SLAC-PUB-1615 August 1975 (T/E)

ELECTRON-POSITRON ANNIHILATION ABOVE 2 GeV AND THE NEW PARTICLES*

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(To be published in Phys. Repts. <u>C</u>, 1975.)

*Work supported by the U.S. Energy Research and Development Agency.

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1. INTRODUCTION AND THEORETICAL FRAMEWORK

1.1 Introduction

In this paper we review experimental results on hadron production in electron-positron annihilations at center-of-mass energies above 2 GeV. Our purpose is to present as complete a picture as we can of the current experimental situation in this field. Therefore where necessary we shall present preliminary results if final results are not available.

At present we have no fundamental theory of hadron production in electron-positron annihilations which directly and simply fits all the existing data. And it would be unproductive for us to attempt to squeeze all the data into an existing theory. Indeed, we hope that in writing this paper we can stimulate the physicist, particularly the young physicist, to take a fresh look at the overall experimental situation; and in doing so to perhaps find a new theoretical direction.

Therefore we do not discuss the details of any of the existing theories nor do we attempt to carry out definitive tests of these theories. We only present the existing theories as a sort of framework upon which to organize the experimental findings. And we only point out in very general terms how particular theories agree or disagree with the data.

1.2 General Dynamics and Kinematics

The most general process for the production of hadrons in e⁺ - e⁻ annihilations is shown in Fig. la. Here the cross hatched region might include a direct electron-hadron interaction. But existing data do

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not demand such an interaction. And if we accept the traditional belief that the electron has only electromagnetic and weak interactions, the dominant process is the exchange of a single, timelike virtual photon between the electronic and the hadronic systems, Fig. 1b. Higher order photon exchange processes, Fig. 1c, may also occur. Although such processes are expected to have cross sections smaller by a factor of the order $\alpha = 1/137$ compared to the single photon exchange process there is no experimental evidence on this point.

Returning to the single photon exchange process, Fig. 1b, we see that all the ignorance hidden in the cross hatched region of the diagram in Fig. 1a has been transferred to the photon-hadron vertex. The basic problem is to find the correct dynamical description of that vertex. Before discussing some models for this vertex, we consider some kinematics.

In the simplest colliding beams situation, the electron and positron have equal, but opposite, momenta in the laboratory system, Fig. 2a. Then the laboratory and center-of-mass system coincide. Designating the energy of each beam by E, we have

$$W = 2E \qquad (l-l)$$

where W is the total energy of the hadronic system. We also use

$$s = W^2 = 4E^2$$
; (1-2)

s is of course also the square of the four-momentum of the timelike virtual photon in Fig. 1b. We note that we use a metric in which the product of two four-vectors is given by $a \cdot b = a^{\mu}b_{\mu} = a^{0}b^{0} - a \cdot b$. When the angle between the two beams, η , is non-zero, Fig. 2b, we have (ignoring the electron mass)

$$s = 2E^{2}(1 + \cos \eta)$$
 (1-3)

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For the general reaction

$$e^+ + e^- \to 1 + 2 + 3 + (1-4)$$

in which N particles designated by 1, 2, 3 ... N are produced, the cross section is 1,2

$$\sigma = \frac{8\pi^4}{s} \sum_{\substack{e^+, e^- \text{ spins} \\ \text{final spins}}} \int_{n=1}^{N} \left(\frac{d^3 p_n}{(2\pi)^3 2E_n} \right) |T_{fi}|^2 \delta^4 (P_f - P_i) \quad (1-5)$$

As usual the summation over spins means an average over the initial states and a summation over the final states. Here, as in the remainder of this paper, we set the electron mass equal to zero. This and all formulas in this paper are in the center-of-mass frame.

Assuming one-photon exchange, Fig. 1b, the matrix element ${\tt T}_{\rm fi}$ has the form

$$T_{fi} = \frac{-e^2 j^{\mu}_{+} - J_{had}, \mu}{e}$$
(1-6)

 $j_{e^+e^-}$ is the leptonic transition current and J_{had} is the four-vector transition current between the vacuum and the final state particles.

In the center-of-mass system, taking the e^+ to be moving along the +z axis and the beams to be unpolarized we obtain a useful simplification of Eqs. 1-5 and 1-6. Noting that the virtual photon four-momentum k has the properties

$$k = (k^{0}, k), k = 0, k^{0} = W, k^{V} J_{had,V} = 0$$
 (1-7a)

we obtain

$$J_{\text{hadron},0} = 0 \qquad (1-7b)$$

and

$$\sigma = \frac{(2\pi)^6 \alpha^2}{s^2} \sum_{\substack{\text{final} \\ \text{spins}}} \int \prod_{n=1}^{N} \left(\frac{d^3 p_n}{(2\pi)^3 2E_n} \right) \left[J_{\text{had},x}^{\dagger} J_{\text{had},x} + J_{\text{had},y}^{\dagger} J_{\text{had},y} \right]$$

 $\times \delta^{l_{\rm f}}(P_{\rm f} - P_{\rm i}) \tag{1-8}$

The subscripts x and y on J_{had} refer to the x and y spatial axis.

We note that the order of magnitude of the cross section is set by α^2 ; $\alpha = 1/137$ is the electromagnetic coupling constant. Furthermore, unless the integral over the current increases with energy, the cross section will decrease at least as rapidly as $1/s^2$ as s increases.

The acceptance of single photon exchange as the dominant process also leads to a strong restriction on the angular distribution of the entire hadronic system because the total angular momentum of the hadronic system is 1. The angular distribution is limited to the terms 1, sin θ , $\cos \theta$, $\sin^2 \theta$, $\cos^2 \theta$, $\sin \theta \cos \theta$ with respect to θ ; and to 1, $\sin \varphi$, $\cos \varphi$ with respect to φ (θ , φ being the spherical angles about the z axis). If the e⁺ and e⁻ beams are unpolarized, as in Eq. 1-8, there will be no φ dependence.^{3,4} For the remainder of this paper we shall ignore polarization effects.

A few examples will illustrate these points. Consider first, just two pseudoscalar particles in the final state, Fig. 3a, such as

$$e^{+} + e^{-} \rightarrow \pi^{+} + \pi^{-} \qquad (1-9a)$$

 or

$$e^{+} + e^{-} \rightarrow K^{+} + K^{-} \qquad (1-9b)$$

Then for Eq. 1-9a

$$\frac{\mathrm{d}\sigma_{+\pi^{-}}}{\mathrm{d}\Omega} = \frac{\alpha^{2} \beta^{3} \sin^{2} \theta |\mathbf{F}_{\pi}(\mathbf{s})|^{2}}{8\mathbf{s}}$$
(1-10)

and a similar equation holds for E. 1-9b. Here $\beta = m/E$ where m is the mass of the π . $F_{\pi}(s)$ is the pion form factor.¹ The total cross section is

$$\sigma_{\pi^{+}\pi^{-}} = \frac{\pi \alpha^{2} \beta^{3} |F_{\pi}(s)|^{2}}{3s}$$
(1-11)

As another example consider the production of just two spin 1/2 point Dirac particles; the only known example being (Fig. 3b)

$$e^{+} + e^{-} \rightarrow \mu^{+} + \mu^{-}$$
 (1-12)

Then

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2 \beta}{4\mathrm{s}} \left[(1 + \cos^2 \theta) + (1 - \beta^2) \sin^2 \theta \right] \qquad (1-13a)$$

In the high energy limit of $\beta \rightarrow 1$

$$\frac{\mathrm{d}\sigma_{\mu\mu}}{\mathrm{d}\Omega} = \frac{\alpha^2(1+\cos^2\theta)}{4\mathrm{s}} \qquad (1-13\mathrm{b})$$

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s} = \frac{21.71}{E^2}$$
 nb (1-13c)

In the last equation E is in GeV. As a final example consider

$$e^{+} + e^{-} \rightarrow \pi^{+} + \pi^{-} + \pi^{0}$$
 (1-14)

 $_{\rm Then}^{(2)}$

$$\frac{d\sigma}{dp_{+}^{0} dp_{-}^{0} d\cos\theta} = \frac{\alpha^{2}}{(2\pi)^{2} 16s} |H(s)|^{2} (p_{+} \times p_{-})^{2} \sin^{2}\theta$$
(1-15)

where p_+ , p_- are the four-vectors of the π^+ and π^- respectively, H(s) is a form factor and θ is the angle between $p_+ \times p_-$ (the normal to the production plane) and the +z axis.

We have already noted that the final state must have total angular momentum 1, assuming one photon exchange. <u>One-photon exchange</u> leads to additional restrictions on the final state:

(a) The final state parity (P) = -1 since parity is conserved in electromagnetic interactions.

(b) The final state isotopic spin (I) is 0 or 1 on the usual assumption that the photon couples almost exclusively to I = 0 or I = 1 states.

(c) The final state charge conjugation number (C) is -1. This prohibits the reaction $e^+ + e^- \rightarrow \pi^0 + \pi^0$, although the reaction is allowed in the two-photon exchange process Fig. 1.c.

(d) If the final state contains only pions, then the G-parity relation, $G = C(-1)^{I}$, demands⁵

odd number of pions if I = 0even number of pions if I = 1 (1-16)

(e) The total final state strangeness (S) is zero.

Of course if detectable hadron production can take place thru the weak interactions, then restrictions (a) through (e) may not apply. In this case the cross hatched area in Fig. 1a would represent the weak interactions. Finally we note that for weak as well as electromagnetic production we believe:

(f) The total final state baryon number (B) is zero.

(g) The total final state charge (Q) is zero.

(h) The total final state electron lepton number (n_{ρ}) is zero.

(i) The total final state muon lepton number $(n_{_{L}})$ is zero.

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We now turn to some models of multihadron production through one-photon exchange.

1.3 The Parton Model

In the parton model⁶ of hadron production we think of the photonhadron vertex as a two step process, Fig. 4,

 $e^+ + e^- \rightarrow parton + antiparton$ (1-17a)

parton + antiparton \rightarrow hadrons (1-17b)

The attractive part of this model is that it makes definite predictions about the size and energy dependence of the total cross section for hadron production, $\sigma_{had}(s)$, if we assume:

1. The partons are point particles with form factors equal to unity.

2. There are a fixed number of kinds of partons with set spins and charges.

3. Free partons cannot exist. Hence every parton-antiparton pair which is produced has a probability of 1 of going into a hadronic final state.

For a parton of mass m, spin 1/2 and charge Qe, e being the unit electric charge, the model predicts 6

$$\sigma_{had}(s) = \frac{4\pi \alpha^2 Q^2}{3s}$$
, $m \ll \sqrt{s/4}$ (1-18a)

If the parton mass m, is close to $\sqrt{s/4}$ we expect that threshold effects will lead to a cross section less than that in Eq. 1-18a. When m is much greater than $\sqrt{s/4}$, we expect the cross section to be much smaller, although virtual parton pairs can still contribute. If there are N types of partons, type n having charge $Q_n e$, all with sufficiently small mass; then

$$\sigma_{\text{had}}(s) = \frac{4\pi \alpha^2 R}{3s} \qquad (1-18b)$$

where (for spin 1/2 partons)

$$R = \sum_{n=1}^{N} Q_n^2$$
 (1-19a)

For later use we note that the numerical form of Eq. 1-18b is

$$\sigma_{had}(s) = \frac{21.71R}{E^2} nb \qquad (1-18c)$$

where E is the electron or positron energy in GeV.

The significance of Eq. 1-18 is simple. The α^2 comes from the electromagnetic coupling constant at each end of the photon line; the 1/s comes from the 1/s² contribution of the photon propagator to the cross section, a power of s being cancelled by vector coupling of the photon. One might argue that most of Eqs. 1-18 is simply a result of one-photon exchange, the significant contribution of the parton model being solely to set the magnitude of R.

Indeed the magnitude of R is so important that R has become an experimental as well as theoretical quantity in its own right. It has become conventional to note Eq. 1-18b is just R times the equation for the cross section of the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$ (Eq. 1-13c) when the muon mass is neglected. We define

$$R(s) = \frac{\sigma_{had}(s)}{\sigma_{\mu\mu}(s)}$$
(19b)

Of course the basic questions are: do partons exist; and if so do they have unit form factors, what are their spins, what are their charges? The only answer we can give at present is that the assumption of their existence and the assignment of certain properties to them does explain a great deal of data.^{1,6} But their existence has not been proven. Indeed the situation may be similar to nineteenth century views of the ether. For a long time the existence of an ether seemed necessary to explain phenomena; the ether was never found; and now we do without it. Perhaps someday we shall do without partons.

The conventional phenomenology is that there are at least three types of quark-partons — the u, d, and s quark with the properties given in Table I. We use the term quark-partons to denote partons with specific spin and internal quantum numbers. If only these quark-partons exist

$$R_{uds} = (2/3)^2 + (1/3)^2 + (1/3)^2 = 2/3$$
 (1-20a)

As we shall see in Sec. 3, the pure quark-parton model fit to the total cross section requires R in the range 3 to 5. Some increase in R can be obtained by accepting the existence of a fourth quark — the charm^{7,8} carrying quark (c) — which has Q = 2/3 (see Table I). Then

$$R_{udsc} = 10/9$$
; (1-20b)

but this is not much of an increase.

To obtain R in the 3 to 5 range we obviously need many more fractional charged quark-partons or integrally charged partons. The first alternative is illustrated by the colored quark-parton scheme.^{8,9} Here an additional three-valued quantum number called color — red, white, and blue — is postulated. Then there are three different u quarks, 3 different d quarks and so forth. Thus

$$R_{uds,color} = 2$$
, $R_{udsc,color} = 10/3$ (1-20c)

The integral charge parton scheme is illustrated by the Han and Nambu model.¹⁰ This model contains 9 quarks, 4 have charge ± 1 and 5 have charge 0. Thus

$$R_{\text{Han-Nambu}} = 4 \tag{1-20d}$$

Even if we do not accept a parton model of e^+ - e^- annihilations very general light cone arguments^{11,12} lead to the same s dependence for $\sigma_{\rm had}$ as is given by Eq. 1-18b, namely

$$\sigma_{had}(s) = \frac{constant}{s}$$
(1-21)

for sufficiently large s. To quote Drell¹² "This [large s] behavior sets in when no large masses are around to impede the approach to the light cone, and it is important to keep in mind that it is not yet clear when that will be." Thus the light cone arguments like the detailed quarkparton model arguments do not tell us when to expect the constant/s behavior. The advantage of the light cone argument is that we can accept the constant/s behavior even if we cannot understand the size of the constant.

To summarize, the parton model predicts that $\sigma_{had}(s)$ will decrease as l/s; unless a threshold for production of higher mass partons is being traversed. In that case there will be an upward step in the cross section proportional to the increase in R. After the step, the cross section will again decrease as l/s. The predictions of the model are not so elegant when it comes to a description of the final hadronic state the process in Eq. 1-17b and in the cross hatched area of Fig. 4. We shall comment on these predictions in the course of discussing the data.

1.4 The Vector Meson Dominance Model

In the vector meson dominance model,¹³⁻¹⁶ Fig. 5, the photon couples at the vertex marked $G_{\gamma V}(s)$ to a vector meson resonance, V, such as the ρ^{0} , ω or φ . The hadronic final states are then simply the decay modes of the V. Of course for a hypothetical high mass vector meson we know nothing <u>a priori</u> about its decay modes — hence the cross hatched area in Fig. 5, once again showing our ignorance. However if we are willing to make an assumption about $G_{\gamma V}(s)$ we can calculate the total hadronic cross section. Indeed we assume $G_{\gamma V}$ is a constant. Then σ_{tot} is described by a simple Breit-Wigner resonance, which in its relativistic form is

$$\sigma_{\text{had}}(s) = \frac{12\pi M_V^2 \Gamma_{ee} \Gamma_V}{s \left[(s - M_V^2)^2 + M_V^2 \Gamma_V^2 \right]}$$
(1-22a)

here M_V is the resonance mass, ${}^\Gamma_V$ is the full width, and ${}^\Gamma_{ee}$ is the partial width for the decay

 $V \rightarrow e^+ + e^-$ (1-23)

given by

$$\Gamma_{ee} = \frac{\alpha^2 M_V}{3(g_v^2/4\pi)}$$
(1-24)

Finally $g_V^2/4\pi$ is a measure of the γ -V coupling; explicitly

$$G_{\gamma V}^{2} = \frac{\alpha}{(g_{V}^{2}/4\pi)}$$
(1-25)

Thus the larger $g_V,$ the weaker the $\gamma\text{-V}$ coupling — a rather unfortunate convention. An alternative form for σ_{had} is

$$\sigma_{\text{had}}(s) = \frac{\frac{l_{4\pi}\alpha^2 M_V^3 \Gamma_V}{(g_V^2/l_{4\pi}) s \left[(s - M_V^2)^2 + M_V^2 \Gamma_V^2\right]}$$
(1-22b)

At s = M_V^2

$$\sigma_{\text{had,max}} = \frac{12\pi}{s} \frac{\Gamma_{\text{ee}}}{\Gamma_{\text{V}}} = \frac{4\pi\alpha^2}{(g_{\text{V}}^2/4\pi)M_{\text{V}}\Gamma_{\text{V}}}$$
(1-22c)

Values of M_V, Γ_V , Γ_{ee} and $g_V^2/4\pi$ for the well known vector mesons are given in Table II.

We shall continue with this model in Sec. 3 where we consider the total cross section data. However we comment now on the large s dependence. If there is a mass M_{max} such that <u>all</u> vector mesons have $M_V < M_{max}$, than unless the higher energy tails of the resonance have a form other than that given by Eq. 1-22a

$$\sigma_{had}(s) = constant/s^3$$
; for $s >> M_{max}$ (1-26)

On the other hand we may postulate an infinite series of vector mesons with ever increasing mass. Thus we may have a slower dependence on s.

$$s_{had}(s) \ge constant/s^n$$
, $n < 3$; as $s \to \infty$ (1-27)

This is obvious. It is also obvious that by adjusting the magnitude of g_V and the mass spectrum of the V's, one can obtain any desired R.

1.5 The Statistical Model for Final States

Assuming one-photon exchange, one might expect the final states in electron-positron annihilation to be at least partially described by statistical model considerations; or at least to be better described by such models than are the final states produced in hadron-hadron collisions. Hadron-hadron collisions are dominated by peripheral effects which produce a strong anisotrophy in the center-of-mass. Most particles move in the forward or backward direction along the line of collisions. However in electron-positron annihilation through one-photon exchange there is center-of-mass isotropy except for the spin l effects discussed in Sec. 1.2. And from a more dynamic point of view, the electron-positron annihilation may be regarded as the formation of a single fireball¹ of energy. The decay of that fireball providing an excellent situation for using statistical ideas.

Returning to Fig. 1b we apply the Fermi statistical model^{1,18} first to the simplified case in which N identical spin O hadrons are produced in the final state. To evaluate the $J^{\dagger}J$ term in Eq. 1-8 we follow Fermi and assume that the energy of the annihilation is contained in a volume Ω . The fundamental assumption is that for N particles in the final state, $J^{\dagger}J$ is proportional to the probability of finding N particles in the volume Ω . Since our normalization convention is that a particle of energy E has a particle density of 2E per unit volume; explicitly

$$J_{had}^{\dagger}J_{had} = A \prod_{n=1}^{N} (2E_{n}\Omega)$$
 (1-28)

where A is a proportionality constant. Equation 1-8 becomes:

$$\sigma_{\rm N} = \frac{(2\pi)^6 \alpha^2 A}{{\rm s}^2 {\rm N!}} \int \prod_{n=1}^{\rm N} \left[\left(\frac{{\rm E}_n \Omega}{(2\pi)^3} \right)^{\frac{{\rm d}^3 {\rm P}_n}{{\rm E}_n}} \right] \, \delta^{l_4} ({\rm P}_{\rm f} - {\rm P}_{\rm i}) \qquad (1-29)$$

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The N! appears in the denominator because there are N identical particles. Following modern treatments¹ of the statistical model we replace the term $E_n \Omega/(2\pi)^3$ by its average value, $1/s_0$, and we retain the Lorentz invariant phase space factors d^3p_n/E_n . Hence

$$\sigma_{N}(s) = \frac{(2\pi)^{6} \alpha^{2} A}{s^{2} N!} \frac{1}{s_{0}^{N}} \int \prod_{n=1}^{N} \left(\frac{d^{3} p_{n}}{E_{n}} \right) \delta^{4}(P_{f} - P_{i}) \qquad (1-30a)$$

and

$$\sigma_{had}(s) = \sum_{n=2}^{\infty} \sigma_{N}(s) \qquad (1-30b)$$

Equations 1-30 predict the energy dependence of the total and topological cross sections, the multiplicity distributions, and the momentum distributions. For example if the particles have zero mass the multiple integral in Eq. 1-30a can be evaluated analytically.¹ Then

$$\int \prod_{n=1}^{N} \left(\frac{d^{3}p_{n}}{E_{n}}\right) \delta^{4}(P_{f} - P_{i}) = \frac{2\pi^{N-1} s^{N-2}}{(N-1)! (N-2)!}$$
(1-31)

and

$$\sigma_{\rm N}(s) = \frac{\alpha^2 B}{{\rm N!}({\rm N-1})! ({\rm N-2})!} \left(\frac{s}{s_0^{\rm I}}\right)^{{\rm N-4}}$$
(1-32)

Here $B = 2(2\pi)^6 A/(\pi s_0^{,4})$ and $s_0^{,} = s_0/\pi$ are constants. This simple model can yield momentum and multiplicity distributions which crudely fit the data. But it cannot give the correct energy behavior. The partial cross sections increase as powers of s for N > 4, yet the data (Sec. 3) show that the total cross section ultimately decreases as s increases.

This is of course a basic problem of this model.

Therefore for actual use in studying $e^+ - e^-$ annihilations we retain only the Lorentz invariant phase space aspects of the model, allowing the matrix element $J^{\dagger}J$ to be extracted as an arbitrary function of (s). Explicitly we replace Eq. 1-30a by

$$\sigma_{N}(s) = C_{N}(s) \int \prod_{n=1}^{N} \left(\frac{d^{3}p_{n}}{E_{n}} \right) \delta^{4}(P_{f} - P_{i})$$
(1-33)

Here $C_N(s)$ is an empirically determined function of s. Of course in the actual data we have not a single type of particle, but many types -- π^+ , π^0 , $\overline{K^+}$, $\overline{K^0}$, p, \overline{p} and so forth. Equation 1-33 must therefore be enlarged to include various kinds of particles. Also charge, strangeness, and baryon number must be conserved. The calculation of $\sigma_N(s)$ under these conditions must be done numerically. The Monte Carlo method devised to fit the data from SPEAR is described in Appendix B.

2. COLLIDING BEAM FACILITY PARAMETERS

Colliding beams facilities are in many ways the most intricate accomplishment of the accelerator builder. And this is not the place to describe these facilities. However it is useful to survey the energy and intensity properties of existing and proposed electron-positron facilities^{19,20} so that the reader can understand the present range of experimental possibilities. In an electron-positron colliding beams facility the beams move in opposite directions in either separate rings, or in different orbits in the same ring. It is only at the interaction regions where the beam orbits intersect and where the particles may collide.

In a colliding beams facility the crucial quantities are the energy, E, of each beam defined in Sec. 1.B, and the luminosity, L. Consider the simple case in which the two beams have equal but opposite momentum. The particles in the beams are not distributed uniformly around the orbit, but collect in bunches. Suppose that a bunch is a cylinder of length ℓ cross sectional area A, and that it contains N particles. When a single electron bunch passes thru a single positron bunch, the number of events produced thru the reaction $e^+ + e^- \rightarrow X$ with cross section σ_X is $N^2 \sigma_X/A$. If there are f bunch collisions per interaction region per second.

Number (
$$e^+ + e^- \rightarrow X$$
) events per second = $\frac{N^2 f \sigma_X}{A}$ (2-1)

It is therefore useful to define the luminosity, L, where

$$L = \frac{N^2 f}{A} \quad cm^{-2} \ sec^{-1}$$

Then

number
$$(e^+ + e^- \rightarrow X)$$
 events per second = $L\sigma_{\chi}$ (2-2)

Existing facilities have actual luminosities in the range of 10^{29} to 10^{32} cm⁻² sec⁻¹. Design goal luminosities for existing and proposed accelerators go as high as 10^{33} cm⁻² sec⁻¹. To get some feeling for these quantities note that the typical total hadronic production cross section, σ_{had} , is 20 nb in the high energy region (Sec. 3). Effective luminosities of 10^{29} to 10^{31} cm⁻² sec⁻¹ then correspond to from 7 to 700

Parameters of existing and proposed electron-positron colliding beam facilities are given in Table III.

3. TOTAL CROSS SECTION

3.1 Data

Figure 6 shows the total cross for hadronic production, σ_{had} as a function of the total energy W (see Appendix C for higher energy data). We observe several kinds of energy dependence.

(a) There are the very narrow resonances, the ω , φ , $\psi(3100)$ and $\psi(3700)$ with full widths of 10 MeV or less.

(b) There are the much broader ρ^0 resonances and the broad resonance-like structure at 4.1 GeV. Incidently, although the ω and ρ^0 appear superimposed in this total cross section curve, they are easily separated experimentally; the dominant decay modes being $\rho^0 \rightarrow \pi^+ + \pi^-$ and $\omega \rightarrow \pi^+ + \pi^- + \pi^0$ respectively (Table II).

(c) Between the resonances is the continuum region which itself has an energy dependence — broadly speaking in the continuum region $\sigma_{\rm had}$ decreases as W increases.

The higher energy data displayed in Fig. 6 are listed in Table IV along with references. We cannot take the space here to describe the various methods used to measure σ_{had} - we can only make a few general comments.

(a) There must be a particle detector in the interaction region which can distinguish $e^+ + e^- \rightarrow hadrons$ event from the very copious Bhabha scattering $e^+ + e^- \rightarrow e^+ + e^-$, from $e^+ + e^- \rightarrow \mu^+ + \mu^-$ events, and from background events produced by beam-gas or beam-vacuum pipe interactions.

(b) The significance of measurements of multiplicity distributions depends upon the solid angle subtended by the detector. The smaller the solid angle the less certain are the multiplicity measurements.

(c) To measure momentum distributions, the detector must have a magnetic field and track chambers.

(d) To determine the particle ratios, the detector must incorporate devices such as scintillation counters for time-of-flight measurements, shower counters, and Cerenkov counters.

These detector requirements cannot be completely satisfied simultaneously. For example, if all the detected particles are to pass thru a Cerenkov counter, the detector must have a relatively small solid angle. Such a detector is illustrated by the DASP double arm spectrometer at DORIS.

A different type of detector is the large solid angle magnetic detector used by the SLAC-LBL collaboration at SPEAR. This detector subtends a 2.8 π sr. solid angle. Most of the higher energy data used in this paper were taken with this detector; hence it is described in detail in Appendix A.

Closely connected to the properties of the detector is the question of how the detector is triggered. That is, under what conditions is the detector instructed to record the event? When studying reactions such as $e^+ + e^- \rightarrow e^+ + e^-$ or $e^+ + e^- \rightarrow \mu^+ + \mu^-$ it is relatively easy to construct a well understood trigger. For $e^+ + e^- \rightarrow$ hadrons events the trigger problem is complex. Some hadronic events may have many charged particles entering the detector and so are easily detected with counters. Other hadronic event may

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have one or zero charged particles entering the detector. Then the event is missed unless neutral particles are detected. Thus a trigger which uses only charged particles will be biased against events with low charged particle multiplicity. For our large magnetic detector at SPEAR we describe the trigger method in Appendix A.

Total hadronic cross section data as well as other hadronic data must be corrected for radiative effects. These effects are caused by the emission of a photon by the incident e⁻ or e⁺, Fig. 7. Then the actual total energy W of the hadronic system is less than 2E. This results in events collected at an energy W₁ being contaminated with events with W < W₁; and $\sigma_{had}(s)$ becomes distorted. A brief discussion of the radiative corrections to σ_{had} is given in Ref. 21.

3.2 Interpretation Below W = 1 GeV

Below W = 1 GeV, σ_{had} is clearly dominated by the production of the ρ^{0} and ω mesons; and the final states correspond to the known decay modes of these vector mesons (Table II). Thus the principle final states are $\pi^{+}\pi^{-}$ and $\pi^{+}\pi^{-}\pi^{0}$. Indeed very little is known about higher multiplicity final states. One of the few measurements²⁷ is $\sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\pi^{0}\pi^{0}) = 11$ nb at 990 MeV.

The vector meson dominance model (Sec. 1.4) provides an elegant explanation of the energy dependence of σ_{had} in this region. Consider the ρ region where the dominant decay mode is

$$e^{+} + e^{-} \rightarrow \rho^{0} \rightarrow \pi^{+} + \pi^{-}$$
(3-1)

Then we can use Eq. 1-22b with $V = \rho$. Replace the Γ_{ρ} in the numerator by $\sigma^2 = \frac{1}{M \rho^3}$

$$\Gamma_{\rho \to \pi^+\pi^-} = \frac{g_{\pi\pi\rho}^{M}\rho^{P}}{48\pi}$$
(3-2)

where $g_{\pi\pi\rho}$ is the $\rho \to \pi^+\pi^-$ coupling constant and β is the pion velocity. Equation 1-22b becomes

$$\sigma_{\text{had}}(s) = \sigma_{\pi^{+}\pi^{-}}(s) = \frac{\pi \alpha^{2} \beta^{3}}{3s} \left(\frac{g_{\pi\pi\rho}^{2}}{g_{\rho}^{2}}\right) \left[\frac{M_{\rho}^{4}}{(s-M_{\rho}^{2})^{2} + M_{\rho}^{2} \Gamma_{\rho}^{2}}\right]$$
(3-3)

Equation 3-3 is of course just Eq. 1-11

$$\sigma_{\pi^{+}\pi^{-}}(s) = \frac{\pi \alpha^{2} \beta^{3} |F_{\pi}(s)|^{2}}{3s}$$
(3-4)

with

$$|F_{\pi}(s)|^{2} = \left(\frac{g_{\pi\pi\rho}^{2}}{g_{\rho}^{2}}\right) \frac{M^{4}}{(s-M_{\rho}^{2})^{2} + M_{\rho}^{2}\dot{r}_{\rho}^{2}}$$
(3-5)

Figure 8a shows the experimental values ¹⁶ of $|F_{\pi}(s)|^2$ extracted using Eq. 3-4. Inserting $M_{\rho} = 770$ MeV, $\Gamma_{\rho} = 125$ MeV into Eq. 3-5, and setting $g_{\pi\pi\rho} = g_{\rho}$, we obtain the curve in Fig. 8a. This shows that the vector meson dominance model works using just the simple form factor of Eq. 3-5. Figure 8b shows a better fit¹⁶ to the data using the Gounaris-Sakurai²⁸ modification of the Breit-Wigner formula with the interference of the $\pi^+\pi^-$ decay mode of the ω taken into account. 3.3 The Region of W = 1 to 2 GeV

The obvious fact in this region is that σ_{had} decreases as W increases. But further interpretation is difficult. First of all vector meson dominance certainly makes some contribution to this region — there are the tails of the ρ , ω and ϕ ; and there is the ρ ' centered at 1600 MeV. But other mechanisms may also contribute. Therefore this region may be difficult to understand using simple theory. Experimentally more data are needed. Also information is needed on the energy threshold behavior of individual hadronic channels.

3.4 Interpretation for W Greater Than 2 GeV

To simplify the discussion of this region we postpone consideration of the ψ particles to Sec. 6. With the ψ particles removed, σ_{had} (Figs. 6 and 9a) shows a roughly monotonic decrease from above 40 nb at 2 GeV to about 18 nb in the 5 GeV range. The only presently known exception to this decrease is a broad enhancement at 4.1 GeV. If we draw a straight line between the 3.8 and 4.8 GeV data points, Fig. 9a, we obtain for this object

> Center ~ 4.1 GeV Height above smooth σ_{had} ~ 12 nb (3-6) Full width at half maximum height ~ 240 MeV

The nature of this enhancement — in particular, is it a resonance — is discussed in Sec. 6.12.3.

To study the energy dependence of σ_{had} we use R defined in Eq. 1-18b. R is listed in Table IV and shown in Fig. 9b. We recall that if R is a constant, σ_{had} varies as 1/s. There is a sequence of observations which can be made on these data.

(a) R is approximately constant at a value of 2.5 from 2 to about3.5 GeV. To within 25%, this behavior agrees with that expected from the parton model for three colored quarks (Eq. 1-20c).

(b) R increases dramatically as W goes from 3.5 to 5 GeV with most of the increase occurring rather suddenly in the neighborhood of 4 GeV. Average values of R are given in Table V. The increase in the average value of R is

$$\Delta R = \langle R \rangle_{4 \text{ to 5 GeV}} - \langle R \rangle_{3 \text{ to 4 GeV}} = (3-7)$$

2.2 ± 0.2 including 4.1 GeV enhancement 1.6 ± 0.2 excluding 4.1 GeV enhancement

Thus the increase occurs whether or not we include the 4.1 GeV enhancement. The fascinating and as yet unanswered question is whether the 4.1 GeV enhancement has anything to do with the increase in R. (See Appendix C for a discussion of the behavior of R at higher energies.)

One possible explanation is that as W increases above 4 GeV, a new set of higher mass quark-partons contributes to hadron production through the diagram in Fig. 4. Labeling these partons by n = N + 1, $N + 2 \dots N'$; the increase in R, Eq. 1-19a, would simply come from

$$\Delta R = \sum_{n=N+1}^{N^{*}} Q_{n}^{2} . \qquad (3-8)$$

In this picture the 4.1 GeV enhancement could be either a threshold effect, or a resonance, or a combination of both.

(c) The vector meson dominance model provides an alternative method of explaining the s behavior of $\sigma_{\rm had}$. Following Sakurai¹⁴ and Greco¹⁵ we write a generalization of Eq. 1-22b as follows.

$$\sigma_{\text{had}}(s) = \frac{4\pi\alpha^2}{s} \sum_{V} \left[\frac{M_V^3 \Gamma_V}{(g_V^2/4\pi) \left[(s - M_V^2)^2 + M_V^2 \Gamma_V^2 \right]} \right]$$
$$\approx \frac{4\pi\alpha^2}{s} \int \left(\frac{2\pi M_V^2}{g_V^2} \right) \frac{M_V \Gamma_V P(M_V^2) dM_V^2}{\left[(s - M_V^2)^2 + M_V^2 \Gamma_V^2 \right]}$$
(3-9)

Here $P(M_V^2)$ is the density of vector meson states. By adjusting $P(M_V^2)$ one can obtain; single resonance behavior such as the 4.1 GeV enhancement; R increasing with s such as occurs in the W = 2 to 5 GeV region; or a constant value of R.

4. MULTIPLICITIES AND PARTICLE RATIOS

4.1 Charged Particle Multiplicities

Having studied the behavior of σ_{had} we begin to investigate the detailed properties of the hadronic states. And one of the simplest properties is the behavior of the average charged particle multiplicity $\langle N_{ch} \rangle_{ee}$ as a function of W. The data discussed here is taken from Refs. 21, 25 and 26. Unfortunately no direct measurements of neutral particle multiplicities are available.

 $\langle N_{ch} \rangle_{ee}$ as a function of W is given in Table IV and Fig. 10a. The ψ 's are excluded, except that the data at W = 3.8 GeV includes the radiative tail of the $\psi(3700)$. As shown in Fig. 10a we can fit $\langle N_{ch} \rangle_{ee}$ by smooth curves of the form:

$$\langle N_{ch} \rangle = a_{1} + b_{1} W$$
; $a_{1} = 2.36, b_{1} = 0.404$ (4-1a)

$$\langle N_{ch} \rangle = a_2 + b_2 \ln W$$
; $a_2 = 1.86, b_2 = 1.56$ (4-1b)

where W is in GeV.

There is no indication that any drastic change occurs in $\langle N_{ch} \rangle_{ee}$ as W increases above 4 GeV to correlate with the change in R (Sec. 3.4). Nor are there drastic changes at other energies.

An immediate question is how does $\langle N_{ch} \rangle_{ee}$ for

$$e^+ + e^- \rightarrow hadrons$$
 (4-2)

compare with $\langle N_{\rm ch} \rangle$ for

$$\pi^{+} + p \rightarrow hadrons$$

$$K^{+} + p \rightarrow hadrons \qquad (4-3)$$

$$p + p \rightarrow hadrons$$

or with proton-antiproton annihilations?

$$p + \bar{p} \rightarrow hadrons (mesons only)$$
 (4-4)

The answer is given in Fig. 10b. As has been discussed by Whitmore,²⁹ $\langle N_{ch} \rangle$ for $\pi^- p$ and pp can be described by a single curve by defining the total initial state kinetic energy in the center-of-mass frame

$$Q = \sqrt{s} - m_a - m_b \qquad (4-5a)$$

for the reaction

$$a + b \rightarrow hadrons$$
 (4-5b)

The masses of a and b are m and m respectively. Then the single formula

$$\langle N_{ch} \rangle = 2.45 + 0.32 \ln Q + 0.53 \ln^2 Q$$
 (4-6)

(curve A in Fig. 10b) using $W_{ee} \equiv Q$, fits both π p and pp multiplicities We see that $\langle N_{ch} \rangle_{ee}$ is very similar in magnitude and W behavior to $\langle N_{ch} \rangle_{\pi^- p}$ and $\langle N_{ch} \rangle_{pp}$ when we use W = Q.

It is hard to say whether or not we should be surprised at the result. We start with very different initial states and the total cross sections differ by a factor of 10^6 ! Yet it appears that once the electromagnetic energy converts into excited hadronic matter, the gross features of the final states in e⁺e⁻ annihilations will be similar to the gross features of the final states produced in hadron-hadron collisions. As we go along this impression will be reinforced. Incidently the deviations of $\langle N_{ch} \rangle_{ee}$ from curve A are not much greater than the deviations from a universal fit to $\langle N_{ch} \rangle$ for just hadron-hadron collisions; such deviations³⁰ are less than $\stackrel{+}{=} 0.3$ units in $\langle N_{ch} \rangle$.

We might also expect that $\langle N_{ch} \rangle_{ee}$ should be very similar to $\langle N_{ch} \rangle_{pp}$ annihilation for the reaction in Eq. 4-4. Unfortunately in higher energy $\bar{p}p$ data it is difficult to separate that reaction from the non-annihilation reaction

 $\bar{p} + p \rightarrow$ nucleon + antinucleon + mesons (4-7) Hence $\langle N_{ch} \rangle_{\bar{p}p}$ contains both reactions. From Abesalashvli <u>et al</u>.³¹ we take the empirical fit.

$$\langle N_{ch} \rangle_{\bar{p}p} = 0.69 + 2.10 \ln W_{\bar{p}p}$$
 (4-8)

In comparing $\langle N_{ch} \rangle_{ee}$ with $\langle N_{ch} \rangle_{\bar{p}p}$ in Fig. 10b we can either set

$$W_{ee} \equiv W_{\overline{p}p}$$
; curve B (4-9a)

or

$$W_{ee} \equiv W_{\overline{p}p} - 2M_{proton}$$
; curve C (4-9b)

Equation 4-9a would be the correct equivalence if only pp annihilation occurred; Eq. 4-9b would be correct if no annihilation occurred. We see that the truth falls in between.

In Fig. 11 we show a model for the $\langle N \rangle_{ch ee}$ multiplicity distributions

$$P(N) = \frac{\sigma_{N \text{ charged had}}}{\sigma_{had}}$$
(4-10)

for W = 3.0 to 4.8 GeV based on data taken by our SIAC-LBL collaboration. These are not the multiplicities seen in the detector, but they are corrected as discussed in Appendix B. The corrections are quite model dependent and the largest correction is for $N_{ch} = 2$. Therefore Fig. 11 is to be taken as a model showing the characteristics decrease in P(N) at large and small N.

Recently the SLAC-IBL magnetic detector collaboration has acquired preliminary data on $\langle N_{ch} \rangle_{ee}$ at higher energies, table 6. This enables us to differentiate between the linear fit of eq.(4.1a) and the logarithmic fit of eq.(4.1b). Over the energy range

$$2.4 \le v \le 7.4$$
 GeV (4.11)

the logarithmic fit

$$\langle N_{ch} \rangle = 1.93 + 1.50 \ln W_{o}$$
, W in GeV, (4.12)

is satisfactory; $\chi^2 = 18$ for 20 degrees of freedom. But the linear fit is poor, yielding $\chi^2 = 28$ for 20 degrees of freedom.

4.2 Particle Fractions

In the large magnetic detector used by the SIAC-LBL collaboration 33 (Appendix A) π 's, K's and \bar{p} 's or p's are separated using time of flight. Therefore there is a momentum upper limit — about 700 MeV/c — above which π 's cannot be separated from K's. And there is a slightly higher upper limit above which p's or \bar{p} 's cannot be separated from π 's or K's. Because of charge symmetry the production of positive or negative particles of the same type must be equal. However in presenting data we use the negative particles because the proton sample is contaminated by protons from beam-gas interactions.

To describe the relative abundance of π , K and \bar{p} we define at a particle momentum p the fractional abundance of type h particle as

$$A_{h}(p) = \frac{number of particles h with momentum p}{number of all negative particles with momentum p}$$
 (4-13)

These fractional abundances are shown in Fig. 12. We observe that as the momentum, p, of a charged particle increases, the probability of it being a K increases roughly linearly. Also

$$A_{K^{-}}(.5 (4-14)$$

quite a high ratio. This fact and the general behavior of the particle fraction is roughly independent of W in this energy range, Fig. 12. The antiproton fraction is a factor of 10 smaller than the K⁻ fraction³⁴ as shown later in Fig. 16.

We know much less about the particle fractions for larger values of p. An intriguing question is whether the K fraction will continue to increase with p. An experiment carried out last year at SPEAR by O'Neill <u>et al</u>.³⁵ reports

$$A_{K^{-}}(p > 1.2 \text{ GeV/c}) = 0.25 \pm 0.08$$
; $W = 4.8 \text{ GeV}$ (4-15)

It has been known for some time that the particle fraction in Fig. 12 are quite similar to the particle fractions found among particles produced near 90° in the center-of-mass in proton-proton collisions.³⁶ In making this comparison p in $e^+ - e^-$ annihilations is replaced by the transverse momentum, p_1 , in p-p collisions. Examples are given in Ref. 34.

5. INCLUSIVE DISTRIBUTIONS AND SCALING

5.1 Single Particle Momentum Distributions

As we begin to study the dynamics of the final hadronic states produced in e⁺ - e⁻ annihilation, we turn to one of the simplest properties of the multi-hadronic final states — the single particle momentum distribution. We define, for charged particles,

$$z = p/p_{max}$$
(5-la)

$$p_{\max}^2 = (W/2)^2 - (\max_{\pi})^2$$
 (5-lb)

where W is as usual the total energy of the hadronic system. We are going to ignore the presence of K's, p's and \bar{p} 's in the data unless otherwise noted. We do this partly for simplicity and partly because we can only separate these particles in the low p region. To start without any theoretical prejudices we first look at the distribution

$$F(z,W) = \frac{1}{\langle N_{ch} \rangle \sigma_{had}} \frac{d \sigma_{had}}{dz}$$
(5-2a)

Thus

$$\int_{0}^{1} F(z, W) dz = 1 \qquad (5-2b)$$

These <u>normalized</u> distributions in z are shown in Fig. 13; and the average value of p for charged particles

$$\langle p_{ch} \rangle = \int_{0}^{1} pF(z,W) dz$$
 (5-3)

as well as $\langle z \rangle$, is shown in Fig. 14.

We observe the following:

(a) $\langle p_{ch} \rangle$ increases slowly as W increase, varying from about 400 MeV/c to about 480 MeV/c in this W range.

(b) The production of low p hadrons is greatly favored.

(c) As W increases, F(z,W) increases in the low z region, and correspondingly decreases in the high z region.

We are immediately struck by the resemblance between F(z,W) and the p₁ distributions found in the multi-hadron final states of hadronhadron collisions. For example in p-p collisions,^{29,30} (for pions and s $\leq 200 \text{ GeV}^2$)

 $\langle p_1 \rangle \sim (0.23 + .051 \ln \sqrt{s}) \text{ GeV/c}$ (5-4)

And $d\sigma_{had}/dp_1$ has a shape,^{1,29} except possibly for the high p_1 tails, roughly like those in Fig. 13.

Thus the e^+ - e^- annihilation single particle momentum distributions bring us back to an old fundamental problem still unsolved in hadronhadron physics! Why in multi-particle final states produced in hadronhadron collisions is there a relatively sharp transverse momentum cutoff? Why are small p_1 values favored? In our e^+ - e^- data we have a similar fundamental problem. Why are small values of p (Figs. 13 and 14) favored? Of course we do not know if the same fundamental mechanism causes the transverse momentum cutoff in hadron-hadron collisions and the relatively low $\langle p_{ch} \rangle$ values in e⁺e⁻ annihilation. If it is the same mechanism, perhaps e⁺e⁻ annihilations — free of peripheral and leading particle effects — offer a better place to attack this problem.

We should point out an obvious difference between hadron-hadron collisions and $e^+ - e^-$ annihilations. In hadron-hadron collisions $\langle p_1 \rangle$ is almost independent of s, and $\langle N_{ch} \rangle$ increases as in s. This can occur because the energy in the \sqrt{s} is taken up mostly by the longitudinal momentum. Hence as \sqrt{s} increases it is the longitudinal momentum which increases drastically. We do not have this freedom in e^+ - e^- annihilations. Ignoring mass effects,

$$\langle N_{all particles} \rangle \langle p_{all particles} \rangle \approx \sqrt{s} = W$$
 (5-5)

Hence if $\langle N_{all particles} \rangle$ increases as $\ln W$, $\langle p_{all particles} \rangle$ must increase as W/ln W. Over the W range being considered we cannot distinguish linear dependence from logarithmic dependence in $\langle p_{ch} \rangle$, Fig. 14.

5.2 Phase Space Model for Single Particle Momentum Distributions

As noted in Sec. 1.5 we can fit the single particle momentum distributions to a phase space model in which the multiplicity distributions are fixed empirically (Appendix B). To see how this occurs we rewrite Eq. 1-33 in the form

$$\sigma_{\rm N}(s) = C_{\rm N}(s) R_{\rm N}(s)$$
 (5-6a)

$$R_{N}(s) = \int \prod_{n=1}^{N} \left(\frac{d^{3}p_{n}}{E_{n}}\right) \delta^{4}(P_{f} - P_{i})$$
(5-6b)

Then for identical particles, the single particle distribution is given by

$$\frac{\mathbb{E} \, d\sigma_{had}(s)}{d^{3}p} = \sum_{n=3}^{\infty} NC_{N}(s)R_{N-1}(s_{r})$$
(5-7a)

where

$$s_r = (\sqrt{s} - E)^2 - p^2$$
 (5-7b)

As discussed in Sec. 5.6 the p distribution is roughly isotopic. Hence for convenience and future use we define

$$\frac{E \, d\sigma_{had}}{p^2 dp} = 4\pi G(s, p) \qquad (5-8a)$$

$$\frac{d\sigma_{had}}{dp} = \frac{4\pi p^2 G(s,p)}{E}$$
(5-8b)

A fit to $d\sigma(s)/dp$ at W = 4.8 GeV, using a phase space model with just pions, is shown in Fig. 40. We see that the fit is quite good equally good fits can be made for the data at other values of W. The fact that the phase space model can provide adequate fits to $d\sigma/dp$ means that we cannot hope to see incisive tests of dynamical theories of $e^+ - e^-$ annihilation in the gross features of the momentum distribution. We have to look at more detailed behavior.
5.3 Feynman Scaling

To the veteran of hadron-hadron inclusive physics, the expression in Eq. 5-8a immediately raises the question, is there Feynman scaling^{1,37} in e^+ - e^- annihilation, analogous to that in hadron-hadron collisions? In hadron-hadron inclusive physics, Feynman scaling predicts that the Lorentz invariant differential cross section

$$\frac{E \, d\sigma_{had}}{d^{3}p} = H(s, p_{\parallel}, p_{1}, \phi) \rightarrow H(x, p_{1})$$

$$x = p_{\parallel}/p_{\parallel max}$$
(5-9)

Thus written in terms of x and p_{\perp} , H is independent of s. Equation 5-9 is only correct for $p \leq 1 \text{ GeV/c}$; at higher p_{\perp} there is additional s dependence.³⁷ Should there be Feynman scaling in $e^+ - e^-$ annihilations? If so, in Eq. 5-8a, should $G(s,p) \rightarrow G(p)$ or should $G(s,p) \rightarrow G(p/p_{max}) = G(z)$? As shown in Fig. 15 for <u>charged</u> particles

$$\frac{E \, d\sigma_{had}}{d^3 p} = G(s,p) \rightarrow G(p) \tag{5-10}$$

is a rough fit to the data; the largest deviations are a factor of two. Thus in this W range there is a rough Feynman scaling in p. Thus the Feynman scaling in p here is analogous to the Feynman scaling in p_1 in hadron-hadron collisions. Integrating Eq. 5-10 we obtain

$$\int G(s, p) d^{3}p = \langle W_{ch} \rangle \sigma_{had}$$
 (5-11)

where $\langle W_{ch} \rangle$ is the average total energy in charged particles. As we expect from Fig. 15 this quantity changes little in this W range.

We should reserve judgement on the significance of this <u>Feynman</u> <u>scaling in p</u> because the data presented here **have** a relatively small W range. The authors know of no simple, elegant explanation for the scaling. Simple arguments such as the fragmentation model³⁸ used in hadron-hadron collisions do not apply here. Indeed as W increases we might expect to see scaling versus $z = p/p_{max}$ rather than versus p for large p. A theory which predicts something close to this will be discussed in the next section.

Figure 16 also shows an interesting connection 34 between p and p₁ in p-p collisions for lower p values. The solid lines are the data of Cronin <u>et al.</u>³⁶ on π production at 90° C.M. in p-p collisions at 200 GeV. Their data are plotted versus p₁ and have been renormalized to show the similarity in the slopes of the inclusive cross sections for pions produced in the two reactions. The e⁺e⁻ data seem to break away from the p-p data at higher momentum.

To conclude this section we return to the question of the relative abundances of K's and \bar{p} . When a quantity proportional to $(E/p^2)(d\sigma/dp)$ is plotted versus E, Fig. 17, the π 's, K's and \bar{p} 's show a remarkable regularity. ³⁴ One observes a distribution

$$\frac{E d\sigma_{had}}{p^2 dp} = \text{constant e}^{-E/E_0}, \quad E_0 = 0.164 \text{ GeV} \quad (5-12)$$

This typically thermodynamic model¹ formulation emphasizes again that statistical considerations are dominant in the gross features of the e⁺ - e⁻ final hadronic states.

5.4 Bjorken Scaling

It is well known^{1,39,40} that the differential cross section for electron-nucleon or muon-nucleon inelastic scattering (Fig. 18a)

$$e \text{ or } \mu + n \rightarrow e \text{ or } \mu + hadrons$$
 (5-13)

can be described by two structure functions W_1 and W_2 . Neglecting the lepton mass

$$\frac{d\sigma}{dq^{2}dq \cdot P} = \frac{\pi \alpha^{2}}{q^{4}E} \left[2|q^{2}| W_{1}(q^{2}, q \cdot P) + (4EE \cdot - |q^{2}|) W_{2}(q^{2}, q \cdot P) \right]$$
(5-14)

Here q and P are the four-momenta of the virtual photon and the incidental nucleon (Fig. 18a) respectively. Thus W_1 and W_2 are in general functions of the Lorentz scalars q^2 and $q \cdot P$. Also E and E' are the laboratory energies of the incident and final lepton, v = E - E', and q^2 is negative in our metric. We note that

$$P \cdot q = M v \tag{5-15}$$

where M is the nucleon mass. The total energy of the hadronic system W_{had} is

$$W_{had}^2 = 2Mv + M^2 + q^2$$
 (5-16)

In general W_1 and W_2 are allowed to be functions of q^2 and v; but as predicted by Bjorken and demonstrated experimentally $^{1,39-42}$

$$MW_{l}(q^{2}, q \cdot P) \rightarrow F_{l}(\omega)$$
 (5-17a)

$$vW_2(q^2, q\cdot P) \rightarrow F_2(\omega)$$
 (5-17b)

$$\omega = \frac{2P \cdot q}{|q^2|} = \frac{2Mv}{|q^2|}$$
(5-17c)

 $l \leq \omega \leq \infty$ (5-17d)

for

$$|q^2| \gtrsim 1(GeV/c)^2$$
 (5-18)

The behavior of W_1 and vW_2 as functions of the single scaling variable ω (Eq. 5-17c) is called Bjorken scaling. Recently this scaling has been demonstrated to hold to within 20% even for higher energy, large $|q^2|$ muon-proton inelastic scattering. ⁴²

Is there an analogous scaling law for $e^+ + e^-$ annihilation

$$e^+ + e^- \rightarrow hadrons$$
 (5-19)

From a very general point of view, ^{11,43} such as light cone algebra, the analogy to Bjorken scaling is the statement, Sec. 1.3, that R is a constant in

$$\sigma_{\text{had}}(s) = \frac{4\pi\alpha^2 R}{3s}$$
(5-20)

We have already discussed the validity of this prediction in Sec. 3.4 and Appendix C.

However if we are willing to use a parton model⁴⁴ then we can construct other analogies to Eq. 5-17. Consider the e^+ - e^- annihilation diagram in Fig. 18b in which

$$e^+ + e^- \rightarrow h + anything$$
, $h = \pi$ or K or n, (5-21)

and the momentum of the h is detected. In analogy to Eqs. 5-15 to 5-17 we define

$$P_{h} = \text{four-momentum of } h \text{ (in Eq. 5-21)} = (E_{h}, p_{h}) \quad (5-22)$$

$$q^{2} = s = W^{2} > 0$$

$$x = \frac{2P_{h} \cdot q}{q^{2}} = \frac{2E_{h}}{\sqrt{s}}$$

$$0 \le x \le 1$$

With θ the angle between p_{mh} and the e^+ - e^- axis, the equation analogous to Eq. 5-14 is

$$\frac{d\sigma_{had}}{dxd\cos\theta} = \frac{3}{2} \sigma_{\mu\mu} x\beta_{h} \left[M_{h} \bar{W}_{lh} (q^{2}, q \cdot P_{h}) + \frac{x\beta_{h}^{2}}{4} \left(\frac{E_{h} \sqrt{s}}{M_{h}} \right) \bar{W}_{2h} (q^{2}, q \cdot P_{h}) \sin^{2} \theta \right]$$
(5-23a)

Here we have used

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s} \tag{5-23b}$$

to emphasize the analogy to equations in Sec.1.3 and to Eq. 5-20. $\beta_{\rm h}$ is the velocity of h.

The analogy to Eqs. 5-17 is

$$M_{h} \bar{W}_{lh}(q^2, q \cdot P_{h}) \rightarrow \bar{F}_{lh}(x)$$
 (5-24a)

$$\left(\frac{\mathbb{E}_{h}\sqrt{s}}{\mathbb{M}_{h}}\right)\tilde{\mathbb{W}}_{2h}(q^{2}, q \cdot \mathbb{P}_{h}) \rightarrow \bar{\mathbb{F}}_{2h}(x)$$
(5-24b)

We shall call this <u>special</u> Bjorken scaling in e⁺e⁻ annihilations, reserving the term Bjorken scaling in e⁺e⁻ annihilations for Eq. 5-20. Equation 5-23 becomes

$$\frac{\mathrm{d}\sigma_{\mathrm{had}}}{\mathrm{d}\mathrm{xd}\,\cos\,\theta} = \frac{3}{2}\,\sigma_{\mu\mu}\,\mathrm{x}\beta_{\mathrm{h}}\,\left[\bar{\mathrm{F}}_{\mathrm{lh}}(\mathrm{x}) + \frac{\mathrm{x}\beta_{\mathrm{h}}^{2}}{4}\,\bar{\mathrm{F}}_{\mathrm{2h}}(\mathrm{x})\,\sin^{2}\,\theta\right] \quad (5-25)$$

The charged pion is the only hadron for which we have sufficient data to make use of Eq. 5-25 throughout the x range. Even so we are not yet prepared to separate \bar{F}_{lh} and \bar{F}_{2h} . Therefore we <u>ignore the low</u> x region and approximate

$$\beta \approx 1$$
 , $x \approx z = p/p_{max}$ (5-26a)

Also as discussed in Sec. 5.6 the angular distribution of the charged particle is almost uniform in $\cos \theta$. With these approximations, Eq. 5-25 reduces to

$$\left(\frac{\mathrm{d}\sigma_{\mathrm{had}}}{\mathrm{d}z}\right)_{\pi} \approx \sigma_{\mu\mu} z \left[3\bar{F}_{l\pi}(z) + \frac{z}{2} \bar{F}_{2\pi}(z)\right]$$
(5-26b)

This can also be written in a form to emphasize the scaling in x (now called z)

$$s\left(\frac{d\sigma_{had}}{dz}\right)_{\pi} = f(z)_{\pi}$$
 (5-26c)

We already know that special Bjorken scaling cannot be true for all z because from Eq. 5-26c

$$\int f(z)_{\pi} dz \approx s \sigma_{had} \langle N_{ch} \rangle = R \langle N_{ch} \rangle$$
 (5-27)

and the right hand side of Eq. 5-27 increases with s. Nevertheless a plot of $sd\sigma_{had}/dz$ versus z is shown in Fig. 19a and 19b. There is perhaps crude special Bjorken scaling in the z $\gtrsim 0.5$ region. In this region we can ignore the pion mass, and write Eq. 5-26c in the more transparent form.

$$\frac{d\sigma_{had}}{dp} \approx 2s^{-3/2} f(2p/\sqrt{s}) ; p \gg m_{\pi}$$
 (5-28)

This prediction is in general quite different from the Feynman scaling in p prediction (ignoring the pion mass again) of Eq. 5-10

$$\frac{d\sigma_{had}}{dp} \approx 4\pi p \ G(p) ; p \gg m_{\pi}$$
 (5-29)

As s increases either Eq. 5-28, special Bjorken scaling, or Eq. 5-29, Feynman scaling, must fail. (Only G(p) having the very special form $G(p) = constant/p^4$ allows both predictions to be true.)

5.E The Neutral Energy Question

At present we know very little about the neutral particle — particularly the π° — momentum distributions and multiplicities produced in e^+ - e^- annihilations. From the statistical picture we might expect

$$\langle \mathbb{N}_{\pi^+} \rangle = \langle \mathbb{N}_{\pi^-} \rangle = \langle \mathbb{N}_{\pi^0} \rangle = \frac{1}{2} \langle \mathbb{N}_{\pi^\pm} \rangle$$
 (5-30a)

$$\langle \mathbf{p}_{\pi^+} \rangle = \langle \mathbf{p}_{\pi^-} \rangle = \langle \mathbf{p}_{\pi^0} \rangle \tag{5-30b}$$

Here $\langle N_{\pi^{\pm}} \rangle = \langle N_{\pi^{\pm}} \rangle + \langle N_{\pi^{-}} \rangle$. The expectation in Eq. 5-30a is reinforced by experience in π^{-} p and pp collisions. In these system ⁴⁵ one calculates $\langle N_{\pi^{\pm}} \rangle$ from the measured $\langle N_{ch} \rangle$ by

$$\langle N_{\pi^{\pm}} \rangle = \langle N_{ch} \rangle - 0.5$$
 , πp (5-31a)

$$\langle N_{\pi} \pm \rangle = \langle N_{ch} \rangle - 14$$
, pp (5-31b)

Table VII shows that Eq. 5-30a is quite well satisfied in $\pi^{-}p$ and pp reactions.

However in e^+ - e^- annihilations we are at present quite uncertain as to whether Eqs. 5-30 are satisfied Our doubts come from a study of the charged energy ratio. We define

$$\Gamma_{\rm ch}(W) = \frac{\text{total charged particle energy}}{\text{total energy}}$$
(5-32)

where the total energy is of course W. If only pions are produced, Eqs. 5-30 predict

$$\Gamma_{\rm ch}(W) = \frac{2}{3}$$
 (5-33a)

But the data, Fig. 20, show

$$\mathbf{F}_{ch}(\mathbf{W}) < \frac{2}{3} \tag{5-33b}$$

This has led to a number of questions being raised about what appears to be an excess of neutral energy.

(a) Are other neutral particles — as unexotic as the η or as exotic as the neutrino — carrying off substantial amounts of energy?

(b) Do the data seem surprising in view of isotopic spin considerations or do they perhaps violate isotopic spin conservation with the photon being limited to I = 0 and 1? As discussed in Ref. 5 the I = 0 state requires Eq. 5-30a be satisfied. But the I = 1 state allows broad limits -- $\langle N_{\pi 0} \rangle$ can be larger than $(\langle N_{\pi^+} \rangle + \langle N_{\pi^-} \rangle)$. Therefore the data do not violate isotopic spin conservation, although they do seem to require a statistically somewhat unlikely distributions in the I = 1 state.

These questions can only be decided with more data. There is at present no direct experimental measurement of the total neutral energy, nor are there measurement of the π^0 or η production. This ignorance, combined with our ignorance of the high momentum \bar{p} spectra, allows us to adjust particle production ratios to fit the data in Fig. 20 without violating isotopic spin conservation, and without introducing exotic neutral particles.

5.6 Single Particle Angular Distributions

As discussed in Sec. 1.2, the single particle angular distribution of the hadrons produced in e^+ - e^- annihilation may have terms as complex as $\sin^2 \theta$ or $\cos^2 \theta$; the distribution depending upon the dynamics. (θ is the angle between the hadron's momentum and the e^+ - e^- axis.) For example, the simple phase space model, Sec. 1.5, predicts an isotopic single particle distribution

$$\frac{d\sigma}{d\cos\theta} = \text{constant}$$
(5-34)

On the other hand, the parton model, Sec. 5.3 and Fig. 4, allows a deviation from isotropy if the $1 + \cos^2 \theta$ behavior of the spin 1/2 partonantiparton pair persists in the single particle angular distribution. Assuming charge symmetry and unpolarized e^+ , e^- beams the most general single particle angular distribution is described by

$$\frac{d\sigma}{d\cos\theta} = 1 + \alpha\cos^2\theta, \ |\alpha| \leq 1$$
 (5-35)

The task of the experimenter is to measure α .

We find^{21,34} that for hadrons with $z \neq 0.5$, α is roughly consistent with 0. Since most hadrons have $z \neq 0.5$, the entire single particle angular distribution is roughly isotopic. We say roughly because the data analysis is not completed, and the lack of perfect isotropy in our apparatus (App. A) complicates the analysis. For hadrons with $z \gtrsim 0.5$ the the analysis is even more complicated because the statistics are poorer and the triggering efficiency (App. A) is less well known. For the $z \gtrsim 0.5$ region α differing from 0 is certainly allowed; and values of α of the order of 1/2 or 1 are as consistent with the <u>preliminary</u> data as are values close to 0.

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6. THE ψ PARTICLES

The past few months since the discovery of ψ particles at SPEAR^{46,47} and BNL⁴⁸ have been the most exhilarating for particle physics in many years. There were several ingredients:

(a) The ψ particles are clearly something new. Very narrow resonances at high energy cannot be accommodated in our conventional understanding of hadronic physics. There must be a new quantum number, selection rule, or dynamical principle.

(b) The experimental evidence for the existence of the ψ particles is unmistakable. There are no three standard deviation effects and no <u>ifs</u> or <u>buts</u>.

(c) The ψ particles are relatively easy to produce in a variety of ways. Already nine additional experimental groups at four laboratories have produced and made measurements on the ψ 's.

(d) The opportunities for future study are great. The study of the decay modes of the ψ 's is an experimenter's dream: in e⁺e⁻ annihilation they are produced copiously, at rest, and with virtually no background. And at all high energy laboratories experimenters are energetically searching for other new particles related to the ψ 's.

(e) The theoretical community is in a state of chaos.⁵⁹ Few theorists could resist the temptation to drop their current problems and to begin sketching out their speculations. In spite of this, it is not yet clear what theoretical framework will eventually incorporate the ψ particles.

In this section we will try to summarize the great amount of information which has been gathered in this short period. As we write this report, we realize that before it is printed new discoveries may clarify the situation, or confuse it even further.

6.1 Production in e⁺e⁻ Annihilation

The $\psi(3095)$ and $\psi(3684)$, (which we will hereafter simply call ψ and ψ), are produced copiously in the reactions

$$e^+e^- \rightarrow \psi \rightarrow hadrons$$
 (6-1)

and

$$e^+e^- \rightarrow \psi^* \rightarrow hadrons$$
 (6-2)

Figures 21 and 22 show the apparent cross sections for reactions 6.1 and 6.2 as measured at SPEAR.⁶⁰ These are only apparent cross sections because in both cases the widths of the resonances are considerably smaller than the experimental resolution.

To obtain apparatus-independent values for the cross sections we integrate over energy to obtain

$$\sum_{\psi} = \int \sigma_{\psi}(E) dE = 9900 \stackrel{+}{-} 1500 \text{ nb} \cdot \text{MeV}$$
 (6-3)

$$\sum_{\psi} = \int \sigma_{\psi} (E) dE = 3700 - 900 \text{ nb} \cdot MeV . \qquad (6-4)$$

These integrated cross sections are corrected for the rather considerable effect of initial state radiation ⁶¹ Results from $ADONE^{50,51}$ for Σ_{ψ} are about 30% lower than Eq. 6-3 after allowing for radiative corrections.

The masses of the ψ and ψ' as determined at the various laboratories are summarized in Table VIII. They are all in good agreement. The mass difference is determined more accurately than either mass. From the agreement between SPEAR and DORIS, we can deduce

$$m_{\psi}$$
, $-m_{\psi} = 590 + 1 \text{ MeV}$. (6-5)

6.2 Total and Leptonic Widths of the ψ

In addition to decaying to hadrons, the ψ has a sizeable decay mode into lepton pairs:

$$\frac{\psi \to e^+ e^-}{\psi \to all} = .069^+ .009$$
(6-6)

< >

and

$$\frac{\psi \rightarrow \mu \mu}{\psi \rightarrow \text{all}} = .069 \pm .009 \tag{6-3}$$

The measurement of these remarkable decays together with the measurement of the total cross section (Eq. 6-3) allows us to calculate the true ψ width in a simple way:

Assume that the ψ has a Breit-Wigner shape.¹ Then for any decay mode, f, the cross section $\sigma_{\psi,f}$ for the reaction

$$e^+e^- \rightarrow \psi \rightarrow f$$
 (6-8)

has an energy dependence similar to that in Eq. 1-22, namely

$$\sigma_{\psi,f} = \frac{\pi(2J+1)}{s} \frac{4m^2 \Gamma_{ee} \Gamma_{f}}{(s-m^2)^2 + m^2\Gamma} .$$
 (6-9)

Here m is the mass of the ψ , J is its spin, Γ_{f} is the partial decay width to the state f, and Γ is the total decay width. Since Γ is very small compared to m, we can expand Eq. 6-9 in W-m to obtain the non-relativisitc form,

$$\sigma_{\psi,f} = \frac{\pi(2J+1)}{m^2} \frac{\Gamma_{ee}\Gamma_{f}}{(W-m)^2 + \Gamma^2/4}$$
(6-10)

For simplicity, in Eq. 6-9 and 6-10 we have ignored radiative effects and

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interference between reaction 6.8 and the direct channel

$$e^+e^- \rightarrow f$$
 (6-11)

These effects can be included in a straight forward way.

Finally, integrating Eq. 6-10 and using J = 1 (see Sec. 6.3), we obtain

$$\Sigma_{\psi,f} = \int \sigma_{\psi,f} \, \mathrm{dW} = \frac{6\pi^2}{m^2} \frac{\Gamma_{ee} \Gamma_{f}}{\Gamma} \,. \tag{6-12}$$

We can now use Eq. 6-12 to obtain all of the widths. In particular,

$$\Gamma_{ee} = \frac{m^2}{6\pi^2} \sum_{\psi,all}$$
(6-13)

and

$$\Gamma = \frac{\sum_{\psi,\text{all}}}{\sum_{\psi,\text{ee}}} \Gamma_{\text{ee}} . \qquad (6-14)$$

Table IX contains the ψ widths as determined at SPEAR.⁶⁰ Radiative and interference effects have been included. Redundent quantities are listed in Table IX to display the proper error correlations. Γ_{had} is the partial width to hadronic states.

The astonishing small width of the ψ , about 70 KeV, is, of course, what makes this particle so remarkable. We will take up the question of the significance of this very narrow width after completing our description of the properties of the ψ particles.

6.3 Quantum Numbers of the ψ

Since the ψ is prduced in e⁺e⁻ annihilation, our first guess is that, like the vector mesons, it couples directly to the photon and thus has the same quantum numbers, $J^{PC} = 1^{--}$. This would not have to be the case, however, if the ψ coupled directly to leptons.

We can determine the quantum numbers directly by observing the interference between the leptonic decays of the ψ ,

$$e^+e^- \rightarrow \psi \rightarrow e^+e^-$$
 (6-15)

and

$$e^+e^- \rightarrow \psi \rightarrow \mu^+\mu^- \tag{6-16}$$

and the direct production of lepton pairs,

$$e^+e^- \rightarrow e^+e^-$$
 (6-17)

and

$$e^+e^- \to \mu^+\mu^-$$
. (6-18)

The amplitude for reaction 6-18 is

$$A(e^+e^- \to \mu^+\mu^-) = \left(\frac{3\pi}{W^2}\right)^{1/2} \left(-\frac{2\alpha}{3}\right), \qquad (6-19)$$

and the amplitude for reaction 6-16 is

$$A(e^+e^- \rightarrow \psi \rightarrow \mu^+\mu^-) = \left(\frac{(2J+1)\pi}{W^2}\right)^{1/2} \frac{\Gamma_{ee}}{m-W-i\Gamma/2} \quad . \tag{6-20}$$

If the ψ has the quantum numbers of the photon, the cross section will have the form

$$\frac{d\sigma}{d\theta} = \frac{9\pi}{8W^2} \left(1 + \cos^2\theta\right) \left| -\frac{2\alpha}{3} + \frac{\Gamma_{ee}}{m - W - i\Gamma/2} \right|^2 . \quad (6-21)$$

The sum of the amplitudes which go into Eq. 6-21 are shown graphically in Fig. 23. As the resonance proceeds around the diagram, it is clear that there will be destructive interference below the resonant energy. The ratio of muon pairs to electron pairs as a function of energy is shown in Fig. 24.⁶⁰ This ratio is used because it is least sensitive to normalization effects and because the electron pairs are expected to have a small constructive interference below the resonance (due to intereference with the spacelike diagram). The data are inconsistent with no interference at the 98% confidence level. This is sufficient to confirm our first guesss, that the quantum numbers are those of the photon, $J^{PC} = 1^{--}$. (Since the SIAC-LBL magnetic detector at SPEAR does not subtend the entire solid angle, there are some technical points to consider which turn out not to matter.⁶⁰)

Figure 25 shows the angular distributions of lepton pairs at the resonance energy. The muon pairs and the electron pairs after sub-traction of the spacelike distribution are consistent with the $1 + \cos^2 \theta$ distribution expected for J = 1.60 A similar result has been reported from DORIS for electron pairs.⁵⁷ These distributions confirm the spin assignment.

Figure 26 shows the front-back charge asymmetry measured as a function of energy. 60 The lack of any large asymmetry confirms the P and C assignments and argues against the possibility of the ψ not being an eigenstate of P or C or of being degenerate with another nearby state having even P or C.

6.4 Hadronic Decays of the ψ

We can determine the isotopic spin of the ψ by observing whether it decays into even or odd numbers of pions (see Eq. 1-16). It turns out that the ψ decays into both even and odd numbers of pions -- a violation of I spin. However, this violation occurs in precisely the way we expect it to occur, and in the way it is required to occur, if the ψ couples to a photon.

Consider the three diagrams in Fig. 27. Figures 27(a) shows the direct decay of the ψ into hadrons, (b) shows the decay of the ψ into hadrons via an intermediate photon, and (c) shows the decay into μ pairs. In (b), the nature of the final state, except for a phase factor, must be the same as the non-resonant final state produced in e⁺e⁻ annihilation at the same energy. This state need not conserve isospin and may be quite different from the state produced by (a). Furthermore, we know what contribution (b) must make because the ratio between (b) and (c) must be the same as it would be if the ψ were not in the diagram, about 2.5.²¹ Thus, from the data in Table IX, we deduce that if the ψ couples to a photon (a) contributes 68% to the width of the ψ , (b) contributes 18%, and the leptonic modes contribute 14%.

To test this hypothesis we want to compare the ratio of all pion state cross sections to μ pair cross section on and off-resonance. This is done in Table X.⁶² The off-resonance data are from runs at 3.0 GeV. The results are consistent with all of the even number of pion production (I = 1) coming from the intermediate photon decay, Fig. 27(b). Most of the odd pion production comes from the direct ψ decay, Fig. 27(a), and the ψ appears to decay directly into a pure I^G = 0⁻ state.

The difference in five pion production on and off-resonance is quite dramatic as shown in Fig. 28. At 3.0 GeV the missing mass recoiling against four charged pions shows no structure while at the ψ a very clear π^0 peak is visible. Here the ψ gives us an unexpected bonus -- by showing us what an **isoscalar** state looks like, we see that non-resonant hadron production is largely isovector.

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Figures 29 shows the $\pi^+\pi^-\pi^0$ invariant mass for five pion decay of the ψ . A peak at the ω mass is visible. (This $\omega\pi\pi$ mode is actually larger than it appears since each event is entered in the graph four times.)

The study of the decays modes of the ψ is just beginning. In addition to giving us information about the ψ , these decays may be useful in studying standard meson spectroscopy. For example, the $\pi\pi$ state in the $\omega\pi\pi$ decay is presumably in a pure I = 0 state.

The decays that have been identified so far are listed in Table XI. 62-65 Where the word "seen" is used, it does not imply that the branching ratio is small, but simply that it has not yet been determined.

6.5 Inclusive Momentum Spectrum from ψ Decays

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We are now studying the single particle inclusive distributions in the hadronic decay modes of the Ψ to compare them with the single particle inclusive distributions in the continuum (section 5.1). Although the rough inclusive distributions are similar--that is, the production of low momentum particles is dominant-- there is the possibility that a detailed comparison of the distributions on-and-off-resonance will reflect differences

in the production mechanisms.

6.6 Total and Leptonic Widths of the ψ '

The widths of the ψ ' can be determined in the same way as those of the ψ were determined (Sec. 6.2). There are a few additional problems

involved in determining the total width:

(a) The electron pair decay is much smaller than the non-resonant electron-positron scattering, so it probably cannot be measured accurately. Electron-muon universality may have to be assumed.

(b) The interference between the direct production of lepton pairs and the ψ ' decay is more important. Thus the ψ ' decay rate into leptons as a function of energy must be understood, (See Sec. 6.7).

(c) The $\psi^{i} \rightarrow \psi$ decays (Sec. 6.8) give approximately back to back leptons from the ψ decay. These must be separated and removed.

Preliminary results for the total and leptonic widths are given in Table XII. More precise values should be available soon.

The width of about a half MeV is larger than that of the ψ , but still quite remarkable. This is particularly so since over half of the ψ ' decays go to a ψ (Sec. 6.8) leaving only 100-300 KeV for decays to normal hadrons.

6.7 Quantum Numbers of the ψ '

Since the ψ ' is produced in e⁺e⁻ annihilation, our first guess is again that it has quantum numbers $J^{PC} = 1^{-}$. This guess is further bolstered by a study of the angular distributions in the $\psi' \rightarrow \psi \pi \pi$ decay (Sec. 6.8).

Our guess could be confirmed by a study of the interference effects in exactly the same way as was done for the ψ , (Sec. 6.3). In fact, the interference effects are expected to be more visible at the ψ ' because the interfering amplitudes are of more comparable size. This study is presently proceeding and results should be available soon.

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6.8 $\psi^{\dagger} \rightarrow \psi$ Decays

The $\psi^{\,\prime}$ decays over half the time into the $\psi,$ primarily via the decay mode

$$\psi' \rightarrow \psi \pi \pi$$
 . (6-21)

These decays are visible in the data in two major ways.⁶⁶ Figure 30 shows the missing mass recoiling against all combinations of $\pi^{+}\pi^{-}$. The ψ is clearly visible and the branching ratio

$$\frac{\psi^{*} \to \psi \pi^{+} \pi^{-}}{\psi \to \text{all}} = 0.32 + 0.04 \tag{6-22}$$

can be determined from it.

Alternatively one can search for inclusive ψ decays by looking for the leptonic decay of the ψ in the ψ ' data. Figure 31 shows the invariant mass distribution of the two highest momentum oppositely charged particles in each ψ ' decay. (The particles are assumed to be muons, and electrons have been eliminated.) There are two well separated peaks, one around 3.7 GeV corresponding to ψ ' decays to μ pairs plus the direct production of μ pairs and one around 3.1 GeV from

$$\psi' \rightarrow \psi + \text{anything} .$$
 (6-23)
 $\downarrow \downarrow \mu^+\mu^-$

These data yield

$$\frac{\psi^{\dagger} \rightarrow \psi + \text{anything}}{\psi^{\dagger} \rightarrow \text{all}} = 0.57 \stackrel{+}{-} 0.08 \tag{6-24}$$

A value of 0.54 $\frac{+}{-}$ 0.10 for this ratio has been reported from DORIS. ⁶⁴

From Eqs. 6-22 and 6-24 we discover that

$$\frac{\psi' \to \psi + \text{anything}}{\psi' \to \psi \pi^+ \pi^-} = 1.78 \stackrel{+}{-} 0.10 \quad . \tag{6-25}$$

(The error in Eq. 6-25 is smaller than the combined errors of Eqs. 6-23 and 6-24 because of correlations in the errors.)

Since the ψ^{\ast} decays via

$$\psi^* \to \psi \pi^+ \pi^- \tag{6-26}$$

we also expect

$$\psi' \rightarrow \psi \pi^0 \pi^0 \quad . \tag{6-27}$$

If these are the only modes for $\psi^{\dagger} \rightarrow \psi$ decays and the ψ^{\dagger} is in a definite state of isospin then we expect Eq. 6-25 to have the values

$$\frac{\psi^{*} \to \psi + \text{anything}}{\psi^{*} \to \psi \pi^{+} \pi^{-}} = 1.00 \text{ for } I = 1$$
(6-28)
(6-28)
(6-28)

(The Clebsch-Gordan coefficients have been corrected for phase space.) Clearly I = 0 is preferred.

If we assume that the ψ' , like the ψ , decays to a pure isoscaler state, the difference between Eqs. 6-28 and 6-25 indicates that there are other $\psi' \rightarrow \psi$ modes than just $\psi' \rightarrow \psi \pi \pi$ with branching ratios of about 8%. What can they be? There is no evidence for

$$\psi' \rightarrow \psi \gamma$$
 (C violating) (6-29)

or

$$\psi^{*} \rightarrow \psi \pi^{0}$$
 (I violating) . (6-30)

Since these modes would be visible in the missing mass recoiling against the μ pairs in reaction 6-23, they must have very small branching ratios if they exist at all.

There is evidence for the mode

$$\Psi^{*} \to \Psi \eta \tag{6-31}$$

and it probably has a branching ratio of a few per cent.

Another candidate mode is

$$\psi' \rightarrow \psi \gamma \gamma$$
 (6-32)

which could occur directly or through an intermediate resonance

$$\psi^{*} \rightarrow X\gamma \tag{6-33}$$

This mode is predicted by models which identify the ψ and ψ ' as a bound state of charmed quarks.⁶⁷ The monochromatic photons from reaction 6-33 are presently being searched for (Sec. 7.2).

A preliminary study of the angular distributions of the decay

$$\psi^{*} \rightarrow \psi \pi^{+} \pi^{-} \qquad (6-34)$$

$$\downarrow^{+} \mu^{-} \text{ or } e^{+} e^{-},$$

indicates that the data are consistent with the hypothesis that the dipion state is a J = 0 state in an S-wave with respect to the ψ . If this is confirmed, it will independently establish the quantum numbers of the ψ ' to be $J^{PC} = 1^{--}$.

The dipion invariant mass distribution, shown in Fig. 32, is puzzling.

There is a suppression of low mass pion pairs which does not seem consistent with the usual parameterizations of the S-wave dipion phase shifts. There is no apparent structure in the $\psi\pi$ mass spectrum which could cause this effect. A detailed study of the Dalitz plot and angular distributions of this decay is in progress.

No review of the ψ particles can be deemed to be complete without a computer reconstruction of what reaction (6-34) looks like in the SIAC-LBL magnetic detector. The obligatory picture is shown in Fig. 33.

6.9 Other Hadronic ψ' Decays

No other hadronic decays have been reported for the ψ '. This in itself may be significant. We would expect the 42 $\frac{+}{-}$ 8% of the ψ ' decays which do not go to ψ 's or leptons to go to hadrons in much the same way as the direct ψ decays (see Table XI). But apparently they go to states with only one missing neutral a much smaller fraction of the time. 6.10 Hadroproduction of ψ Particles

The ψ was independently discovered at ${\rm BNL}^{48}$ in the reaction

$$p + Be \rightarrow \psi + anything$$
 (6-34)
 \downarrow, e^+e^-

at 28.5 GeV. Later the ψ was produced at FNAL⁵⁶ in the reaction

n + Be
$$\rightarrow \psi$$
 + anything (6-35)
 $\downarrow \rightarrow \mu^{+}\mu^{-}$

at 250 GeV. The cross sections for these reactions are model dependent but several things are clear:

(a) The cross section for ψ production, corrected for branching ratios, is of order 10⁻³¹ cm²/ nucleon at FNAL energies.

(b) The cross section drops around two orders magnitude as one goes to BNL energies.

(c) Since the ratio of resonant to non-resonant cross sections for the production of lepton pairs in these experiments is much higher than those measured in e^+e^- annihilation, the ψ is not produced via an intermediate virtual photon. This is not surprising since we know the ψ has direct decays to hadrons, (Sec. 6.4).

Hadroproduction of the ψ ' has not been reported.⁶⁹

6.11 Photoproduction of the ψ Particles

Photoproduction of the ψ and ψ ' has been measured at SIAC^{57,58} and FNAL.⁵⁶ At SIAC a preliminary measurement of the cross section for

$$\gamma + Be \rightarrow \psi + anything$$
 (6-36)
 $\downarrow \rightarrow \mu^+\mu^-$

per nucleon and corrected for branching ratios is $3.7 \begin{array}{c} +2.2 \\ -1.5 \end{array}$ nb at 18 GeV. ⁵⁷ At FNAL the same cross section corrected for coherent effects is about 13 nb at about 150 GeV. An upper limit of 1 nb at 11 GeV was set at Cornell.⁷⁰ Quantitative results on ψ ' photoproduction are not yet available.

With the aid of vector dominance (Sec. 1.4) we can use these results to extract the values of the ψ -nucleon total cross section. In doing this one has to assume that the ψ - γ coupling, g_{ψ} , is the same at $q^2 = 0$ and $q^2 = m_{\psi}^2$; this assumption could be wrong by a large factor. It is also assumed that reaction 6-36 describes a diffractive quasi-clastic reaction. The ψ -nucleon total cross section, σ (ψ N), is obtained from the relation

$$\sigma^{2}(\Psi \mathbb{N}) = 16\pi \frac{g_{\Psi}^{2} \left[\frac{d\sigma(\gamma \mathbb{N} \to \Psi \mathbb{N})}{dt} \right]_{t=0}}{\text{t}}$$
(6-37)

The values for $\sigma(\psi N)$ for both the SLAC and FNAL results are about 1 mb. A comparison of vector meson and ψ photoproduction cross sections, coupling constants, and total nucleon cross sections is given in Table XIII. ^{71,72} The ψ -nucleon total cross section is at least an order of magnitude smaller than the other vector meson cross sections, but it still an order of magnitude larger than the γ -nucleon total cross section. The ψ - γ coupling constant, however, is comparable to those of the vector mesons.

6.12 Conclusions and Unanswered Questions

6.12.1 Are the ψ Particles Hadrons?

Whether the ψ particles participate in the strong interactions (as we know them) is a question which cannot be answered yet. However, except for their narrow widths, there is no evidence against the hypothesis that they are hadrons and there are several bits of evidence in favor:

- (a) P and C appear to be good quantum numbers.
- (b) The ψ does not appear to couple directly to leptons.
- (c) The ψ 's decay directly to hadrons in a definite state of isotopic spin.
- (d) The ψ-nucleon total cross section, as determined from diffractive photoproduction measurements, is about 1 mb.

6.12.2 Why are the ψ Particles Long Lived?

If we assume the ψ 's are hadrons, then there are two general classes of theories which can suppress the decay widths.^{73,74} One possibility is

that the ψ 's strong decay is exactly forbidden by its possession of a new non-additive quantum number. Various color models are examples of this. The other possibility is that the ψ 's strong decay is inhibited. by a dynamical principle based on the existence of new additive quantum numbers. Charm is an example of this case in which the ψ 's have zero charm quantum number but are composed of charmed quarks.

If the ψ 's are colored states then presumably they will decay primarily through photon emission on the grounds "what the photon bringeth the photon taketh away."⁷⁵ So far we have not seen any strong evidence for radiative decays. For example, the $2\pi^{+}2\pi^{-}\pi^{0}$ decay discussed in Sec. 6.4 could contain some contamination from $2\pi^{+}2\pi^{-}\gamma$. However, the observation of a large contribution from $\omega\pi\pi$ and $\rho\pi\pi\pi$ and the position of the missing mass peak in Fig. 28 indicate the $2\pi^{+}2\pi^{-}\gamma$ decay cannot be a major one.

In the charm model the ψ 's are narrow because of a dynamical principle known as Zweig's rule.⁷³ This phenomenological rule states that processes in which initial quark pairs cannot appear on different final state particles are suppressed. This is illustrated in Fig. 34. The φ meson decay into 3π (Fig. 34a) is suppressed relative to the decay into $K\bar{K}$ (Fig. 34b). In the case of the ψ there are no decays like $\varphi \rightarrow K\bar{K}$ because the ψ is below threshold for charmed meson pair production. Decays such as $\psi' \rightarrow \psi\pi\pi$ (Fig. 34c) are also inhibited but presumably not as strongly as ψ' decays to ordinary hadrons.

The verification of the charm model will probably require the discovery of charmed hadrons with weak decays.⁷⁶ In Sec. 7 we will discuss the searches for these particles in e^+e^- annihilation which have been done or are in progress.

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6.12.3 What is the Enhancement Around 4.1 GeV?

The enhancement around 4.1 GeV (Sec. 3.4) could be either a resonance or a threshold phenomena related to the increase in R. There is currently no evidence that the enhancement is actually a resonance. To establish it as a resonance it will probably be necessary to find at least one channel with a clear resonant behavior.

If the entire enhancement is attributed to a resonance, then the partial width to electron pairs, Γ_{ee} , is about 4 KeV, or about twice that of the ψ '. Given this large leptonic width, an explanation which assigns the ψ , ψ ', and the 4.1 GeV enhancement all to be radial excitations of a charmed quark pair seems unlikely.

We have observed three phenomena, each of which in itself is extraordinary:

(a) The ψ particles,

(b) the broad enhancement around 4.1 GeV, and

(c) the increase in R from around 2.5 to 5.

It would be even more extraordinary if these phenomena were not all related. Thus, at first, we should seek explanations which explain all three in a natural way.

7. SEARCH FOR NEW PARTICLES

7.1 Narrow Vector Mesons

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Narrow vector meson states like the ψ can be discovered by raising the energy of the colliding beams a few MeV every few minutes. This is precisely how the ψ ' was discovered.⁴⁷ A systematic scan has been done at SPEAR, in the region 3.2 to 5.9 GeV.⁷⁸ The data in 1.88 MeV steps are shown in Fig. 35. Other than the ψ ' there is no evidence for resonances with integrated cross sections greater than one-quarter that of the ψ '. The search at SPEAR will eventually be extended to masses of about 8 GeV.

A search in the region 1.9 GeV to 3.1 GeV is in progress by two groups at ADONE. 65 In the regions which have been searched so far (1.915 to 2.045, 2.205 to 2.544, and 2.966 to 3.090 GeV) there is no significant structure at the level of one-sixth the cross section of the ψ .

7.2 Monochromatic Photons

Charm particle models predict that there should be many different angular momentum states of the charmed quark-anti-quark pair. Some of these states should be reached by monochromatic photon emission from the ψ' .⁶⁷ The most likely transitions are shown in Fig. 36.

Several experiments which would be sensitive to these photons are in progress. One experiment at SPEAR uses NaI crystals to measure the photon energy and it is sensitive to photons with energies above 50 MeV. Preliminary results indicate that these are no monochromatic photons with energies above 200 MeV with a branching ratio of more than 4%.⁷⁹

7.3 Charmed Mesons

An extensive search has been made for charmed mesons produced in e⁺e⁻ annihilation at 4.8 GeV.⁸⁰ The search looked for narrow peaks in in inclusive two and three body state invariant mass distributions in various modes. No significant peaks were found. The results are shown in Table XIV. The mass region 1.85 to 2.4 is the relevant one for charmed mesons.

To interpret these data we first have to estimate the amount of expected charm meson production. From the usual quark charges, we would estimate that 40% of the events at 4.8 GeV should contain a pair of charmed mesons in addition to any ordinary mesons that may be present, (Sec. 1.3). This gives a cross section of about 15 nb for inclusive charmed meson production. There are three types of charmed mesons which will decay weakly. All other charmed mesons will decay into these. Thus, for each type we expect a cross section of about 5 nb. The limits in Table XIV range from about .1 to .5 nb or from about 2% to 10% in branching ratio.

These limits do not rule out charm models but they make them uncomfortable. Conventional models⁷⁶ seem to predict branching ratios into some of these modes from 2 to 5 times higher than the limits.

7.4 Direct Lepton Production

Another way to search for charmed mesons (and also heavy leptons⁸¹) is to look for "direct" muons or electrons produced in multihadronic events. There are several problems, some of which are unique to e⁺e⁻ annihilation:

(a) There are backgrounds from ordinary hadron decays which must be subtracted. For electrons, there are π^0 Dalitz decays and electrons produced by photons converting in the beam pipe. For muons, there are π and K decays.

(b) Lepton identification is difficult at low energy. Here is one case where being in the center of mass does not help. For example if you wish to reduce the pion punch-through background to 0.7% (5 absorption lengths), then it is necessary to use enough absorber so that only muons of over 1.2 GeV momentum will penetrate.

(c) There are potential problems from highly radiative e and μ pair production and from the two-photon processes $e^+e^- \rightarrow e^+e^-e^+e^-$, $e^+e^- \rightarrow e^+e^- + hadrons$.

So far there has been no evidence for direct electron or muon production at about the 5% level. Attempts are now in progress to increase the sensitivity of these searches to the 1 to 2% level.

Evidence has been seen for anomalous muon production in neutrino interactions.⁷⁷

ACKNOWLEDGEMENT

Much of the data reviewed in this paper is the work of our colleagues at the Lawrence Berkeley Laboratory and at the Stanford Linear Accelerator Center who are our collaborators in the Large Magnetic Detector at SPEAR. We are deeply indebted to them and hope that this paper provides some picture of the scope of their accomplishments.

APPENDIX A

The SIAC-LBL Magnetic Detector at SPEAR

The majority of data discussed in this report came from the SLAC-LBL Magnetic Detector at SPEAR. We include here a description of this detector and its trigger so that the reader can understand its capabilities and limitations. In Appendix B, we will discuss the Monte Carlo techniques which must be employed to interpret the data taken in this detector. A.1 The Detector

A drawing of the detector is shown in Fig. 37. The magnetic field is provided by a solenoidal coil 3.6 m long and 3.3 m in diameter. The field of approximately 4 kG is longitudinal to the beam axis and is uniform to about $\pm 2\%$.

We will start at the interaction region and describe in turn each element through which a produced particle passes. The beam pipe has a mean radius of 8 cm and is made of .15 mm thick corrugated stainless steel. The average effective thickness, due to the corrugations, is .20 mm.

Originally there were two semi-cylindrical scintillators forming a complete cylinder about the beam pipe at a radius of 13 cm. These "pipe counters" were 3 mm thick and 91 cm long. They were part of the trigger and served primarily to reduce triggers from cosmic rays. In September 1974, the original counters were replaced by four semicylindrical scintillators forming two nesting cylinders at 11 and 13 cm radii. The new counters are each 7 mm thick and 36 cm long. The reduction in length was made possible by an increase in SPEAR rf from 51 to 358 MHz and a consequential reduction in the effective bunch length

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from typically 25 cm to 6 cm (fwhm).

There are two cylindrical proportional chambers around the beam pipe at 17 and 22 cm radii. Each consists of 512 sense wires parallel to the beam axis and cathode strips perpendicular to the beam axis. The chambers are 51 and 81 cm long, respectively. The structural material is polystyrene foam so that there is little material in the beam. These chambers were installed in January 1975 and have not been used in the analysis of data data presented in this report. Their main use will be to help identify weak decays of K° , Λ , etc. They may eventually be used to obtain a more general trigger.

The main tracking elements of the detector are four modules of concentric cylinderical magnetostrictive spark chambers at radii of 66, 91, 111 and 135 cm. Each module consists of four cylinders of wires with the wires set at $+2^{\circ}$, -2° , $+4^{\circ}$ and -4° with respect to the beam axis. The tracking algorithms require sparks in three of the four modules and thus the angular acceptance is normally defined by the 2.68 m length of the third chamber. Neglecting the finite length of the interaction regions and the curvature of tracks in the magnetic field, these chambers track particles over $.70 \times 4\pi$ sterrad solid angle. The rms momentum resolution for a 1 GeV/c track is about 15 MeV/c. Because of the high degree of redundancy (three out of four modules, two out of four wires per module required) these chambers are highly efficient in tracking particles. The inefficiency has been estimated at 1% per track or less. The structural support for the chambers consist of six, 6 mm wall, 5 cm diameter, aluminum posts at a radius of 79 cm, and a 1.3 cm thick aluminum cylinder at a radius of 1.49 m. These support posts can be major sources of multiple

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scattering; for this reason the analysis programs normally require that at least two particles in each event do not pass through a support post so that a good vertex for the event can be found. These posts subtend about 6% of the solid angle so this requirement reduces the effective solid angle of the detector somewhat.

Particles produced at angles from 16 to 42° relative to the positron beam will be tracked by a set of four "end-cap chambers". These are magnetostrictive wire spark chambers with the "wires" etched on printed circuit boards. The traces are either radial or circular so that the azimuthal and polar angle of the particle are read directly. These chambers extend the solid angle for tracking particles to $.83 \times 4\pi$ sterrad. They have been installed on only one side of the detector due to interference with the proportional chamber cables. They were first installed in April 1974 and have not yet been extensively used in the data analysis.

Immediately beyond the aluminum cylinder supporting the spark chambers are a cylindrical array of 48 2.5 cm thick plastic scintillator trigger counters. They are 2.61 m long and are viewed by 5 cm photomultiplier tubes from both ends. They are part of the trigger and provide time-offlight information with an rms resolution of about 0.5 ns. This is sufficient to separate pions from kaons up to a momentum of 600 MeV/c and kaons from protons up to a momentum of 1100 MeV/c. The solid angle subtended by these counters is $.65 \times 4\pi$ sterrad.

Next a particle will pass through the 9 cm aluminum solenoid coil and enter a cylindrical array of 24 lead-plastic scintillator shower counters. The 3.10 m long counters are made of five layers, each layer containing 6.4 mm of lead and 6.4 mm of scintillator. The counters are

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viewed from each end by a 13 cm photomultiplier tube. They are part of the trigger and have the primary function of discriminating between electrons and hadrons. They also have been used to a limited extent to detect photons. The plastic scintillators in the shower counters were inadvertently scratched during assembly of the counters. As a result the attenuation length was reduced from 145 cm to typically 75 cm and consequently there has been some reduction in efficiency for minimum ionizing particles near the center of the counters. Triggering efficiencies will be discussed in more detail below.

Beyond the shower counters are a 20 cm thick iron flux return and, until recently, two planar magnetostrictive wire spark chambers to detect muons. The flux return is an adequate hadron filter for most purposes⁸² but has limited sensitivity for searching for muon production in multiparticle final states. For this reason we began installing additional hadron absorbers on top of the detector in January 1975. The absorbers are 33 cm thick barite-loaded concrete slabs with a density of 3.28 g/cm³. The original muon chambers have been rearranged so that there are chambers after each set of two slabs. Four slabs have been installed to data and the fifth and sixth slabs are scheduled for installation in July 1975.

A.2 The Trigger

The trigger rate of the magnetic detector is limited to a few triggers per second by the time required to recharge the spark chamber pulsing system. To achieve this low a trigger rate, it has been found necessary to require a pipe counter and two sets of trigger counters with associated shower counters to fire. The shower counters are set to fire on minimum ionizing particles.

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There are two problems with this trigger. First, almost all measurements are biased by the trigger in a nontrivial way. For example, assume one wishes to measure a single particle inclusive cross section. The physical measurement one actually makes is that cross section times the probability that another charged particle is detected in the detector. In order to correct for such measurement biases, it is necessary to construct models of the final state and perform Monte Carlo simulations. This process will be discussed in Appendix B.

The second problem is that the shower counters are not fully efficient for the detection of charged particles. There are two aspects to this problem. First, the counters themselves are inefficient due to edge effects where counters meet and due to the short attenuation length discussed above. Second, low energy hadrons tend to interact or range out before reaching the shower counters.

Figure 38 shows the efficiency for the shower counters to fire on cosmic rays as a function of z, the distance from the center of the counter.⁸³ At large values of |z| the inefficiency is due to edge effects; near z = 0 the effects of light attenuation are apparent.

Figure 39 shows the apparent efficiency for low energy hadrons to fire a shower counter as a function of momentum.⁸³ These efficiencies are determined from the multi-hadronic data. Since a particle must pass through a minimum of 40 g/cm² of material before reaching an active element of the shower counter, pions with momenta below 200 MeV/c will range out due to ionization loss. This effect is apparent in Fig. 39. The non-zero efficiency below 200 MeV/c is presumably due to nuclear interactions producing π^{0} 's, gamma rays (from π^{0} 's produced in the e⁺e⁻ interaction) firing a counter toward which a charge particle is heading, and electrons in the
multi-hadronic sample from π^0 Dalitz decay and photon conversion in the beam pipe and pipe counter. The apparent efficiency above 200 MeV/c is in reasonable agreement with a calculation of the effect of nuclear interactions in the material in front of the shower counters.⁸⁴

Clearly, we can understand the triggering efficiency of the detector only to the extent that we understand these shower counter efficiencies.

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APPENDIX B

Monte Carlo Simulations of the Hadronic Final State

As discussed in Appendix A.2, the two charged particle trigger requirement and the limited solid angle of the SIAC-LBL magnetic detector necessitates a Monte Carlo type simulation of the detector and hadronic final state in order to determine the detector efficiency. In this appendix we will take the determination of the total hadronic cross section as an example.²¹ We will first briefly sketch the analysis procedure and then discuss the different models which have been used and the sensitivity of the efficiency to them.

B.1 The Analysis Procedure

From the events which triggered the detector, a sample of hadronic events was selected in such a way as to minimize backgrounds from other processes. Events must have had at least two charge tracks which satisfied the hardware trigger, missed the spark chamber support posts, and formed a vertex in the fiducial region. If only two charged tracks were present in the detector some additional requirements were applied: The tracks must not have given large pulse heights in the shower counters, must have had coplanarity angle with the beam of between 20 and 160°, and must have had momenta greater than 300 MeV/c. These requirements were imposed to reduce backgrounds from the leptonic processes $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow e^+e^- \rightarrow e^+e^-e^+e^-$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$.

Monte Carlo simulations were then performed to obtain a sample of simulated events which resembled the observed events. This process will be discussed in the next section.

One of the outputs of the Monte Carlo simulation was a matrix of efficiencies, $\epsilon_{i,j}$, for observing j charged particles given that i were

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produced. These efficiencies were used in an overdetermined set of simultaneous equations which were solved to obtain multiplicities and detection efficiencies. We will refer to this last step as the unfold.

Finally, corrections were made for events which failed to make a vertex, for contamination from beam-gas interactions and the two-photon leptonic processes, and for radiative effects.

B.2 The Monte Carlo Simulation

The Monte Carlo simulation is not straight forward in that we must construct a complete model of the final state. This is clearly an iterative process. In practice we never understand the final state perfectly, so we construct a variety of models, all of which fit the data reasonably, and study our sensitivity to the models. Fortunately, the detector efficiencies are not very sensitive to the details of the models.

We have used three general models: an all-pion model, a jet model, and a heavy particle model. Below we will briefly indicate how each is generated, how well they reproduce some aspects of the data, and how efficiencies determined by these models differ. We will also investigate how the unfold procedure modifies these results.

The all pion model assumes that only pions are produced in the final state. To generate each event a total multiplicity is first selected from a Poisson distribution. Then the distribution into charged and neutral pions is selected from a bionomial distribution of n-l particles where n is the chosen multiplicity. The final particle is chosen to conserve charge. States containing all neutral pions are discarded since they are forbidden by charge conjugation invariance. Finally, the event is generated according to invariant phase space.⁸⁵

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There are two free parameters, the mean of the Poission distribution and the charged to neutral ratio in the binomial distribution. These parameters are adjusted so that the simulation and the data agree on the observed mean multiplicity and median charged particle momentum for events in which three or more charged particles were detected.

The jet model is identical to the all pion model except for the insertion of a matrix element squared of the form

$$|\mathbf{M}|^2 \propto e^{-\Sigma p_{\text{li}}^2/R}$$
(B-1)

where p_{\perp} is the transverse momentum to a jet axis, the summation is over all particles and R is a parameter which is set so that the average transverse momentum is about 350 MeV/c. The jet axis is given an angular momentum of the form $1 + \alpha \cos^2 \theta$ where θ is the angle to incident beams. This form is the most general allowed by one photon exchange. In a spin 1/2 parton model, $\alpha = 1$, and in a spin 0 parton model, $\alpha = -1$. We have studied both, although the data seem to prefer $\alpha \gtrsim 0$ (see Section 5.6). The angular anisotropy generated by this model is mild enough to be consistent with the data.

The heavy particle model is the same as the all pion model except that in addition to pions, we include etas, kaons, and nucleons. The distributions of the different types of particles are chosen according to multinomial distributions subject to the constraints of strangeness, baryon number, and charge conservation. It was found that reasonable agreement with all of the preliminary data was obtained by simply setting an equal probability for each type of particle except that the ratio of π^0 's and η 's to charged π 's was allowed to vary to fit the observed median charged momentum. The heavy particle production is thus inhibited only by the smaller volume of available phase space. The resulting multiplicity distributions are given in Table XV. Note that the values given in this table represent something between an extreme case and a reasonable case calculation and should not be construed as measurements.

Figure 40 shows a comparison of the observed and simulated momentum distributions for events with three or more detected charged particles at center of mass energy 4.8 GeV for the all pion model. The distributions are in reasonable agreement over the entire range except for the region above 2 GeV/c. The discrepancy in this region is most likely due to long tails of the momentum resolution function which are not included in the simulation. The heavy particle model and the jet model give almost identical spectra to that of the all pion model.

Figure 41 shows the detection efficiency as a function of the number of produced charged particles. Results are shown for $\sqrt{s} = 3.0$ and 4.8 GeV for the all pion model. (The other models given similar results.) The most important dependence is the relatively low efficiency for the detection of two-prong events. Clearly, it is quite important to determine what fraction of the events come from two-prongs. This is the function of the unfold procedure.

In Fig. 42 we can see to what extent the Monte Carlo and the unfold reproduce the observed multiplicity distribution. At 4.8 GeV the Monte Carlo's Poisson total multiplicity distribution yields a charged multiplicity which is in reasonably good agreement with the data; however, at 3.0 GeV it tends to overestimate the number of observed two-prongs. In both cases, the unfold improves the agreement, but still exhibits a systematic deviation from the data: it overestimates the even prongs and underestimates the odd

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prongs. This means that the Monte Carlo is not properly simulating some aspect of either the detector or the final state. Figure 42 shows results for the all pion model, but all the models exhibit the same even-odd effect.

Finally Table XVI contains a comparison of the detection efficiency and charged multiplicity as determined by the Monte Carlos and unfolds for the different models. The efficiencies determined by the unfolds average 15% higher than those determined by the Monte Carlos at 3.0 GeV and 4% higher at 4.8 GeV. Similarly, the multiplicities from the unfolds are 8% higher at 3.0 GeV and 4% higher at 4.8 GeV. Taking the unfolds alone, the efficiencies differ among the various models by 10% at 3.0 GeV and by 8% at 4.8 GeV. The multiplicities differ by only 2 to 3%.

These results given some indication of the sensitivity of the cross sections and multiplicities to assumed models of the final state. It should be remembered, however, that this is not the only source of experimental error.

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APPENDIX C. Higher Energy Total Cross Sections

After this paper was written, <u>preliminary</u> data from the SIAC-LBL magnetic detector collaboration on σ_{had} and R (see section 3.4) at higher energy was presented by C.C. Morehouse at the April, 1975 meeting of the American Physical Society and by G.J. Feldman at the International Conference on High Energy Physics, Palermo, June, 1975. This data is given in Table 17 and shown in Fig. 43, an extension of Fig. 9. We emphasize that it is preliminary. The outstanding characteristic of this higher energy data is that R is roughly constant in the W region between 4.5 GeV and 7.4 GeV; and that roughly constant value of R is between 5. and 6. Thus there is <u>no</u> evidence that this high value of R, compared to the quark models discussed in Sec. 1.3, is an intermediate energy phenomena. That is, there is <u>no</u> evidence that as W increases, R is about to decrease and begin to approach a lower asymptotic value. Indeed, the data is compatible with R still increasing in this region.

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TABLE I

The u, d, s are the conventionally accepted quarks, the c separated by the dashed lines is proposed but there is no evidence for its existence comparable to the evidence for the u, d or s. I, I_z , Q, B, Y, S and C are the isotopic spin, z component of the isotopic spin, charge, baryon number, hypercharge, strangeness and charm.

Name	u	đ	S	С
Other Name	q	n	λ	י P [†]
I	1/2	1/2	0	
Iz	+1/2	-1/2	0	I 0
ନ	+2/3	-1/3	-1/3	+2/3
В	1/3	1/3	ı/3	1/3
Y	1/3	1/3	-2/3	1 O
S	0	0	-1	· 0
с . с	0	0	0	 ' 1

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TABLE II

Properties of Vector Mesons 17

Name	ρΟ	ω	φ	ρ'
Mass (MeV)	770	783	1020	1600
$I^{G}(J^{P})$	1 ⁺ (1 ⁻)	0 (1)	0 (1)	1 ⁺ (1 ⁻)
Γ(MeV)	150	10.0	4.2	400
Γ_{ee} (KeV)	6.5	0.76	1.34	
Γ _{ee} /Γ	4.3 × 10 ⁻⁵	7.6×10^{-5}	3.2×10^{-4}	
$\Gamma_{\mu\mu}/\Gamma$	6.7 x 10 ⁻⁵		2.5×10^{-4}	
$g_{\rm v}^2/4\pi$	2.56 ± .27	18.4 ± 2.0	11.0 ± 0.9	· •
hadronic	π ⁺ π ⁻ 100%	π ⁺ π ⁻ π ⁰ 90.0%	к ⁺ к ⁻ 46.6%	4π domi
decay modes		π ^ο γ 8.7%	к _I K _s 34.6%	
		$\pi^+\pi^-$ 1.3%	π ⁺ π ⁻ π ⁰ 15.8%	
		·	ηγ 3.0%	

`

π dominant

TABLE III

Parameters of electron-positron colliding beams facilities.

Name	Location	Status	Maximum Total Energy (GeV)	Туре
ACO	Orsay	operating	1.1	single ring
ADONE	Frascati	operating	3.1	single ring
DCI	Orsay	under construction	3.6	two rings, four beams
CEA	Cambridge	no longer operating	5.0	rebuilt synchrotron
VEPP-3	Novosibirsk	testing	4.0	single ring
VEPP-4	Novosibirsk	under construction	10 14.	single ring
SPEAR	SIAC	operating	~ 9.0	single ring
DORIS	DESY	operating	~ 9.0	two rings
EPIC	Rutherford	proposed	28.0	single ring
PEP	SIAC-LBL	proposed	30.0	single ring
PETRA	DESY	proposed	38.0	single ring

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TABLE IV

Values of $\sigma_{\rm had};$ R (defined in 1.C) and $\langle N_{\rm ch} \rangle$

Total Energy W(GeV)	σ _{had} (nb)	R	$\langle N_{ch} \rangle$	Ref.	Facility
1.2 1.3 1.4 1.5 1.6 1.7 1.85 1.9 1.94 1.9 1.9 1.9 2.1 2.6 8 3.0	$218 \pm 108 \\ 305 \pm 88 \\ 100 \pm 70 \\ 148 \pm 26 \\ 135 \pm 25 \\ 126 \pm 18 \\ 73 \pm 15 \\ 71 \pm 14 \\ 68 \pm 21 \\ 53 \pm 18 \\ 54.6 \pm 3.3 \\ 42 \pm 24 \\ 33 \pm 14 \\ 18 \pm 9 \\ 29 \pm 7 \\ 29 \pm 7 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\ 108 \\$	$3.6 \pm 1.8 \\ 5.9 \pm 1.7 \\ 2.3 \pm 1.6 \\ 3.8 \pm 0.7 \\ 4.0 \pm 0.7 \\ 4.2 \pm 0.6 \\ 2.9 \pm 0.6 \\ 3.0 \pm 0.6 \\ 2.9 \pm 0.9 \\ 2.4 \pm 0.8 \\ 2.77 \pm 0.17 \\ 2.8 \pm 1.6 \\ 2.6 \pm 1.1 \\ 1.6 \pm 0.8 \\ 3.0 \pm 0.7 \\ 3.$		22	ADONE
1.35 1.65 1.98 2.8 3.0	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$		23	ADONE
2.6 2.8 3.0	18 ± 5 17 ± 5 14 ± 5	1.4 ± 0.4 1.5 ± 0.5 1.5 ± 0.5		24	ADONE
4.0 5.0	26 ± 6 21 ± 5	4.7 ± 1.1 6.0 ± 1.5	4.2 ± 0.6 4.3 ± 0.6	25 26	CEA
468012333 222333333 33333444444680 4444445	31.8 \pm 3.6 32.5 \pm 4.4 29.4 \pm 4.1 23.3 \pm 2.0 22.5 \pm 3.4 21.4 \pm 2.3 18.9 \pm 2.6 18.7 \pm 2.6 18.7 \pm 2.6 19.1 \pm 2.2 19.7 \pm 1.7 24.5 \pm 3.3 31.8 \pm 2.2 19.7 \pm 1.7 24.5 \pm 2.9 19.6 \pm 2.9 15.3 \pm 1.9 15.3 \pm 1.9 18.2 \pm 1.5 17.7 \pm 1.5	2.11 \pm 0.24 2.53 \pm 0.34 2.65 \pm 0.37 2.41 \pm 0.21 2.49 \pm 0.27 2.37 \pm 0.38 2.52 \pm 0.27 2.37 \pm 0.33 2.49 \pm 0.38 2.85 \pm 0.33 3.28 \pm 0.28 4.51 \pm 0.61 6.15 \pm 0.40 5.02 \pm 0.40 3.73 \pm 0.40 5.09 \pm 0.43	$\begin{array}{c} 3.31 \pm 0.12\\ 3.18 \pm 0.15\\ 3.37 \pm 0.18\\ 3.55 \pm 0.04\\ 3.55 \pm 0.21\\ 3.89 \pm 0.12\\ 3.84 \pm 0.19\\ 3.93 \pm 0.19\\ 4.00 \pm 0.17\\ 3.87 \pm 0.05\\ 3.90 \pm 0.20\\ 4.04 \pm 0.17\\ 4.00 \pm 0.17\\ 4.00 \pm 0.10\\ 4.02 \pm 0.18\\ 4.40 \pm 0.24\\ 4.62 \pm 0.23\\ 4.31 \pm 0.04\\ 4.32 \pm 0.09\end{array}$	21	SPEAR

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TABLE V

Average values of R = $\sigma_{had}/\sigma_{\mu\mu}$. The 4.1 GeV enhancement is excluded by using the dashed line in Fig. 9b.

W(GeV)	Comment	R
2.0 to 3.0	· · · · · · · · · · · · · · · · · · ·	2.49 ± .09
3.0 to 4.0	excludes V's, includes 4.1 GeV enhancement	2.85 <mark>+</mark> .11
3.0 to 4.0	excludes V's, excludes 4.1 GeV enhancement	2.7711
4.0 to 5.0	includes 4.1 GeV enhancement	5.04 * .19
4.0 to 5.0	excludes 4.1 GeV enhancement	4.36 ± .16

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	Table 6		
Preliminary measure	ements on $\langle N_{ch} \rangle_{ee}$	at higher	energies
W(GeV)	$\langle N_{ch} \rangle$		
6.0	4.6±0.2		
6.2	4.3±0.2		•
6.8	4.7±0.3		
7.4	4.9+0.2		

TABLE VII

 $\langle N_{\pi^{\pm}} \rangle = \langle N_{\pi^{\pm}} \rangle + \langle N_{\pi^{-}} \rangle$ and $\langle N_{\pi^{0}} \rangle$ in π^{-} p and pp collisions

System	p _{lab} (GeV/c)	$\langle n_{\pi^{\pm}} \rangle$	 ⁿ ⁰ 	$\langle N_{\pi^0} \rangle / \langle N_{\pi^{\pm}} \rangle$
π¯ϼ	9.9	3.11 ± 0.06	1.48 ± 0.21	0.48
π¯р	18.5	3.90 ± 0.06	1.80 ± 0.08	0.46
π¯р	40.0	5.12 ± 0.04	2.51 ± 0.06	0.49
ЪЪ	19.0	2.62 ± 0.02	1.36 ± 0.12	0.52
pp	20.5	6.25 ± 0.17	3.19 ± 0.32	0.51

TABLE VIII

Masses of the ψ Particles

Laboratory	Ref.	$m_{\psi}(MeV)$	m _ψ ,(MeV)
SIAC (SPEAR)	60	3095 ± 4	3684 * 5
DESY (DORIS)	54	3090 ± 31	3680 ± 37
Frascati (ADONE)	49	3100 (error not given)	

TABLE IX

ż

Widths and Branching Ratios of the ψ

Г _{ее}	4.8 ± 0.6 KeV
Γ _{μμ}	4.8 ± 0.6 KeV
Γ_{had}	59 <mark>+</mark> 14 KeV
Γ	69 <u>†</u> 15 KeV
r _{ee} /r	0.069 ± 0.009
$\Gamma_{\mu\mu}/\Gamma$	0.069 ± 0.009
$\Gamma_{\rm had}/\Gamma$	0.86 ± 0.02
Γ_{μ}/Γ_{e}	1.00 ± 0.05

TABLE X

Comparison of all pion state production to μ pair production at 3.0 GeV and the $\psi.$

state

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. . .

 $\frac{3.0}{\frac{n\pi}{3.0}}$ σψ σΨ

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 $2\pi^{+} 2\pi^{-}$ 0.82 ± 0.22
 $2\pi^{+} 2\pi^{-} \pi^{0}$ > 5.2

 $3\pi^{+} 3\pi^{-}$ 1.10 ± 0.54
 $3\pi^{+} 3\pi^{-} \pi^{0}$ > 4.5

TABLE 11

Decay modes of the V. Data from the SIAC-LBL collaboration [62], if no other reference given.

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Decay modes identified

mode	comment	\$	ref.
+ - e c		6.9 ± 0.9	
μμ-		6.9 + 0.9	
व्य		0.23 ± 0.05	62,64
۸Ĩ		seen	
ηγ		> 0.15 < 1.8	64 65
+-0 ялл	Dominantly $p\pi$ (1.3 \div 0.3%)	seen	
21+21-	Via intermediate y	0.4 ± 0.1	
π ⁺ π ⁻ K ⁺ K ⁻		0.5 ± 0.2	
ѫ⁺ѫ⁻ рҏ҄	•	seen	
2π ⁺ 2π ⁻ π ⁰	Including $\omega \pi \pi$ (0.8 \div 0.3%) and $\rho \pi \pi \pi$ (1.2 \pm 0.4%) I = 0 implies B. R. $(\pi^{+}\pi^{-}3\pi^{\circ}) = 1/2$ B. R. $(2\pi^{+}2\pi^{-}\pi^{\circ})$.	4.0 ± 1.0	
ѫ ⁺ ѫ ⁺ ѫ ⁻ ҝ ⁰ ₈ к ⁺		seen	
3x ⁺ 3x ⁻	Via intermediate 7	0.4 - 0.2	
2x ⁺ 2x ⁻ K ⁺ K ⁻		0.3 [±] 0.1	
π [±] π [±] π ⁻ π ⁻ K ⁰ _κ ⁺		seen	
3π ⁺ 3π ⁻ π ⁰		2.9 ± 0.7	
$4\pi^{+}4\pi^{-}\pi^{0}$		0.9 - 0.9	
	Decay modes searched for	and not seen	
mode	comment	upper limit \$	ref
+- x x	G violating	< 0.03	.64

 x⁺x⁻
 < 0.06</td>
 64

 x⁰x⁰/_s
 < 0.02</td>

 77
 Forbidden for vector meson
 < 0.35</td>
 63

 x⁰y
 < 0.55</td>
 63,65

Pa

TABLE XII

Widths of the ψ^*

 $\Gamma_{ee} = \Gamma_{\mu\mu}$ (equality assumed) $2.2 \pm 0.5 \text{ KeV}$

Г

400 + 400 = 200 KeV

TABLE XIII

Properties of the vector mesons and ψ photoproduction cross sections, $d\sigma(\gamma N \rightarrow VN)dt = A \exp(-b|t|)$. The γ -V coupling constants $g_V^2/4\pi$ are obtained from e⁺e⁻ colliding beam measurements and the total cross sections, $\sigma(VN)$, are obtained from Eq. 6-37.

Particle	Photon Energy (GeV)	$\sigma(\gamma N \rightarrow VN)$ (nb)	b (GeV/c) ⁻²	ε _γ /4π	σ(VN) (mb)
ρ ^Ο	9.3	13,500 ± 500	6.5	2.3 ± 0.3	23 ± 3
ω	9.3	1,800 ± 300	6.6 <u>+</u> 1.1	18.4 ± 1.8	24 ± 3
φ	9.3	550 ± 70	4.6 ± 0.7	12.2 + 1.0	9 1 1 .
Ψ	150	~ 13	~ 4	11.5 ± 1.4	~ 1

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TABLE XIV

Limits on narrow width resonance production at W = 4.8 GeV The upper limits are for inclusive cross sections in nb and are at the 90% confidence level.

Decay Mode	Mass Region (GeV/c)			
	1.5 - 1.85	1.85 - 2.4	2.4 - 4.0	
- + + Κ ⁺ π π	0.51	0.49	0.19	
- + + π π π	0.48	0.38	0.18	
κ ^ο π ⁺	0.26	0.27	0.09	
	0.54	0.33	0.09	
κ ⁺ π ⁺	0.25	0.18	0.08	
κ <mark>0</mark> π ⁺ π ⁻	0.57	0.40	0.27	
π ⁺ π ⁻	0.13	0.13	0.09	
к+ к-	0.23	0.12	0.10	

TABLE XV

Multiplicities generated by the heavy particle Monte Carlo. The values given should not be construed as measurements.

Particle	Average Multiplicity	
	$\sqrt{s} = 3.0 \text{ GeV}$	\sqrt{s} = 4.8 GeV
π^+ or π^-	1.26	1.48
π ⁰	1.44	2.25
η	. 36	.64
K^+ , K^- , K^0 , or $\overline{K^0}$.17	.27
p, p, n, or n	.013	.03

5.05

7.05

total

J

TABLE XVI

Detection efficiencies and charged multiplicities determined by various Monte Carlo simulations and unfolds.

Model	\sqrt{s} = 3.0 GeV		\sqrt{s} = 4.8 GeV	
	e	n ch	E	n ch
All pion Monte Carlo	. 382	3.38	.558	4.19
All pion unfold	•437	3.56	.585	4.39
Heavy particle Monte Carlo	. 386	3.32	.562	4.32
Heavy particle unfold	.416	3.51	•571	4.38
Jet model (α = 1) Monte Carlo	. 364	3.20	•533	4.10
Jet model ($\alpha = 1$) unfold	.420	3.50	.563	4.34
Jet model (α = -1) Monte Carlo	· 379	3.09	.587	4.12
Jet model (α = -1) unfold	.457	3.52	.606	4.26

Table 17

Preliminary data on σ_{had} and R at higher energies

from SIAC-LBL magnetic detector collaboration.

Total Energy W(GeV)	σ _{had} (nb)	R
5.6	14.7 ± 2.1	5.3 ± 0.8
6.0	12.0 ± 1.8	5.0 ± 0.7
. 6.2	12.7 ± 1.9	5.6 ± 0.8
6.8	10.2 ± 1.5	5.5 ± 0.8
7.4	9.4 ± 1.4	5.9 ± 0.9

Figure Captions

Fig. l	Hadron production by e^+ - e^- annihilation: (a) the general
	diagram; (b) the one-photon exchange diagram; (c) the
	two-photon exchange diagram.
Fig. 2	Kinematics of colliding beams intersecting at: (a) zero angle;
	and (b) an angle η.
Fig. 3	Feynman diagrams for the reactions: (a) $e^+ + e^- \rightarrow \pi^+ + \pi^-$,
	$e^+ + e^- \rightarrow K^+ + K^-$; and (b) $e^+ + e^- \rightarrow \mu^+ + \mu^-$.
Fig. 4	The parton model for $e^+ + e^- \rightarrow hadrons$.
Fig. 5	The vector meson dominance model for $e^+ + e^- \rightarrow hadrons$.
Fig. 6	The total cross section, σ_{had} , as a function of W = \sqrt{s} . Data
	sources: below 1.2 GeV, Ref. 16; triangles, Ref. 22; open
	circles, Ref. 23; squares, Ref. 24; crosses, Refs. 25 and 26;
	closed circles and ψ data, Ref. 21.
Fig. 7	Radiative decay diagram.
Fig. 8	Fits to σ_{n} in $ ho$ peak region. See text for explanation.
Fig. 9	(a) σ_{had} versus W; (b) $R = \sigma_{had}^{\prime}/\sigma_{\mu\mu}^{\prime}$ versus W. References
	give in caption of Fig. 6.
Fig 10	(a) The average charged particle multiplicity ${\langle { m N}_{ m ch} { angle}_{ m ee}}$ for
	$e^+ + e^- \rightarrow hadrons;$ (b) comparison of $\langle N_{ch} \rangle_{ee}$ with $\langle N_{ch} \rangle$
	for hadron-hadron collisions. See text for significance of
	curves A, B, C. Data from Ref. 21.
Fig. 11	A model for the multiplicity distribution $P\left(N_{ch}\right)$ for $e^{+} + e^{-} \rightarrow$
	hadrons. For a discussion of the dependence of these models on

the data see the text and App. B.

- Fig. 12 Particle fractions for K (open circles) and π (closed circles); Ref 33.
- Fig. 13 The single charged particle momentum distribution F(z) versus z where $z = p/p_{max}$. F(z) is normalized to $\int_{0}^{1} F(z)dz = 1$. Data from Ref. 21.
- Fig. 14 (a) $\langle p \rangle$ for charged particles versus W; (b) $\langle z \rangle = \langle p/p_{max} \rangle$ for charged particles versus W. Data from Ref. 21.
- Fig. 15 $E d\sigma/d^3p$ for charged particles versus p. Data from Ref. 21.
- Fig. 16 E $d\sigma/d^3p$ for charged particles versus p. The lines show E $d\sigma/d^3p$ versus p_1 for charged pions produced in p-p collisions at 200 GeV. $e^+ - e^-$ data from Ref. 34, p-p data from Ref. 36.
- Fig. 17 $(E/p^2)(\Delta N/\Delta p)$ for π^- , K^- , and \bar{p} versus the particle energy. From Ref. 34.
- Fig. 18 Comparison of one-photon exchange diagrams for (a) $e + n \rightarrow e + hadrons$ and (b) $e^+ + e^- \rightarrow hadron h + other hadrons.$
- Fig. 19 $s(d\sigma_{had}/dz)$ for charged particles versus z. Data from Ref. 21.
- Fig. 20 $\langle W_{ch} \rangle / W$, the ratio of the average total charged particle energy versus the total energy. Data from Ref. 21.
- Fig. 21 The total cross section for $e^+ e^- \rightarrow hadrons$ in the region of the ψ .
- Fig. 22 The total cross section for $e^+ e^- \rightarrow hadrons$ in the region of the ψ^{\bullet} .
- Fig. 23 Schematic drawing of the amplitude for production of μ pairs in the ψ region assuming that the ψ has the same quantum numbers as the photon. A_{QED} is the amplitude for direct production of μ pairs far below and far above the resonant energy.

A is the amplitude at the point of maximum destructive min interference below the resonant energy.

- Fig. 24 The ratio of μ pair yield to e pair yield in the region of the ψ for $|\cos \theta| \leq .6$. The dashed line gives the expected ratio for no interference while the solid line gives the expected ratio for full interference. Radiative and resolution effects are included.
- Fig. 25 (a) The angular distribution of electron pairs for the energy range 3.0944 to 3.0956 GeV. The open squares show the result of subtracting the expected contribution from the direct production and scattering of electron pairs. (b) The angular distribution of muon pairs for the same energy region. The lines represent $1 + \cos^2 \theta$.
- Fig. 26 The front-back asymmetry for μ pair production in the region of the ψ . The asymmetry is defined as the number of positive muons produced in the direction of the incident positron minus the number of negative muons produced in the direction of the incident positron all divided by the number of muon pairs.
- Fig. 27 Feynman diagrams for (a) the direct ψ decay to hadrons, (b) the ψ decay to hadrons via an intermediate photon, and (c) the ψ decay to μ pairs.
- Fig. 28 The invariant mass squared recoiling against four charged pions at (a) 3 GeV and (b) the ψ .
- Fig. 29 The invariant mass of $\pi^+ \pi^- \pi^0$ combinations from the decay $\psi \rightarrow 2\pi^+ 2\pi^- \pi^0$. Each event is entered in the plot four times.

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- Fig. 30 The distribution of missing mass recoiling against all pairs of oppositely charged particles at the ψ '.
- Fig. 31 The distribution of the $\mu^+ \mu^-$ invariant mass for the highest momentum oppositely charged particle pair from each ψ^* event. Electron pairs are excluded.
- Fig. 32 The distribution of $\pi^+ \pi^-$ invariant mass from the decay $\psi^* \rightarrow \psi \pi^+ \pi^-$. The curve shows the product of phase space times the geometrical acceptance.
- Fig. 33 A computer reconstruction of the decay $\psi^{\dagger} \rightarrow \psi \pi \pi$ where $\psi \rightarrow e^{\dagger} e^{-}$ from the SIAC-LBL magnetic detector at SPEAR. The event is seen in the x-y projection where z is the beam and magnetic field direction. The closed rectangles represent trigger and shower counters which fired. (For a description of the detector see Appendix A.)
- Fig. 34 Quark diagrams for the decays (a) $\phi \rightarrow 3\pi$, (b) $\phi \rightarrow K\bar{K}$, and (c) $\psi^{\dagger} \rightarrow \psi \pi \pi$.
- Fig. 35 The relative cross section for $e^+e^- \rightarrow hadrons$ in 1.88 MeV steps. Fig. 36 The most likely gamma ray transitions in the charm model.
- Fig. 37 Cross sectional view of the SIAC-LBL magnetic detector at SPEAR. The positron beam enters from the right and the electron beam enters from the left.

- Fig. 38 The efficiency for the shower counters to fire on cosmic ray muons as a function of the distance from the center of the counter.
- Fig. 39 The apparent efficiency of the shower counters to fire on hadrons as a function of hadron momentum.
- Fig. 40 A comparison of the observed momentum spectrum for events with three or more charged particles detected at 4.8 GeV to the allpion model Monte Carlo simulation.
- Fig. 41 The efficiency for detecting a hadronic event as a function of the charged mulitplicity of the event at 3.0 and 4.8 GeV.
- Fig. 42 A comparison of the observed charge particle multiplicity to the Monte Carlo simulation and the unfold for the all-pion model at (a) 3.0 GeV and (b) 4.8 GeV.



Fig. 1



Fig. 2



(b) 2681A3

Fig. 3











Fig. 6







Fig. 8



Fig. 9


Fig. 10



Fig. 11







Fig. 13



e j

Fig. 14



Fig. 15



Fig. 16







Fig. 18



Fig. 19







•••



Fig. 22





Fig. 24



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Fig. 26



Fig. 26



Fig. 27













Fig. 32



Fig. 33



Fig. 34



Fig. 35







Fig. 38



Fig. 39





Fig. 41





