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ANGULAR DISTRIBUTION OF PARTICLES FROM TRANSVERSELY POLARIZED $e^+ e^-$ COLLIDING BEAMS*

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ABSTRACT

The effects of the transversely polarized electron and positron beams in the e^+e^- colliding beam experiment are discussed.

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In this note we consider the effects of the transversely polarized electron and positron beams[1] in the e^+e^- colliding beam experiment. V. N. Baier and his co-workers[2] have discussed many of the cross sections with two-body final states. We have expanded their treatments and generalized their results to arbitrary inclusive cross sections in which only one particle in the final state is detected.

We consider processes which contain only a time-like photon in the intermediate state. The reactions $e^+ + e^- \to e^+ + e^-$ and $e^+ + e^- \to 2\gamma$ are exceptions which will be dealt with later. Let us consider an arbitrary process in which only one final particle, whose four momentum is denoted by $p = (E, \vec{p})$, is detected. We choose the direction of the incident electron to be the z-axis and that of the magnetic field $\vec{H}$ to be the x-axis, as shown in Fig. 1. The cross section for an arbitrary process mediated by one time-like photon exchange can be written as

$$d\sigma = \frac{d^3p}{2E(2\pi)^2} \frac{e^4}{\sqrt{(p_+ \cdot p_-)^2 - m^4}} \frac{1}{q^4} L^{\mu\nu} W_{\mu\nu} \quad . \tag{1}$$

$L^{\mu\nu}$ is the tensor formed by the initial electron-positron current:

$$L_{\mu\nu} = \frac{\mathrm{Tr}}{4} \frac{\left(1 - s_+ \gamma_5 \gamma_x\right)}{2} (\not{p}_+ - m)\gamma_\mu \frac{\left(1 - s_- \gamma_5 \gamma_x\right)}{2} (\not{p}_- + m)\gamma_\nu \quad , \tag{2}$$

where s_- and s_+ are, respectively, the polarization of the electrons and positrons in the direction of the magnetic field. The tensor $L_{\mu\nu}$ has only two non-vanishing components in the limit $m = 0$. They are

$$L_{xx} = \frac{q^2}{8} (1 - s_+ s_-) \tag{3}$$

and

$$L_{yy} = \frac{q^2}{8} (1 + s_+ s_-) \, . \tag{4}$$

In the colliding beam machine one has in general[1]

$$s_+ = -s_- = 0.924\,[1 - \exp(-t/\tau)]\ , \tag{5}$$

where the characteristic time τ is given by

$$\tau = \frac{6.075 \times 10^4}{E^2 H^3}\ \text{minutes} \cdot \text{GeV}^2 \cdot (\text{kG})^3\ . \tag{6}$$

At time $t = 0$, $L_{xx} = L_{yy}$, since the e^+ and e^- beams are initially unpolarized. As t becomes large compared with τ, s_+ and s_- reach their maximum values $s_\pm = \pm 0.924$. When this occurs L_{xx} dominates over L_{yy}. L_{xx} can be regarded as the intensity of a virtual photon beam completely polarized in the x direction. Similar interpretation can be made for L_{yy}. It is also interesting to observe that if one beam is polarized and the other is unpolarized then no polarization effect is observable.[3]

Next we consider the tensor for the final state $W_{\mu\nu}$. Since q and p are the only two available vectors in the problem, gauge invariance and Lorentz invariance demand that

$$W_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - q_{\mu\nu}\right) W_1 + \frac{1}{M^2}\left(p_\mu - \frac{(p\cdot q)}{q^2} q_\mu\right)\left(p_\nu - \frac{(p\cdot q)}{q^2} q_\nu\right) W_2\ , \tag{7}$$

where W_1 and W_2 are functions of q^2 and $p\cdot q$. The situation is very similar to the structure functions in the electron scattering[4] except in this case W_2 is not positive definite while W_1 is. When dealing with $W_{\mu\nu}$ it is more convenient to use a coordinate system[5] where the direction of $\vec{p}$ is the z' axis. In this system the only nonvanishing components of $W_{\mu\nu}$ are diagonal ones:

$$W_{z'z'} = W_1 + \frac{p^2}{M^2} W_2 \equiv W_0 \tag{8}$$

and

$$W_{x'x'} = W_{y'y'} \equiv W_1\ , \tag{9}$$

where x' and y' can be in any direction perpendicular to the z' axis as can be seen readily from (7). In terms of W_0 and W_1, Eq. (1) reduces to

$$\frac{d\sigma}{dx\, d\Omega} = \frac{\alpha^2 \beta x}{4q^2}\left[W_1 + (W_0 - W_1)\frac{\sin^2\theta}{2}(1 + \xi^2 \cos 2\phi)\right] \tag{10}$$

where $\xi^2 \equiv - s_+ s_-$, $\beta = p/E$ is the velocity/c of the detected particle, and $x = 2q\cdot p/q^2 = E/\text{Energy of the incident electron}$.

W_0 and W_1 are

$$W_0 = \sum_{\substack{\text{all final states} \\ \text{except } \vec{p}}} (2\pi)^3 \delta^4(p + \ldots - q) \,|\langle p \ldots | j_{z'}(0) | 0 \rangle|^2 \tag{11}$$

and

$$W_1 = \sum_{\substack{\text{all final states} \\ \text{except } \vec{p}}} (2\pi)^3 \delta^4(p + \ldots - q) \,|\langle p \ldots | j_t(0) | 0 \rangle|^2 \tag{12}$$

j_t is $j_{x'}$, $j_{y'}$, or $(j_{x'} \pm i j_{y'})/\sqrt{2}$, all of which give the same answers because of (7).

When the final state contains only two particles (p and p'), W_1 and W_0 contain a δ function, hence it is convenient to introduce form factors $G_1(q^2)$ and $G_0(q^2)$:

$$W_1(q^2, 2q\cdot p) = \delta\left[(q-p)^2 - m_{p'}^2\right] G_1(q^2) \tag{13}$$

and

$$W_0(q^2, 2q\cdot p) = \delta\left[(q-p)^2 - m_{p'}^2\right] G_0(q^2) \quad , \tag{14}$$

where

$$G_0(q^2) = \sum_{\lambda\lambda'} |\langle p\lambda, p'\lambda' | j_{z'}(0) | 0 \rangle|^2 \quad , \tag{15}$$

$$G_1(q^2) = \sum_{\lambda\lambda'} |\langle p\lambda, p'\lambda' | j_t(0) | 0 \rangle|^2 \quad . \tag{16}$$

λ and λ' are respectively the helicities of p and p'. In this case (10) becomes the simplified expression

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^4}\,\beta\left[G_1(q^2) + \left\{G_0(q^2) - G_1(q^2)\right\}\frac{\sin^2\theta}{2}(1 + \xi^2\cos 2\phi)\right] . \tag{17}$$

If the undetected particle p' is a resonance with a width Γ instead of being a stable particle, then the δ function in (13) and (14) must be replaced by a Breit-Wigner function

$$\delta\left[(q-p)^2 - m_{p'}^2\right] \rightarrow \frac{m_{p'}\Gamma}{\pi}\,\frac{1}{\left[(q-p)^2 - m_{p'}^2\right]^2 + \Gamma^2 m_{p'}^2} \tag{18}$$

which produces a bump in the x distribution with a peak at

$$x = 1 - (m_{p'}^2 - m_p^2)/q^2 . \tag{19}$$

Since the matrix element of $j_{z'}$ transforms like Y_1^0, we see from (11) that W_0 represents the probability that the final state is in the angular momentum state $|J, J_{z'}> = |1, 0>$. Similarly the matrix element of $\pm(j_{x'} \pm ij_{y'})/\sqrt{2}$ transforms like $Y_1^{\pm 1}$. Hence from (12) we see that W_1 represents the probability that the final state to be in the state $|J, J_{z'}> = |1, \pm 1>$. In the scattering (spacelike photon) case, W_0 is proportional to the scalar photon cross section[6] σ_s and W_1 is proportional to the transverse photon cross section[6] σ_T. When the final state is the particle-antiparticle two-body state, its S channel equivalent is elastic scattering. In this case our G_0 becomes $\propto G_e^2$, the square of the coulomb form factor,[7,8] and our G_1 becomes $\propto G_m^2$, the square of the magnetic form factor.[7,8]

Let us proceed to calculate W_0 and W_1 corresponding to different final states.

$e^+ + e^- \to \pi^+ + \pi^-$

In this case, the matrix element of current can be written as

$$<p,p'|j_i(0)|0> \;=\; (p-p')_i\, F(q^2) \;=\; 2p_i\, F(q^2) \tag{20}$$

where $F(q^2)$ is the pion form factor normalized such that $F(0) = 1$. Since the direction of p is chosen as the z' axis we immediately obtain, using (15), (16), and (20),

$$G_1 \;=\; 0 \tag{21}$$

and

$$G_0 \;=\; 4p^2 F^2(q^2)\,. \tag{22}$$

Thus, when the polarization is complete ($\xi^2 = 1$), we have

$$\frac{d\sigma}{d\Omega} \;=\; \frac{\alpha^2}{4q^2}\,\beta^3 \sin^2\theta \cos^2\phi\; F^2(q^2) \tag{23}$$

$e^+ + e^- \to \mu^+ + \mu^-$

$$<pp'|j_\mu(0)|0> \;=\; \bar{u}(p)\gamma_\mu v(p')\,.$$

Ignoring the mass of the muon, we have

$$G_1 \;=\; 8E^2 \tag{24}$$

and

$$G_0 \;=\; 0\,. \tag{25}$$

When the polarization is complete ($\xi^2 = 1$), we have

$$\frac{d\sigma}{d\Omega} \;=\; \frac{\alpha^2}{2q^2}(1-\sin^2\theta \cos^2\phi)\,. \tag{26}$$

These two examples are two extremes. For the spin 0 case the component of angular momentum along $\vec{p}$ is zero, hence only G_0 contributes to the cross section and the ϕ distribution is maximum at $\phi = 0$ and π but minimum at $\phi = \pi/2$. For the spin-$\frac{1}{2}$ case (zero mass and no anomalous magnetic moment) exactly the

opposite is true, namely, the component of angular momentum[5] along $\vec{p}$ is ± 1, hence only G_1 contributes to the cross section and the ϕ distribution is maximum at $\phi = \pi/2$ but minimum at $\phi = 0$ and π. All other possibilities are intermediate between these two extremes. For example, when $G_0 = G_1$ the angular distribution is isotropic.

$e^+ + e^- \rightarrow$ (spin 0, parity -) + (spin J, parity $(-1)^J$) or
$e^+ + e^- \rightarrow$ (spin 0, parity +) + (spin J, parity $(-1)^{J+1}$)

The case $J = 0$ is forbidden by conservation of angular momentum and parity. We show in the following that when $J \geq 1$, the conservation of parity and angular momentum also demand that $G_0 = 0$. Hence the angular distribution is exactly the same as that for $e^+ + e^- \rightarrow \mu^+ + \mu^-$. The proofs for the two cases are similar. Therefore let us prove only the first one. We call the particle with spin 0 and parity - the pion, whose momentum is denoted by p. The momentum and the helicity of the other particle are denoted by p' and λ'. A particle with natural parity, spin J, and momentum p' can be represented by a completely symmetric tensor of rank J, $F_{\mu_1,\mu_2,\ldots,\mu_J}$ satisfying $g_{\mu_1\mu_2} F_{\mu_1,\mu_2,\ldots,\mu_J} = 0$ and $p'_{\mu_1} F_{\mu_1,\mu_2,\ldots,\mu_J} = 0$. Since the pion has odd parity, the matrix element must be of the form

$$< p, p'\lambda' | j_\mu(0) | 0 > \; = \; \epsilon_{\mu\nu\alpha\beta} q_\nu p_\alpha F_{\beta,\mu_2,\ldots,\mu_J} q_{\mu_2} \cdots q_{\mu_J} \, . \tag{27}$$

But q_ν is nonzero only when $\nu = 0$, and p_α is nonzero only when $\alpha = 0, z'$. Hence, when $\mu = z'$, the matrix element is zero. This proves our assertion. From (27), we also see immediately that J cannot be zero.

Another way of proving the above assertion is to use the operator[8] Y, which changes a state into its mirror image, the mirror being in the x'z' plane. The operator Y does not change the direction of momenta $\vec{p}$ and $\vec{p}'$ but it changes the signs of the helicities of particles:

$$Y|\vec{p}'\,J\lambda'\rangle = \eta_{p'}(-1)^{J-\lambda'}|\vec{p}'\,J-\lambda'\rangle$$
$$Y|\vec{p}\,\text{spin }0\rangle = \eta_p|\vec{p}\,\text{spin }0\rangle \tag{28}$$

where $\eta_{p'} = (-1)^J$ and $\eta_p = -1$ are parities of particles p' and p respectively. We also have

$$Y^{-1}\,j_{z'}(0)Y = j_{z'}(0)\,.$$

Thus

$$\langle\vec{p}\,\text{spin }0,\ \vec{p}'\,J\lambda'|j_{z'}(0)|0\rangle$$
$$= \langle\vec{p}\,\text{spin }0,\ \vec{p}'\,J\lambda'|Y^{-1}\,j_{z'}(0)Y|0\rangle$$
$$= (-1)^{2J+1}\delta_{\lambda'0}\langle\vec{p}\,\text{spin }0,\ \vec{p}'\,J\lambda'|j_{z'}(0)|0\rangle \tag{29}$$

Hence $\langle\vec{p}\,\text{spin }0,\ \vec{p}'\,J\lambda'|j_{z'}(0)|0\rangle = 0$.

<u>$e^+ + e^- \to p + \bar{p}$ (A pair of spin 1/2 particles)</u>

The matrix element is

$$\langle p\,p'|j_\mu(0)|0\rangle = \bar{u}(p)\left[A\gamma_\mu + (p'-p)_\mu B\right]v(p') \quad , \tag{30}$$

where

$$A = F_1 + F_2\kappa = G_m$$
$$B = F_2\kappa/(2M) = (G_m - G_e)/\left[2M(1+\tau)\right]$$
$$\kappa = 1.79, \qquad \tau = -q^2/(4M^2), \qquad F_1(0) = F_2(0) = 1 \quad .$$

After taking the spin average, we obtain

$$G_1 = 8E^2G_m^2 \tag{31}$$

and

$$G_0 = 8M^2G_e^2 \quad . \tag{32}$$

$e^+ + e^- \to V^+ + V^-$ (A pair of spin 1 particles)[9]

$$\langle p p' | j_\mu(0) | 0 \rangle$$
$$= F(q^2)\left\{(p'-p)_\mu\left[g_{\alpha\beta}\left(1+\frac{1}{2}\lambda q^2/M^2\right) - \lambda M^{-2} q_\alpha q_\beta\right] + (1+\kappa+\lambda)\left(g_{\alpha\mu} q_\beta - g_{\beta\mu} q_\alpha\right)\right\} \epsilon^\alpha (\epsilon'^\beta{}_*) \tag{33}$$

where κ and λ are functions of q^2. At $q^2=0$, they are related to the magnetic dipole moment by

$$\mu = e(1+\kappa+\lambda)/(2M)$$

and the electric quadrupole moment by

$$Q = -e(\kappa-\lambda)/M^2 \quad .$$

ϵ and ϵ' are polarization vectors for $V^-(=p)$ and $V^+(=p')$ respectively. If the coupling is minimal, then $\lambda=0$ but κ is arbitrary.[10] In the gauge theory which has Yang-Mills type of coupling, we have $\lambda=0$ and $\kappa=1$. $F(q^2)$ is the charge form factor normalized such that $F(0)=1$. In this case G_1 and G_0 defined in (14), (15) and (16) can be shown to be

$$G_1 = 8p^2\gamma^2 (1+\kappa+\lambda)^2 F^2(q^2) \quad , \tag{34}$$

and

$$G_0 = \left[8p^2(1+2\lambda\gamma^2)^2 + 4p^2(1+2\kappa\gamma^2)^2\right] F^2(q^2) \quad , \tag{35}$$

where $\gamma=E/M$.

We note that at high energies, $\gamma>>1$, the angular distribution is similar to that of $e^+ + e^- \to \pi^+ + \pi^-$ as long as $\lambda\neq0$ or $\kappa\neq0$. On the other hand when $\lambda=\kappa=0$ we have $G_0 \ll G_1$ which means the angular distribution is similar to that of $e^+ + e^- \to \mu^+ + \mu^-$.

Bhabha Scattering ($e^+ + e^- \to e^+ + e^-$)

There are two Feynman diagrams contributing to this process. One is the annihilation diagram and the other contains the exchange of a space-like photon.

The square of the latter diagram dominates the cross section and it is not affected by the transverse beam polarization. Hence the overall polarization effect is small for this cross section. The cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8q^2} \frac{1}{(1-\cos\theta)^2} \left[(3+\cos^2\theta)^2 + \xi^2 \sin^4\theta \cos 2\phi\right] \tag{36}$$

where $\xi^2 = -s_+ s_-$.

Thus the polarization increases $d\sigma/d\Omega$ at ϕ near 0 and π and decreases it at ϕ near $\pi/2$ and $3\pi/2$. This is more like the pion case than the muon case. The reason is that the polarization affects both the square of the annihilation diagram and the interference term between the two diagrams. The interference term dominates over the square of the annihilation diagram in the spin dependent terms (similar to the spin independent term). The interference term has the opposite sign from the square of the annihilation diagram. Hence the effect is opposite that of the muon pair case. In Baier's paper[2] the sign in front of ξ^2 is wrong. This error was first found experimentally at SPEAR. J. D. Bjorken and this author have independently derived Eq. (36), thus confirming the experimentalists' conclusion.

$\underline{e^+ + e^- \to 2\gamma}$

The angular distribution for this cross section is ($m_e = 0$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2}\left[\frac{1+\cos^2\theta}{\sin^2\theta} - \xi^2 \cos 2\phi\right] , \tag{37}$$

where $\xi^2 = -s_+ s_-$. This expression agrees with the one given by Baier.[2] The effect of the transverse polarization is to decrease the cross section near $\phi = 0$ and π but to increase the cross section near $\phi = \pi/2$. Thus the effect is similar to the case for $e^+ + e^- \to \mu^+ + \mu^-$, except that in the two-photon reaction it is less pronounced.

Remarks

1. As can be seen from (10), in principle no new information can be gained from using the transversely polarized beams, because when $\xi^2 = 0$ we can still obtain W_0 and W_1 by measuring the θ dependence of the cross section. However in practice the transversely polarized beams are very useful for separating W_0 and W_1. For example, at SPEAR the range of θ is from 45° to 135°, hence $\sin^2\theta$ varies only from 0.5 to 1.0, whereas the range of ϕ is 2π. Hence the factor $(1 + \xi^2 \cos 2\phi)$ varies from $(1 + \xi^2)$ to $(1 - \xi^2)$. In addition, the radiative corrections affect the θ dependence more than the ϕ dependence of the cross section.

2. In order to obtain W_1 and W_0 for fixed q^2 and $x \equiv 2\, q\cdot p/q^2$, we do not need the information on detailed angular distributions. Let us define two integrated cross sections: H is the cross section integrated with respect to ϕ from $\pi/4$ to $3\pi/4$ and from $5\pi/4$ to $7\pi/4$ and $\cos\theta$ from -a to a. We may choose the normalization so that

$$H = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi \int_0^a d\cos\theta \; \frac{16q^2}{\pi a \alpha^2} \; \frac{d\sigma}{\beta x \, dx \, d\Omega}$$

$$= W_1 + (W_0 - W_1)(1 - \frac{a^2}{3})(\frac{1}{2} - \xi^2 \frac{1}{\pi}) \, . \tag{38}$$

V is the cross section integrated with respect to ϕ from $-\pi/4$ to $\pi/4$ and from $3\pi/4$ to $5\pi/4$ and $\cos\theta$ from -a to a. We may choose the normalization so that

$$V = \int_0^{\frac{\pi}{4}} d\phi \int_0^a d\cos\theta \, \frac{16q^2}{\pi a \alpha^2} \, \frac{d\sigma}{\beta x \, dx \, d\Omega}$$

$$= W_1 + (W_0 - W_1)(1 - \frac{a^2}{3})(\frac{1}{2} + \xi^2 \frac{1}{\pi}) \, . \tag{39}$$

From (38) and (39), we may solve for W_1 and W_0 in terms of H and V:

$$W_1 = H\left(\frac{\pi}{4\xi^2} + \frac{1}{2}\right) - V\left(\frac{\pi}{4\xi^2} - \frac{1}{2}\right) \tag{40}$$

and

$$W_0 = V\left[\frac{\pi}{4\xi^2}\frac{\left(1 + \frac{a^2}{3}\right)}{\left(1 - \frac{a^2}{3}\right)} + \frac{1}{2}\right] - H\left[\frac{\pi}{4\xi^2}\frac{\left(1 + \frac{a^2}{3}\right)}{\left(1 - \frac{a^2}{3}\right)} - \frac{1}{2}\right] , \tag{41}$$

where $\xi^2 = - s_+ s_-$ and a is the cut in $\cos\theta$, $|\cos\theta| < a$. Another useful relation between (W_1, W_0) and (H, V) is the asymmetry in ϕ:

$$A_\phi \equiv \frac{H-V}{H+V} = \frac{\xi^2 \frac{2}{\pi} A_W (1 - \frac{a^2}{3})}{1 + \frac{a^2}{3} A_W} , \tag{42}$$

where A_W is the asymmetry in W,

$$A_W = \frac{W_1 - W_0}{W_1 + W_0} \, . \tag{43}$$

The maximum value of A_ϕ is obtained when $W_0 = 0$,

$$A_\phi^{max} = \frac{\xi^2 \frac{2}{\pi} (1 - \frac{a^2}{3})}{1 + \frac{a^2}{3}} , \tag{44}$$

which must be satisfied by the muon pair, hence it can be used to determine the polarization ξ^2. The minimum value of A_ϕ is obtained when $W_1 = 0$:

$$A_\phi^{min} = -\xi^2 \frac{2}{\pi} . \tag{45}$$

Preliminary experimental results[11] indicate that for pion inclusive reactions at $q_0 = 7.4$ GeV and with 3 or more prongs, $A_\phi = (H-V)/(H+V)$ is positive and consistent with being A_ϕ^{max} when $0.6 < x < 0.9$.

3. Let us consider in some detail the relationship between σ_s and σ_T used in the inelastic electron scattering and our W_0 and W_1. σ_s and σ_T in electron scattering are defined by

$$\sigma_T = \frac{4\pi^2\alpha}{k} \sum_{\substack{\text{all final}\\ \text{states}}} (2\pi)^3 \delta^4(p_f-q-p) \,|\, \langle \text{final states} \,|\, j_x(0) \,|\, p \rangle \,|^2 \tag{46}$$

and

$$\sigma_s = \frac{4\pi^2\alpha}{k} \sum_{\substack{\text{all final}\\ \text{states}}} (2\pi)^3 \delta^4(p_f-q-p) \,|\, \langle \text{final states} \,|\, \epsilon_{11}^\mu \, j_\mu(0) \,|\, p \rangle \,|^2 \tag{47}$$

where $k = (p\cdot q + \frac{1}{2}q^2)/M$ and ϵ_{11} is a four vector with components

$$(\epsilon_{11}^0, \epsilon_{11}^1, \epsilon_{11}^2, \epsilon_{11}^3) = (q_z, 0, 0, q_0)/(-q^2)^{\frac{1}{2}} .$$

The direction of the spatial part of q is chosen as the z-axis. Because of gauge invariance, we have $j_0 = j_z q_z/q_0$ and

$$\epsilon_{11}^\mu j_\mu = (q_z j_0 - q_0 j_z)/(-q^2)^{\frac{1}{2}} = j_z(-q^2)^{\frac{1}{2}}/q_0 . \tag{48}$$

In general $-q^2/q_0^2$ is a very small number. Hence even if the matrix element of j_z is comparable to that of j_x we will obtain a very small value for $R \equiv \sigma_s/\sigma_T$. In other words, the smallness of R in electron scattering does not necessarily reflect the smallness of the matrix element of j_z. It can simply reflect the fact that there is a near cancellation between the scalar

and longitudinal matrix elements due to gauge invariance. In contrast to this there is no such cancellation in the colliding beam experiment because here the scalar component of the current is zero due to gauge invariance.

W_0 and W_1 are not necessarily related to σ_s and σ_T unless they are dominated by the same set of Feynman diagrams such as in the parton model. In hadron physics there are many processes which contribute to the total cross section at high energies. We expect that the diagram which dominates the total cross section in one kinematical region is not necessarily the one which dominates in the other. Hence one can not analytically continue the total cross section from one kinematical region to another. Yet it is interesting that experimentally $\sigma_s/\sigma_T \sim 0$ and $W_0/W_1 \sim 0$ (for large x) and both facts are consistent with a parton interpretation.

4. It is interesting to observe that, in all the expressions for the angular distributions given in this paper [see Eqs. (10), (17), (36), and (37)], the ϕ dependence always occurs in the form $\xi^2 \cos 2\phi$. In processes involving only one time-like photon exchange the origin of this combination can be seen easily from Eqs. (1), (2), (3), (4), and (7), which show that ξ^2 dependent term must be proportional to

$$\xi^2(p_x^2-p_y^2)W_2 = \xi^2 p^2 \sin^2\theta \cos 2\phi \, W_2 \ .$$

Let us try to understand why the combination $\xi^2 \cos 2\phi$ also occurs in the reaction $e^+ + e^- \rightarrow e^+ + e^-$ and $e^+ + e^- \rightarrow 2\gamma$ from some general principles. The absence of polarization effect when only one beam is polarized is due to invariance under time reversal and parity and neglect of radiative corrections as shown in Ref. 3. Thus the effect of polarization must occur bilinearly in $\vec{s}_+$ and $\vec{s}_-$ and the cross section must have the form

$$\sigma(\phi) = A + B(\vec{s}_+ \cdot \vec{s}_-) + C(\vec{s}_+ \cdot \vec{p})(\vec{s}_- \cdot \vec{p}) + D\vec{s}_+ \cdot (\vec{p} \times \vec{p}_e)\vec{s}_- \cdot (\vec{p} \times \vec{p}_e)$$

$$= A + (\vec{s}_+ \cdot \vec{s}_-)(B + Cp^2 \sin^2\theta \cos^2\phi + Dp^2 \sin^2\theta \sin^2\phi) \tag{49}$$

where $\vec{s}_+$ and $\vec{s}_-$ are polarization vectors of initial positrons and electrons, respectively, $\vec{p}$ and $\vec{p}_e$ are momenta of the detected particle and the incident electron, respectively. A, B, C, and D are functions of q^2, $q \cdot p$, and θ.

In all the processes considered the spin dependent parts come only from the states of the initial e^+e^- system of opposite helicity. These are denoted respectively by $|\rightrightarrows\rangle$ and $|\leftleftarrows\rangle$, where the arrows denote the spin directions of the particles, and are characterized by having the z-component of the angular momentum +1 and -1 ,

$$J_z |\rightrightarrows\rangle = |\rightrightarrows\rangle \tag{50}$$

and

$$J_z |\leftleftarrows\rangle = - |\leftleftarrows\rangle \ . \tag{51}$$

The other states, $|\leftrightarrows\rangle$ and $|\rightleftarrows\rangle$, do not contribute to the coefficients B, C, and D in the limit $m_e^2/E^2 \to 0$. These spin antiparallel combinations contribute only to the square of space-like photon exchange diagram in the $e^+ + e^- \to e^- + e^+$ reaction.

Let us consider two transversely polarized states denoted by $|\uparrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ which represent respectively a state with spin of both particles in the x direction and a state with spin of one particle in the x direction and the other in the -x direction. It is easy to show that

$$|\uparrow\uparrow\rangle = \frac{1}{i\sqrt{2}} (|\rightrightarrows\rangle + |\leftleftarrows\rangle) + \ldots \tag{52}$$

and

$$|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}} (|\rightrightarrows\rangle - |\leftleftarrows\rangle) + \ldots \ , \tag{53}$$

where "..." represents the states $|\rightleftarrows>$ and $|\leftrightarrows>$ which do not contribute to the polarization dependent part of the cross section. We next apply the rotation operator $R = \exp\left(\frac{-i\pi}{2} J_z\right)$ on both sides of (52). This yields $R|\uparrow\downarrow> = |\uparrow\uparrow>$, which implies that in Eq. (49) we have

$$\sigma_{\uparrow\uparrow}(\phi) = \sigma_{\uparrow\downarrow}(\phi + \tfrac{\pi}{2}) \,. \tag{54}$$

From this we obtain $B = -\frac{1}{2}(C+D)p^2 \sin^2\theta$ and the desired result follows immediately.

The functional form $\xi^2 \cos 2\phi$ has the properties that it is symmetric with respect to reflections in both the yz and xz planes. Integration of this term with respect to ϕ gives zero. Hence the cross section for unpolarized beams can be obtained from that for an arbitrarily polarized beam by simply taking the ϕ average of the latter.

5. In the parton model one assumes that a pair of on-the-mass-shell partons are first produced by a photon far above the threshold. These high energy partons then decay into pions. When x is large the pion must be emitted almost parallel to the parent parton due to energy momentum conservation. Hence our z' axis must almost coincide with the direction of motion of the parent parton when x is large and thus $W_1 >> W_0$. Since there is no evidence of spin 1/2 particles accompanying each large x event, the parton pair must annihilate each other in the final states. As far as the author knows there is no calculation which demonstrates that the states $J_{z'} = \pm 1$ are actually favored over the state $J_{z'} = 0$ when partons annihilate each other into multipions ($n \geq 3$). (The direction of an energetic pion is defined as the z' axis.)

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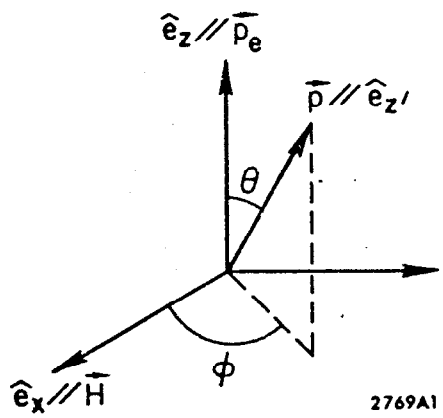

Fig. 1

The coordinate system used to describe the polarization and the angular distribution. The incident electron is moving in the direction $\hat{e}_z$, the magnetic field is pointing toward $\hat{e}_x$, and $\vec{p}$ is the momentum of the detected particle whose direction is chosen as z' axis when dealing with the hadronic matrix element.