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THE IMAGINARY PART OF THE HELICITY NONFLIP C=-1 EXCHANGE AMPLITUDES IN ELASTIC SCATTERING BETWEEN 6 AND 14 GeV *

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ABSTRACT

Using accurate data on the differential cross sections for elastic scattering of particles and antiparticles on protons from 6 to 14 GeV, the projection of the C=-1 exchange amplitude onto the dominant diffractive amplitude is measured. Strong peripherality in impact parameter space is observed for all the measured amplitudes with peak values scaling with the interaction radius. The zero of the imaginary part of the C=-1 exchange amplitude in $K^{\pm}p$ scattering moves to smaller t values as the energy increases from 6 to 14 GeV. The $p^{\pm}p$ exchange amplitude is more peripheral than the $\pi^{\pm}p$ and $K^{\pm}p$ counterparts and an unexpected energy dependence is observed.

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Complete amplitude analysis of a hadronic reaction requires extensive measurements of a complete set of observables. Since such measurements require a prohibitive experimental effort in most cases, it is very important to look for alternate methods to extract only a few amplitudes for a limited set of measurements. New accurate measurements [1] of particle and antiparticle elastic scattering on protons permit a study of the C=-1 exchange amplitudes through their interference with the dominant diffractive amplitude. The data comprise measurements of $K^{\pm}p$ scattering at 6.4, 10.4 and 14 GeV, and $\pi^{\pm}p$ and $p^{\pm}p$ at 10.4 GeV. Our analysis will be focused on the energy dependence of the $K^{\pm}p$ amplitudes and on the comparison between $X^{\pm}(\equiv \pi^{\pm}, K^{\pm}, p^{\pm})$ projectiles at 10.4 GeV. Large statistics and a relative normalization uncertainty of less than 0.5% between $X^{\pm}$ cross sections have been achieved.

We decompose each s-channel helicity amplitude for $X^{\pm}p \to X^{\pm}p$ into a (C=+1) diffractive amplitude P and two nondiffractive amplitudes R^{+} and R^{-} corresponding respectively to C=+1 and C=-1 t-channel exchanges. The difference between elastic $X^{\pm}p$ differential cross sections results from the interference between the even (C=+1) and odd (C=-1) amplitudes

$$\frac{d\sigma}{dt}(X^{-}p) - \frac{d\sigma}{dt}(X^{+}p) = 4 \sum_{\lambda\lambda'} \mathrm{Re}\left\{\left(P_{\lambda\lambda'} + R^{+}_{\lambda\lambda'}\right) R^{-*}_{\lambda\lambda'}\right\} \tag{1}$$

where λ, λ' are the helicities of the initial and final proton states. Projecting all amplitudes on the even helicity nonflip amplitude $(P_{++} + R^{+}_{++})$, one gets

$$\frac{d\sigma}{dt}(X^{-}p) - \frac{d\sigma}{dt}(X^{+}p) = 4\left(P_{++} + R^{+}_{++}\right)\left(R^{-}_{++}\right)_{\parallel} + 4\mathrm{Re}\left\{\left(P_{+-} + R^{+}_{+-}\right) R^{-*}_{+-}\right\} \tag{2}$$

where $\parallel$ denotes the parallel component. At high energy $P_{++} + R^{+}_{++}$ $(\simeq P_{++})$ becomes the dominant amplitude and the second term in the above sum can be neglected to first order. $(R^{-}_{++})_{\parallel}$ can therefore be extracted from the elastic

differential cross section through

$$\left(R^-_{++}\right)_{\parallel} \simeq \Delta_{X(s,t)} = \frac{\frac{d\sigma}{dt}(X^-p) - \frac{d\sigma}{dt}(X^+p)}{\sqrt{8\left[\frac{d\sigma}{dt}(X^-p) + \frac{d\sigma}{dt}(X^+p)\right]}} \tag{3}$$

Notice that only the projected amplitude, $(R^-_{++})_{\parallel}$, is measured and in order to extract $\mathrm{Im}\,R^-_{++}$, one needs to know the phase ϕ_+ of $(P_{++} + R^+_{++})$ and $\mathrm{Re}\,R^-_{++}$ as a function of t. The phase ϕ_+ is known at t=0 from Coulomb interference measurements [2] but away from t=0 it is not accessible to experiment and therefore methods incorporating analyticity [3] are needed. At energies above 6 GeV and small t-values, $\mathrm{Im}\,R^-_{++}$ can be well approximated by $(R^-_{++})_{\parallel}$, for most aspects of the analysis, due to the small difference between ϕ_+ and 90°: for example in $K^{\pm}p$ scattering ϕ_+ at t=0 decreases from 96° to 93.5^o when the energy increases from 6.4 to 14 GeV. However the precise location of the zeroes of $(R^-_{++})_{\parallel}$ and $\mathrm{Im}\,R^-_{++}$ can differ significantly due to this small phase rotation.

Let us discuss now the approximation resulting from dropping the second term in eq. (3). It is known experimentally that the I_t=0 amplitude is mostly helicity-conserving [4] and that therefore the contribution of the Pomeron and exchange of f quantum numbers (I_t=0, C=+1) to the second term will be very small [5]. This result is expected to hold for KN and NN scattering. There is however another contribution for these two processes coming from I_t=1, C=+1 exchange (A_2 quantum numbers) which can be evaluated since it is related to the charge-exchange reactions. For example, in KN scattering we have

$$4\mathrm{Re}\left[A_2(\rho+\omega)^*\right] \simeq 4\mathrm{Re}\left[A_{2+-}\,\rho^*_{+-}\right] \simeq \frac{1}{4}\left[\frac{d\sigma}{dt}(K^-p \to \bar{K}^o n) - \frac{d\sigma}{dt}(K^+p \to K^o p)\right] \tag{4}$$

This contribution is known experimentally to be small as expected from exchange degeneracy.

We now proceed to discuss our data. In fig. 1 the particle-antiparticle cross section differences $\Delta_X(t)$ are shown for all three pairs of processes. They are all qualitatively very similar with a forward peak, a zero around $-t = 0.2$ GeV^2 and a negative branch extending to a second zero beyond $-t = 1$ GeV^2: these properties are characteristic of a peripheral amplitude in impact parameter space. Such an amplitude can be represented in t-space with a form [6]

$$\left(R^-_{++}\right)_{\parallel} = A\, e^{Bt} J_0 (R\sqrt{-t}) \qquad (5)$$

corresponding, in impact parameter space, to a distribution peaked at b=R with a width $\Delta b = \sqrt{2B}$. We have used this expression to fit all our measured distributions. In fig. 1 the fits are shown with a solid curve whereas the extrapolation of the fit function outside the fitting interval is indicated with a dashed line. In all cases the fit is good and except for $p^{\pm}p$ the data follows expression (5) even for $-t > 1$ GeV^2. The values for the fitted parameters are shown in table 1. We have also plotted in fig. 1 the values $\Delta_X(0)$ computed from total-cross sections [7] and real parts [2], [8] through the optical theorem: they agree very well with the extrapolation of our data at t=0 which is an independent check of our relative normalization.

<u>The energy dependence of $\Delta_K(t)$</u>

In fig. 2a we examine the s dependence of $\Delta_K(t=0)$ given by A in the expression (5): it shows a steady decrease in our energy region which can be parametrized by $s^{-0.52}$. It is interesting to compare this behavior to the values obtained directly at t=0 from total cross sections alone using the optical theorem.

$$\mathrm{Im}\, R^-_{++} = \frac{1}{8\sqrt{\pi}} (\sigma_- - \sigma_+)$$

and the values for $\Delta_K(0)$ obtained from total cross sections and known real-to-imaginary ratios, α,

$$\left(R^-_{++}\right)_{\parallel}(t=0) \simeq \Delta_K(0) = \frac{\left(1+\alpha_-^2\right)\sigma_-^2 - \left(1+\alpha_+^2\right)^2 \sigma_+^2}{8\sqrt{2\pi}\,\sqrt{\left(1+\alpha_-^2\right)\sigma_-^2 + \left(1+\alpha_+^2\right)\sigma_+^2}}$$

where $\alpha_\pm \equiv \alpha(K^\pm p)$ and $\sigma_\pm \equiv \sigma_T(K^\pm p)$. In addition to the agreement between our data and the optical point normalization of $\Delta_K(0)$ discussed above, we notice that $(R^-_{++})_{\parallel}$ at t=0 becomes essentially identical to $\mathrm{Im}\, R^-_{++}$ in our energy range and therefore we expect only small corrections in going from the measured phase-rotated amplitude to the purely imaginary component. The correction is largest in our experiment at 6.4 GeV and would become very significant at lower energies.

Figure 2b shows the s dependence of the parameter B, characterized by a strong shrinkage of R^-_{++} from 6 to 14 GeV with $\alpha' \simeq 1.2\ \mathrm{GeV}^{-2}$. Seen in the impact parameter space, this effect corresponds to a widening of the peripheral peak: the FWHM, Δb, grows from 0.5 f at 6.4 GeV to 0.9 f at 14 GeV. Since this trend runs counterwise to the idea of peripherality at higher energies—unless the peak value R moves to larger values as energy increases—it is very interesting to look at the s dependence of R as measured in our experiment. However, rather than taking the R values from the global fit using eq. (5), it is more precise to use the values for the directly measured crossover point [1].

The s dependence of the zero of $\mathrm{Im}\, R^-_{++}$ in $K^\pm p$ scattering

The results for the "raw" crossover point, t_c, as measured from the differential cross sections, are shown in fig. 3 and indicate a slight decrease of $|t_c|$ from 6 to 14 GeV. It is our purpose here to translate this effect into the variation of the zero of $\mathrm{Im}\, R^-_{++}$, t_0. In order to do that we have to evaluate the

small corrections implied in eq. (2). Denoting the amplitudes by the corresponding t-channel particle quantum numbers, we have:

$$\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p) = 4\,\mathrm{Im}\,(P+f)_{++}\,\mathrm{Im}\,R^-_{++} + T_1 + T_2 + T_3$$

$$T_1 = 4\,\mathrm{Re}\,(P+f)_{++}\,\mathrm{Re}\,R^-_{++}$$

$$T_2 = 4\sum_{\lambda\lambda'}\mathrm{Re}\,(A_2R^{-*})_{\lambda\lambda'}$$

$$T_3 = 4\,\mathrm{Re}\left[(P+f)_{+-}\,R^{-*}_{+-}\right]$$

where $R^- \equiv \rho+\omega$.

T_2 is a very small term which can be extracted very reliably from charge-exchange measurements [9], as noted above. T_3 is also very small and can be estimated from πN amplitudes at 6 GeV [10]. Both T_2 and T_3 have the effect of moving the observed crossover to smaller values and their combined shift is $-0.004\ \mathrm{GeV}^2$ at 6.4 GeV. By far the largest correction comes from the term T_1 and it is also the least known. The real parts are known at t=0 but their study as a function of t requires the use of analyticity. As a guide to estimate this effect we use πN amplitudes [11] derived from data and fixed-t dispersion relations with a rotated $(P+f)_{++}$ amplitude in order to obey the t=0 KN real part. In using these amplitudes we make the assumption that ρ_{++} exchange in πN scattering behaves similarly to $(\rho+\omega)_{++}$ exchange in KN scattering [12]. The effect on the crossover point is again a shift to smaller values: we obtain shifts of 0.03, 0.02, 0.015 GeV^2 at respectively 6.4, 10.4 and 14 GeV. Determinations of the crossover zero at lower energies [10] suffer from even bigger corrections rising to about 0.07 GeV^2 at 3 GeV, rendering the extraction of the zero of $\mathrm{Im}\,R^-_{++}$ quite uncertain. The values of t_0 obtained from our experiment are also

shown in fig. 3: As the energy increases from 6 to 14 GeV, the $|t_0|$ values decrease by 0.027 GeV^2. The zero of $(R^-)_{\parallel}$ is unambiguously measured and moves by 0.012 GeV^2, while the rest of the effect for $\mathrm{Im}\,R^-_{++}$ comes from the T_1 correction. If we extrapolate the variation of $t_0^{-\frac{1}{2}}$ linearly with ln s to higher energies where $t_c \simeq t_0$, we expect a crossover position of $\sim$0.17 GeV^2 in elastic $K^{\pm}p$ scattering at 100 GeV. Such an s dependence of R is strong enough to preserve the peripherality of $\mathrm{Im}\,R^-_{++}$ even if the slope B continues to shrink at the same rate, therefore confirming the idea of strong absorption and peripherality as a property of (vector) exchange amplitudes. It is also interesting to note that the rate of increase of R between 6 and 14 GeV is quite similar to the shrinkage of the average forward slope of the K^+p differential cross section: since the latter quantity is related to the size of the diffractive KN interaction radius, it seems to confirm the naive picture of a classical strong absorption inside the interaction radius.

Comparison between π, K, p amplitudes at 10.4 GeV

At 10.4 GeV a very interesting comparison of Δ_π, Δ_K and Δ_p can be performed and since the corrections to $\Delta_X(t)$ to obtain $\mathrm{Im}\,R^-_{++}$ are small at 10 GeV, the data on $\Delta_X(t)$ can be compared directly.

We first remark that Δ_π and Δ_K are very similar in shape: the slopes and the radii agree between the two processes indicating a rather close similarity of the impact parameter profiles. We note that this result provides some justification for using R^- amplitudes from πN analyses to correct the crossover position, as discussed in the preceding section. More importantly, it shows

again that the optical picture of strong absorption holds since πN and KN have a rather similar interaction radius.

On the other hand $\Delta_p(t)$ is characterized by quite different parameters. First, the radius R is substantially bigger, corresponding to a smaller $|t_c|$ value. The t_c values between $Kp, \pi p$ and pp scale approximately with the inverse of the average forward elastic slope. The slope of $\Delta_p(t)$ is much smaller than the value obtained for π or K data: it corresponds to a more peripheral amplitude centered at 1.25 f with a FWHM of only 0.45 f. A more intriguing result is the observation that B_p antishrinks as energy increases from 3-6 GeV [10] to 10.4 GeV, resulting in an increased peripherality. Such a behavior is unexpected and it will be very interesting to investigate this effect at higher energies.

Because of the different zero positions, resulting from different absorption and size, we conclude that naive quark model ideas like ω-exchange universality will not hold between KN and NN process, whereas SU(3) symmetry tests of ρ and ω exchanges in πN and KN scattering are approximately satisfied due to more similar absorption effects.

Conclusion

We have extracted the projection of the helicity nonflip C=-1 exchange amplitude onto the dominant $I_t=0$ helicity nonflip amplitude in elastic processes between 6 and 14 GeV. Small corrections are made to relate these projections to the imaginary part of the C=-1 amplitudes. We observe strong peripherality in all the measured amplitudes with the peak value in impact parameter space following simple geometrical ideas (i.e., $R \sim B_{el}$). The zero of the KN amplitude moves to smaller t values as the energy increases in a way roughly given by the interaction radius in our 6 - 14 GeV range. This variation is strong

enough to preserve the peripherality of the amplitude even though shrinkage tends to widen its profile. The exchange amplitude measured in $p^{\pm}p$ scattering is very peripheral and antishrinkage is observed resulting in an increasing peripherality as the energy grows.

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TABLE 1

The parameters, A, B and R, from eq. (5) obtained from fits to the measured amplitudes, $\Delta_X(t)$. The crossover positions t_c and t_0 are obtained from a local fit (see ref. [1]).

Reaction	Momentum (GeV)	t range used in the fit	A ($\sqrt{mb}$/GeV)	B (GeV^{-2})	R (GeV^{-1})	t_c [a] (GeV^2)	t_0 (GeV^2)
$K^{\pm}p$	6.4	.02 - .6	.684 ± .015	.56 ± .14	5.13 ± .04	.219 ± .005	.253
$K^{\pm}p$	10.4	.02 - .8	.542 ± .010	1.46 ± .08	5.15 ± .03	.211 ± .004	.234
$K^{\pm}p$	14	.02 - .8	.466 ± .008	1.56 ± .06	5.13 ± .03	.209 ± .004	.226
$\pi^{\pm}p$	10.4	.02 - .8	.241 ± .011	1.45 ± .18	5.02 ± .08	.231 ± .007	.251
$p^{\pm}p$	10.4	.02 - .6	1.220 ± .016	.43 ± .05	6.24 ± .02	.146 ± .002	--

[a] The errors listed are statistical only. Systematic uncertainties are typically less than half as small, as discussed in ref. [1].

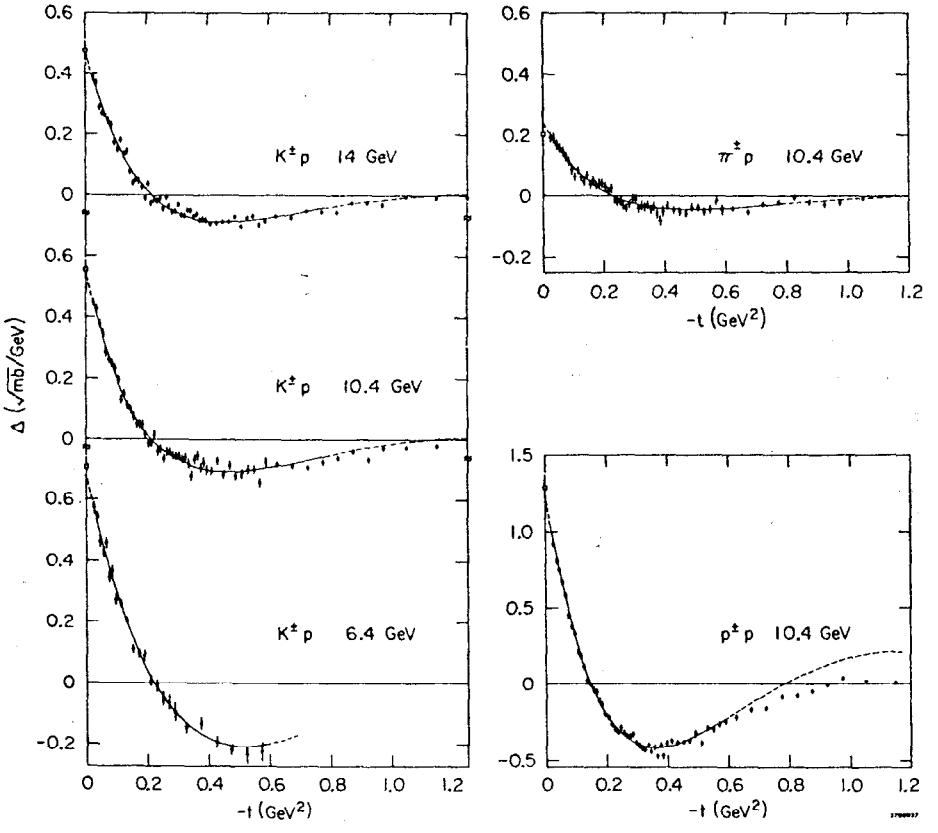

Fig. 1

The measured amplitudes $\Delta_X(t)$ as defined by eq. (3) in the text for the scattering of $K^{\pm}p$ at 6.4, 10.4 and 14 GeV, and $\pi^{\pm}p$ and $p^{\pm}p$ at 10.4 GeV. The lines represent the fit using expression (5) with the dashed portion corresponding to the extrapolation of the function outside the fit interval.

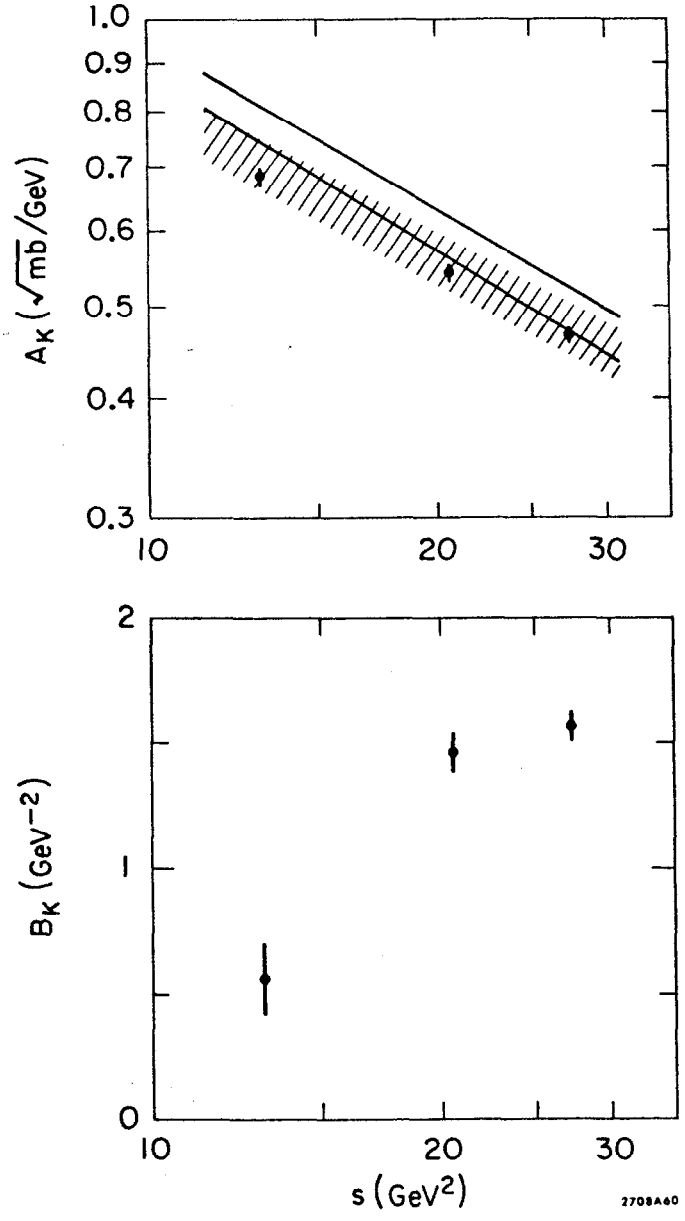

Fig. 2

The energy dependence of the parameters A and B from eq. (5) for $\Delta_K(t)$ at 6.4, 10.4 and 14 GeV. The top band in Fig. 2a corresponds to the value at t=0 of Im R^-_{++} from total cross section measurements, whereas the hatched band indicates the values for the normalization A obtained from total cross sections and real-to-imaginary ratios.

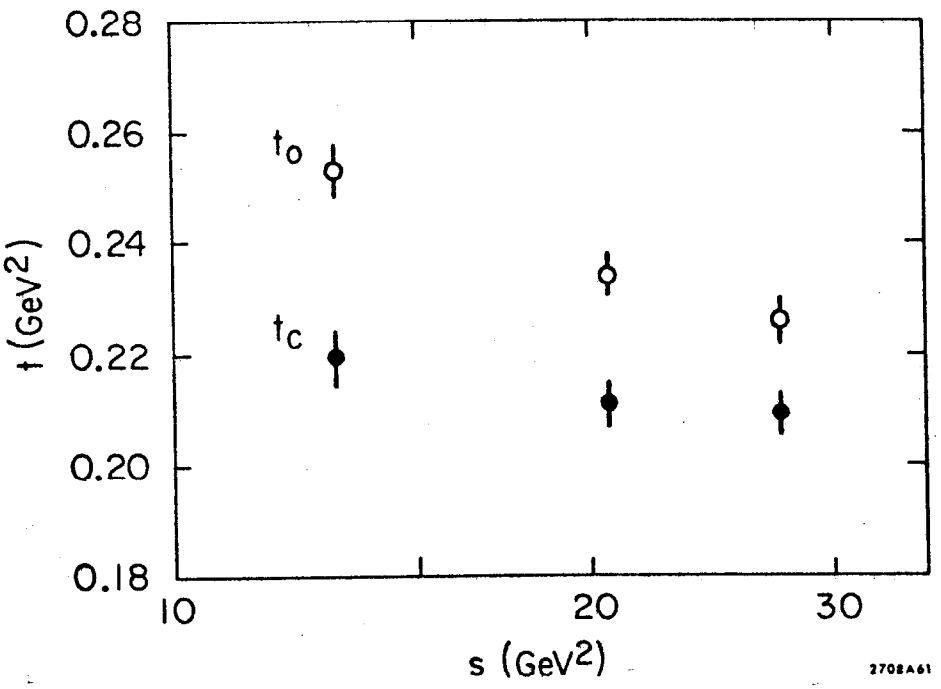

Fig. 3

The energy dependence of the "raw" crossover point, t_c, as measured from a local fit of the $K^{\pm}p$ differential cross sections [1] and the extracted zero of $\mathrm{Im}\, R^{-}_{++}$, t_0.