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COLOR-SYMMETRY BREAKING AND THE BARYON SPECTRUM II*

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ABSTRACT

Recently I proposed that the forces between quarks of two of the colors in the colored-quark model are of shorter range than forces involving quarks of the third color, and showed that if this color-symmetry breaking mechanism is strong, the favored baryon SU(6) representations and parities are $\underline{56}^{+}$ and $\underline{70}^{-}$, in agreement with experimental indications. In this paper it is shown that this prediction also results if the symmetry breaking is small, in which case the sign of the symmetry breaking is irrelevant. I discuss a possible experimental way to test the validity of the mechanism and measure the approximate size of the symmetry breaking. A useful list of orthonormal quark-model wave functions of specific energies and symmetries is given.

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I. INTRODUCTION

The existing data on baryon resonances suggest that the even- and odd-parity baryons correspond exclusively to the SU(6) representations $\underline{56}$ and $\underline{70}$, respectively.[1] This contradicts the one-triplet quark model with Bose statistics and harmonic-oscillator forces (symmetric quark model) for all but the two lowest energy levels.[2] For example, the symmetric quark model predicts even-parity resonances corresponding to the representations $\underline{56}$, $\underline{70}$, and $\underline{20}$ at the second excited level.

I call the "color-symmetric" quark model the model with three triplets of quarks, with color SU(3) symmetry as well as the usual SU(3) symmetry, Fermi statistics, harmonic-oscillator forces, and the identification of observed hadrons with color singlets. This model predicts the same hadron spectrum as the symmetric quark model. However, there is no experimental evidence that the proposed color symmetry is exact or nearly exact. One would expect color-symmetry breaking to lead to octet admixtures in the predominantly singlet states identified with the hadrons. However, the unobserved, predominantly octet states may still be either at very high masses or nonexistent. It has been shown in a recent paper that large color-symmetry breaking can lead to a predicted baryon spectrum of the SU(6) representations and parities 56^+ and 70^-, in agreement with experimental indications.[3] The main purpose of the present paper is to show that this result also follows if the color-symmetry breaking is weak. In Sec. IV I discuss briefly the possibility of measuring the size of the symmetry breaking.

Since I am concerned with representations predicted in the color-symmetric quark model that have not been observed, I neglect mass differences between states of the same SU(6) $\times$ O(3) representation [where O(3) refers to the quark

orbital motion]. Exact SU(6) symmetry is assumed.

It is helpful to review some basic facts concerning hadron spectra in the color-symmetric quark model. The quark-antiquark states that constitute the mesons correspond to the SU(6) representations 35 and 1. These both are observed in the same mass region. Consequently, if SU(6) symmetry breaking is neglected, it is reasonable to consider the $\underline{35}$ and $\underline{1}$ meson states to be degenerate. This implies that the force in the cross channel not only is SU(6) symmetric, but transforms as an SU(6) singlet.

When one considers the three-quark baryon states, the statistics of the quarks comes into play. The lowest orbital state is completely symmetric. If this state is a pure color singlet, and so is totally antisymmetric in the color indices, Fermi statistics requires that the SU(6) wave function must correspond to the symmetric $\underline{56}$ representation, in agreement with experiment. Similar reasoning implies that the first excited orbital state corresponds to the mixed-symmetry SU(6) representation $\underline{70}$.

Of course it is possible that the quark-quark potential contains a piece that does not transform as an SU(6) singlet. This would affect the relative masses of different predicted three-quark states. However, it is observed experimentally that the 70-fold representation corresponding to the first excited quark-model states lies approximately midway between the 56-fold ground state and its first Regge recurrence, as expected in the color-symmetric quark model. This supports the evidence from the meson spectrum that the potential is an SU(6) singlet. Therefore I make the usual assumption that the potential depends only on color and orbital quantum numbers, so that it is an SU(6) singlet. The predicted baryon representation structure depends on the symmetry of the orbital and color wave functions.

If color symmetry is broken, the wave function is no longer completely antisymmetric in the color indices. Because of the requirement of Fermi statistics this affects the symmetry of the rest of the wave function and hence influences the predictions concerning which SU(6) representations occur.

The basic procedure for the calculation is given in Sec. II and the results are listed in Sec. III. The orbital wave functions for the lowest baryon levels are listed in terms of relative coordinates in the Appendix. These are needed in the calculation, and may be useful as well to other calculations involving modifications of the color-symmetric quark model for baryons.

II. PROCEDURE

The baryons are nonrelativistic, three-quark states. The quark binding forces are transmitted by the exchange of an octet of vector gluons coupled to the color indices of the quarks.[4] For calculational purposes, the radial dependence of the quark-quark potential is taken to be that of a harmonic oscillator. The possible effect of deviations from this assumption is discussed in Sec. IV.

The colors are labeled r, w, and b (for red, white, and blue), and the Hamiltonian is assumed invariant to the SU(2) of the colors red and white. Thus the color-symmetry breaking is taken to be analogous to the observed breaking of ordinary SU(3). The breaking of ordinary SU(3) is neglected.

The gluon-exchange potential is a sum of two-body potentials $V = V_{\alpha\beta} + V_{\beta\alpha} + V_{\gamma\alpha}$, where α, β, and γ are the three quarks in the baryon. It is assumed that $V_{\mu\gamma}$ is the sum of a short-range part V_s and a long-range part V_ℓ, i.e.,

$$V_{\mu\gamma} = \sum_{i=1}^{3} J_i^\mu J_i^\nu U_s(r_{\mu\nu}) + \sum_{i=4}^{8} J_i^\mu J_i^\nu U_\ell(r_{\mu\nu}) , \qquad (1)$$

where $r_{\mu\nu} = |\vec{r}_\mu - \vec{r}_\nu|$, J_i^μ is the i'th Hermitean generator of SU(3), operating in the color space of the quark μ, and J_1, J_2, and J_3 are the generators of the rw SU(2) subgroup of color SU(3). The configuration-space potentials U_s and U_ℓ are decreasing functions of $r_{\mu\nu}$; they would be positive if Yukawa potentials were used. Exact color symmetry corresponds to $U_s = U_\ell$. A simple possible cause of range difference between U_s and U_ℓ is mass splitting of the vector-gluon octet transmitting the forces, if the three gluons coupled as the generators of the rw SU(2) subgroup are heavier than the other gluons.[5]

I also allow the mass of the blue quark M_b to be different from M_{rw}, the common mass of the red and white quarks. The kinetic energy operator for the quark μ is

$$T_\mu = -(\hbar^2/2M_\mu)\nabla_\mu^2 \quad , \tag{2}$$

$$M_\mu = M_{rw}(\delta_{\mu r} + \delta_{\mu w}) + M_b\delta_{\mu b} \, . \tag{3}$$

In R1 it was assumed that the breaking of color symmetry is so large that the red and white quarks in a baryon are very tightly bound, so that there are no orbital excitations in the relative red-white coordinate. The excited baryons then correspond to orbital excitations of the blue quark around the red-white center of mass. The baryons have a simple quark-diquark structure.

However, there is no reason to believe that the color-symmetry breaking is so large. I take the alternate view in this paper. It is assumed that the color-symmetry breaking is small enough to be treated by second-order perturbation theory. The unperturbed states are color singlets. Since the perturbation is symmetric in the red-white interchange, the perturbed states are singlets in the rw SU(2). Thus I need not consider color decuplet states, only singlets and octets.

In order to apply perturbation theory, I rewrite the potential of Eq. (1) in the form

$$V_{\mu\nu} = V_{0,\mu\nu} + V'_{\mu\nu} \quad ,$$

$$V_{0,\mu\nu} = \sum_{i=1}^{8} J_i^\mu J_i^\nu \, U_0(r_{\mu\nu}) \quad , \tag{4}$$

$$V'_{\mu\nu} = \left[\sum_{i=1}^{3} J_i^\mu J_i^\nu - \frac{3}{8} \sum_{i=1}^{8} J_i^\mu J_i^\nu \right] U'(r_{\mu\nu}) \tag{5}$$

where U_0 and U' are given by

$$U_0 = \frac{5}{8} U_\ell + \frac{3}{8} U_s \quad , \tag{6}$$

$$U' = U_s - U_\ell \quad . \tag{7}$$

The total unperturbed potential $V_{0,\alpha\beta} + V_{0,\beta\gamma} + V_{0,\gamma\alpha}$ transforms as a color singlet and the perturbing potential $V'_{\alpha\beta} + V'_{\beta\alpha} + V'_{\gamma\alpha}$ transforms as a color octet state. This may be shown by making use of the equation $\sum_{i=1}^{8} J_i^\mu J_i^\nu = \frac{1}{2}[C(t) - C(\mu) - C(\nu)]$, where C(i) is the eigenvalue of the SU(3) quadratic Casimir operator for the representation i, and t is the $\mu\nu$ representation.[6]

In a similar fashion, the kinetic energy operator of Eq. (2) may be separated into unperturbed and perturbed parts,

$$T_\mu = T_{0,\mu} + T'_\mu \quad ,$$

such that the unperturbed kinetic energy $T_{0,\alpha} + T_{0,\beta} + T_{0,\gamma}$ transforms as a color singlet and to first order in the quark mass difference the perturbed kinetic energy $T'_\alpha + T'_\beta + T'_\gamma$ transforms as a color octet. The exact definition of $T_{0,\mu}$ and T'_μ will be given later.

The baryon states are color singlets to zero'th order. Since the perturbing Hamiltonian transforms as an octet state there is no first-order contribution to the energy. The energy correction to lowest order is the second order term $\sum_j |H'_{jn}|^2/(E_n - E_j)$. Only octet intermediate states j will contribute.

Since the octet unperturbed states are of high mass compared to singlet states, the second-order energy contributions to the hadron states are negative. Therefore, the favored SU(6) representations for a particular oscillator level are those with the largest second-order energy contributions.

I list here some needed properties of functions of mixed symmetry. All permutations of the three variables α, β, and γ may be obtained from products

of the two transpositions $(\alpha\beta)$ and $(\alpha\gamma)$. If a function $A(\alpha\beta\gamma)$ is of mixed symmetry, there are two components $A^{\lambda,\alpha\beta}$ and $A^{\rho,\alpha\beta}$ with the permutation properties,[7]

$$\begin{aligned} (\alpha\beta)\, A^{\lambda,\alpha\beta} &= A^{\lambda,\alpha\beta} \quad , \quad (\alpha\gamma)\, A^{\lambda,\alpha\beta} = -\tfrac{1}{2} A^{\lambda,\alpha\beta} - \tfrac{1}{2}\sqrt{3}\, A^{\rho,\alpha\beta} \\ (\alpha\beta)\, A^{\rho,\alpha\beta} &= -A^{\rho,\alpha\beta} \quad , \quad (\alpha\gamma)\, A^{\rho,\alpha\beta} = \tfrac{1}{2} A^{\rho,\alpha\beta} - \tfrac{1}{2}\sqrt{3}\, A^{\lambda,\alpha\beta} \end{aligned} \tag{8}$$

The symmetry properties of quadratic combinations are needed also. If A and B are functions of mixed symmetry in three variables, and C is bilinear in A and B, the symmetry properties of C are

$$\begin{aligned} C^{s} &= A^{\rho}B^{\rho} + A^{\lambda}B^{\lambda} \quad , \quad C^{a} = A^{\lambda}B^{\rho} - A^{\rho}B^{\lambda} \quad , \\ C^{\lambda} &= A^{\rho}B^{\rho} - A^{\lambda}B^{\lambda} \quad , \quad C^{\rho} = A^{\lambda}B^{\rho} + A^{\rho}B^{\lambda} \quad , \end{aligned} \tag{9}$$

where the superscripts s and a denote totally symmetric and totally antisymmetric functions. If A is of mixed symmetry and B is antisymmetric,

$$C^{\lambda} = A^{\rho}B^{a} \quad , \quad C^{\rho} = -A^{\lambda}B^{a} \quad . \tag{10}$$

That part of the wave function having to do with ordinary SU(3), spin, and orbital angular momentum is denoted by ϕ. The total wave function ψ is antisymmetric in the three quarks α, β, and γ, so for a color singlet ϕ is completely symmetric. For a color singlet state n, ψ_n may be written $\psi_n = \left(\frac{1}{6}\right)^{\frac{1}{2}} \sum_P \tau^P \alpha_r \beta_w \gamma_b \phi_n^s(\alpha\beta\gamma)$, where the superscript of ϕ denotes the symmetry; r, w, and b are the color indices; the sum is over the six permutations of α, β, and γ; and the signature factor τ^P is one for even permutations and minus one for odd permutations. Since ϕ^s is completely symmetric, the quark indices of ϕ may be replaced with color indices, i.e.,

$$\psi_n = \left(\frac{1}{6}\right)^{\frac{1}{2}} \sum_P \tau^P \alpha_r \beta_w \gamma_b \phi_n^s(rwb) \quad . \tag{11}$$

A color octet wave function ψ_j is of mixed color symmetry, so it follows from Fermi statistics and the combination rules of Eqs. (9) and (10) that ϕ_j is of mixed symmetry. The only states j for which H'_{jn} may be nonzero (with n a color singlet) are composed of one red, one white, and one blue quark. It is convenient to write the color-octet states in terms of $\phi^{\lambda,rw}$ and $\phi^{\rho,rw}$ rather than in terms of $\phi^{\lambda,\alpha\beta}$ and $\phi^{\rho,\alpha\beta}$. Since ϕ_n^s and H' are both invariant to transposing the red and white quarks, there is no contribution to H'_{jn} from $\phi^{\rho,rw}$. The contributing color-octet wave functions may be written,

$$\psi_j = \left(\frac{1}{6}\right)^{\frac{1}{2}} \sum_P \tau^P \alpha_r \beta_w \gamma_b \phi_j^{\lambda,rw}(rwb) \quad . \tag{12}$$

If the $\phi_j^{\lambda,rw}$ in this equation is written in terms of $\phi^{\lambda,\alpha\beta}$ and $\phi^{\rho,\alpha\beta}$, one can see that the color wave function is a pure octet.

I consider next the matrix elements of $V' = V'_{\alpha\beta} + V'_{\beta\gamma} + V'_{\gamma\alpha}$. The operators $\sum_{i=1}^{3} J_i^\mu J_i^\nu$ and $\sum_{i=1}^{8} J_i^\mu J_i^\nu$ of Eq. (5) are diagonal in the representation of color wave functions symmetric and antisymmetric in μ and ν. The color-singlet, unperturbed wave functions ψ_n are antisymmetric in each pair of quarks. Therefore, only the eigenvalues corresponding to antisymmetric pairs are contained in V'_{jn}. By using Eqs. (11) and (12) one may write,

$$V'_{jn} = \langle \phi_j^{\lambda,rw} \, , \, (V'_{rw} + V'_{rb} + V'_{wb}) \, \phi_n \rangle \quad . \tag{13}$$

In this equation V'_{ij} includes the appropriate eigenvalue for the state antisymmetric in i and j. If V' from Eq. (5) is substituted into Eq. (13), the result is[6]

$$V'_{jn} = \langle \phi_j^{\lambda,rw} \, , \left[-\frac{1}{2} U'(r_{rw}) + \frac{1}{4} U'(r_{rb}) + \frac{1}{4} U'(r_{wb}) \right] \phi^s \rangle \quad . \tag{14}$$

A convenient set of variables is

$$\vec{R} = \left[M_{rw}(\vec{r}_r + \vec{r}_w) + M_b \vec{r}_b \right] / (2M_{rw} + M_b) \quad ,$$

$$\vec{\lambda} = (\vec{r}_r + \vec{r}_w - 2\vec{r}_b)/\sqrt{6} \quad , \tag{15}$$

$$\vec{\rho} = (\vec{r}_r - \vec{r}_w)/\sqrt{2} \quad .$$

$\vec{R}$ is the center-of-mass coordinate. It is easy to show that $\vec{\lambda}$ and $\vec{\rho}$ have the mixed-symmetry properties defined in Eq. (8), corresponding to the labels λ, λ,rw and ρ,rw.

The total kinetic energy operator T, expressed in these variables, is

$$T = -\frac{1}{2}\hbar^2\left[(2M_{rw} + M_b)^{-1}\nabla_R^2 + (\tfrac{1}{3}M_{rw}^{-1} + \tfrac{2}{3}M_b^{-1})\nabla_\lambda^2 + M_{rw}^{-1}\nabla_\rho^2\right] . \tag{16}$$

The quark masses may be written in terms of the average mass M_0 and the mass difference M', i.e.,

$$M_{rw} = M_0 + \tfrac{1}{3}M' \quad , \qquad M_b = M_0 - \tfrac{2}{3}M' \, .$$

I expand the mass terms in Eq. (16) to first order in M'/M_0. The singlet or unperturbed part of T is given by

$$T_0 = -\tfrac{1}{2}(\hbar^2/M_0)(\tfrac{1}{3}\nabla_R^2 + \nabla_\lambda^2 + \nabla_\rho^2) \, . \tag{17}$$

The matrix element T'_{jn} of the octet part of T is

$$T'_{jn} = -\hbar^2(M'/6M_0^2)\langle\phi_j^{\lambda,rw}, (\nabla_\lambda^2 - \nabla_\rho^2)\phi_n^s\rangle \, . \tag{18}$$

I take the configuration-space dependence of the potentials of Eq. (1) to be of the harmonic-oscillator type, and define an average force constant k_0 and a force-constant difference k' by the equations

$$U_s(r) = -\tfrac{1}{2}(k_0 + \tfrac{5}{8}k')r^2 \quad , \qquad U_\ell(r) = -\tfrac{1}{2}(k_0 - \tfrac{3}{8}k')r^2 \, .$$

The minus signs are included since U_s and U_ℓ are defined to be decreasing with increasing distance.

For a color singlet state n, the matrix element $V_{0,nn}$ is $k_0(\lambda^2 + \rho^2)$, where the components of V_0 are defined by Eq. (4). The octet-singlet matrix elements

of the perturbing potential, Eq. (14), may be written

$$V'_{jn} = \frac{3}{8} k' \langle \phi_j^{\lambda,rw} , (\rho^2 - \lambda^2)\phi_n^s \rangle . \tag{19}$$

For simplicity I assume that the symmetry-breaking parameters in T' and V' are related by the equation

$$(M'/M_0) = - (9/8)(k'/k_0). \tag{20}$$

With this assumption the ratio of the λ^2 and ∇_λ^2 terms (or of the ρ^2 and ∇_ρ^2 terms) in H' is the same as in H_0.

III. CALCULATION AND RESULTS

It was shown in Sec. II that the contributing matrix elements of the perturbing Hamiltonian are given by the sum of Eqs. (18) and (19), where j and n are color octet and singlet states. It is convenient to write ϕ in terms of direct products UB, where U is an SU(6) [spin and ordinary SU(3)] wave function, and B depends on the relative orbital coordinates $\vec{\lambda}$ and $\vec{\rho}$. I denote states of the symmetric and antisymmetric SU(6) representations 56 and 20 by U^s and U^a, and the two components of a mixed-symmetry 70 state by U^λ and U^ρ. The possible symmetries of the orbital wave functions B are determined by the combination rules, Eqs. (9) and (10). If U corresponds to the 56 or 20, ϕ_n^s and ϕ_j^λ must be of the types,

$$\underline{56}\text{ case:}\quad \phi_n^s = U^s B_n^s\ ,\quad \phi_j^\lambda = U^s B_j^\lambda\ . \tag{21}$$

$$\underline{20}\text{ case:}\quad \phi_n^s = U^a B_n^a\ ,\quad \phi_j^\lambda = U^a B_j^\rho\ . \tag{22}$$

All mixed components are of one of the types λ,rw or ρ,rw, but the rw indices have been suppressed. If the SU(6) representation is 70, there is only one type of color-singlet state ϕ_n^s, but there are three types of color-octet states, corresponding to the three possible types of symmetry of B.

$$\underline{70}\text{ case:}\quad \phi_n^s = (1/\sqrt{2})\,(U^\rho B_n^\rho + U^\lambda B_n^\lambda)\ , \tag{23}$$

$$\phi_j^{s\lambda} = U^\lambda B_j^s\ , \tag{24a}$$

$$\phi_j^{a\lambda} = U^\rho B_j^a\ , \tag{24b}$$

$$\phi_j^{m\lambda} = (1/\sqrt{2})(U^\rho B_j^\rho - U^\lambda B_j^\lambda)\ . \tag{24c}$$

The first superscript of ϕ is the symmetry of B, and m denotes mixed symmetry.

The perturbation H' is SU(6) symmetric and so does not mix SU(6) representations. The matrix elements of H' in the 56 and 20 cases are of the forms,

$$H'^{56}_{jn} = \langle B^{\lambda}_{j}, H'B^{s}_{n} \rangle \ , \tag{25}$$

$$H'^{20}_{jn} = \langle B^{\rho}_{j}, H'B^{a}_{n} \rangle \ . \tag{26}$$

In the case of the $\underline{70}$ representation, the three possible types of H'_{jn} are

$$H'^{s70}_{jn} = (1/\sqrt{2})\langle B^{s}_{j}, H'B^{\lambda}_{n} \rangle \ , \tag{27a}$$

$$H'^{a70}_{jn} = (1/\sqrt{2})\langle B^{a}_{j}, H'B^{\rho}_{n} \rangle \ , \tag{27b}$$

$$H'^{m70}_{jn} = \tfrac{1}{2}(\langle B^{\rho}_{j}, H'B^{\rho}_{n} \rangle - \langle B^{\lambda}_{j}, H'B^{\lambda}_{n} \rangle) \ , \tag{27c}$$

where the first superscript of H' is the symmetry of the intermediate orbital state.

In order to compute the matrix elements of H' one can express the orbital wave functions B in the representation of the rectilinear harmonic-oscillator excitations, i.e., in terms of the quantum numbers $N(\rho_i)$ and $N(\lambda_i)$, where i denotes x, y, and z. The total oscillator level N is given by $N = N(\rho) + N(\lambda)$, where $N(\rho) = N(\rho_x) + N(\rho_y) + N(\rho_z)$, etc. The unperturbed energy E_0 is equal to $\hbar\omega(N+3)$, where $\omega = (2k_0/M_0)^{\frac{1}{2}}$. Because of the assumption of Eq. (20), H' is diagonal in the $N(\rho_i) - N(\lambda_i)$ representation. The diagonal matrix elements are

$$\langle N'_j s \,|H'|\, N'_n s \rangle = \delta_{N's} \tfrac{3}{8}(k'/k_0)\hbar\omega\,[N(\rho) - N(\lambda)] \ , \tag{28}$$

where $\delta_{N's}$ is the product of δ's for all six $N(\rho_i)$ and $N(\lambda_i)$.

In order to compute the H'_{jn} for a quark-model level N and orbital angular momentum L, one needs the orbital wave functions classified by their symmetries. I will consider all states up to and including N = 3, and the L = N = 4 states. The relevant wave functions have been obtained from the results of Karl and Obryk, and are listed in the Appendix.[7]

The unperturbed energies of all states of a given N are equal. The degeneracy is broken by the second-order perturbation term $\sum_j |H'_{jn}|^2/(E_n-E_j)$. The intermediate states j are color octets, so E_n-E_j is negative. I assume that color-octet states of the same N are degenerate. Since the perturbation connects only singlet and octet states of the same N, the energy denominator (E_n-E_j) is the same for all states of the same quark-model level. The energetically favored states of a particular N are those with the largest factor $\sum_j |H'_{jn}|^2$. The sum includes all relevant states of the proper symmetry, and in the case of the representation 70, the different symmetry possibilities shown in Eqs. (27a)-(27c).

The relevant $\sum_j |H'_{jn}|^2$ may be computed by using Eqs. (25) through (28) and the list of orbital wave functions in the Appendix. In making these calculations one may use the fact that all the following polynomials D satisfy the normalization condition of the Appendix:

$$\sqrt{2}\, Y^{\rho}_{+} Y^{\rho}_{+}\,,\quad 2Y^{\lambda}_{+} Y^{\rho}_{+}\,,\quad \tfrac{4}{3}^{\frac{1}{2}} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\,,\quad 2Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\,,\quad \tfrac{2}{3}^{\frac{1}{2}} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\,,$$

$$\tfrac{8}{3}^{\frac{1}{2}} Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\,,\ \text{and}\ 2Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\,.$$

I define the operator $\Delta = N(\rho) - N(\lambda)$, whose eigenvalues are $N(\rho) - N(\lambda)$ in the $N(\rho_i) - N(\lambda_i)$ representation. It is seen from Eq. (28) that H'_{jn} is proportional to Δ_{jn}, so that the quantity $\sum_j |\Delta_{jn}|^2$ is a measure of the amount by which the color-symmetry breaking lowers the energy of the state n. The values of Δ and $|\Delta|^2 = \Delta^2$ for the states under consideration are listed in Table I.

The notation is similar to that of Eqs. (25) to (27c). The numerical superscript of Δ is the SU(6) representation, which is of the same symmetry as the orbital wave function of the singlet state n. In the cases of the representations 56 and 20, the intermediate state must be of mixed orbital symmetry, while in

the 70 case, the orbital symmetry of the intermediate state is given as the first superscript. Subscripts are given only for those N and L for which there are two orbital states of mixed symmetry. In these cases the states are denoted by α and β, as in the Appendix. The two subscripts of Δ refer to the intermediate and initial states, respectively. For a particular state n, if there are none or only one intermediate state that may contribute, only Δ^2 is given. The Δ^2 include the sum over intermediate states, and $(\Delta_\alpha^{70})^2$ denotes $\sum_j (\Delta_{j\alpha}^{70})^2$.

It is seen from the results of Table I that when there are two states of intermediate orbital symmetry for a particular N and L, the states α and β are the proper ones to diagonalize H to second order, i.e., $H'_{\alpha j} H'_{j\beta}$ is zero for all states j.

The results concerning which representations are favored are essentially the same as when the color-symmetry breaking is large.[3] When N = 2 (L = 2 or 0) and when N = 4, the representation 56 is favored, and when N = 3 (L = 3 or 1), the representation 70 is favored. When there are two states of the 70 of the same N and L, the energy correction is small for one of them.

IV. CONCLUSIONS

Color-symmetry breaking removes the degeneracy of the color-symmetric quark model. It is assumed here that the symmetry breaking is sufficiently great so that the comparatively unfavored states do not appear in the same mass region as the favored states. The type of deviation considered is essentially one in which the forces between red and white quarks are of range different from that of forces involving blue quarks.

In R1 it was shown that if the symmetry-breaking is large and of such sign that baryon wave functions consist of a tightly bound red-white diquark, with a comparatively loosely bound blue quark, the favored states are of the SU(6) representations and parities $\underline{56}^{+}$ and $\underline{70}^{-}$. This is in agreement with experimental observation. One of the main conclusions of this paper is that this result remains the same if the symmetry-breaking is sufficiently weak that one may use eigenfunctions of a color-symmetric Hamiltonian as unperturbed wave functions.

For simplicity it was assumed in Sec. II that the quark mass-splitting is such that only states of the same total harmonic-oscillator level are connected by the perturbation. However, some calculations have been done for levels up to $N = 2$, in which this mass-splitting is not included; the results concerning which states are favored are not changed.

Since the symmetry breaking of Sec. II contributes only in second order, the effect does not depend on the sign of the perturbation. The results would be the same if the red-white force were of relatively long range. However, if this were the sign and the symmetry breaking were large, the wave function would not be so simple as the quark-diquark structure used in R1.

The difference between the Δ^2 for the favored and unfavored states in Table I is the same order as the size of Δ^2 for the favored states. Clearly, if the

symmetry breaking is very weak, the unfavored states (such as the $\underline{70}$ for L = N = 2) should be observed experimentally. It was pointed out in R1 that, for strong symmetry breaking, some unfavored $\underline{56}^-$ and $\underline{70}^+$ states should appear for sufficiently high N. Thus, if color-symmetry breaking is responsible for the observed deviations from the predictions of the color-symmetric quark model, some $\underline{70}^+$ and $\underline{56}^-$ states should appear for high N. If this occurs, whether or not such states occur also at low N will be an indication of the strength of the symmetry breaking. If the model of R1 is essentially correct, one would also expect that states involving orbital excitation of the red-white di-quark should appear at high masses.

Whether the symmetry breaking is large or small, the predicted energy gap between the N = 0 ($\underline{56}$) and N = 1 ($\underline{70}$) levels is greater than that between the N = 1 ($\underline{70}$) and N = 2 ($\underline{56}$) levels.[8] This prediction is difficult to test experimentally because of the mass differences between states of the same level. However, if one considers only zero-strangeness states of the same Regge sequence, and measures in terms of either mass or mass-squared, the N = 0-1 difference is larger than the N = 1-2 difference.[9]

There is another possible source of degeneracy breaking of such states as the N = L = 2, $\underline{56}$ and $\underline{70}$ states of the color-symmetric quark model. The radial dependence of the two-quark potential may be more nearly Coulomb type than oscillator type. This would cause the (1s)(1d) state to be lower than the $(1p)^2$ state, where the two orbital angular momenta refer to the ρ and λ coordinates. This could break the degeneracy of the color-symmetric model, since the ratio of these two states is not the same for all N = 2 states. On the other hand, it is reasonable to assume that the two-quark radial dependence is the same as that between a quark and an antiquark in a meson. No such deviation from oscillator

dependence is observable in the meson spectrum, since many $N = 0$, 1, and 2 states are nearly exchange degenerate, i.e., lie close to a linear Regge trajectory. Therefore, it is not reasonable to assume that deviation from oscillator dependence makes a large effect on the baryon spectrum either.

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APPENDIX

This Appendix contains a list of all the orbital wave functions B for the harmonic oscillator levels up to and including N = 3, and for the N = L = 4 states. The B are related to polynomials D of order N in $\vec{\lambda}$ and $\vec{\rho}$ by the equation

$$B = (q^{\frac{1}{2}N+3}\pi^3)^{-\frac{1}{2}} D \exp\left[-\tfrac{1}{2}(\lambda^2 + \rho^2)/q\right] \quad , \quad q = (\hbar/M_0\omega) \quad .$$

Since $\lambda^2 + \rho^2$ is invariant to all permutations of the three quarks, the symmetry properties of D are those of B. The list is given in Table II; $D^i_{N,L}$ corresponds to the quark-model level N, orbital angular momentum L, symmetry i, and maximum value of the z component of L. If there are two states of the same N, L, and symmetry, they are distinguished by a third subscript, α or β. The functions $Y^{\lambda,\rho}_+$ and $Y^{\lambda,\rho}_0$ are defined by $Y^{\lambda}_+ = -(1/\sqrt{2})(\lambda_x + i\lambda_y)$, $Y^{\lambda}_0 = \lambda_z$, etc. The λ and ρ components of a function of mixed symmetry are listed together, separated by a comma.

Polynomials of specific symmetries and the appropriate orders in $\vec{\lambda}$ and $\vec{\rho}$ are given in Table 3 of Karl and Obryk.[7] These polynomials are not convenient for calculations, however, because some of them correspond to mixtures of different energy levels, they are not all orthogonal, and they are not normalized uniformly. Each polynomial D of Table II corresponds to only one energy level N, and the corresponding B are all orthogonal and normalized by the condition

$$\int d^3\rho \int d^3\lambda\, |B|^2 = 1 \, .$$

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3. Richard H. Capps, Phys. Rev. Letters 33, 1637 (1974). This paper is referred to here as R1.

4. The fact that this mechanism can lead to color singlets as the lightest states is shown by Y. Nambu in Preludes in Theoretical Physics, A. de-Shalit, H. Feshbach, and L. Van Hove, eds. (North-Holland Publishing Co., Amsterdam, 1966), p. 133.

5. The potential of Eq. (1) is of the same form as that of R1; the red, white, and blue quarks correspond to the symbols A, B, and C, respectively, of R1.

6. The normalization of the generators used here is such that the value of the quadratic Casimir operator for the fundamental triplet representation of SU(3) is $\frac{4}{3}$.

7. G. Karl and E. Obryk, Nucl. Phys. B8, 609 (1968).

8. In R1, where the orbital wave functions are simple, the amount by which a state is favored is proportional to the color-singlet probability of the state. This probability is 1, $\frac{1}{2}$, and $\frac{1}{2}$ for N = 0(56), 1(70), and 2(56) states, respectively.

9. This effect is present if one compares states of maximum angular momentum, i.e., the $\Delta_{\frac{3}{2}}^{+}(1232)$, $N_{\frac{5}{2}}^{-}(1678)$, and $\Delta_{\frac{7}{2}}^{+}(1950)$, or if one compares the states in the nucleon sequence, $N_{\frac{1}{2}}^{+}(938)$, $N_{\frac{3}{2}}^{-}(1525)$, and $N_{\frac{5}{2}}^{+}(1685)$.

TABLE I

VALUES OF Δ AND Δ^2 FOR LEVELS UP TO N = L = 4

$N = 0$: $(\Delta^{56})^2 = 0$

$N = 1$, $L = 1$: $(\Delta^{70})^2 = 1$

$N = 2$, $L = 2$:

$(\Delta^{56})^2 = 4$ $\quad \Delta^{s70} = \sqrt{2}$ $\quad \Delta^{m70} = 0$ $\quad (\Delta^{70})^2 = 2$

$N = 2$, $L = 1$: $(\Delta^{20})^2 = 0$

$N = 2$, $L = 0$:

$(\Delta^{56})^2 = 4$ $\quad \Delta^{s70} = \sqrt{2}$ $\quad \Delta^{m70} = 0$ $\quad (\Delta^{70})^2 = 2$

$N = 3$, $L = 3$:

$(\Delta^{56})^2 = 3$ $\quad (\Delta^{20})^2 = 3$ $\quad \Delta^{s70} = \left(\frac{3}{2}\right)^{\frac{1}{2}}$

$\Delta^{a70} = \left(\frac{3}{2}\right)^{\frac{1}{2}}$ $\quad \Delta^{m70} = 2$ $\quad (\Delta^{70})^2 = 7$

$N = 3$, $L = 2$: $(\Delta^{70})^2 = 1$

$N = 3$, $L = 1$:

$\Delta^{56}_{\alpha s} = \sqrt{3}$ $\quad \Delta^{56}_{\beta s} = 0$ $\quad (\Delta^{56})^2 = 3$

$\Delta^{20}_{\alpha a} = \sqrt{3}$ $\quad \Delta^{20}_{\beta a} = 0$ $\quad (\Delta^{20})^2 = 3$

$\Delta^{s70}_{s\alpha} = \left(\frac{3}{2}\right)^{\frac{1}{2}}$ $\quad \Delta^{a70}_{a\alpha} = \left(\frac{3}{2}\right)^{\frac{1}{2}}$ $\quad \Delta^{m70}_{\alpha\alpha} = -2$

$\Delta^{m70}_{\alpha\beta} = \Delta^{m70}_{\beta\alpha} = 0$ $\quad (\Delta^{70}_{\alpha})^2 = 7$

$\Delta^{s70}_{s\beta} = 0$ $\quad \Delta^{a70}_{a\beta} = 0$ $\quad \Delta^{m70}_{\beta\beta} = 1$ $\quad (\Delta^{70}_{\beta})^2 = 1$

$N = 4$, $L = 4$:

$\Delta^{56}_{\alpha s} = (12)^{\frac{1}{2}}$ $\quad \Delta^{56}_{\beta s} = 0$ $\quad (\Delta^{56})^2 = 12$

$\Delta^{s70}_{s\alpha} = \sqrt{6}$ $\quad \Delta^{m70}_{\alpha\alpha} = 0$

$\Delta^{m70}_{\alpha\beta} = \Delta^{m70}_{\beta\alpha} = 2$ $\quad (\Delta^{70}_{\alpha})^2 = 10$

$\Delta^{s70}_{s\beta} = 0$ $\quad \Delta^{m70}_{\beta\beta} = 0$ $\quad (\Delta^{70}_{\beta})^2 = 4$

TABLE II

ORTHONORMAL POLYNOMIALS D CORRESPONDING TO SPECIFIC SYMMETRIES AND OSCILLATOR LEVELS

N = 0 and 1:

$$D^{S}_{0,0} = 1$$

$$D^{\lambda,\rho}_{1,1} = \sqrt{2}\, Y^{\lambda}_{+} \,,\, \sqrt{2}\, Y^{\rho}_{+}$$

N = 2:

$$D^{S}_{2,2} = Y^{\rho}_{+} Y^{\rho}_{+} + Y^{\lambda}_{+} Y^{\lambda}_{+}$$

$$D^{\lambda,\rho}_{2,2} = (Y^{\rho}_{+} Y^{\rho}_{+} - Y^{\lambda}_{+} Y^{\lambda}_{+}) \,,\, 2\, Y^{\lambda}_{+} Y^{\rho}_{+}$$

$$D^{a}_{2,1} = \sqrt{2}\, (Y^{\lambda}_{+} Y^{\rho}_{0} - Y^{\rho}_{+} Y^{\lambda}_{0})$$

$$D^{S}_{2,0} = \left(\frac{1}{3}\right)^{\frac{1}{2}} (\rho^2 + \lambda^2 - 3q)$$

$$D^{\lambda,\rho}_{2,0} = \left(\frac{1}{3}\right)^{\frac{1}{2}} (\rho^2 - \lambda^2) \,,\, \left(\frac{1}{3}\right)^{\frac{1}{2}} 2\vec{\lambda}\cdot\vec{\rho}$$

N = 3:

$$D^{S}_{3,3} = \left(\frac{1}{3}\right)^{\frac{1}{2}} (-\, Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} + 3\, Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+})$$

$$D^{a}_{3,3} = \left(\frac{1}{3}\right)^{\frac{1}{2}} (Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} - 3\, Y^{\rho}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+})$$

$$D^{\lambda,\rho}_{3,3} = (Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} + Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+}) \,,\, (Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} + Y^{\rho}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+})$$

$$D^{\lambda,\rho}_{3,2} = \left(\frac{8}{3}\right)^{\frac{1}{2}} (Y^{\lambda}_{+} Y^{\rho}_{0} - Y^{\rho}_{+} Y^{\lambda}_{0})\, Y^{\rho}_{+} \,,\, \left(\frac{8}{3}\right)^{\frac{1}{2}} (Y^{\rho}_{+} Y^{\lambda}_{0} - Y^{\lambda}_{+} Y^{\rho}_{0})\, Y^{\lambda}_{+}$$

$$D^{S}_{3,1} = \left(\frac{1}{5}\right)^{\frac{1}{2}} \left[(\rho^2 - \lambda^2)\, Y^{\lambda}_{+} + 2\vec{\lambda}\cdot\vec{\rho}\, Y^{\rho}_{+}\right]$$

$$D^{a}_{3,1} = \left(\frac{1}{5}\right)^{\frac{1}{2}} \left[(\rho^2 - \lambda^2)\, Y^{\rho}_{+} - 2\vec{\lambda}\cdot\vec{\rho}\, Y^{\lambda}_{+}\right]$$

$$D^{\lambda,\rho}_{3,1,\alpha} = \left(\frac{1}{15}\right)^{\frac{1}{2}} \left[(3\lambda^2 + \rho^2 - 10q)\, Y^{\lambda}_{+} + 2\vec{\lambda}\cdot\vec{\rho}\, Y^{\rho}_{+}\right] \,,\, \left(\frac{1}{15}\right)^{\frac{1}{2}} \left[(3\rho^2 + \lambda^2 - 10q) Y^{\rho}_{+} + 2\vec{\lambda}\cdot\vec{\rho}\, Y^{\lambda}_{+}\right]$$

$$D^{\lambda,\rho}_{3,1,\beta} = \left(\frac{4}{3}\right)^{\frac{1}{2}} \left[(-\rho^2 + q) Y^{\lambda}_{+} + \vec{\lambda}\cdot\vec{\rho}\, Y^{\rho}_{+}\right] \,,\, \left(\frac{4}{3}\right)^{\frac{1}{2}} \left[(-\lambda^2 + q) Y^{\rho}_{+} + \vec{\lambda}\cdot\vec{\rho}\, Y^{\lambda}_{+}\right]$$

TABLE II (cont.)

$N = 4,\ L = 4$:

$$D^{S}_{4,4} = \frac{1}{2}\left(Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} + Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} + 2Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\right)$$

$$D^{\lambda,\rho}_{4,4,\alpha} = \left(\frac{1}{3}\right)^{\frac{1}{2}}\left(Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} - Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+}\right),\left(\frac{4}{3}\right)^{\frac{1}{2}}\left(Y^{\rho}_{+} Y^{\rho}_{+} + Y^{\lambda}_{+} Y^{\lambda}_{+}\right) Y^{\lambda}_{+} Y^{\rho}_{+}$$

$$D^{\lambda,\rho}_{4,4,\beta} = \left(\frac{1}{12}\right)^{\frac{1}{2}}\left(-Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} - Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} + 6Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+}\right),$$

$$\left(\frac{4}{3}\right)^{\frac{1}{2}}\left(Y^{\lambda}_{+} Y^{\rho}_{+} Y^{\rho}_{+} Y^{\rho}_{+} - Y^{\rho}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+} Y^{\lambda}_{+}\right)$$