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PARITY VIOLATION IN ATOMS*

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ABSTRACT

Recent theoretical predictions for parity violating effects in atoms are reviewed. Order of magnitude estimates are given for the effect of weak neutral currents in various electronic and muonic atoms. The observation of any of these polarization effects will not only establish the existence of a parity violating interaction between charged leptons and the nucleus, but also determine the sign of the coupling – an important constraint for theory.

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The recent experimental observations of a new kind of weak interaction in high energy neutrino interactions (the neutral current) have led to an intense search for parity violation effects in atoms. The possibility of such phenomena was in fact first suggested by Zel'dovich more than a decade ago in the context of the discovery of parity violation in weak interactions. Recent theoretical work has been focused on finding the most advantageous atomic states for each of the various parity violating effects. Light and heavy atoms of the electronic and muonic kind have different advantages and disadvantages for the various effects. We give here order of magnitude estimates for the very simplest amplitudes which have been computed and refer the reader to the literature for more detailed considerations.

An important advance in weak interaction theory made by Weinberg and Salam consists in the construction, for the first time, of viable field theoretic models which do not violate any known general principle (causality, relativity, unitarity). In fact, these developments may foreshadow a unified theory of weak and electromagnetic interactions. A general feature of a large class of models proposed is the required existence of so-called "neutral current" interactions, weak processes in which the charge of the leptons remains unchanged during the interaction. Thus, unlike the usual weak Fermi interaction, where a neutrino transforms into a muon, ($\nu_\mu \to \mu$), here $\nu \to \nu$, $e \to e$, $\mu \to \mu$. Stimulated by these theories, the recent experiments at CERN and FNAL have shown that in high energy neutrino (and antineutrino) interactions with nuclei, the neutral current events ($\nu_\mu \to \nu_\mu$) occur at a rate consistent with their weak nature. The question which arises is whether electrons (or muons) would couple in a similar way to nucleons.[a] All theories (which we know) that allow high energy neutrino neutral currents generally also require similar electron (or muon) parity violating couplings at low or high energies.[b] Electrons (or muons) in atoms are the ideal place to search for these interactions. There is some circumstantial evidence from the relative rate of ν and $\bar{\nu}$ processes about the parity violating (V-A) character of these interactions. However, direct evidence is still lacking on this point and atomic phenomena may well be the first to settle the issue of parity violation by neutral currents.

As their name suggests, (low energy) weak interactions are very feeble when compared to electromagnetic couplings. Let us recall that the weak coupling constant G, introduced by Fermi, is $\sim 10^{-5}\, M_p^{-2}$ where M_p is the proton mass. If the neutral current interaction H_W between an electron and a nucleus of Z protons and (A-Z) neutrons has the same coupling constant G, has short range, and conserves angular momentum, then, up to a numerical factor of order unity[c]

$$H_W = G\,\frac{\vec{\sigma}\cdot\vec{p}}{m}\,\delta^3(\vec{r})\,A + G\delta^3(\vec{r})\,A\,. \tag{1}$$

Here the first term is parity violating and the second parity conserving.[d] We have assumed, for simplicity, equal coupling to protons and neutrons (isoscalar interaction).

The first order shift of the energy levels of an atom is due in first order to

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[a] Three events of the type $\nu_\mu e^- \rightarrow \nu_\mu e^-$ have also been reported. If these are real they demonstrate that electrons do take part in interactions of this form.

[b] An exception is theories in which the neutrino charge radius squared is of order G.

[c] In Eq. (1), m, $\frac{1}{2}\vec{\sigma}$, $\vec{p}$, $\vec{r}$ are the mass, spin, momentum, and position of the electron.

[d] We are ignoring any dependence on the nuclear spin. Such effects are discussed by Feinberg and Chen.

the parity conserving part of H_W and is $G(mZ\alpha)^3 A$.[e] This is 10^{-7} times the Lamb shift $\sim m\alpha(Z\alpha)^4$ and therefore much too small to identify or distinguish from finite size effects. One is thus forced to find phenomena where the electromagnetic coupling makes no contribution and thus do not obscure the contribution of weak interactions.

Since the interaction (1) is parity violating it can mix any $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ states of a hydrogenic atom. The parity violating matrix element $\langle S_{\frac{1}{2}} | H_W | P_{\frac{1}{2}} \rangle$ is $\sim G(Z\alpha)(mZ\alpha)^3 A$. For a hydrogenic atom the important mixing is that between the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ states which are only split by the Lamb shift $\sim m\alpha(Z\alpha)^4$, so that

$$\text{"Wrong parity" amplitude in the } 2S_{\frac{1}{2}} \text{ state of hydrogen} \sim (Gm^2)\frac{A}{\alpha} \tag{2}$$

The small component of wrong parity is revealed in transition amplitudes which are forbidden if parity is conserved. For example, a small electric dipole amplitude E1 now connects the ground state $1S_{\frac{1}{2}}$ to the excited state $2S_{\frac{1}{2}}$:

$$E1 \sim \left(\frac{e}{mZ\alpha}\right)\left(Gm^2\right)\frac{A}{\alpha} \tag{3}$$

Here $(e/mZ\alpha)$ is a typical electric dipole for an atom. In the absence of the parity violating interaction the states $2S_{\frac{1}{2}}$ and $1S_{\frac{1}{2}}$ are connected through a single photon only by an M1 magnetic dipole amplitude which is suppressed relative to a normal M1 amplitude (because of the orthogonality of the two space wave functions 2S and 1S), by a factor $(Z\alpha)^2$:

$$M1 \sim \left(\frac{e}{m}\right)(Z\alpha)^2 \tag{4}$$

Here the first factor, (e/m), is the spin magnetic moment of the electron.

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[e] The expectation value of $\delta^3(\vec{r})$ is $|\psi(0)|^2$ for which we take the inverse volume of the atom, $(mZ\alpha)^3$. For a many electron atom $|\psi(0)|^2$ is measured in hyperfine structure, as noted by Fermi and Segré. The evaluation of $\vec{\sigma}\cdot\vec{p}\,\delta^3(r)$ in the first term, which involves $\psi_1(0)\,\psi_2'(0)$, has been carried out by Bouchiat and Bouchiat.

The simultaneous contribution of E1 and M1 amplitudes between the same pair of states can be observed in a number of ways. If the upper state $2S_{\frac{1}{2}}$ is populated equally in the two $J_z = \pm\frac{1}{2}$ substates, the photons emitted in the transition $2S_{\frac{1}{2}} \rightarrow 1S_{\frac{1}{2}}$ are slightly circularly polarized (Curtis Michel 1965, Bouchiat and Bouchiat 1974). The preference of an unoriented sample to emit one kind of circularly polarized light constitutes direct evidence for parity violation. The expected magnitude of the circular polarization P of the photons, due to the impurity of the ground state, is:

$$P = \frac{N_L - N_R}{N_L + N_R} = 2\,\frac{E1}{M1} \sim \frac{Gm^2}{\alpha(Z\alpha)^3}\,A \tag{5}$$

In this equation $N_{L(R)}$ is the number of left- (right-) hand circularly polarized photons emitted. For hydrogen $Z = A = 1$, Eq. (5) predicts $P \sim 10^{-3}$, and the effect decreases rapidly for heavier atoms. However, for low Z atoms the branching ratios strongly favor the two photon decay of the $2S_{\frac{1}{2}}$ state and thus make the experiments difficult (see Feinberg and Chen for details). In any case it is difficult in practice to achieve high densities of atomic hydrogen.

We now turn to many electron atoms in which there is no approximate degeneracy between $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ states. For these atoms, to find the amplitude of a $P_{\frac{1}{2}}$ state mixed into an $S_{\frac{1}{2}}$ state we divide by the typical energy difference $m(Z\alpha)^2$ to get:

$$\begin{array}{c}\text{"Wrong parity" amplitude}\\ \text{in an } S_{\frac{1}{2}} \text{ state}\end{array} \sim \left(Gm^2\right)(Z\alpha)^2 A \tag{6}$$

as a representative order of magnitude. Thus the circular polarization P is now

$$P \simeq \frac{2E1}{M1} \simeq \frac{(e/mZ_{eff}\alpha)(Gm^2)(Z\alpha)^2 A}{(e/m)(Z_{eff}\alpha)^2} \tag{7}$$

Bouchiat and Bouchiat have considered in detail the case of the $6S_{\frac{1}{2}}$ to $7S_{\frac{1}{2}}$ transition in the Cs atom, for which they argue that the circular polarization P may be as large as 10^{-4}. The transition is due to an outer electron, for which $Z_{eff} \approx 1$ both in the magnetic amplitude (so that $M1 \sim 10^{-4}$ e/m) and in the first bracket of the E1 amplitude. On the other hand, the wavefunction near the nucleus is still governed by $Z \approx 55$.

Curtis Michel and Bouchiat and Bouchiat have also estimated the effect of the electron-electron neutral current interactions and have found that they are typically not as important as the electron-nucleus interaction in mixing different parity states. This is apparently due to the Coulomb repulsion of electrons, which makes the short range weak interaction less effective. As Gorshkov and Labzovsky point out, the mixing of higher ($L \neq 0$) orbitals is sensitive only to the electron-electron interaction.

The branching ratios for the sensitive transitions in many-electron atoms are still forbidding but these difficulties can be circumvented in absorption experiments as we shall discuss below.

It is clear from (5) that by going to muonic atoms one gains at least a factor of $(m_\mu/m_e)^2 \sim 10^4$ insensitivity to parity violation over the corresponding electronic hydrogenic atoms. This was pointed out independently by Bernabeu, Erikson and Jarlskog, by Feinberg and Chen, and by Moskalev. These authors also emphasized the advantages of working with muonic atoms of low Z, e.g., Li and Be, in which the states $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ are nearly degenerate due to the opposing effects of the contributions of vacuum polarization and of the finite size of the nucleus. Due to the near degeneracy of these states the mixing due to H_W becomes large. For a muonic atom of low Z, Bernabeu et al. and Moskalev predict $P \sim 10^{-2}$. This large a value of P is quite promising for experimental searches. As pointed out by all three groups the interaction H_W will also give rise to parity violating decay correlations between the muon spin and the momentum of the photon emitted. The all-important question in observing these effects is the branching ratio of the single photon decay of the $2S_{\frac{1}{2}}$ state. The competing processes are 2 photon decay, Auger effect[f] (de-excitation of $2S_{\frac{1}{2}}$ to $1S_{\frac{1}{2}}$ with simultaneous electron expulsion), and de-excitation by Stark mixing due to external electrons. These effects are discussed in detail by Feinberg and Chen. The possibility of using higher orbital states ($P_{3/2}$, $D_{3/2}$) in muonic atoms, which may be parity mixed for finite sized nuclei, has also been proposed. So far we have discussed only the emission of radiation.

The parity violations discussed above take place also in absorption. This is feasible only with many electron atoms for which high density samples are easily made. If a sample of randomly oriented atoms is irradiated with unpolarized light of the frequency corresponding[g] to the transition $nS_{\frac{1}{2}} \rightarrow n'S_{\frac{1}{2}}$, then one kind of circularly polarized light will be absorbed preferentially compared to the other kind. The transmitted wave will thus become partially polarized. If we denote by σ_L (σ_R) the absorption cross section of left (right) circularly polarized light, then by virtue of time-reversal invariance (which we assume in H_W),

$$\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{N_L - N_R}{N_L + N_R} = P \tag{8}$$

where the circular polarization P has been defined and estimated in (5) and (7).

[f] If all electrons were expelled only the two photon decay would produce a background.

[g] This experiment is discussed in detail by Bouchiat and Bouchiat. They have pointed out the advantages of tunable lasers in these experiments.

The practical advantage of working in absorption is that one does not have to compete against unwanted processes with larger branching ratios. The preferential absorption of one kind of photon over the other kind can be ascertained by monitoring the fluorescence arising from the excited $S_{\frac{1}{2}}$ state.

Recalling that by the optical theorem σ_L (and σ_R) are related to the imaginary part of forward scattering amplitude f_L (and f_R) of left- (and right-) handed polarized light, a nonvanishing P implies a corresponding inequality $\mathrm{Im}\, f_L(0) \neq \mathrm{Im}\, f_R(0)$, and therefore in general $\mathrm{Re}\, f_L(0) \neq \mathrm{Re}\, f_R(0)$. The difference of the real parts will cause a plane polarized beam of light to rotate its plane of polarization when traversing a medium composed of free atoms. This effect, known as optical activity, is familiar for light propagation through media composed of handed molecules. The effect for free atoms, where the parity violation is due to weak interactions, was conjectured in 1959 by Zel'dovich, who restricted his considerations to hydrogen atoms. Recently Khriplovich has drawn attention to heavy atoms where the effect is possibly detectable. The rotatory power ϕ(rad/cm) of a sample of N atoms/cm^3 for light of wavelength λ is

$$\phi = \frac{1}{2} N \cdot \lambda \cdot \mathrm{Re}\left[f_L(0) - f_R(0) \right] \qquad (9)$$

where $f_{R(L)}(0)$ is the forward scattering amplitude for right- (left-) handed light waves by the atoms in the sample. The difference between f_R and f_L is proportional to the wrong parity amplitude(6); otherwise the scale of the scattering amplitude of light by an electron in an atom is (e^2/mc^2) so that:

$$\phi = \frac{1}{2} N \cdot \lambda \cdot (e^2/mc^2)\, Gm^2\, \alpha^2\, Z^2\, A \qquad (10)$$

For a gas of 10^{19} $T\ell$ atoms/cm^3 (with $Z \sim 80$, $A \sim 200$) at a wavelength λ of 10^{-4} cm, formula (7) gives $\phi \sim 10^{-8} \mathrm{cm}^{-1}$. This is within a factor of 10 of the more accurate value of Khriplovich, who also considers relativistic corrections ignored here.

The observation of any of these polarization effects will not only establish the existence of a parity violating interaction between charged leptons and the nucleus, but also determine the sign of the coupling - an important constraint for theory. Comparisons of these effects in electronic and muonic atoms will test the electron-muon universality of the neutral weak current. It should however be noted that the usual beta decay weak interaction even without neutral currents predicts parity violation in atoms but with a reduced effective coupling αG (relative to G in Eq. (1)). This occurs because of the distribution in space of the charge of the electron due to the weak interactions, the same effect which gives weak corrections to the gyromagnetic ratio of the leptons and the charge form factor of the neutrino.

Many experiments are presently under way to search for some of these effects. The discovery of any effect will illuminate our knowledge of the weak interactions.

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