12) a fit over a wide energy region. For the integration in Eq. (10), we need $E \frac{d\sigma_H}{d^3p}$ only for $E_{lab} = 300$ GeV.

$\pi^- + p \to \pi^0 + X$ in the large p_T region seems to be better described by*

$$E \frac{d\sigma'}{d^3p} = 2.5 \cdot 10^4 \cdot (p^2 + 6)^{-6.5} \epsilon^3 \quad [\mathrm{mb/GeV}^2] \tag{13}$$

Assuming that $\pi^- + p \to \pi^- + X$ is described by a similar cross section and allowing for a factor 4 in Eq. (10) due to the contribution of other intermediate fragments (mainly from π^0, but also from p, π^+, and kaons), we get r(A), as shown in Fig. 1.

It is, of course, important to extract some general properties of the model: $c_0(p_T)$ is expected to decrease with increasing energy, since the hard component is expected to increase faster than the soft component. Therefore, r(A) should increase in the transition region $\sim 30\%$ for $p_T = 2$ GeV and $E_{lab} = 500$ GeV.

The detailed energy dependence of $c_2(p_T)$ reflects, of course, the (unknown) energy dependence of $\frac{d\sigma'}{d^3p}$. If, e.g., $\frac{d\sigma'}{d^3p}$ increases with energy far more than $\frac{d\sigma_H}{d^3p}$, $c_2(p_T)$ (and therefore r) will increase.

For $\pi^\pm$, π^0, and K^+ we expect the double scattering contribution to be of the same order of magnitude. Since $\frac{d\sigma_H}{d^3p}$ $(p + p \to K + X)$ is slightly smaller than $\frac{d\sigma_H}{d^3p}$ $(p + p \to \pi + X)$, this could explain why r(A) is slightly larger for K^+ than for pions. (It is only the ratio of double to single scattering which matters.) For K^-, protons, and antiprotons, which require $c_2 \approx .4(K^-)$, $\approx 1\cdot$ (p and $\bar{p}$), we have to assume that proton production by intermediate fragments (e.g., pions) falls less steeply in p_T than the direct production cross section (this is, in fact, predicted by the CIM).

*We do not mean to give by Eq. (13) a fit over a wide energy region. Furthermore, we ignore any angular dependence apart from ϵ^3, since the main contribution in the integral comes from $\theta_{CM} \sim 45^\circ$–$90^\circ$, and the observed angular dependence in this region is only weak. Also, due to the preliminary nature of the data, Eq. (13) should only be considered as an order-of-magnitude estimate.

On the side opposite to the large p_T particle, we expect that any jet structure, which might show up in pp collisions, is masked by the 50% - 100% double scattering contribution. This is true, even if the fragments of the jet would have little interaction with the nucleus. However, the coplanarity of the event should still be preserved.

III. OTHER EXPLANATIONS

Pumplin and Yen[14] have attempted to explain the increase of the effective exponent in Eq. (1) by double scattering. They, however, did not consider the behavior in the transition region and furthermore took the same form—including the "soft part"—for the first and second scattering, therefore ending up with r(A) too large. Apart from that, they used the equivalent of Eq. (8a), although both collisions are inelastic. Some authors[4,5] have proposed that the large p_T secondaries might be formed out of constituents of different nucleons, therefore $\frac{d\sigma}{d^3p} \sim c_1 A + c_2 A^2 + \ldots\ldots.$. Since $r(W_i)/r(T_i) > 1$, this would imply that constituents out of nucleons, separated more than several fermi, act coherently to form a large p_T secondary. Furthermore, a power series in A with positive coefficients disagrees with the data.

IV. THE EFFECT OF FERMI MOTION

In this context, it might be worthwhile to discuss the influence of fermi motion in the nucleus. Only the motion parallel to the beam affects the cross section. Assuming for the moment uniform distribution in the fermi sphere,

$$\frac{dn}{dp_z}(P_z) = \pi P^2_{max} \left(1 - P_z^2/P^2_{max}\right) / \frac{4\pi}{3} P^3_{max} \tag{14}$$

The maximum fermi momentum P_{max} of protons and neutrons is determined by their respective densities[15,16]

$$P_{max} \approx m \frac{1}{3} \left(\frac{n}{A}\right)^{1/3} \tag{15}$$

where m denotes the nucleon mass. $\sqrt{s}$ changes due to this quite significantly: $\sqrt{s} \to \sqrt{s}\left(1 + \frac{1}{2}\frac{P_z}{m}\right)$ and therefore $x_T \to x_T\left(1 - \frac{1}{2}\frac{P_F}{m}\right)$ for fixed p_T.

Let us take for the sake of easy calculation[17]

$$E \frac{d\sigma}{d^3p} \sim p_T^{-8}\, e^{-13x_T} \tag{16}$$

After averaging over the fermi sphere, we find that the cross section changes by a factor f:

$$f = \frac{3}{\alpha^2}\left(\mathrm{ch}\,(\alpha) - \frac{1}{\alpha}\,\mathrm{sh}\,(\alpha)\right) = 1 + \frac{\alpha^2}{10}\,(1 + \alpha^2\, 0.036 + \ldots) \tag{17}$$

where $\alpha = 13x_T \cdot P_{max}/2m$. For n = A/2 and $x_T = .5$, $f \approx 1.08$; for x_T near 1, f = 1.35. Note that Eq. (17) only introduces a slight (< 1%) A dependence due to the variation of proton and neutron densities (which implies a change in the fermi momentum of protons and neutrons).

The above treatment ignores correlations between the nucleons in configuration space which make the wave functions less smooth and gives, therefore, only a lower limit on the kinetic energy.[15] It is hard to see, however, how this should explain the rise of r from titanium to tungsten, since one would expect these correlations to be only of short range.

V. CONCLUSIONS

Separation of the inclusive cross section for pion production into a "soft part" $\sim A^{.85} c e^{-6p_T}$, and a "hard part" $\sim A\, E\, d\sigma_H/d^3p$, where $E\, d\sigma_H/d^3p$ is independently determined from the cross section, explains the rise of r to 1, since the second term gets more important with increasing p_T.

Inclusion of double scattering gives a term $\sim A^{4/3}$, which—together with the single scattering term $\sim A$—fits the data as well as a simple power law. Also for the other particles, a form $A + c(p_T)A^{4/3}$ describes the A dependence of the cross section surprisingly well.

The correct order of magnitude of the double scattering term is given by the form of the cross section for the first and second scattering, as described in Eq. (12) and (13).

However, a number of questions remain unsolved: Why should hard partons have so little interaction with nuclear matter? Equivalently, why can we ignore absorption? To what extent is the decomposition of the cross section into soft and hard parts reasonable, and what is the detailed form of $\frac{d\sigma_H}{d^3p}$ for $x_{\parallel} > .5$?

Fermi motion increases the cross section by at least 30% for x_T near 1. It is hard to see, however, how this could give a significant contribution to the observed A dependence.

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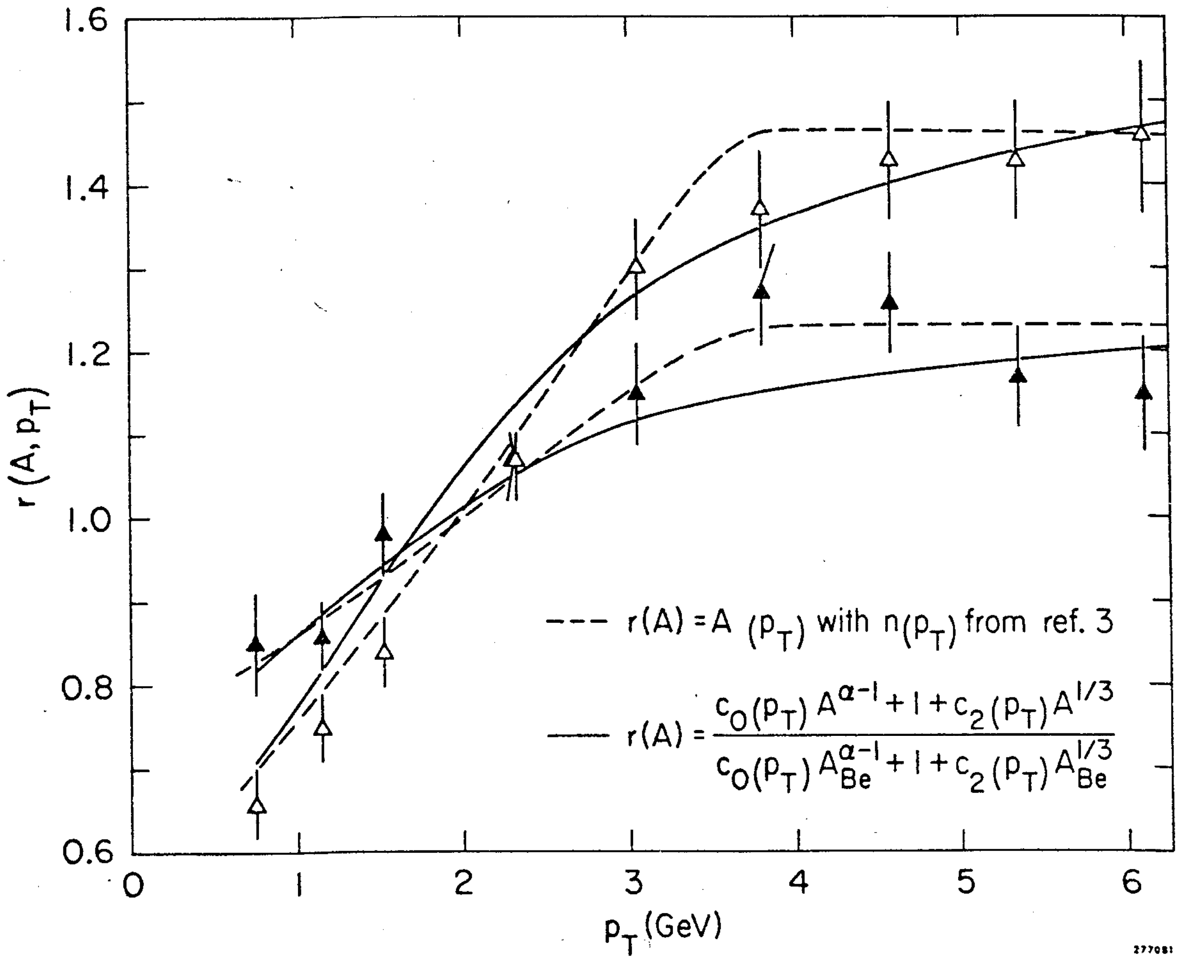
Figure Captions

1. Ratio of invariant cross section per nucleon for π^- production from W(Δ) and T_i, compared to those from Be, plotted vs p_T. The broken line is given by Eq. (1), with $n(p_T)$ taken from Ref. 3. The solid line is given by Eq. (11).
2. Schematic picture of the double scattering process inside a nucleus.
3. Decomposition of the invariant cross section for π production from protons into a soft and a hard component. The experimental points are from
   (a) Cronin *et al.*,[3] (E_{lab} = 300 GeV, reduced to A = 1 by Eq. (1)).
   (b) Banner *et al.*,[10] ($\sqrt{s}$ = 23.2 GeV).
   (c) Alpen *et al.*,[10] ($\sqrt{s}$ = 23.4 GeV).



r(A, p_T)
1.6
1.4
1.2
1.0
0.8
0.6
0
1
2
3
4
5
6
p_T (GeV)
r(A) = A (p_T) with n(p_T) from ref. 3
$r(A) = \frac{c_0(p_T)\,A^{\alpha-1} + 1 + c_2(p_T)\,A^{1/3}}{c_0(p_T)\,A_{Be}^{\alpha-1} + 1 + c_2(p_T)\,A_{Be}^{1/3}}$
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Fig. 1

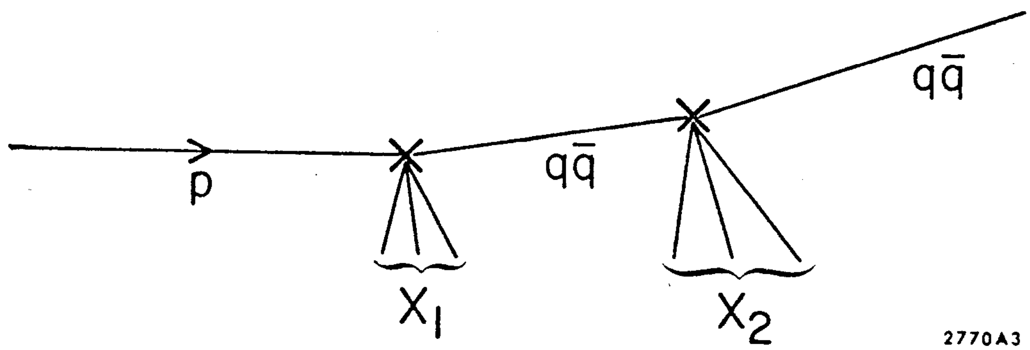
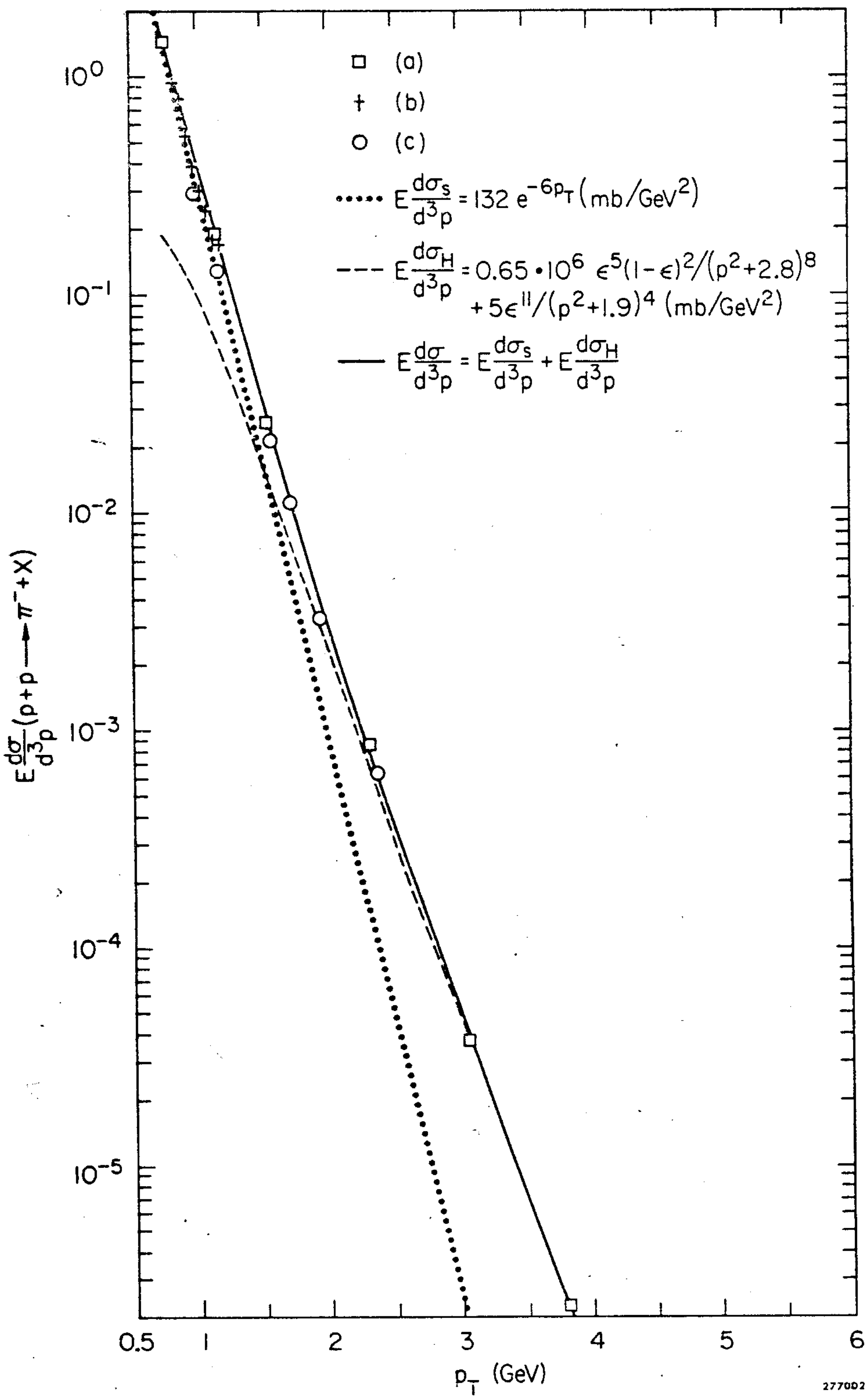


Fig. 2



(a)
(b)
(c)
$E\frac{d\sigma_s}{d^3p} = 132\, e^{-6p_T}$ (mb/GeV^2)
$E\frac{d\sigma_H}{d^3p} = 0.65 \cdot 10^6\, \epsilon^5(1-\epsilon)^2/(p^2+2.8)^8 + 5\epsilon^{11}/(p^2+1.9)^4$ (mb/GeV^2)
$E\frac{d\sigma}{d^3p} = E\frac{d\sigma_s}{d^3p} + E\frac{d\sigma_H}{d^3p}$
$E\frac{d\sigma}{d^3p}(p+p \longrightarrow \pi^- + X)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.5
1
2
3
4
5
6
p_T (GeV)
277002

Fig. 3