SLAC-PUB-1618 August 1975 (T)

## STRONGLY COUPLED FIELDS: II. INTERACTIONS OF THE YUKAWA-TYPE\*

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## ABSTRACT

We investigate explicitly the large coupling limit of the renormalizable interactions of particles of spin 1/2 and spin 0. This is done with a previously formulated theory of strong coupling. The principal result of the investigation is the complete decoupling of spin 1/2 particles from those of spin 0 in the large coupling regime. The applications of our findings to semiclassical models of quark confinement and to Bjorken scaling are discussed.

(Submitted for publication.)

<sup>\*</sup>Work supported by the U.S. Energy Research and Development Administration.

### INTRODUCTION

This paper is a continuation of a previous work (hereafter referred to as A) on the behavior of renormalizable quantum field theory in the large coupling limit. The subject of the present discussion will be the renormalizable interactions of spin zero and spin 1/2 particles, with an eye toward models of quark confinement and Bjorken scaling. Before taking up these matters, let us, in the interest of continuity, begin by reviewing briefly the connection of the discussion here with that of A.

In A, we formulated a general theory of the large coupling limit of renormalizable quantum field theory. The formulation was based upon Feynman's path space<sup>2</sup> approach to quantum field theory. It (the formulation) is related to but significantly different from the formal approach of Hori<sup>3</sup> to the strong coupling limit of quasi-"trilinear" interactions, i.e., interactions of the form  $\phi^*\phi_{\chi}$ , etc. (For a detailed comparison of our approach with that of Hori, see Appendix I.)

What we showed in A is that the connected Green's functions of renormalizable quantum field theory have discussible limits as the coupling tends to infinity. The problem of regularization was also treated in detail and a procedure was introduced which permits the interpretation of the large coupling limit in complete analogy with the familiar small coupling limit. This was illustrated explicitly in the case of the scalar field with quartic self-coupling. This theory of the scalar field with quartic self-coupling will be interpreted by itself in more detail in a later work.<sup>5</sup> In what follows, as we have stated above, we shall consider the renormalizable interactions of this scalar field and its pseudoscalar analogue with fermions.

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A motivation for considering such interactions is the rather recent<sup>4</sup> work on the behavior of such interactions in the so-called semiclassical approximation. More, precisely, it has been argued that, in this approximation, Lagrangian field theories of the type

$$\mathscr{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - H(\phi^2 - f^2)^2 + \overline{\psi}(i \not\partial - G \phi) \psi$$
(1.1)

where  $\phi$  is a scalar field and  $\psi$  is one of spin 1/2, have, for G, H>> 1, a bound state of  $\psi$  surrounded by a meson cloud and that the rest energy of this state is much less than the rest energy of a free  $\psi$  quantum, that is to say, much less than Gf, presuming spontaneously broken symmetry. This result has been put forward as a model of quark confinement.

In the following section, we shall show that upon application of our approach to strong coupling to the theory (1.1), one obtains the rather remarkable result that, as G,  $H \rightarrow \infty$ , the spin 1/2 field decouples completely from the field of spin zero and behaves as a free fermion! The meson field then interacts with itself according to a renormalizable effective generating functional for its connected vertices. We shall exhibit this functional explicitly. The detailed interpretation of this meson system will be taken up elsewhere.<sup>5</sup>

We shall, therefore, find that the semiclassical solutions of Ref. 4 are not characteristic of strict quantum field theory. The problem of quark confinement thus a remains at large in the strict theory of fields. This problem will also be taken up elsewhere.

The decoupling result can also be established for pseudoscalar mesons  $\phi'$ , interacting with  $\psi$  via  $iG\bar{\psi}\gamma_5\psi\phi'$ . This is shown in Section III. Hence, it appears that this decoupling is a general feature of Yukawa-type interactions.

Although quite amazing, our result gives a natural picture of Bjorken scaling in quantum field theory, presuming large effective couplings at large momentum transfer. This picture is in complete agreement with our previous work<sup>6</sup> on the partial differential equations of renormalizable field theory, where we found that Bjorken scaling was quite consistent with a theory of strong interactions for which the origin of coupling constant space is not attractive<sup>7</sup> at large momentum transfer.

### II. INTERACTIONS BETWEEN SCALARS AND FERMIONS

We consider here the theories of the type (1.1)

$$\mathscr{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - H(\phi^2 - f^2)^2 + \overline{\psi}(i \not\partial - G \phi) \psi$$
(2.1)

We shall show that in the limit G, H >> 1, this theory becomes one in which  $\psi$  is a free noninteracting fermion of mass |Gf|. In order to establish this result, we shall use the methods developed in A.

Namely, we first represent, after convention, the generating functional Z for the connected Green's functions of (2.1) as

$$e^{iZ} = \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \mathscr{D}\phi \exp i \int d^{4}x \left[\mathscr{L} + J\phi + \bar{\eta}\psi + \bar{\psi}\eta\right]$$
(2.2)

where J and  $\bar{\eta}$  and  $\eta$  are the respective sources of  $\phi$ ,  $\psi$ , and  $\bar{\psi}$ . As usual, we find it convenient to shift  $\phi$  to  $\phi$ +f to obtain

$$e^{iZ} \equiv \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \mathscr{D}\phi \exp i \int d^{4}x \left\{ 1/2 \left( \partial_{\mu}\phi \partial^{\mu}\phi - 8 \operatorname{Hf}^{2}\phi^{2} \right) - \operatorname{H}\phi^{4} - 4 \operatorname{Hf}\phi^{3} + J(\phi+f) + J(\phi+f) + \bar{\psi} \left[ i \not{\partial} - G(\phi+f) \right] \psi + \bar{\eta}\psi + \bar{\psi}\eta \right\} .$$
(2.3)

The principal idea of A is to exploit the fact that Z has an expansion in negative powers of its couplings which should be meaningful as they (the couplings) become large. This exploitation is to be effected by isolating an appropriate set of expansion operators for the large coupling limit. Recall that for the small coupling limit, the expansion operators are the obvious:

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$$\mathrm{H}\phi^4$$
,  $4\mathrm{Hf}\phi^3$ ,  $\mathrm{G}ar{\psi}\psi\phi$  .

For large coupling, the situation is not so transparent.

Following the discussion in A, we shall rewrite (2.3) with the aid of the following identities:

$$\begin{split} \exp i \int d^{4}x \left[ \bar{\psi} i \partial \psi + \bar{\eta} \psi + \bar{\psi} \eta \right] &= \int \mathscr{D}\psi_{1} \mathscr{D}\bar{\psi}_{1} \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \exp i \int d^{4}x \left[ \bar{\psi}_{1} i \partial \psi_{1} \right. \\ &\quad + \bar{\lambda}(\psi_{1} - \psi) + (\bar{\psi}_{1} - \bar{\psi})\lambda + \bar{\eta}\psi_{1} + \bar{\psi}_{1}\eta \right] \end{split} \tag{2.4a} \\ & \exp i \int d^{4}x \left[ \frac{1}{2} \left( \partial_{\mu} \phi \partial^{\mu} \phi - 8 H f^{2} \phi^{2} \right) - 4 H f \phi^{3} + J(\phi + f) - G \bar{\psi}\psi(\phi + f) \right] \\ &= \int \mathscr{D}\kappa_{1} \mathscr{D}\pi_{1} \mathscr{D}\pi_{2} \mathscr{D}\kappa_{2} \exp i \int d^{4}x \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8 f^{2} H \phi^{2} \right) - 4 H f \kappa_{2}^{3} + J(\kappa_{1} + f) - G \bar{\psi}\psi(\kappa_{2} + f) \right. \\ & \left. + \pi_{1}(\kappa_{1} - \phi) + \pi_{2}(\kappa_{2} - \phi) \right] \end{aligned} \tag{2.4b}$$

$$\exp -i \int d^4 x \ H \phi^4 \equiv \int \mathscr{D}\sigma \ \exp i \int d^4 x \left[ \sigma^2 + 2 \sqrt{H} \ \sigma \phi^2 \right] \quad . \tag{2.4c}$$

Equation (2.3) is thus the same as

$$\begin{split} \exp i \mathbf{Z} &= \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \,\mathscr{D}\phi \,\mathscr{D}\pi_1 \,\mathscr{D}\kappa_1 \,\mathscr{D}\pi_2 \,\mathscr{D}\kappa_2 \,\mathscr{D}\sigma \,\mathscr{D}\psi_1 \,\mathscr{D}\bar{\psi}_1 \,\mathscr{D}\lambda \,\mathscr{D}\bar{\lambda} \\ & \exp i \int d\mathbf{x} \left[ \bar{\psi}_1 \, i \mathscr{D}\psi_1 + \bar{\lambda}(\psi_1 - \psi) + (\bar{\psi}_1 - \bar{\psi})\lambda + \bar{\eta}\psi_1 + \bar{\psi}_1 \eta \right. \\ & \left. + \frac{1}{2} \left( \partial_\mu \kappa_1 \partial^\mu \kappa_1 - 8f^2 H \phi^2 \right) - 4Hf \kappa_2^3 + \sigma^2 + 2\sqrt{H}\sigma \phi^2 \\ & \left. + J(\kappa_1 + f) - G\bar{\psi}\psi(\kappa_2 + f) + \pi_1(\kappa_1 - \phi) + \pi_2(\kappa_2 - \phi) \right] \end{split}$$
(2.5)

The appropriate expansion operators for G,  $H \to \infty\,$  may now be isolated by the shifts

$$\phi \rightarrow \phi + \frac{\pi_1 + \pi_2}{4\sqrt{H}(\sigma - 2f^2\sqrt{H})}$$

$$\psi \rightarrow \psi - \frac{\lambda}{G(\kappa_2 + f)}$$

$$\psi \rightarrow \bar{\psi} - \frac{\bar{\lambda}}{G(\kappa_2 + f)} \qquad (2.7)$$

There results

$$\begin{split} \exp i \mathbf{Z} &\equiv \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \mathscr{D}\phi \, \mathscr{D}\pi_{1} \mathscr{D}\kappa_{1} \mathscr{D}\pi_{2} \mathscr{D}\kappa_{2} \mathscr{D}\sigma \mathscr{D}\psi_{1} \mathscr{D}\bar{\psi}_{1} \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \\ & \exp i \int d^{4}x \Biggl[ \bar{\psi}_{1} i \mathscr{J}\psi_{1} + \bar{\lambda}\psi_{1} + \bar{\psi}_{1}\lambda + \bar{\eta}\psi_{1} + \bar{\psi}_{1}\eta \\ & + \frac{1}{2} \Bigl( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8f^{2} H \phi^{2} \Bigr) - 4Hf\kappa_{2}^{3} \\ & + J(\kappa_{1} + f) - G\bar{\psi}\psi(\kappa_{2} + f) + \pi_{1}\kappa_{1} + \pi_{2}\kappa_{2} + \sigma^{2} \\ & + 2\sqrt{H}\sigma \phi^{2} + \frac{\bar{\lambda}\lambda}{G(\kappa_{2} + f)} - \frac{(\pi_{1} + \pi_{2})^{2}}{8\sqrt{H}(\sigma - 2f^{2}\sqrt{H})} \Biggr] \quad . \end{split}$$
(2.8)

The integral over  $\mathscr{D}\psi \mathscr{D}\overline{\psi}$  can now be done. We find (see Appendix II)

$$\int \mathcal{D}\psi \,\mathcal{D}\bar{\psi} \,\exp \,-\mathrm{i}\int \mathrm{d}^4 x \,\,\mathrm{G}(\kappa_2 + \mathrm{f})\bar{\psi}\psi \,\equiv \,\exp \,+\, \frac{\int \mathrm{d}^4 x}{\Delta x} \,\,\log \,\mathrm{G}^4(\kappa_2 + \mathrm{f})^4 \tag{2.9}$$

where  $\Delta x$  is the measure of each set in an appropriate uniform covering of spacetime and is absorbed in any systematic renormalization of the theory. This point about renormalization has been discussed in A and will be demonstrated more explicitly elsewhere.<sup>5</sup> We need not be concerned with it for our purposes here.

Upon introducing (2.9) into (2.8) and expanding in powers of 1/G and  $1/\sqrt{H}$  we find

$$\begin{split} \exp i \mathbf{Z} &= \sum_{\mathbf{n},\mathbf{m}} \frac{1}{\mathbf{n}! \mathbf{m}!} \left( \frac{\mathbf{i}!}{\mathbf{G}} \right)^{\mathbf{m}} \left( \frac{-\mathbf{i}}{8\sqrt{\mathbf{H}}} \right)^{\mathbf{n}} \prod_{j=1}^{\mathbf{m}} \int d^{4}\mathbf{x}_{j} \frac{\delta^{2}}{\delta \eta_{1}(\mathbf{x}_{j}) \delta \tilde{\eta}_{1}(\mathbf{x}_{j})} \\ &= \frac{\int_{-\infty}^{\infty} d\beta_{j}}{(\beta_{j}+\mathbf{f}-\mathbf{i}\cdot\mathbf{c})} \frac{\int_{-\infty}^{\infty} d\alpha_{j}}{2\pi} \int d^{4}\mathbf{y}_{\ell} \int_{-\infty}^{\infty} d\mu_{\ell} \mu_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{(\nu_{\ell}-2\mathbf{f}^{2}\sqrt{\mathbf{H}+\mathbf{i}\cdot\mathbf{c}})} \\ &= \frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \int \mathscr{D}\psi_{1} \mathscr{D}\bar{\psi}_{1} \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \mathscr{D}\kappa_{1} \mathscr{D}\kappa_{2} \mathscr{D}\pi_{1} \mathscr{D}\pi_{2} \mathscr{D}\sigma \mathscr{D}\phi \\ &= \exp\left\{ \mathbf{i} \int d^{4}\mathbf{x} \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8\mathbf{f}^{2}\mathbf{H}\phi^{2} \right) - 4\mathbf{f}\mathbf{H}\kappa_{2}^{3} \\ &+ J(\kappa_{1}+\mathbf{f}) + \sigma^{2} + 2 \sqrt{\mathbf{H}} \sigma\phi^{2} - \frac{\mathbf{i}}{\Delta \mathbf{x}} \log \mathbf{G}^{4}(\kappa_{2}+\mathbf{f})^{4} \\ &+ \pi_{1}\kappa_{1} + \pi_{2}\kappa_{2} + \bar{\psi}_{1}\mathbf{i}\vartheta\psi_{1} + \bar{\eta}\psi_{1} + \bar{\psi}_{1}\eta + \bar{\lambda}(\psi_{1}+\eta_{1}) \\ &+ (\bar{\psi}_{1}+\bar{\eta}_{1})\lambda \right] + \mathbf{i} \sum \alpha_{j}(\beta_{j}-\kappa_{2}(\mathbf{x}_{j})) \\ &+ \mathbf{i} \sum \rho_{\ell}(\mu_{\ell}-\pi_{1}(\mathbf{y}_{\ell})-\pi_{2}(\mathbf{y}_{\ell})) \\ &+ \mathbf{i} \sum \xi_{\ell}(\mathbf{v}_{\ell}-\sigma(\mathbf{y}_{\ell})) \right\} \Big|_{\eta_{1}, \eta_{1}=0} . \end{split}$$

$$(2.10)$$

Effecting the integrals over  $\kappa_2$  and  $\pi_2$ , we see that at the m-th order in 1/G and n-th order in  $1/\sqrt{H}$  we have the ratio

$$\mathbf{R} = \frac{\int \mathscr{D}\kappa_{2}\mathscr{D}\pi_{2} \exp i\left\{\int d^{4}x \left[\pi_{2}\kappa_{2} \cdot \frac{i}{\Delta x} \log \mathbf{G}^{4}(\kappa_{2}+\mathbf{f})^{4}-4\mathbf{f}\mathbf{H}\kappa_{2}^{3}\right] - \sum_{i}\alpha_{j}\kappa_{2}(\mathbf{x}_{j}) - \sum_{\ell}\rho_{\ell}\pi_{2}(\mathbf{y}_{\ell})}{\int \mathscr{D}\kappa_{2}\mathscr{D}\pi_{2} \exp i \int d^{4}x \left[\pi_{2}\kappa_{2} \cdot \frac{i}{\Delta x} \log \mathbf{G}^{4}(\kappa_{2}+\mathbf{f})^{4}-4\mathbf{f}\mathbf{H}\kappa_{2}^{3}\right]}$$
$$= \prod_{\ell=1}^{m} \left(\frac{\rho_{\ell}/\Delta x + \mathbf{f}}{\mathbf{f}}\right)^{+4} \exp -i4\mathbf{f}\mathbf{H}\rho_{\ell}^{3}/(\Delta x)^{2} \quad , \qquad (2.11)$$

almost everywhere. (See Appendix III.) The RHS of this last equation is independent of  $\{x_j\}$  and  $\{\alpha_j\}$ . This permits the remaining integrals over  $\{\alpha_j\}$  and  $\{\beta_j\}$  to be done trivially. There results

$$\begin{split} \exp i\mathbf{Z} &= \sum_{\mathbf{n}, \mathbf{m}=0}^{\infty} \frac{1}{\mathbf{m}!\mathbf{n}!} \left(\frac{+\mathbf{i}}{\mathbf{Gf}}\right)^{\mathbf{m}} \left(\frac{-\mathbf{i}}{8\sqrt{\mathbf{H}}}\right)^{\mathbf{n}} \prod_{j=1}^{\mathbf{n}} \prod_{\ell=1}^{\mathbf{n}} \int d\mathbf{x}_{j} \frac{\delta^{2}}{\delta\eta_{1}(\mathbf{x}_{j})\delta\bar{\eta}(\mathbf{x}_{j})} \\ &\int d^{4}\mathbf{y}_{\ell} \int_{-\infty}^{\infty} d\mu_{\ell} \mu_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \left(\frac{\rho_{\ell}/\Delta\mathbf{x}+\mathbf{f}}{\mathbf{f}}\right)^{\mathbf{+}4} \frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left(\nu_{\ell}-2\mathbf{f}^{2}\sqrt{\mathbf{H}}+\mathbf{i}\epsilon\right)} \\ &\frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \int \mathcal{D}\psi_{1} \mathcal{D}\overline{\psi}_{1} \mathcal{D}\lambda \mathcal{D}\overline{\lambda} \mathcal{D}\kappa_{1} \mathcal{D}\pi_{1} \mathcal{D}\sigma \mathcal{D}\phi \\ &\exp i\left\{\int d^{4}\mathbf{x} \left[\frac{1}{2}\left(\partial_{\mu}\kappa_{1}\partial^{\mu}\kappa_{1}-8\mathbf{f}^{2}\mathbf{H}\phi^{2}\right)+\mathbf{J}(\kappa_{1}+\mathbf{f})+\sigma^{2}\right. \\ &+ 2\sqrt{\mathbf{H}} \sigma\phi^{2}+\pi_{1}\kappa_{1}+\bar{\psi}_{1}\mathbf{i}\phi\psi_{1}+\bar{\eta}\psi_{1}+\bar{\psi}_{1}\eta \\ &+ \bar{\lambda}(\psi_{1}+\eta_{1})+(\bar{\psi}_{1}+\bar{\eta}_{1})\lambda\right] - 4\mathbf{H} \sum_{\ell} \rho_{\ell}^{3}/(\Delta\mathbf{x})^{2} \\ &+ \sum_{\ell} \rho_{\ell}(\mu_{\ell}-\pi_{1}(\mathbf{y}_{\ell})) + \sum_{\ell} \xi_{\ell}(\nu_{\ell}-\sigma(\mathbf{y}_{\ell}))\right\}\Big|_{\eta_{1}=0=\bar{\eta}_{1}} . \end{split}$$

We may next sum easily over all orders in 1/Gf, obtaining

$$\begin{split} \exp i\mathbf{Z} &\equiv \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-i}{8\sqrt{H}}\right)^{n} \prod_{\ell=1}^{n} \int d^{4}\mathbf{y}_{\ell} \int_{-\infty}^{\infty} d\boldsymbol{\mu}_{\ell} \, \boldsymbol{\mu}_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\boldsymbol{\rho}_{\ell}}{2\pi} \\ &\left(\frac{f}{\rho_{\ell}/\Delta \mathbf{x} + \mathbf{f} - \mathbf{i} \, \epsilon}\right)^{-4} \frac{\int_{-\infty}^{\infty} d\boldsymbol{\nu}_{\ell}}{\left(\boldsymbol{\nu}_{\ell} - 2\mathbf{f}^{2}\sqrt{\mathbf{H}} + \mathbf{i} \epsilon\right)} \frac{\int_{-\infty}^{\infty} d\boldsymbol{\xi}_{\ell}}{2\pi} \\ &\int \mathcal{D}\psi_{1} \mathcal{D}\bar{\psi}_{1} \mathcal{D}\lambda \mathcal{D}\bar{\lambda} \, \mathcal{D}\bar{\lambda} \, \mathcal{D}\kappa_{1} \, \mathcal{D}\pi_{1} \mathcal{D}\sigma \, \mathcal{D}\phi \quad \exp \mathbf{i} \left\{\int d^{4}\mathbf{x} \left[\frac{1}{2} \left(\partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8\mathbf{f}^{2}\mathbf{H}\phi^{2}\right) \right. \right. \\ &\left. + \mathbf{J}(\kappa_{1} + \mathbf{f}) + \sigma^{2} + 2 \, \sqrt{\mathbf{H}} \, \sigma\phi^{2} \\ &\left. + \pi_{1}\kappa_{1} + \bar{\psi}_{1}\mathbf{i}\partial\psi_{1} + \bar{\eta}\psi_{1} + \bar{\psi}_{1}\eta + \bar{\lambda}\psi_{1} + \bar{\psi}_{1}\lambda + \frac{\bar{\lambda}\lambda}{\mathrm{Gf}} \right] \\ &\left. + \sum_{\ell} \rho_{\ell}(\boldsymbol{\mu}_{\ell} - \pi_{1}(\mathbf{y}_{\ell})) + \sum_{\ell} \xi_{\ell}(\boldsymbol{\nu}_{\ell} - \sigma(\mathbf{y}_{\ell})) - 4\mathbf{f}\mathbf{H} \sum_{\ell} \rho_{\ell}^{3}/(\Delta \mathbf{x})^{2} \right\} . \end{split}$$
(2.13)

Finally, we may effect the integrals over  $\psi_1$ ,  $\overline{\psi_1}$ ,  $\lambda$  and  $\overline{\lambda}$  by the usual shifts: We first shift

$$\psi_{1} \rightarrow \psi_{1} - \int dy \ S_{F}(x-y) (\eta+\lambda) \ (y)$$

$$\bar{\psi}_{1} \rightarrow \bar{\psi}_{1} - \int dy (\bar{\eta}+\bar{\lambda}) \ (y) \ S_{F}(y-x) \quad , \qquad (2.14)$$

$$i \not = S_{F}(x-y) = \delta(x-y)$$
 . (2.15)

This gives

$$\begin{split} \mathbf{K}_{\mathbf{F}} &\equiv \int \mathscr{D}\psi_{1} \mathscr{D}\bar{\psi}_{1} \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \ \exp \,\mathbf{i} \int \mathrm{d}^{4}\mathbf{x} \left[ \bar{\psi}_{1} \mathbf{i} \mathscr{D}\psi_{1} + (\bar{\eta} + \bar{\lambda})\psi_{1} + \bar{\psi}_{1}(\eta + \lambda) + \frac{\bar{\lambda}\lambda}{\mathrm{Gf}} \right] \\ &\equiv \int \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \ \exp \,\mathbf{i} \int \mathrm{d}^{4}\mathbf{x} \left[ -\int \mathrm{d}^{4}\mathbf{y}(\bar{\eta} + \bar{\lambda}) \left( \mathbf{x} \right) \ \mathbf{S}_{\mathbf{F}} \left( \mathbf{x} - \mathbf{y} \right) (\eta + \lambda) \left( \mathbf{y} \right) + \frac{\bar{\lambda}\lambda}{\mathrm{Gf}} \right] \quad . \end{split}$$
(2.16)

From the simple shifts

$$\lambda \to \lambda - \eta$$
 ,  $\bar{\lambda} \to \bar{\lambda} - \bar{\eta}$  (2.17)

we then obtain

$$\begin{split} \mathbf{K}_{\mathrm{F}} &= \int \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \,\exp\,\mathbf{i}\int \mathrm{d}^{4}\mathbf{x} \left[ -\int \mathrm{d}\mathbf{y}\,\bar{\lambda}(\mathbf{x})\,\mathbf{S}_{\mathrm{F}}(\mathbf{x}-\mathbf{y})\lambda(\mathbf{y}) + \frac{\bar{\lambda}\lambda - \bar{\eta}\lambda - \bar{\lambda}\eta + \bar{\eta}\,\eta}{\mathrm{Gf}} \right] \\ &= \int \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \,\exp\,\mathbf{i} \frac{\int \mathrm{d}^{4}\mathbf{k}}{(2\pi)^{4}} \left[ -\bar{\lambda}(-\mathbf{k})\,\frac{1}{\mathbf{k}+\mathbf{i}\,\epsilon}\,\lambda(\mathbf{k}) + (\bar{\lambda}(-\mathbf{k})\,\lambda(\mathbf{k}) - \bar{\eta}(-\mathbf{k})\,\lambda(\mathbf{k}) - \bar{\lambda}(-\mathbf{k})\eta(\mathbf{k}) \right. \\ &+ \left. \bar{\eta}(-\mathbf{k})\,\eta(\mathbf{k}) \right) / \mathrm{Gf} \right] \quad, \end{split}$$
(2.18)

where we have passed into momentum space

$$\eta(\mathbf{x}) \equiv \frac{\int d^4 \mathbf{k}}{(2\pi)^4} e^{-i\mathbf{k}\cdot\mathbf{x}} \eta(\mathbf{k})$$

$$\bar{\eta}(\mathbf{x}) \equiv \frac{\int d^4 \mathbf{k}}{(2\pi)^4} e^{-i\mathbf{k}\cdot\mathbf{x}} \bar{\eta}(\mathbf{k}) \qquad (2.19)$$

Therefore, the substitutions

$$\lambda(\mathbf{k}) \to \lambda(\mathbf{k}) + \frac{1}{\mathrm{Gf}} \left( \frac{1}{\mathrm{Gf}} - \frac{1}{\mathbf{k} + \mathbf{i} \epsilon} \right)^{-1} \eta(\mathbf{k})$$
  
$$\bar{\lambda}(\mathbf{k}) \to \bar{\lambda}(\mathbf{k}) + \frac{\bar{\eta}(\mathbf{k})}{\mathrm{Gf}} \left( \frac{1}{\mathrm{Gf}} - \frac{1}{\mathbf{k} + \mathbf{i} \epsilon} \right)^{-1}$$
(2.20)

allow us to write

$$K_{\rm F} \equiv \exp i \frac{\int d^4 k}{(2\pi)^4} \left[ -\frac{\bar{\eta} (-k)}{G^2 f^2} \left( \frac{1}{Gf} - \frac{1}{k+i\epsilon} \right)^{-1} \eta(k) + \frac{\bar{\eta} (-k) \eta(k)}{Gf} \right]$$
$$= \exp -i \frac{\int d^4 k}{(2\pi)^4} \bar{\eta}(-k) \frac{1}{k-Gf+i\epsilon} \eta(k) \quad .$$
(2.21)

The fundamental result of this communication is thus

$$\begin{split} \exp i\mathbf{Z} &\equiv \left[ \exp -i \frac{\int d^{4}k}{(2\pi)^{4}} \,\bar{\eta}(-k) \frac{1}{k'-Gf+i\epsilon} \,\eta(k) \right] \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{8\sqrt{H}} \right)^{n} \\ &= \prod_{\ell=1}^{n} \int d^{4}y_{\ell} \,\int_{-\infty}^{\infty} d\mu_{\ell} \,\mu_{\ell}^{2} \,\frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \left( \frac{f}{\rho_{\ell}/\Delta x + f-i\epsilon} \right)^{-4} \,\frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left( \nu_{\ell} - 2f^{2}\sqrt{H} + i\epsilon \right)} \\ &= \frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \,\int \mathscr{D}\kappa_{1} \,\mathscr{D}\pi_{1} \,\mathscr{D}\sigma \,\mathscr{D}\phi \,\exp\left\{ i \int d^{4}x \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8f^{2}H\phi^{2} \right) \right. \right. \\ &+ J(\kappa_{1}+f) + \sigma^{2} + 2\sqrt{H}\sigma\phi^{2} + \pi_{1}\kappa_{1} \right] + i \sum_{\ell} \rho_{\ell}(\mu_{\ell} - \pi_{1}(y_{\ell})) \\ &+ i \sum_{\ell} \xi_{\ell}(\nu_{\ell} - \sigma(y_{\ell})) - i \,4fH \,\sum_{\ell} \rho_{\ell}^{3}/(\Delta x)^{2} \right\} \,. \end{split}$$

$$(2.22)$$

From (2.22), it is obvious that the large coupling limit of the theory (2.1) consists of a free fermion of mass Gf and a system of mesons interacting renormalizably in a manner which corresponds to an effective generating functional  $Z_{eff}(J)$  given by

$$\begin{split} \exp \mathrm{iZ}_{\mathrm{eff}}(\mathbf{J}) &= \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{-\mathrm{i}}{8\sqrt{\mathrm{H}}} \right)^{n} \prod_{\ell=1}^{n} \int \mathrm{d}^{4} \mathbf{y}_{\ell} \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\mu}_{\ell} \, \boldsymbol{\mu}_{\ell}^{2} \frac{\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}_{\ell}}{2\pi} \left( \frac{\mathrm{f}}{\rho_{\ell}/\Delta \mathbf{x} + \mathrm{f} - \mathrm{i}\,\epsilon} \right)^{-4} \\ &= \frac{\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\nu}_{\ell}}{\left( \boldsymbol{\nu}_{\ell} - 2\mathrm{f}^{2}\sqrt{\mathrm{H}} + \mathrm{i}\,\epsilon \right)} \frac{\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\xi}_{\ell}}{2\pi} \int \mathcal{D}_{\kappa_{1}} \mathcal{D}_{\pi_{1}} \mathcal{D}_{\sigma} \mathcal{D}_{\phi} \\ &= \exp \left\{ \mathrm{i} \int \mathrm{d}^{4} \mathbf{x} \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - 8\mathrm{f}^{2} \mathrm{H}\phi^{2} \right) + \mathrm{J}(\kappa_{1} + \mathrm{f}) + \sigma^{2} + 2\sqrt{\mathrm{H}}\sigma\phi^{2} + \pi_{1}\kappa_{1} \right] \\ &\quad + \mathrm{i} \sum \rho_{\ell} (\boldsymbol{\mu}_{\ell} - \pi_{1}(\mathbf{y}_{\ell})) + \mathrm{i} \sum \boldsymbol{\xi}_{\ell} (\boldsymbol{\nu}_{\ell} - \sigma(\mathbf{y}_{\ell})) - 4\mathrm{i}\,\mathrm{fH} \sum \rho_{\ell}^{3}/(\Delta \mathbf{x})^{2} \right\} \end{split}$$

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$$=\sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-i}{8\sqrt{H}}\right)^{n} \prod_{\ell=1}^{n} \int d^{4}y_{\ell} \int_{-\infty}^{\infty} d\mu_{\ell} \mu_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \left(\frac{f}{\rho_{\ell}/\Delta x + f - i\epsilon}\right)^{-4}$$

$$=\frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left(\nu_{\ell}-2f^{2}\sqrt{H}+i\epsilon\right)} \frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \left(\frac{2i^{3/4}(\Delta x)^{1/4}\sqrt{\pi/\xi_{\ell}}}{\Gamma(1/4)}\right)$$

$$=\exp i \left[\sum_{\ell} \left(\xi_{\ell}\nu_{\ell} + \rho_{\ell}(\mu_{\ell}+J(y_{\ell})) - 4f H\rho_{\ell}^{3}/(\Delta x)^{2}\right) - \frac{1}{4}\sum_{\ell,\ell}\xi_{\ell}\xi_{\ell}, \delta(y_{\ell}-y_{\ell})\right]$$

$$+\frac{1}{2}\sum_{\ell,\ell}\rho_{\ell}\rho_{\ell}, \frac{\int d^{4}k}{(2\pi)^{4}} k^{2} \cos k \cdot (y_{\ell}-y_{\ell}) + \int dx Jf\right], \qquad (2.23)$$

where in making this last step we have used the results of A. The sole effect of  $\bar{\psi}\psi\phi$  in the presence of  $(\phi^2-f^2)^2$  is to modify this last interaction to the form (2.23). Equation (2.23) should be compared <sup>5</sup> with the result (3.21) of A for the theory

$$\mathscr{L} = \frac{1}{2} \left( \partial_{\mu} \phi \partial^{\mu} \phi - m^{2} \phi^{2} \right) - g \phi^{4} \quad .$$
 (2.24)

To repeat, the fermion is free in the large coupling limit of (2.1).

#### III. INTERACTIONS OF PSEUDOSCALARS AND FERMIONS

We turn next to theories of the type

$$\mathscr{L} = \frac{1}{2} \left( \partial_{\mu} \phi \partial^{\mu} \phi - m_{\phi}^{2} \phi^{2} \right) - H \phi^{4} + \bar{\psi} (i \partial - m_{\psi}) \psi - i G \bar{\psi} \gamma_{5} \psi \phi \quad .$$
(3.1)

Again, we shall be able to establish the complete decoupling of  $\psi$  from  $\phi$  as H, G  $\rightarrow\infty$  .

Our starting point is still the path integral expression for the generating functional for connected Green's functions

$$\exp i \mathbf{Z} = \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}\phi \quad \exp i \int d^4 x \left[ \mathcal{L} + J\phi + \bar{\eta}\psi + \bar{\psi}\eta \right] \quad . \tag{3.2}$$

We shall proceed here in precise analogy with the discussion in Section II.

Namely, from the analogues of the identities (2.4) we obtain

$$\begin{split} \exp \mathrm{iZ} &= \int \mathscr{D}\psi \, \mathscr{D}\bar{\psi} \, \mathscr{D}\phi \, \mathscr{D}\kappa_1 \, \mathscr{D}\pi_1 \, \mathscr{D}\kappa_2 \, \mathscr{D}\pi_2 \, \mathscr{D}\sigma \, \mathscr{D}\psi_1 \, \mathscr{D}\bar{\psi}_1 \, \mathscr{D}\lambda \, \mathscr{D}\bar{\lambda} \\ & \exp \mathrm{i} \int \mathrm{d}^4 \mathrm{x} \left[ \frac{1}{2} \left( \partial_{\mu} \kappa_1 \partial^{\mu} \kappa_1 - \mathrm{m}_{\phi}^2 \phi^2 \right) + \sigma^2 + 2\sqrt{\mathrm{H}} \phi^2 \sigma \right. \\ & + \mathrm{J}\kappa_1 + \bar{\psi}_1 (\mathrm{i} \mathscr{D} - \mathrm{m}_{\psi}) \psi_1 + \bar{\lambda} (\psi_1 - \psi) + (\bar{\psi}_1 - \bar{\psi}) \lambda \\ & + \bar{\psi}_1 \eta + \bar{\eta} \psi_1 - \mathrm{i} \mathrm{G} \bar{\psi} \gamma_5 \psi \kappa_2 + \pi_2 (\kappa_2 - \phi) + \pi_1 (\kappa_1 - \phi) \right] \,. \end{split}$$
(3.3)

From the shifts

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$$\phi \rightarrow \phi + \frac{\pi_1 + \pi_2}{4\sqrt{H} \left(\sigma - m_{\phi}^2 / 4\sqrt{H}\right)}$$

$$\psi \rightarrow \psi + i\gamma_5 \lambda / G\kappa_2 \qquad (3.4)$$

$$\bar{\psi} \rightarrow \bar{\psi} + i\bar{\lambda}\gamma_5 / G\kappa_2$$

we obtain

It can be shown<sup>3</sup> that

$$\int \mathscr{D}\psi \,\mathscr{D}\bar{\psi} \, \exp i \int d^4x \, (-iG\bar{\psi}\gamma_5\psi\kappa_2) \equiv \exp + \frac{\int d^4x}{\Delta x} \log G^4\kappa_2^4 \quad . \tag{3.6}$$

(See Appendix II.) Hence, following the arguments leading from (2.10) to (2.13) we obtain

$$\begin{split} \exp i \mathbb{Z} &= \sum_{\mathbf{m},\mathbf{n}} \frac{1}{\mathbf{m}! \mathbf{n}!} \left( \frac{1}{G} \right)^{\mathbf{m}} \left( \frac{-i}{8\sqrt{H}} \right)^{\mathbf{n}} \prod_{j=1}^{\mathbf{m}} \prod_{\ell=1}^{n} \int d^{4}\mathbf{x}_{j} \frac{\delta}{\delta\eta(\mathbf{x}_{j})} \gamma_{5} \frac{\delta}{\delta\overline{\eta}(\mathbf{x}_{j})} \\ &= \frac{\int d\beta_{j}}{\beta_{j}-i\overline{\epsilon}} \frac{\int_{-\infty}^{\infty} d\alpha}{2\pi} \int d^{4}\mathbf{y}_{\ell} \int_{-\infty}^{\infty} d\mu_{\ell} \mu_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left(\nu_{\ell}-\mathbf{m}_{\phi}^{2}/4\sqrt{H}\right)} \\ &= \frac{\int_{-\infty}^{\infty} d\xi_{j}}{2\pi} \int \mathscr{D}\psi_{1} \mathscr{D}\overline{\psi}_{1} \mathscr{D}\lambda \mathscr{D}\lambda \mathscr{D}\kappa_{1} \mathscr{D}\kappa_{2} \mathscr{D}\pi_{1} \mathscr{D}\pi_{2} \mathscr{D}\sigma \mathscr{D}\phi \\ &= \exp i \left\{ \int d^{4}\mathbf{x} \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - \mathbf{m}_{\phi}^{2} \phi^{2} \right) + \mathbf{J}\kappa_{1} + \sigma^{2} + 2\sqrt{H}\sigma\phi^{2} \right. \\ &\quad - \frac{i}{\Delta \mathbf{x}} \log \mathbf{G}^{4} (\kappa_{2}-i\overline{\epsilon})^{4} + \pi_{1}\kappa_{1} + \pi_{2}\kappa_{2} \\ &\quad + \overline{\psi}_{1}(i \mathscr{D}-\mathbf{m}_{\psi})\psi_{1} + \overline{\lambda}\psi_{1} + \overline{\psi}_{1}\lambda + \overline{\psi}_{1}\eta + \overline{\eta}\psi_{1} + \overline{\eta}_{1}\lambda + \overline{\lambda}\eta_{1} \right] \\ &\quad + \sum \alpha_{j}(\beta_{j}-\kappa_{2}(\mathbf{x}_{j})) + \sum \rho_{\ell}(\mu_{\ell}-\pi_{1}(\mathbf{y}_{\ell}) - \pi_{2}(\mathbf{y}_{\ell})) \\ &\quad + i \sum \xi_{\ell}(\nu_{\ell}-\sigma(\mathbf{y}_{\ell})) \right|_{\eta_{1}=0=\overline{\eta}_{1}} . \end{split}$$

At the m-th order in 1/G and the n-th order in  $1/\sqrt{H}$  we have the ratio

$$\frac{\int \mathscr{D}\kappa_{2} \mathscr{D}\pi_{2} \exp i\left\{\int d^{4}x \left[\pi_{2}\kappa_{2} - \frac{i}{\Delta x} \log(G(\kappa_{2} - i\tilde{\epsilon}))^{4}\right] - \sum \alpha_{j}\kappa_{2}(x_{j}) - \sum \rho_{\ell}\pi_{2}(y_{\ell})\right\}}{\int \mathscr{D}\kappa_{2} \mathscr{D}\pi_{2} \exp i \int d^{4}k \left[\pi_{2}\kappa_{2} - \frac{i}{\Delta x} \log(G(\kappa_{2} - i\tilde{\epsilon}))^{4}\right]} = \prod_{\ell=1}^{m} \frac{(i\tilde{\epsilon})^{-4}}{\left(\rho_{\ell}/\Delta x - i\tilde{\epsilon}\right)^{-4}} , \text{ almost everywhere } .$$
(3.8)

Inserting this into (3.7) and summing through all orders in 1/G, we find

The integrals over  $\left\{\lambda,\bar{\lambda}\right\}$  can now be effected by the shifts

$$\begin{split} \lambda &\to \lambda - \overline{\epsilon} \ \mathrm{G}\gamma_5 \psi_1 \\ \bar{\lambda} &\to \bar{\lambda} - \overline{\epsilon} \ \mathrm{G}\bar{\psi}_1 \gamma_5 \end{split} \tag{3.10}$$

We find

$$\int \mathscr{D}\lambda \mathscr{D}\bar{\lambda} \exp i \int d^4x \left[ \frac{\bar{\lambda}\gamma_5 \lambda}{\bar{\epsilon}G} + \bar{\lambda}\psi_1 + \bar{\psi}_1 \lambda \right]$$
  
$$\equiv \exp -i \int d^4x \ \bar{\epsilon} \ G\bar{\psi}_1 \gamma_5 \psi_1 \quad . \tag{3.11}$$

Introducing (3.11) into (3.9), we have

$$\begin{split} \exp i \mathbf{Z} &= \int \mathscr{D}\psi_{1} \mathscr{D}\overline{\psi}_{1} \exp i \int d^{4}x \left[ \overline{\psi}_{1} (i \mathscr{J} - \mathbf{m}_{\psi} - \overline{\epsilon} \mathbf{G} \gamma_{5}) \psi_{1} + \overline{\eta} \psi_{1} + \overline{\psi}_{1} \eta + \overline{\psi}_{1} \eta \right] \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{-i}{4\sqrt{H}} \right)^{n} \prod_{\ell=1}^{n} \int d^{4}y_{\ell} \int_{-\infty}^{\infty} d\mu_{\ell} \mu_{\ell}^{2} \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \left( \frac{i\overline{\epsilon}}{\rho_{\ell} / \Delta x - i\overline{\epsilon}} \right)^{-4} \\ &= \frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left( \nu_{\ell} - \mathbf{m}_{\phi}^{2} / 4\sqrt{H} \right)} \frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \int \mathscr{D}\kappa_{1} \mathscr{D}\pi_{1} \mathscr{D}\sigma \mathscr{D}\phi \\ &= \exp i \left\{ \int d^{4}x \left[ \frac{1}{2} \left( \partial_{\mu}\kappa_{1} \partial^{\mu}\kappa_{1} - \mathbf{m}_{\phi}^{2} \phi^{2} \right) + J\kappa_{1} + \sigma^{2} + 2\sqrt{H} \sigma \phi^{2} + \pi_{1}\kappa_{1} \right] \\ &+ \sum \rho_{\ell} (\mu_{\ell} - \pi_{1}(y_{\ell})) + \sum \xi_{\ell} (\nu_{\ell} - \sigma(y_{\ell})) \right\} . \end{split}$$
(3.12)

Taking the limit  $\bar{\epsilon} \downarrow 0$  in the first factor we see that the fermion functional is just that of a free spin 1/2 particle. Furthermore, the integrals over  $\kappa_1$ ,  $\pi_1$ ,  $\sigma$  and  $\phi$  may be done in the same way that they were effected in Eq. (2.23). We therefore have

$$\begin{split} \exp i \mathbf{Z} &= \left[ \exp -i \frac{\int_{\mathbf{d}}^{\mathbf{d}} \mathbf{k}}{(2\pi)^{4}} \, \bar{\eta}(-\mathbf{k}) \, \frac{1}{\mathbf{k} - \mathbf{m}_{\mathbf{f}}^{\mathbf{+}\mathbf{i}\,\epsilon}} \, \eta(\mathbf{k}) \right] \\ &\qquad \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{-i}{4\sqrt{\mathbf{H}}} \right)^{n} \prod_{\ell=1}^{n} \int_{\mathbf{d}}^{\mathbf{d}} \mathbf{y}_{\ell} \, \int_{-\infty}^{\infty} d\mu_{\ell} \, \mu_{\ell}^{2} \, \frac{\int_{-\infty}^{\infty} d\rho_{\ell}}{2\pi} \left( \frac{\bar{\epsilon}}{\rho_{\ell}/\Delta \mathbf{x} - i\bar{\epsilon}} \right)^{-4} \\ &\qquad \frac{\int_{-\infty}^{\infty} d\nu_{\ell}}{\left( \nu_{\ell} - \mathbf{m}_{\phi}^{2}/4\sqrt{\mathbf{H}} \right)} \, \frac{\int_{-\infty}^{\infty} d\xi_{\ell}}{2\pi} \left( \frac{i^{3/4} (\Delta \mathbf{x})^{1/4} \, \sqrt{\pi/\xi_{\ell}}}{\Gamma(1/4)} \right) \\ &\qquad \exp i \left[ \sum_{\ell=1}^{n} \left\{ \xi_{\ell} \nu_{\ell} + \rho_{\ell} (\mu_{\ell} + \mathbf{J}(\mathbf{y}_{\ell})) \right\} - \frac{1}{4} \, \sum_{\ell, \ell'=1}^{n} \, \xi_{\ell} \xi_{\ell'} \delta(\mathbf{y}_{\ell} - \mathbf{y}_{\ell'}) \\ &\qquad + \frac{1}{2} \, \sum_{\ell, \ell'=1}^{n} \, \rho_{\ell} \rho_{\ell'} \, \frac{d^{4} \mathbf{k}}{(2\pi)^{4}} \, \mathbf{k}^{2} \, \cos \, \mathbf{k} \cdot (\mathbf{y}_{\ell} - \mathbf{y}_{\ell'}) \right] \, . \end{split}$$
(3.13)

Again, the spin 1/2 particle completely decouples from the meson and behaves as a free particle. This is the desired result.

### IV. DISCUSSION

The detailed discussion of the effective meson systems was not presented here in the interest of clarity. These systems will be discussed in detail in one of the later works, with an eye toward a better understanding of the so-called "kink"<sup>8</sup> mode solution of the respective classical equations in the context of the strict theory of quantum fields.

The results above, which were obtained by use of the path-space approach to quantum field theory, suggest that in the operator approach there exists a canonical transformation of the field variables which, at least for large couplings, makes the decoupling of spin 1/2 and spin 0 fields manifest. We have not been able to find such a transformation.

The question naturally arises as to the validity of the decoupling result for small values of coupling. This question can not be answered on the basis of the work above, because our expansions in inverse powers of coupling are only valid as asymptotic expansions when the couplings are large. Just how large is "large" is unknown to us.

The lack of a "bag" state in the Yukawa model is in agreement with the work of Sawyer<sup>9</sup> in two Minkowsky dimensions, where he has found that, in a certain approximation, the quantum corrections to the vacuum essentially undo the lowering in energy of the one fermion "bag" state relative to the free fermion state.

Finally, we should remark that the other large confinement project, that of Chodos <u>et al.</u>,  $^{10}$  is known<sup>11</sup> to be intimately related to the approach of Ref. 4. Indeed, most aspects of the model of Ref. 10 can be shown<sup>11</sup> to be obtainable

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as a certain variant of the approach of Ref. 4. From this fact it follows that our decoupling result would tend to indicate that the model of Ref. 10 is not stable against strict second quantization. Of course, a more complete treatment of this model would involve considering it directly in the path space approach. We have not attempted to do this.

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## APPENDIX I: COMPARISON WITH THE APPROACH OF HORI

In this appendix we should like to examine more closely the relationship between our approach to strong interactions and that of Hori. We have already pointed out the essential difference between the two approaches in (A). Here, we shall make this point of difference more explicit.

Specifically, we already pointed out in A that the main difference in the two approaches lies in the isolation of small parts of the Lagrangian as the coupling tends to  $\infty$ . We isolate this small part in a manner which permits the usual "free" part of the Lagrangian to be integrated out completely. Hori, on the other hand, uses the "free" Lagrangian as his expansion operator. The question naturally arises as to the detailed numerical relation between his work and ours.

We investigate this issue here by considering a simple theory of trilinear type,

$$\mathscr{L} = \partial_{\mu} \phi^* \partial^{\mu} \phi - m_1^2 \phi^* \phi - g \phi^* \phi \chi + \frac{1}{2} \left( \partial_{\mu} \chi \partial^{\mu} \chi - m_2^2 \chi^2 \right) \quad . \tag{AI. 1}$$

Hori has examined this theory with his method for  $g \rightarrow \infty$  and found, for example, that

$$<0|T^{*}(\phi(x)\phi^{*}(y))|_{0}>_{c} = 0(1/g^{2}, m^{2}/g^{2})$$
 (AI. 2)

We shall show here that in our approach (AI. 2) does not hold true.

Our starting point is again

$$\exp iZ = \int \mathscr{D}\phi \, \mathscr{D}\phi^* \, \mathscr{D}\chi \exp i \int d^4x \left[ \mathscr{L} + J_1^* \phi + \phi^* J_1 + J_2 \chi \right] \quad . \tag{AI.3}$$

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In the by-now familiar way, we use identities of the type (2.4) to rewrite (AI.3) as

$$\begin{split} \exp \mathbf{i} \mathbf{Z} &\equiv \int \mathcal{D}\phi \, \mathcal{D}\phi^* \mathcal{D}\chi \, \mathcal{D}\pi_1^* \mathcal{D}\pi_1 \, \mathcal{D}\kappa_1 \, \mathcal{D}\kappa_1^* \mathcal{D}\pi_2 \, \mathcal{D}\kappa_2 \, \mathcal{D}\pi_3 \mathcal{D}\kappa_3 \\ & \exp \mathbf{i} \int d^4 \mathbf{x} \left[ \partial_\mu \kappa_1^* \partial^\mu \kappa_1 - \mathbf{m}_1^2 \kappa_1^* \kappa_1 + \frac{1}{2} \left( \partial_\mu \kappa_2 \partial^\mu \kappa_2 - \mathbf{m}_2^2 \chi^2 \right) \right. \\ & - \mathbf{g} \phi^* \phi \kappa_3 + \mathbf{J}_1^* \kappa_1 + \kappa_1^* \mathbf{J}_1 + \mathbf{J}_2 \kappa_2 + \pi^* (\kappa_1 - \phi) + \pi_1 (\kappa_1^* - \phi^*) \\ & + \pi_2 (\kappa_2 - \chi) + \pi_3 (\kappa_3 - \chi) \right] \, . \end{split}$$
(AI. 4)

The shifts

$$\phi \rightarrow \phi - \frac{\pi_1}{g\kappa_3}$$

$$\phi^* \rightarrow \phi^* - \frac{\pi_1^*}{g\kappa_3}$$
(AI. 5)

$$\chi \rightarrow \chi - \frac{\pi_2 + \pi_3}{2m_2^2}$$

allow us to write, as  $g, m_2 \rightarrow \infty$  with  $m_{1,2}^2/g \rightarrow 0$ ,

$$\begin{split} \exp i \mathbf{Z} &\equiv \int \mathcal{D}\phi \, \mathcal{D}\phi^* \, \mathcal{D}\chi \, \mathcal{D}\pi_1 \, \mathcal{D}\pi_1^* \, \mathcal{D}\kappa_1 \, \mathcal{D}\kappa_1^* \, \mathcal{D}\pi_2 \, \mathcal{D}\kappa_2 \, \mathcal{D}\pi_3 \, \mathcal{D}\kappa_3 \\ & \exp i \int d^4 x \left[ \partial_\mu \kappa_1^* \partial^\mu \kappa_1 - m_1^2 \kappa_1^* \kappa_1 + \frac{1}{2} \left( \partial_\mu \kappa_2 \partial^\mu \kappa_2 - m_2^2 \chi^2 \right) \right. \\ & - g \phi^* \phi \kappa_3 + J_1^* \kappa_1 + \kappa_1^* J_1 + J_2 \kappa_2 + \pi_1^* \kappa_1 + \pi_1 \kappa_1^* + \pi_2 \kappa_2 + \pi_3 \kappa_3 \\ & + \frac{\left( \pi_2 + \pi_3 \right)^2}{2m_2^2} + \frac{\pi_1 \pi_1^*}{g \kappa_3} \right] \end{split}$$

$$\begin{split} &= \sum_{n_{1},n_{2}=0}^{\infty} \frac{1}{n_{1}!} \frac{1}{n_{2}!} \left(\frac{i}{g}\right)^{n_{1}} \left(\frac{i}{2m_{2}^{2}}\right)^{n_{2}} \frac{1}{n_{1}} \frac{1}{n_{2}} \prod_{j=1}^{n_{2}} \prod_{k=1}^{n_{2}} \prod_{j=1}^{n_{2}} \prod_{j=1}^{n_{2}$$

where  $a \downarrow 0$  has been introduced to define the ratio

$$\mathbf{R} = \frac{\int \mathscr{D}\phi \mathscr{D}\phi^* \mathscr{D}\chi \mathscr{D}\pi_3 \mathscr{D}\kappa_3 \exp i \int d^4 \mathbf{x} \left[ -\frac{\mathbf{a}}{2} \phi^* \phi - \mathbf{g} \phi^* \phi \kappa_3^{+} \pi_3 \kappa_3 \right] - \sum \xi_{\ell} \pi_3 (\mathbf{y}_{\ell}) - \sum \lambda_j \kappa_3 (\mathbf{x}_j)}{\int \mathscr{D}\phi \mathscr{D}\phi^* \mathscr{D}\chi \mathscr{D}\pi_3 \mathscr{D}\kappa_3 \exp i \int d^4 \mathbf{k} \left[ -\frac{\mathbf{a}}{2} \phi^* \phi - \mathbf{g} \phi^* \phi \kappa_3^{+} \pi_3 \kappa_3 \right]}$$
(AI. 7)

for, the result (AI. 6) has been divided by the denominator of this last expression. This denominator diverges for a 10. Hence, we have divided exp iZ by too "large" a factor when a=0. This fact presents no special problem, since such a division as that implied by (AI. 7) is without physical significance. The quantity a would simply be absorbed into any systematic definition of the theory. This is not necessary for our present purposes.

To order 1/g we have

$$\exp iZ = 1 + \frac{i}{g} \int d^4x \int_{-\infty}^{\infty} d\beta \int_{-\infty}^{\infty} d\bar{\beta} \left( \frac{\beta^2 + \bar{\beta}^2}{-i\epsilon} \right) \frac{\int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\bar{\alpha}}{(2\pi)^2}$$
$$\exp i \left[ \alpha\beta + \bar{\alpha}\bar{\beta} + \alpha \operatorname{Re} J_1(x) + \bar{\alpha} \operatorname{Im} J_1(x) + \frac{1}{4} \Delta_{\mathrm{F}}^{-1}(0) (\alpha^2 + \bar{\alpha}^2) \right] + 0 \left( \frac{1}{\mathrm{gm}_2^2} \right)$$
(AI.8)

where  $\Delta_{F}^{-1}(x)$  is the inverse of Feynman's function

$$\Delta_{\rm F}^{-1}({\rm x}) \equiv \frac{\int {\rm d}^4 {\rm k}}{(2\pi)^4} \, ({\rm k}^2 - {\rm m}^2) \, \cos \, {\rm k} \cdot {\rm x} \quad . \tag{AI.9}$$

Let the charged field  $\phi$  be represented by

$$\phi = (\phi_1 + i\phi_2) / \sqrt{2}$$

where  $\boldsymbol{\phi}_1 \text{ and } \boldsymbol{\phi}_2$  are hermitean. Then, it follows from (AI.8) that

$$\begin{aligned} -2 < 0 | \mathrm{T}^{*}(\phi_{1}(\mathbf{x}) \phi_{1}(\mathbf{y})) | 0 \rangle &= \frac{\delta^{2} \mathrm{i}\mathbf{Z}}{\delta \mathrm{Re} \mathrm{J}_{1}(\mathbf{x}) \delta \mathrm{Re} \mathrm{J}_{1}(\mathbf{y})} \Big|_{\mathrm{J}^{*}_{\mathrm{I}} = \mathrm{J}_{1} = 0 = \mathrm{J}_{2}} \\ &= \frac{\delta(\mathbf{x} - \mathbf{y})}{\mathrm{g} \epsilon} \int \mathrm{d}\beta \, \mathrm{d}\bar{\beta} \frac{\mathrm{d}\alpha \, \mathrm{d}\bar{\alpha}}{(2\pi)^{2}} \left(\beta^{2} + \bar{\beta}^{2}\right) \\ &\alpha^{2} \exp \mathrm{i} \left[\alpha\beta + \bar{\alpha}\bar{\beta} + \frac{\Delta_{\mathrm{F}}^{-1}}{4} \left(\alpha^{2} + \bar{\alpha}^{2}\right)\right] + 0\left(\frac{1}{\mathrm{gm}_{2}^{2}}\right) \\ &= -\frac{\delta(\mathbf{x} - \mathbf{y})}{\mathrm{g} \epsilon} \int \mathrm{d}\beta \, \mathrm{d}\bar{\beta} \frac{\mathrm{d}\alpha \, \mathrm{d}\bar{\alpha}}{(2\pi)^{2}} \left(\frac{\partial^{2}}{\partial\mu^{2}} + \frac{\partial^{2}}{\partial\mu^{2}}\right) \\ &\alpha^{2} \exp \mathrm{i} \left[\beta(\alpha - \mu) + \bar{\beta}(\bar{\alpha} - \bar{\mu}) + \frac{1}{4} \Delta_{\mathrm{F}}^{-1}\left(0\right) \left(\alpha^{2} + \bar{\alpha}^{2}\right)\right] \Big|_{\mu, \bar{\mu} = 0} + 0\left(\frac{1}{\mathrm{gm}_{2}^{2}}\right) \\ &= -\frac{\delta(\mathbf{x} - \mathbf{y})}{\mathrm{g} \epsilon} \left(\frac{\partial^{2}}{\partial\mu^{2}} + \frac{\partial^{2}}{\partial\bar{\mu}^{2}}\right) \mu^{2} \exp \mathrm{i} \frac{\Delta_{\mathrm{F}}^{-1}}{4} \left(\mu^{2} + \bar{\mu}^{2}\right) \Big|_{\mu, \bar{\mu} = 0} \\ &+ 0\left(\frac{1}{\mathrm{gm}_{2}^{2}}\right) \\ &= -\frac{2\delta(\mathbf{x} - \mathbf{y})}{\mathrm{g} \epsilon} + 0\left(\frac{1}{\mathrm{gm}_{2}^{2}}\right) , \qquad (AI. 10) \end{aligned}$$

which disagrees with Hori's result (AI.2). The result (AI.10) diverges as  $\epsilon \downarrow 0$ , implying that  $\epsilon$  itself must be involved in the definition of the parameters of the strict theory. However, this does not concern us here.

The origin of the difference in numerical results between our work and that of Hori is clearly the role of the gradient terms. In our approach, no assumption is made about the size of these operators. In Hori's approach, the operators are presumed small. Since our formulation can be put in complete analogy with the small coupling limit, we feel our approach is indeed a reliable approach. The result (AI. 10) would then tend to cast doubt on Hori's method of handling the "free" Lagrangian, i.e., handling gradients. In any event, we shall not be concerned further with this approach of Hori.

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### APPENDIX II: YUKAWA FUNCTIONALS

In this appendix, we shall establish Eq. (2.9). Equation (3.6) can be derived by completely similar reasoning. We do this in the interest of completences, since the result is essentially well-known<sup>3</sup> to some.

The starting point is the familiar dynamical principle of Schwinger.  $^{12}$  Namely, the functional  $^{3}$ 

$$\Omega = \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \exp i \int dx \left[ -g\bar{\psi}\psi\phi + \bar{\eta}\psi + \bar{\psi}\eta \right]$$

satisfies

$$\frac{\partial \Omega}{\partial g} = \int \mathscr{D}\psi \, \mathscr{D}\bar{\psi} \left( -i \int d^4 x \, \bar{\psi}\psi \phi \right) \exp i \int d^4 x \left[ -g \bar{\psi}\psi \phi + \bar{\eta}\psi + \bar{\psi}\eta \right]$$
$$= -i \int d^4 x \, \phi \, \frac{\delta^2}{\delta \eta \left( x \right) \, \delta \bar{\eta} \left( x \right)} \, \Omega \quad . \tag{AII. 1}$$

In order to take advantage of (AII. 1), we observe that the shifts

$$\psi \to \psi + \eta/g\phi \tag{AII. 2}$$
  
$$\bar{\psi} \to \bar{\psi} + \bar{\eta}/g\phi$$

allow us to write

$$\Omega = \int \mathscr{D}\psi \, \mathscr{D}\overline{\psi} \, \exp \, i \int d^4 x \left( -g \overline{\psi} \psi \phi + \overline{\eta} \eta \, /g \phi \right) \quad . \tag{AII. 3}$$

Then, defining

$$\Gamma(\mathbf{g}\phi) \equiv \int \mathscr{D}\psi \, \mathscr{D}\bar{\psi} \, \exp \, \mathbf{i} \int \mathrm{d}^4 \mathbf{x} (-\mathbf{g}\bar{\psi}\psi\phi) \tag{AII.4}$$

and substituting (AII. 3) into (AII. 1) we find

$$\frac{\partial \Gamma}{\partial g} \exp i \int d^{4}x \ \bar{\eta}\eta/g \phi - \frac{i\Gamma}{g^{2}} \int d^{4}x \ \bar{\eta}\eta/\phi \ \exp i \int d^{4}x \ \bar{\eta}\eta/g \phi$$

$$= -i\Gamma \int d^{4}x \left[ \frac{\delta}{\delta\eta(x)} \frac{i\eta(x)}{g\phi} \exp i \int d^{4}x \ \bar{\eta}\eta/g \phi \right] \phi$$

$$= -i\Gamma \int d^{4}x \left[ \frac{4i\delta(0)}{g\phi} + i(-i) \frac{\bar{\eta}\eta}{g^{2}\phi^{2}} \right] \phi \ \exp i \int d^{4}x \ \bar{\eta}\eta/g \phi$$

$$= \left( 4\Gamma \int d^{4}x \frac{\delta(0)}{g} - \frac{i\Gamma}{g^{2}} \int d^{4}x \ \bar{\eta}\eta \right) \exp i \int d^{4}x \ \bar{\eta}\eta/g \phi \quad . \tag{AII.5}$$

This gives

$$\frac{\partial \Gamma}{\partial g} = 4\Gamma \int d^4 x \ \frac{\delta(0)}{g} \quad . \tag{AII.6}$$

Furthermore, explicitly, we also have

$$g\frac{\partial\Gamma}{\partial g} = \int d^4x \ \phi \ \frac{\delta}{\delta\phi} \ \Gamma \quad . \tag{AII. 7}$$

Equations (AII. 6) and (AII. 7) immediately give (2.9):

$$\Gamma = \exp \frac{\int dx}{\Delta x} \log g^4 \phi^4 \quad . \tag{AII.8}$$

It should be pointed out that the result (3.6), which is implied by the result (AII.8), disagrees with the analogous result of Hori<sup>3</sup> insofar as the sign of the argument of the exponential is concerned.

# APPENDIX III: EVALUATION OF MEASURE ZERO CONTRIBUTIONS

In establishing the results in Sections II and III, we have ignored contributions from measure zero sets. Here, we shall show that this is legitimate.

It's sufficient to consider the situation at order 1/G and 1/ $\sqrt{H}$ . We have from (2.10)

$$\begin{split} \frac{\int_{\mathrm{dxdy}}}{8\mathrm{G}\sqrt{\mathrm{H}}} & \frac{\delta^{2}}{\delta\eta_{1}(\mathbf{x}_{j})\delta\bar{\eta}(\mathbf{x})} \frac{\int_{-\infty}^{\infty}\mathrm{d}\beta}{\beta^{\mathrm{H}\mathrm{f}\mathrm{-1}\epsilon}} \frac{\int_{-\infty}^{\infty}\mathrm{d}\alpha}{2\pi} \int_{-\infty}^{\infty}\mathrm{d}\mu \ \mu^{2} \ \frac{\int_{-\infty}^{\infty}\mathrm{d}\rho}{2\pi} \\ & \frac{\int_{-\infty}^{\infty}\mathrm{d}\nu}{\left(\nu-2\mathrm{f}^{2}\sqrt{\mathrm{H}}+\mathrm{i}\epsilon\right)} \frac{\int_{-\infty}^{\infty}\mathrm{d}\xi}{2\pi} \int \mathcal{D}\psi_{1}\mathcal{D}\bar{\psi}_{1}\mathcal{D}\lambda\mathcal{D}\bar{\lambda} \ \mathcal{D}\kappa_{1}\mathcal{D}\kappa_{2}\mathcal{D}\pi_{1}\mathcal{D}\pi_{2}\mathcal{D}\sigma\mathcal{D}\phi \\ & \exp\mathrm{i}\left\{\int\mathrm{d}^{4}\mathbf{x}\left[\frac{1}{2}\left(\partial_{\mu}\kappa_{1}\partial^{\mu}\kappa_{1}-8\mathrm{f}^{2}\mathrm{H}\phi^{2}\right)-4\mathrm{f}\mathrm{H}\kappa_{2}^{3}+\mathrm{J}(\kappa_{1}+\mathrm{f})+\sigma^{2}\right. \\ & + 2\sqrt{\mathrm{H}}\ \sigma\phi^{2}-\frac{\mathrm{i}}{\Delta\mathrm{x}}\ \log\ \mathrm{G}^{4}(\kappa_{2}+\mathrm{f})^{4}+\pi_{1}\kappa_{1}+\pi_{2}\kappa_{2}+\bar{\psi}_{1}\mathrm{i}\vartheta\psi_{1} \\ & +\bar{\eta}\psi_{1}+\bar{\psi}_{1}\eta+\bar{\lambda}(\psi_{1}+\eta_{1})+(\bar{\psi}_{1}+\bar{\eta}_{1})\lambda\right]+\alpha(\beta-\kappa_{2}(\mathrm{x})) \\ & +\rho(\mu-\pi_{1}(\mathrm{y})-\pi_{2}(\mathrm{y}))+\xi(\nu-\sigma(\mathrm{y}))\Big\}\Big|_{\eta_{1}},\bar{\eta}_{1}=0 \end{split} \tag{AIII. 1}$$

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In evaluating (AIII. 1) we used the result (2.11) almost everywhere. The relevant exceptional case is that region where x=y. This region contributes

$$\frac{\Delta \mathbf{x} \int d\mathbf{y}}{8G\sqrt{H}} \frac{\delta}{\delta \eta_{1}(\mathbf{y})} \frac{\int_{-\infty}^{\infty} d\beta}{\beta + \mathbf{f} - \mathbf{i}\epsilon} \frac{\int_{-\infty}^{\infty} d\alpha}{2\pi} \int_{-\infty}^{\infty} d\mu \ \mu^{2} \frac{\int_{-\infty}^{\infty} d\rho}{2\pi} \frac{\int_{-\infty}^{\infty} d\nu}{(\nu - 2\mathbf{f}^{2}\sqrt{H} + \mathbf{i}\epsilon)}$$

$$\frac{\int_{-\infty}^{\infty} d\xi}{2\pi} \frac{2\mathbf{i}^{3/4} (\Delta \mathbf{x})^{1/4} \sqrt{\pi/\xi}}{\Gamma(1/4)} \exp \mathbf{i} \left[ \xi \nu + \rho (\mu + \mathbf{J}(\mathbf{y})) + \alpha \beta - \frac{1}{4} \xi^{2} \delta(0) + \frac{1}{2}\rho^{2} \Delta_{\mathbf{F}}^{-1}(0) + \int d\mathbf{x} \ \mathbf{J}\mathbf{f} \right]$$

$$\int \mathcal{D}\pi_{2} \mathcal{D}\kappa_{2} \exp \mathbf{i} \left\{ \int d\mathbf{x} \left[ \pi_{2}\kappa_{2} - 4H\mathbf{f}\kappa_{2}^{3} - \frac{\mathbf{i}}{\Delta \mathbf{x}} \log \mathbf{G}^{4}(\kappa_{2} + \mathbf{f})^{4} \right] - \alpha \kappa_{2}(\mathbf{y}) - \rho \pi_{2}(\mathbf{y}) \right\}$$

$$\int \mathcal{D}\psi_{1} \mathcal{D}\overline{\psi}_{1} \mathcal{D}\lambda \mathcal{D}\overline{\lambda} \exp \mathbf{i} \int d\mathbf{x} \left[ \overline{\psi}_{1} \mathbf{i} \overline{\psi}\psi_{1} + \overline{\eta}\psi_{1} + \overline{\psi}_{1}\eta + \overline{\lambda}(\psi_{1} + \eta_{1}) + (\overline{\psi}_{1} + \overline{\eta}_{1})\lambda \right] \quad (AIII.2)$$

The ratio (2.11) becomes

$$\begin{split} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\pi_{2}(\mathbf{y}) d\kappa_{2}(\mathbf{y}) \exp i \left[ \Delta \mathbf{x} \pi_{2}(\mathbf{y}) \kappa_{2}(\mathbf{y}) - 4\Delta \mathbf{x} \operatorname{Hf} \kappa_{2}^{3}(\mathbf{y}) \right. \\ \left. - i \log \operatorname{G}^{4}(\kappa_{2}(\mathbf{y}) + \mathbf{f})^{4} - \alpha \kappa_{2}(\mathbf{y}) - \rho \pi_{2}(\mathbf{y}) \right] / \\ \left. \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\pi_{2}(\mathbf{y}) d\kappa_{2}(\mathbf{y}) \exp i \left[ \Delta \mathbf{x} \pi_{2}(\mathbf{y}) \kappa_{2}(\mathbf{y}) \right. \\ \left. - 4 \operatorname{Hf} \Delta \mathbf{x} \kappa_{2}^{3}(\mathbf{y}) - i \log \operatorname{G}^{4}(\kappa_{2}(\mathbf{y}) + \mathbf{f})^{4} \right] \\ \left. = \frac{\left( \rho / \Delta \mathbf{x} + \mathbf{f} \right)^{4} \Delta \mathbf{x}}{\mathbf{f}^{4} 2 \pi} \frac{2\pi}{\Delta \mathbf{x}} \exp \left( -i \alpha \rho / \Delta \mathbf{x} - i4 \operatorname{Hf} \rho^{3} / (\Delta \mathbf{x})^{2} \right) \\ \left. = \frac{\left( \rho / \Delta \mathbf{x} + \mathbf{f} \right)^{4}}{\mathbf{f}^{4}} \exp \left[ -i \alpha \rho / \Delta \mathbf{x} - i4 \operatorname{Hf} \rho^{3} / (\Delta \mathbf{x})^{2} \right] \end{split}$$
(AIII. 3)

Thus, (AIII.2) is equivalent to

$$\begin{split} \frac{\Delta x}{8Gf} &= \frac{1}{\sqrt{H}} \int dy \; \frac{\delta^2}{\delta \eta_1(y) \; \delta \bar{\eta}_1(y)} \quad \int_{-\infty}^{\infty} d\mu \; \mu^2 \; \frac{\int_{-\infty}^{\infty} d\rho}{2\pi} \; \frac{(\rho / \Delta x + f)^3}{f^3} \\ &= \frac{\int_{-\infty}^{\infty} d\nu}{\nu - 2f^2 \sqrt{H} + i\epsilon} \; \frac{\int_{-\infty}^{\infty} d\xi}{2\pi} \; \frac{2i^{3/4} (\Delta x)^{1/4} \sqrt{\pi/\xi}}{\Gamma(1/4)} \\ &= \exp i \left[ \xi \nu + \rho (\mu + J(y)) - \frac{1}{4} \; \xi^2 \; \delta(0) + \frac{1}{2} \; \rho^2 \Delta_F^{-1}(0) + \int dx \; Jf \; -4 \; Hf \; \rho^3 / (\Delta x)^2 \right] \\ &\quad + \left. \frac{\int d^4 k}{(2\pi)^4} \; \left\{ \bar{\eta}_1 \not k \; \eta_1 - \bar{\eta} \eta_1 - \bar{\eta}_1 \eta \right\} \right]_{\eta_1 = \bar{\eta}_1 = 0} \; . \end{split}$$
 (AIII.4)

This should be compared to

$$\frac{1}{8\mathrm{Gf}\sqrt{\mathrm{H}}} \int \mathrm{dx} \,\mathrm{dy} \,\frac{\delta^2}{\delta \eta_1(\mathbf{x}) \,\delta \bar{\eta}_1(\mathbf{x})} \int_{-\infty}^{\infty} \mathrm{d\mu} \,\mu^2 \,\frac{\int_{-\infty}^{\infty} \mathrm{d\rho}}{2\pi} \frac{(\rho/\Delta \mathbf{x}+\mathbf{f})^4}{\mathbf{f}^4}$$

$$\frac{\int_{-\infty}^{\infty} \mathrm{d\nu}}{\nu - 2\mathbf{f}^2 \sqrt{\mathrm{H}} + \mathbf{i}\epsilon} \frac{\int_{-\infty}^{\infty} \mathrm{d\xi}}{2\pi} \frac{2\mathbf{i}^{3/4} (\Delta \mathbf{x})^{1/4} \sqrt{\pi/\xi}}{\Gamma(1/4)}$$

$$\exp \mathbf{i} \left[ \xi \nu + \rho(\mu + \mathbf{J}(\mathbf{y})) - \frac{1}{4} \xi^2 \,\delta(0) + \frac{1}{2} \rho^2 \Delta_{\mathrm{F}}^{-1}(0) + \int \mathrm{dx} \,\mathrm{Jf} - 4 \,\mathrm{Hf} \,\rho^3/(\Delta \mathbf{x})^2 + \frac{\int \mathrm{d}^4 \mathbf{k}}{(2\pi)^4} \left\{ \bar{\eta}_1 \,\mathbf{k} \,\eta_1 - \bar{\eta} \eta_1 - \bar{\eta}_1 \eta \right\} \right|_{\eta_1 = \bar{\eta}_1 = 0} \cdot (\mathrm{AIII.5})$$

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Evaluation of the functional derivatives gives us

$$\frac{\Delta x}{8Gf} \frac{1}{\sqrt{H}} \int d^{4}y \left[\bar{\eta}(y) \eta(y)\right] \int_{-\infty}^{\infty} d\mu \ \mu^{2} \frac{\int_{-\infty}^{\infty} d\rho}{2\pi} \frac{(\rho/\Delta x + f)^{3}}{f^{3}}$$

$$\frac{\int_{-\infty}^{\infty} d\nu}{\nu - 2f^{2}\sqrt{H}} \frac{\int_{-\infty}^{\infty} d\xi}{2\pi} \frac{2i^{3/4} (\Delta x)^{1/4} \sqrt{\pi/\xi}}{\Gamma(1/4)}$$

$$\exp i \left[\xi\nu + \rho(\mu + J(y)) - \frac{1}{4}\xi^{2} \delta(0) + \frac{1}{2}\rho^{2} \Delta_{\mathrm{F}}^{-1}(0) + \int \mathrm{dx} J(x)f - 4 \operatorname{Hf} \rho^{3}/(\Delta x)^{2}\right]$$
(AIII. 6)

to be compared with

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$$\frac{1}{8Gf} \frac{1}{\sqrt{H}} \int d^{4}x \left[ \bar{\eta}(x) \eta(x) \right] \int d^{4}y \int_{-\infty}^{\infty} d\mu \ \mu^{2} \frac{\int_{-\infty}^{\infty} d\rho}{2\pi} \frac{(\rho/\Delta x + f)^{4}}{f^{4}} \\ \frac{\int_{-\infty}^{\infty} d\nu}{\nu - 2f^{2}\sqrt{H}} \frac{\int_{-\infty}^{\infty} d\xi}{2\pi} \frac{2i^{3/4} (\Delta x)^{1/4} \sqrt{\pi/\xi}}{\Gamma(1/4)} \\ \exp i \left[ \xi \nu + \rho(\mu + J(y)) - \frac{1}{4} \xi^{2} \delta(0) + \frac{1}{2} \rho^{2} \Delta_{\mathrm{F}}^{-1}(0) + \int d^{4}x \ \mathrm{Jf} - 4 \ \mathrm{Hf} \rho^{3}/(\Delta x)^{2} \right] .$$
(AIII. 7)

Thus, it can easily be seen that the former contribution (AIII.6) is negligible compared with the latter (AIII.7) as  $\Delta x \rightarrow 0$ .

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