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CONSTITUENT THEORY OF LARGE MASS PRODUCTION*

C. T. Sachrajda† and R. Blankenbecler
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

It is argued that the study of the production of a pair of particles with large invariant mass can provide information on the dynamics of hadronic constituents in a similar way to the study of large transverse momentum reactions. A general framework for the analysis and interpretation of these reactions is developed in terms of hadron-irreducible subprocesses and the constituent interchange model. Counting rules are developed that predict the energy and mass dependence of the cross section. In particular, the production of a massive lepton pair or hadron pair in reactions with photon or hadron beams is discussed. A particularly interesting process that is discussed involves the production of a massive hadron pair in electron-positron annihilation. Its importance in measuring properties of generalized structure functions is emphasized.

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†Harkness Fellow.

I. INTRODUCTION

The recent discovery[1] of very narrow large-mass resonances has generated considerable interest in the theory of the production of lepton pairs at very high mass in hadron-hadron collisions. In a superficially quite different vein, recent experimental and theoretical developments in large transverse momentum single particle processes[2] has, as its next logical extension, the correlated production of a pair of hadrons with large invariant mass (each of which has a large p_T). In this paper, we shall present a unified treatment of these processes using the physical picture and assumptions of the constituent interchange model. We shall develop dimensional counting-type rules that completely characterize the mass and energy dependence of the cross sections.

The best known theoretical description of large mass lepton pair production is the Drell-Yan model,[3] which utilizes the annihilation of two point-like constituents into a heavy photon as a basic process. However, an upper bound derived for this model by Einhorn and Savit showed that it was much too small to fit the data.[4] The discovery of the $\psi(3.1)$ and its subtraction from the cross section improved this situation considerably, but it still appears that the Drell-Yan prediction for the lepton pair continuum could be low by perhaps an order of magnitude. There is however considerable uncertainty in interpreting the data, so that the real discrepancy may be smaller.[5] One possibility, briefly discussed in a SLAC workshop,[6] is that there are important basic processes in addition to the one assumed by Drell-Yan. Several different basic processes will be explicitly discussed here, and the characterization of a general process will be given. If the results of the analysis of the large transverse production of a single particle are any guide, several basic processes may well be important, even ones that are nonleading when compared to the Drell-Yan process.[7]

In this paper we shall use for reasons of simplicity a spinless quark model with dimensionless coupling constants. Since we are primarily interested in the scaling behavior and not the angular distributions, this should be permissible. With this model, we shall discuss first the production of a large mass lepton pair in hadron-hadron collisions, and a quark counting formula will be given for the general case. Then the production of a large mass hadron pair will be discussed in a simplified model motivated by the large transverse momentum models. This result will be applied to the production of a massive hadron pair in electron-positron collisions, $e^+e^- \rightarrow H_1H_2X$. The discussion will stress the important threshold and crossing properties that can be determined by measuring this type of process and its crossed analogue, $H_1H_2 \rightarrow \ell^+\ell^-\bar{X}$. Finally, the possible effect of the CIM selection rules for allowed interactions between constituents will be discussed and the importance of experimental information on this point will be stressed.

The derivations of our results will be simplistic and naive in the extreme—and hence probably useful. We shall calculate the lowest order diagrams in a renormalizable theory with scalar quarks and assume that the logarithmic corrections do not accumulate in higher order so as to modify our essentially dimensional results. Our experience with the analysis of large p_T reactions is support for this point of view. We shall not explicitly discuss the production of narrow resonances, such as the ψ's, but they could be included by using the methods and basic processes developed here together with a model of such resonances.[8]

Let us now turn to a derivation of the lepton pair mass distribution for a general hard scattering model.

II. DECOMPOSITION OF LEPTON PAIR PRODUCTION

In this section, a probabilistic formula for the production of a large mass lepton pair will be derived. This result is similar to the formulation used to describe high transverse momentum processes in hard scattering and parton models.[2] The reaction $AB \to \ell^+\ell^-X$ is decomposed as illustrated in Fig. 1. The final fragmentation states of the projectile A are denoted by $(A\bar{a}) = (\alpha)$, and similarly for B, $(B\bar{b}) = (\beta)$. For simplicity, these will be treated as one particle states of a given mass, but an integration over a mass spectrum reflecting their true multiparticle character is implied.

The cross section is written as

$$\frac{d\sigma}{d^4Q} = \int |M(AB \to \ell^+\ell^-X)|^2 \, dp \, \delta^4(Q-\ell^+-\ell^-)/2\lambda(s,A^2,B^2) \quad , \qquad (2.1)$$

where

$$\lambda^2(s,t,u) = s^2+t^2+u^2 - 2(st+su+tu) \quad ,$$

and

$$dp = \prod_i \frac{d^4p_i}{(2\pi)^3} \, \delta^{(+)}\left(p_i^2 - m_i^2\right) (2\pi)^3 \, \delta^4(P_A+P_B-Q-\Sigma p_i)$$

$$\frac{d^4\ell^+ \, d^4\ell^-}{(2\pi)^6} \, \delta^+(\ell^{+2} - m^2) \, \delta^+(\ell^{-2} - m^2) \quad ,$$

where the p_i label the final state hadron momenta. The matrix element is written as an incoherent sum over contributing intermediate states

$$|M(AB \to \ell^+\ell^-X)|^2 = \sum_{ab,\,d} \psi_A^2\left(p_a^2\right) \psi_B^2\left(p_b^2\right) |M(ab \to \ell^+\ell^-d)|^2 \quad , \qquad (2.2)$$

where

$$\psi_A^2\left(p_a^2\right) = \phi_A^2\left(p_a^2\right)\left(p_a^2 - m_a^2\right)^{-1} \quad ,$$

and ϕ_A is the vertex function describing the breakup $A \to a+\alpha$, where a is off-shell.

Defining finite momentum frame variables as

$$P_A = \left(P + \frac{A^2}{4P},\ \vec{0}_T,\ P - \frac{A^2}{4P}\right)$$

$$P_B = \left(P + \frac{B^2}{4P},\ \vec{0}_T,\ -P + \frac{B^2}{4P}\right)$$

$$p_\alpha = \left((1-x_a)P + \frac{\alpha^2+k_a^2}{4(1-x_a)P},\ -\vec{k}_a,\ (1-x_a)P - \frac{\alpha^2+k_a^2}{4(1-x_a)P}\right) ,$$

then the phase space factor achieves the form

$$\int \frac{d^4p_\alpha}{(2\pi)^3}\,\delta^+\left(p_\alpha^2 - \alpha^2\right) = \int \frac{d^2k_a}{2(2\pi)^3} \int_0^1 \frac{dx_a}{(1-x_a)} .$$

Defining the probability distribution function in x_a and $\vec{k}_a$ as

$$G_{a/A}\left(x_a, k_a^2\right) \equiv \frac{x_a}{2(2\pi)^3(1-x_a)}\,\psi^2\left(p_a^2\right) , \tag{2.3}$$

where

$$p_a^2 = m_a^2 + x_a\left[A^2 - \frac{k_a^2+m_a^2}{x_a} - \frac{k_a^2+\alpha^2}{1-x_a}\right] ,$$

and introducing the cross section for the basic subprocess $ab \to \ell^+\ell^- d$, one can write

$$\frac{d\sigma}{d^4Q}(AB \to \ell^+\ell^- X) = \sum_{ab,\,d} \int dx_a\, d^2k_a\, dx_b\, d^2k_b\, G_{a/A}\left(x_a, k_a^2\right) G_{b/B}\left(x_b, k_b^2\right)$$

$$\frac{\lambda\left((p_a+p_b)^2,\ p_a^2,\ p_b^2\right)}{x_a x_b\, \lambda(s, A^2, B^2)}\ \frac{d\sigma}{d^4Q}(ab \to \ell^+\ell^- d) . \tag{2.4}$$

If the distribution functions damp the off-shell behavior sufficiently rapidly, and if s is sufficiently large, then

$$\frac{\lambda\left((p_a+p_b)^2,\, p_a^2,\, p_b^2\right)}{x_a x_b\, \lambda(s, A^2, B^2)} \sim \frac{x_a x_b s - \vec{k}_a \cdot \vec{k}_b}{x_a x_b s} \sim 1 \quad ,$$

and the cross section formula considerably simplifies with the basic subprocess being effectively on-shell.

Finally, the cross section for the production of a lepton pair with $(\text{mass})^2 = Q^2$ can be achieved by integrating over $\vec{Q}$ and x_Q. The final result is

$$\frac{d\sigma}{dQ^2}(AB \to \ell^+\ell^- X) = \sum_{ab,\,d} \int dx_a\, dx_b\, G_{a/A}(x_a)\, G_{b/B}(x_b)$$

$$\frac{d\sigma}{dQ^2}\left(ab \to \ell^+\ell^- d;\; s' = x_a x_b s,\, Q^2\right) \qquad (2.5)$$

which will be used in our later discussions.

The general behavior of the distribution functions have been discussed elsewhere.[2] The results that will be needed here are

$$\begin{aligned} G_{a/A}(x) &\sim x^{-\alpha(0)} && x \sim 0 \\ &\sim (1-x)^{g_{a/A}} && x \sim 1 \end{aligned}$$

where

$$g_{a/A} \equiv 2n(A\bar{a}) - 1 \quad ,$$

and $n(A\bar{a})$ is the minimum number of elementary fields in the state $(A\bar{a})$. In the next section, the limiting behavior of the basic processes will be discussed. This behavior, in the limit $Q^2 \to s$, will be parametrized by terms of the form $(\epsilon = 1-\tau,\ \tau = Q^2/s)$

$$Q^4 \frac{d\sigma}{dQ^2}(s, Q^2) = s^{-n}(Q^2)^{-m}\, \epsilon^f \sigma_0 \quad . \qquad (2.6)$$

The general cross section arising from this basic process is then

$$Q^4 \frac{d\sigma}{dQ^2} = \sigma_0 (Q^2)^{-m} \int dxdy\ G_{a/A}(x)\ G_{b/B}(y) \left(1 - \frac{\tau}{xy}\right)^f (xy\ s)^{-n}\ \theta(xy-\tau)$$

$$\equiv (Q^2)^{-N} \epsilon^H \Sigma_0(\epsilon) \tag{2.7}$$

where $N = n+m$, and if

$$G_{a/A}(x) \propto \frac{1}{x} (1-x)^{g(a/A)}$$

$$G_{b/B}(x) \propto \frac{1}{x} (1-x)^{g(b/B)}$$

then

$$H = f + g(a/A) + g(b/B) + 2$$

and

$$\Sigma_0(\epsilon) \propto \sigma_0\, \tau^n \int_0^1 dwdz\ \frac{w^f (1-w)^{g(b/B)}\ z^{1+g(b/B)+f}\ (1-z)^{g(a/A)}}{(\tau + wz)^{1+g(b/B)}\ (\tau + wz\,\epsilon)^{1+n+f}}$$

which is finite at $\tau = 0$ and 1, and is slowly varying for $0 < \tau < 1$.

III. BASIC MECHANISMS FOR MASSIVE LEPTON PAIR PRODUCTION

In this section we study the various mechanisms which may be responsible for massive lepton pair production. We start by evaluating $Q^4 d\sigma/dQ^2$ for the subprocess $(a+b) \to (\ell^+\ell^-)d$ (cf. Fig. 1) and later we shall substitute this, together with the appropriate structure functions for the initial state hadrons, into Eq. (2.5) to obtain the full differential cross section.

It is convenient to begin by performing the integrations over the lepton momenta ℓ^+, ℓ^-; the remaining integrations can then be viewed as corresponding to the cross section for producing a heavy photon of mass M. In all of the processes to be considered below, the photon-quark interaction is labeled as in Fig. 2, and the integral over the lepton variables is of the type

$$I = \sum_{\text{spins}} \int d^4\ell^+ \, d^4\ell^- \, \delta^+(\ell^{+2}-m^2)\, \delta^+(\ell^{-2}-m^2)\, \delta^4(\ell^+ + \ell^- - Q)$$

$$\times \, |\bar{u}(\ell^-)\, (\not{q} - \not{k})\, v(\ell^+)|^2 \tag{3.1}$$

The lepton masses m can be neglected with respect to Q^2, and the integral and spin sum can be evaluated

$$I = \frac{2}{3}\pi\lambda^2(Q^2, q^2, k^2) \ . \tag{3.2}$$

We are now ready to calculate the cross section for the various relevant subprocesses.

A. $q + \bar{q} \to \gamma$

This is the familiar Drell-Yan process (see Fig. 3a). Using Eq. (3.2) we see that

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{\pi}{3}\lambda\left(Q^2, p_a^2, p_b^2\right) \delta(s-Q^2) \tag{3.3}$$

where $s = (p_a + p_b)^2$. As stated in Section II, the assumption is made that the structure functions strongly damp the off-shell behavior of a and b so that we can neglect p_a^2, p_b^2 with respect to Q^2, and obtain

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{\pi}{3}\,\delta(1-Q^2/s) \quad . \tag{3.4}$$

B. $\underline{q+q \rightarrow q+q+\gamma}$

This is the simplest process of the bremsstrahlung class of models, in which the massive photon is emitted off a heavy timelike quark (see Fig. 3b). Whether such quark-quark scattering terms exist in nature (or at least at present energies), is questionable since they would give a $p_\perp^{-4}$ behavior for the inclusive pion distribution and a s^{-8} behavior of the fixed angle pp cross section, neither of which is observed. Nevertheless we calculate its behavior. Again we neglect p_a^2, p_b^2 with respect to Q^2, and also set the final state quark masses (μ) equal to zero eventually. Then

$$Q^4 \frac{d\sigma}{dQ^2}(B) = \frac{1}{2s} \int \frac{d^4k_1\, d^4k_2\, d^4Q'}{\left[(Q+k_1)^2 - \mu^2\right]^2}\, \delta^4(p_a + p_b - k_1 - k_2 - Q') \cdot$$
$$\cdot\; \delta^{(+)}\left(k_1^2-\mu^2\right) \delta^+\left(k_2^2-\mu^2\right) \delta^+(Q'^2-Q^2)\, \frac{2\pi}{3}\, \lambda^2\left((Q'+k_1)^2, Q'^2, \mu^2\right) \quad . \tag{3.5}$$

It is now convenient to introduce 1 in the form

$$1 = \int d^4q\, \delta^4(Q + k_1 - q) \quad . \tag{3.6}$$

The k_1 integration in (3.5) can now be written as

$$\begin{aligned} I &= \int d^4k_1\, \delta\left(k_1^2-\mu^2\right) \delta\left((q-k_1)^2 - Q^2\right) \\ &= \frac{\pi}{2} \left(1 - Q^2/q^2\right) \quad . \end{aligned} \tag{3.7}$$

where we have neglected μ^2. Note that

$$\lambda^2(q^2, Q^2, \mu^2) \cong q^4(1 - Q^2/q^2)^2 \quad , \tag{3.8}$$

and as q^2 approaches its kinematic limit, the right hand side of (3.8) vanishes. Because of this factor, $(1 - Q^2/q^2)^2$, models of the bremsstrahlung type will always have two more powers of ϵ than naively expected from phase space.

Substituting (3.6), (3.7), and (3.8) into (3.5) we have

$$Q^4 \frac{d\sigma}{dQ^2}(B) = \frac{\pi^2}{6s} \int d^4k_2\, d^4q\, \delta^4(p_a + p_b - k_2 - q)\, \delta^+\left(k_2^2 - \mu^2\right) (1 - Q^2/q^2)^3 \quad . \tag{3.9}$$

Again it is convenient to introduce a factor of unity, this time in the form

$$1 = \int dM^2\, \delta(q^2 - M^2) \quad . \tag{3.10}$$

The k_2 integration is now given by

$$\begin{aligned} &= \int d^4k_2\, d^4q\, \delta^4(p_a + p_b - k_2 - q)\, \delta^+(q^2 - M^2)\, \delta^+\left(k_2^2 - \mu^2\right) \\ &= \frac{\pi}{2}(1 - M^2/s) \quad , \end{aligned} \tag{3.11}$$

and thus

$$Q^4 \frac{d\sigma}{dQ^2}(B) = \frac{\pi^3}{12s} \int_{Q^2}^{s} dM^2 (1 - Q^2/M^2)^3 (1 - M^2/s) \quad . \tag{3.12}$$

Changing variables of integration from M^2 to z by the transformation

$$M^2 = Q^2 + \epsilon s z, \qquad \epsilon = 1 - Q^2/s \quad , \tag{3.13}$$

we find

$$Q^4 \frac{d\sigma}{dQ^2}(B) = \frac{\pi^3}{12} \epsilon^5 \int_0^1 dz\, z^3 (1-z) \left[(1-\epsilon) + \epsilon z\right]^{-3} \quad . \tag{3.14}$$

Both similarities and differences can now be seen in comparison with the large $p_\perp$ case. Since here we do not need to worry about "balancing" the large

Q^2, every quark or particle in the final state can be thought of as a "spectator." Thus we expect three powers of ϵ on the right hand side of (3.14) from the two final state quarks, and indeed we do get these, together with two extra factors of ϵ from the right hand side of (3.8). Notice also that the $1/Q^2$ (or $1/s$) behavior is the same as in the Drell-Yan case.

C. $\underline{q - \pi \to q - \gamma}$

This is the simplest example of a process with more than 2 initial state quarks (Fig. 3c). This process also clearly has an annihilation type graph with a Drell-Yan basic process. Since it was computed in (A), only the bremsstrahlung graph will be computed here. For this process (under the usual assumptions) we have

$$Q^4 \frac{d\sigma}{dQ^2}(C) = \frac{g^2}{2s} \int d^4k \, d^4Q' \, \delta^+(k^2-\mu^2) \, \delta^+(Q'^2-Q^2) \, \delta^4(p_a+p_b-k-Q')$$

$$\frac{1}{s^2} \cdot \frac{2\pi}{3} \lambda^2(s, Q^2, \mu^2) \quad ,$$

or

$$Q^4 \frac{d\sigma}{dQ^2}(C) = \frac{\pi^2}{6}\left(\frac{g^2}{s}\right) \epsilon^3 \quad . \tag{3.15}$$

We see that the ϵ behavior is as expected—one power of ϵ from the single spectator and an additional two from the spin sum and integration over the lepton variables. There is now a factor $1/s$ in the right hand side of (3.15) which was not present in process (A) or (B); it will become clear below that every time we increase the number of quark fields in the initial state of the subprocess by one, $Q^4 \, d\sigma/dQ^2$ gains an additional factor of $1/s$.

D. <u>$q+M \to q+q+\bar{q}+\gamma$</u>

The cross section for this process (see Fig. 3d) is given by

$$Q^4 \frac{d\sigma}{dQ^2} \propto \left(\frac{g^2}{s}\right) \epsilon^7 J_d \tag{3.16}$$

where

$$J_d = \int_0^1 dz\, dx\, z^3(1-z)^3\, x(1-x)\, [1-\epsilon z]^{-3}\, [1-\epsilon zx]^{-1} \quad .$$

Again the ϵ and s behavior are as expected.

D. <u>$q+(2q) \to q+q+q+\gamma$</u>

This reaction (see Fig. 3e) can be calculated by similar methods to those used to evaluate the above processes, and one finds that

$$Q^4 \frac{d}{dQ^2} \propto \left(\frac{g^2}{s}\right) \epsilon^7 J_e \quad , \tag{3.17}$$

where

$$J_e = g^2 \int_0^1 dz\, dw\, dv\, z^3(1-z)^3\, w^2 v\, [1-\epsilon z(1-wv)]^{-1}\, [1-\epsilon z]^{-1} [x-\epsilon z(x+w(1-x)-wv)]^{-2},$$

and x is the fraction of the momentum of the diquark system carried by one of its quarks. Both the ϵ and s behavior are as expected from the previous discussion.

F. <u>$M+\bar{M} \to \gamma$</u>

This mechanism using physical intermediate states has been proposed by Chu and Koplik.[9] They treated the heavy photon in analogy to the ρ-meson and showed that such a mechanism is not inconsistent with the data of Christianson <u>et al.</u>,[14] when the ψ contribution is subtracted out. For this process (see

Fig. 3f), we find

$$Q^4 \frac{d\sigma}{dQ^2} \propto \left(\frac{g^2}{s}\right)^2 \delta(1 - Q^2/s) \quad . \tag{3.18}$$

This again complies with our expected s and ϵ behavior.

G. $\underline{(2q)+(2q) \to q+q+q+q+\gamma}$

As a final example we present the process $(2q)+(2q) \to 4q+\gamma$ for which Fig. 3g gives one of the contributions. For this diagram we find

$$Q^4 \frac{d\sigma}{dQ^2} \propto \left(\frac{g^2}{s}\right)^2 \epsilon^9 J_g \tag{3.19}$$

where

$$J_g = \int_0^1 \frac{dz\, dw\, dv\, du\; z^4 (1-z)^3 (1-w) v \log(1-\epsilon zvu)}{[1-\epsilon z]^3 [1-\epsilon zw][x-\epsilon zv]^2 [xy-\epsilon zvu]^2 \epsilon} \quad ,$$

and x and y are the momentum fractions present in the two incident diquark systems. The other diagrams for $(2q)+(2q) \to 4q+\gamma$ give a similar behavior.

We are now in a position to state our general rules for the subprocess $a+b \to \ell^+\ell^- + n_f q$ which follow essentially from dimensional analysis. They can be summarized in the form

$$Q^4 \frac{d\sigma}{dQ^2} \propto \left(\frac{g^2}{s}\right)^{n_i-2} \epsilon^{2n_f+\gamma} \quad , \tag{3.20}$$

where n_i is the number of quark fields in $a+b$. γ is defined to be (-1) when the massive photon results from the annihilation of 2 nearly on-shell constituents, such as in the Drell-Yan process, and is defined to be (+ 1) when the photon is bremsstrahlunged off a massive timelike quark.

IV. THE EXCLUSIVE-INCLUSIVE CONNECTION

In Section III we considered basic mechanisms for the production of massive lepton pairs in which, apart from the lepton pair, a certain number of free on-shell quarks are produced. In this section we study the consequences of requiring some or all of the final state quarks to bind into hadrons. Before presenting some simple examples we would like to recall the analogous results in the case of the production of particles at large transverse momentum. The smooth connection between the simple particle inclusive and exclusive cross sections was discussed by Bjorken and Kogut.[10]

The cross section (inclusive and exclusive) for the production of particles at large transverse momentum can be characterized as a sum of terms each of which is a product of a power of ϵ and a power of $1/p_\perp^2$. Each such term arises from a particular subprocess. ϵ here is defined to be $(1 - 4p_\perp^2/s)$. The power of ϵ, F say, for a particular subprocess is given by the expression

$$F = 2n_s - 1 \quad ,$$

where n_s is the number of quarks not participating in the central subprocess. The power of $(1/p_\perp^2)$ is given simply by the Brodsky-Farrar counting rules:[11] for each additional quark participating in the central subprocess (whether this quark is an initial or final state leg), there is an extra power of $1/p_\perp^2$. It is found that each time the number of spectators is decreased by one, and hence the power of ϵ is decreased by two, the power of $1/p_\perp^2$ is also increased by two. Thus each lost factor of ϵ is "compensated" for by an extra factor of $1/p_\perp^2$.

Below it will be shown that a similar, but different, connection exists for the production of a massive lepton pair (see discussion after Eq. (3.14)). We start by presenting a few simple examples.

A. $\underline{q+q \to (2q)+\gamma}$

The (2q) system has a fixed mass m and consists of two on-shell quarks (Fig. 4a). Although this example is quite unphysical, it demonstrates the consequences of fixing the mass of a number (in this case 2) quarks. For this process we have

$$sQ^4 \frac{d\sigma}{dQ^2\, dm^2} \propto \int d^4k_1\, d^4k_2\, d^4Q'\, \delta^+\left(k_1^2-\mu^2\right)\delta^+\left(k_2^2-\mu^2\right)\delta^+(Q'^2-Q^2)$$
$$\delta^4(k_1+k_2+Q'-P) \times \left[(Q'+k_1)^2-\mu^2\right]^{-2} \lambda^2\left((Q+k_1)^2, Q'^2, \mu^2\right) \quad . \tag{4.1}$$

This integral can be performed by the techniques introduced in Section III and gives the following result:

$$Q^4 \frac{d\sigma}{dQ^2 dm^2} \propto \frac{\epsilon'^3}{s} \int_0^1 dz\; z\left[1-\epsilon+z\epsilon'\right]^{-2} \quad , \tag{4.2}$$

where

$$\epsilon' = \frac{1}{s}\left[(\sqrt{s}-m)^2 - Q^2\right]$$

and, as before, $\epsilon = 1 - Q^2/s$. For sufficiently small m^2, ϵ' is approximately equal to ϵ. We can integrate the right hand side of (4.2) over m^2 to obtain (3.14), a result which can be understood naively by noting that the range of the m^2 integration is $\epsilon^2 s$.

From Eq. (4.2) it can be seen that fixing the mass of the final state two quark system has reduced the powers of ϵ by 2 (from 5 to 3), so that for the purpose of counting these powers, the di-quark pair can be treated as one particle. The price that is paid for this reduction in powers of ϵ is a factor of $1/s$ which is present in (4.2) but not in (3.14). As will be shown below this result can be generalized as follows: each time the number of hadrons (whether quarks or bound systems) in the final state is reduced by one due to additional binding (keeping the number of quark fields constant), the power of ϵ is reduced by two and the power of $1/s$ is increased by one.

We now turn to a more physical example.

B. $q+\bar{q} \to M+\gamma$

After carrying out the loop integration (see Fig. 4b), the amplitude for this process is proportional to (modulo factors of log s, which in the spirit of dimensional counting we neglect) $g\bar{u}(\ell^-)\gamma \cdot k\, v(\ell^+)/Q^2$. Squaring this amplitude and integrating over the appropriate phase space we obtain

$$Q^4 \frac{d\sigma}{dQ^2} \propto \left(\frac{g^2}{s}\right) \epsilon^3 \quad . \tag{4.3}$$

This result is in agreement with the behavior expected from the discussion given after example A.

C. $q+(2q) \to (3q)+\gamma$

The (3q) system has a fixed mass m and consists of three on-shell quarks. A typical diagram for this process is shown in Fig. 4c. This diagram can be evaluated by the techniques of Section III and gives the result (to leading order in ϵ and $1/s$)

$$Q^4 \frac{d\sigma}{dQ^2 dm^2} \propto \frac{m^2}{s^3} \epsilon^3 \quad . \tag{4.4}$$

The other diagram of the same order for this process gives the same behavior. Again this result is in agreement with the discussion presented after example A.

As a final example we present the following.

D. $q + (2q) \rightarrow B + \gamma$

The baryon state (B) has a fixed mass m. A typical diagram for this process is shown in Fig. 4d. One finds for this diagram

$$Q^4 \frac{d\sigma}{dQ^2} \propto \frac{g^2 h^2}{s^3} \epsilon^3 \quad , \qquad (4.5)$$

again as expected. The baryon wave function is characterized by the constant h which has dimensions of mass^2.

By considering examples similar to those presented above, it is seen that the formula (3.20) can be simply generalized when certain sets of the final particles bind. It simply gains an additional factor of

$$\left(g^2/\epsilon^2 Q^2\right)^{\Sigma(n_{bnd}-1)} \quad , \qquad (4.6)$$

where n_{bnd} is the number of quarks in each of the bound states. The examples in Section III all corresponded to $n_{bnd} = 1$ so that this factor was unity. In the next section we shall use Eqs. (3.20) and (4.6) together with the convolution formula (2.5) to classify hadronic and photonic production of massive lepton pairs.

V. CHARACTERIZATION OF CROSS SECTIONS

In this section, a general cross section formula giving the ϵ and Q^2 behavior for any given basic process will be exhibited. It will then be used to give a characterization of the cross sections for different incident beams on nucleon targets. These characterization formulae are meant to be used as a guide to possible behaviors of the cross sections - the dominant terms in a certain kinematic regime must be determined by experiment. If the results on large transverse momentum reactions are used as a guide, several different basic processes are probably important.

Collecting together the results derived in Sections II, III, and IV, the general cross section behavior for each basic process can be written in the form (valid for small ϵ):

$$Q^4 \frac{d\sigma}{dQ^2} \propto \epsilon^{\gamma+2n_f} \left(\frac{g^2}{Q^2}\right)^{n_a+n_b-2} \left(\frac{g^2}{\epsilon^2 Q^2}\right)^{\Sigma(n_{bnd}-1)} , \qquad (5.1)$$

where

$$n_f = n(\bar{a}A) + n(\bar{b}B) + n(d)$$

is the total number of final state quark fields present. If there is a composite exclusive state arising from the basic process, then n_{bnd} is the number of quarks that are bound up in it ($n_{bnd} = 2$ for meson and 3 for baryon, and if only free quarks are present, $n_{bnd} = 1$). This is then summed over each bound state present. The parameter γ can be determined by examining the topology of the graph - for an annihilation-type graph, $\gamma = -1$, and for a bremsstrahlung-type graph, $\gamma = +1$. The last factor in this equation, which depends on n_{bnd}, simply reflects the fact that if the final quarks bind among themselves, one gains in phase space, i.e. fewer powers of ϵ, but loses in powers of Q^2, corresponding to the necessary presence of hadronic form factors in this case.

Let us first apply this formula to derive a familiar result. If one considers a pure annihilation process, then it will be possible to derive the time-like behavior of hadronic form factors. In this stiuation $A = \overline{B}$, $\gamma = -1$, $n_f = 0$, $n_{bnd} = 1$, and $n_a = n_d = n_A = n_B$. The formula becomes

$$Q^4 \frac{d\sigma}{dQ^2} \sim \epsilon^{-1} \left(\frac{g^2}{Q^2}\right)^{2(n_B-1)} \sim \delta(\epsilon)\ |F_B(Q^2)|^2 ,$$

which is the usual dimensional counting result for the form factor.

Let us now consider the process $pp \to \ell^+\ell^- X$. Using the previous formulae, the general cross section arising from the basic processes A, B, ... of Section II takes the form (valid for small ϵ and large Q^2)

$$Q^4 \frac{d\sigma}{dQ^2} \cong \epsilon^{11} \Big\{ A + FQ^{-4} + C\epsilon^2 Q^{-2} + D\epsilon^6 Q^{-2} + E\epsilon^2 Q^{-2} + B\epsilon^2 + \ldots \Big\} \quad . \tag{5.2}$$

The relative normalization constants A, B, ... also label the basic subprocess. Each factor of Q^2 on the right-hand side of this equation should be replaced by $(Q^2 + M^2)$, where M^2 is a typical hadronic mass, in phenomenological applications. If the CIM fits to large transverse momentum data are used as a guide, one might expect that the dominant terms are A, C, and E, but only experiment can decide in this new regime. Finally, note that if the final three quarks in basic process E bind to form a baryon or baryonic resonance, which should be important at lower energies and small ϵ, the above E term is multiplied by the factor $(\epsilon^2 Q^2)^{-2}$.

Now consider the process $\pi p \to \ell^+\ell^- X$. Using the same notation as above (the values of A, B, ... should be different of course for each type of beam particle), the cross section becomes

$$Q^4 \frac{d\sigma}{dQ^2} = \epsilon^5 \left\{ A + (F + F_1\epsilon^4)Q^{-4} + (C + C_1\epsilon^4)\epsilon^2 Q^{-2} + E\epsilon^6 Q^{-2} + \ldots \right\} , \tag{5.3}$$

where the F and C terms have no bremsstrahlung from the incoming beam meson and F_1 and C_1 involve a radiated $q\bar{q}$ pair. Again the E term is multiplied by $(\epsilon^2 Q^2)^{-2}$ if the final quarks bind to form a baryon.

The final hadronic process that will be explicitly characterized is the reaction $\bar{p}p \to \ell^+\ell^- X$. In our standard notation, the cross section is characterized by the terms

$$Q^4 \frac{d\sigma}{dQ^2} = \epsilon^7 \left\{ A + F\epsilon^4 Q^{-4} + \bar{F}\epsilon^{-4} Q^{-4} + B\epsilon^6 + \ldots \right\} , \tag{5.4}$$

where the $\bar{F}$ term arises from the unusual basic process $(qq) + (\bar{q}\bar{q}) \to \gamma \to \ell^+\ell^-$.

VI. EFFECTIVE POWER ANALYSIS

It has proven very useful in analyzing data on large transverse momentum processes to extract the effective power behaviors in ϵ and p_T^2.[10] It should also prove helpful to make an analogous type of analysis in the case of large mass production. The results of such an analysis should be very sensitive to trends in the data and to the dominant basic process and should allow a simple comparison with different theories. The effective powers here are defined as (compare Eq. (2.7))

$$H_{eff} \equiv \epsilon \frac{\partial}{\partial \epsilon} \ln \left(Q^4 \frac{d\sigma}{dQ^2} \right) \Bigg|_{Q^2} \qquad (6.1)$$

$$N_{eff} \equiv -Q^2 \frac{\partial}{\partial Q^2} \ln \left(Q^4 \frac{d\sigma}{dQ^2} \right) \Bigg|_{\epsilon} , \qquad (6.2)$$

where the Q^2 derivative is taken at fixed ϵ, and the ϵ derivative (which is equivalent to an energy derivative) is taken at fixed Q^2. By taking differences of cross sections between beam particle and antiparticle, it may be possible to isolate quasielastic peaks, which show up as a maximum in ϵ at fixed Q^2, or a vanishing of H_{eff}. For more details, see Ref. 2.

These effective powers are very useful in determining the dominant basic subprocesses, since, according to Eq. (5.1), each such process predicts

$$H_{eff} \sim H \equiv \gamma + 2n_f - 2\Sigma(n_{bnd} - 1)$$

and

$$N_{eff} \sim N \equiv n_a + n_b - 2 + \Sigma(n_{bnd} - 1) . \qquad (6.3)$$

VII. PHOTOPRODUCTION OF A MASSIVE LEPTON PAIR

In this section we derive the dimensional counting rules for the photoproduction of a massive lepton pair. The vector dominance model (VDM) in which the incoming photon couples to a virtual vector meson which then scatters off the target nucleon provides only a subset of the processes present in the constituent interchange model. A sample set of diagrams which contribute to the photoproduction of a heavy lepton pair is presented in Fig. 5. Eq. (2.5) still applies where $G_{a/A}(x_a)$ is now the photon structure function, and $d\sigma/dQ^2$ for the central process is calculated as in the case of two initial state hadrons. The ϵ and Q^2 behavior of the cross section for these diagrams is

$$Q^4 \frac{d\sigma}{dQ^2} (\gamma N \to \ell^+ \ell^- X) = \epsilon^4 (a + b\epsilon^4 + c\epsilon^4 Q^{-2} + dQ^{-6} + e\epsilon^2 Q^{-4} + + f\epsilon Q^{-2} + g\epsilon Q^{-4} + \ldots) , \tag{7.1}$$

where the lower case letters label both the normalization and subprocess in Fig. 5. If we characterize the cross section in the usual way by

$$Q^4 \frac{d\sigma}{dQ^2} \propto \epsilon^H (Q^2)^{-N} , \tag{7.2}$$

then the value of N is still given by Eq. (5.1), since it does not depend on the initial state particles. The power of ϵ, H, is however slightly different from that in Eq. (5.1) because of the point-like coupling in the photon structure function. It is now given by

$$H = 2n_s^h + n_s^{em} + \gamma \tag{7.3}$$

where n_s^{em} is the number of spectator quarks arising from a point electromagnetic coupling, and n_s^h is the number of spectator quarks arising through hadronic couplings. This distinction arises due to the spin structure of the photon, and is discussed in detail in reference 12.

Of the processes represented in Fig. 5, only (g) and (h) are present in the VDM. However, analogous processes to (a)-(e) all exist in the vector dominance picture. The quark or antiquark is now not taken directly from the photon but from the vector meson which is coupled to the photon. The Q^2 and ϵ behavior of these terms is exactly the same as that found in meson-proton scattering. If the results of the CIM fit to large p_T photoprocesses are used as a guide, the process labeled by e is expected to be important at medium energies. It can be thought of as proton scattering from a photon target. Phenomenologically it may be difficult to distinguish the VMD processes from the non-VMD processes, since they differ only by a single power of ϵ. Hence one has to be careful about drawing any conclusions about $VN \rightarrow \ell^+\ell^-X$ from $\gamma N \rightarrow \ell^+\ell^-X$.

VIII. PRODUCTION OF A HADRONIC-PAIR WITH LARGE INVARIANT MASS

In this section we generalize our discussion to the production of a hadronic pair at large invariant mass. Experiments which are set up to measure the invariant mass distribution of lepton pairs in hadronic collisions can also measure many purely hadronic events in which two of the final state hadrons are observed in the detectors. Below we derive a set of dimensional counting rules for the differential cross section $d\sigma/dM^2$ for such inclusive processes. A study of these distributions can thus be used to obtain information about the interactions of hadronic constituents. The reaction $AB \to CDX$, where the CD system has a large invariant mass, is decomposed as in Fig. 6, analogously to the lepton pair case. We are not considering those pairs that have a large mass due just to rapidity differences but to those in which a large p_T is involved.

In addition to this mechanism, in which the large mass pair has its origins in a large p_T basic reaction, one could also consider the production and few particle decay of a massive fireball. A fireball in our language is a massive coherent state composed of several constituents. There are additional processes which could produce a hadron pair with a large invariant mass, and which cannot be decomposed as in Fig. 6; however, all these processes involve (at least) off shell propagators, so that away from the resonance region, they are expected to be small if the hadronic structure functions sharply damp the off-shell behavior of the constituents. Their behavior can be calculated in a similar way to those discussed below.

Under the usual assumption that the structure functions of A and B strongly damp the off-shell and transverse momentum dependences of a and b respectively, an analogous equation to (2.5) can be derived, which reads

$$\frac{d\sigma}{dQ^2}(AB \to CDX) = \sum_{ab,e} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{d\sigma}{dQ^2}(ab \to CDe;\, s' = x_a x_b s, Q^2) \tag{8.1}$$

It now remains to calculate the basic process $d\sigma/dQ^2(ab \to CDe)$. We have

$$s\frac{d\sigma}{dQ^2}(ab \to cde) \propto \int |Mab \to cd|^2 \, \psi_C^2(p_c^2)\, \psi_D^2(p_d^2) dp_C dp_D dp_r dp_q$$

$$\delta^4(P_a + P_b - p_C - p_D - p_r - p_q)\delta^+(Q^2 - (p_C + p_D)^2) \;. \tag{8.2}$$

We shall work in the rest frame of the $(a + b)$ system, and neglect particle masses, so that $p_a = (P,\ 0,\ P)$, $p_b = (P,\ 0,\ -P)$, and $P = \sqrt{s}/2$. "Finite momentum frame" variables are now introduced in terms of which

$$p_C = (xP + \frac{\vec{r}^{\,2}}{4xp},\ \vec{r},\ xP - \frac{\vec{r}^{\,2}}{4xp})$$

$$p_r = \left((z-1)xP + \frac{[(z-1)\vec{r} + \vec{k}]^2}{4(z-1)xP},\ \ (z-1)\vec{r} + \vec{k},\ (z-1)xP - \frac{[(z-1)\vec{r} + \vec{k}]^2}{4(z-1)xp}\right) \tag{8.3}$$

$$p_D = \left(yP + \frac{\vec{q}^{\,2}}{4yp},\ \vec{q},\ yP - \frac{\vec{q}^{\,2}}{4yP}\right)$$

$$p_q = \left((w-1)yP + \frac{[(w-1)\vec{q} + \vec{\ell}]^2}{4(w-1)yP},\ (w-1)\vec{q} + \vec{\ell},\ (w-1)yP - \frac{[(w-1)\vec{q} + \vec{\ell}]^2}{4(w-1)yP}\right)$$

In these variables, one has the simple formulae

$$p_c^2 = \vec{k}^2/(z-1)$$

and (8.4)

$$p_d = \vec{\ell}^{\,2}/(w-1)$$

This choice of frame aids in decoupling the final state integrations.

Under the assumption that ψ_C and ψ_D suppress the off-shell and large transverse momentum behavior of c and d, the energy and momentum conservation equations can be written,

$$\begin{aligned} z\vec{r} + w\vec{q} &= 0 \\ zx + wy &= 1 \end{aligned} \tag{8.5}$$

and

$$\frac{z\vec{r}^{\,2}}{x} + \frac{w}{y}\vec{q}^{\,2} = s \ .$$

Using these relations it can be shown that

$$Q^2 = (p_C + p_D)^2 \cong s/wz \ . \tag{8.6}$$

The integration over x, y, $\vec{r}$, and $\vec{q}$ is now just the usual 2-body phase space, and after this is performed we are left with

$$Q^4 \frac{d\sigma}{dQ^2}(ab \to CDe) = s^2 \int_1^\infty dw\, dz\, z\tilde{G}_{C/c}\left(\frac{1}{z}\right) w\, \tilde{G}_{D/d}\left(\frac{1}{w}\right) \cdot \frac{d\sigma}{dt}(ab \to cd)\delta(wz - s/Q^2), \tag{8.7}$$

where the fractional momentum distributions for decay are given by

$$\tilde{G}_{C/c}(x) \equiv \int \frac{d^2k}{(x^{-1}-1)} \psi_c^2(p_c^2) \ .$$

Equations (8.7) and (8.1) can now be used to write the cross section for the process AB → CDX in the form

$$Q^4 \frac{d\sigma}{dQ^2}(AB \to CDX) \propto \epsilon^H (Q^2)^{-N} \ . \tag{8.8}$$

The value of N is given by the dimensional counting rules for $d\sigma/dt$ for fixed angle scattering,

$$N = n(a) + n(b) + n(c) + n(d) - 4 \ , \tag{8.9}$$

where n(a) is the number of quark fields in a, etc., while H is given by

$$H = 2n_f - 1 \ , \tag{8.10}$$

where n_f is the number of independent particles (quarks or bound systems) in the final state excluding those in C and D.

As an example, we consider the reaction $pp \to (\pi\pi)X$ where the 2π system has invariant mass Q. Several diagrams for this process are shown in Fig. 7. The differential cross section for this reaction is then given by

$$Q^4 \frac{d\sigma}{dQ^2} = \epsilon^{11}(a+bQ^{-4}+cQ^{-8}+dQ^{-8}+\ldots) + \epsilon^{15}(e+\ldots) , \quad (8.11)$$

where the normalization constants a, b, etc., also label the subprocesses depicted in Fig. 7. There are clearly other potentially relevant subprocesses; however, in the absence of data we present explicit expressions only for the above sample. If the results of CIM fits to large p_T single particle inclusive are used as a guide, a should be small and the dominant terms should be b and c. The d process may be important at low energies.

The cross section for the processes $pp \to (p\pi)X$ and $pp \to (p\bar{p})X$, for example, are expected to be different from the $(\pi\pi)$ case, not only because the dominant subprocesses may be different, but even if they are the same, the ϵ behavior is predicted to be different. It will be very interesting to compare such reactions at small ϵ, where our predictions take their simplest form. The cross section corresponding to other subprocesses, and also reactions with other trigger particles, can be simply calculated by using the above rules. In this purely hadronic case, it is very important to compare the cross section behavior for differing beam and final state particles, and to analyze the data using the effective power analysis described in Section VI to aid in its interpretation.

IX. PRODUCTION OF A PAIR OF HADRONS WITH LARGE INVARIANT MASS IN e^+e^- COLLISIONS

In this section we present our final example, the inclusive production of a pair of hadrons with large invariant mass in e^+e^- collisions. A detailed study of this reaction will be presented elsewhere;[13] here we restrict the discussion to the presentation of some examples and simply note that there is no uncertainty in describing the initial stages of the process in the constituent model. This reaction can be thought of as the crossed process to inclusive lepton-pair production in hadronic collisions; there is a one-to-one correspondence between the diagrams of the two reactions. There is, however, an important practical difference in that, whereas in hadronic collisions one is restricted to using a nucleon target, in e^+e^- collisions it is possible, and indeed easier, to have both the trigger particles be mesons. Because of the small value of the antiquark distribution function in the proton, it is likely that the Drell-Yan model is more important in meson-nucleon collisions (or meson-meson collisions if they were possible) than in nucleon-nucleon collisions. Similarly, one expects that in e^+e^- collisions, massive meson pairs, for example, will be produced by a "reversed" Drell-Yan process in which the mesons are produced by the cascade (see Fig. 8)*

$$e^+e^- \to \gamma \to q_1 + \bar{q}_2 \to (M_1 + x_1) + (M_2 + \bar{x}_2) \quad . \tag{9.1}$$

It is evident that Eq. (8.7) is still valid, where $d\sigma/dt$ is now given by

$$\frac{d\sigma}{dt}\,(e^+e^- \to q\bar{q}) \sim \frac{\lambda^2(s,\, m_c^2,\, m_d^2)}{s^4} \tag{9.2}$$

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* Production by means of other mechanisms can be calculated by the methods of the previous sections and will be discussed in detail in Ref. 13.

Assuming that the hadron structure functions $\tilde{G}$ suppress the off-shell behavior of c and d so that $\lambda(s, m_c^2, m_d^2) \sim s$, the appropriate cross section is given by

$$Q^4 \frac{d\sigma}{dQ^2} \propto \frac{Q^2}{s} \int dw\, dz\, \tilde{G}_{C/c}\left(\frac{1}{z}\right) \tilde{G}_{D/d}\left(\frac{1}{w}\right) wz\, \delta\left(wz - \frac{s}{Q^2}\right) \quad , \tag{9.3}$$

where Q^2 is the mass of the (C+D) system. In terms of ϵ and s this cross section is written in the scaling form

$$Q^4 \frac{d\sigma}{dQ^2} = \epsilon^H \left(\frac{Q^2}{s}\right) f\left(\frac{Q^2}{s}\right) \quad , \tag{9.4}$$

where $H = 2n_f - 1$ and n_f is again the number of final quarks not in the large mass system C + D. For small values of ϵ, $f(Q^2/s)$ is a slowly varying function, but in general it can contain explicit factors of (Q^2/s) that directly reflect the large z behavior of $\tilde{G}(1/z)$.

Using Eq. (9.2) it is now possible to carry out the usual ϵ and Q^2 analyses for different trigger particles, and this will be done in Ref. 13. For example, if the trigger particles are K^+K^-, $\pi^+\pi^-$, or $K^+\pi^-$, i.e. nonexotic, then we have

$$Q^4 \frac{d\sigma}{dQ^2}(K^+K^-, \pi^+\pi^-, \text{or } K^+\pi^-) = \frac{Q^2}{s} \epsilon^3 f\left(\frac{Q^2}{s}\right) \quad . \tag{9.5}$$

These processes are represented diagrammatically in Fig. 9a. However, when the trigger particles are $\pi^+\pi^+$, $K^+\pi^+$, i.e. exotic, as in Fig. 9b, one finds the quite different result,

$$Q^4 \frac{d\sigma}{dQ^2}(K^+\pi^+) \sim \frac{Q^2}{s} \epsilon^7 g\left(\frac{Q^2}{s}\right) \quad . \tag{9.6}$$

If distributions such as these in (9.5) and (9.6) are found to agree with the data for small ϵ, then much interesting information about the $\tilde{G}$ structure functions can be obtained by examining various combinations of trigger particles.

Thus it seems most likely that massive hadron pair production in e^+e^- collisions will provide valuable insight into the nature of quark dynamics. The information gained on the fractional momentum decay distributions is very important since it can be independently checked in purely hadronic reactions at large p_T and since $\tilde{G}$ is related to the ordinary structure functions G by crossing.[2] Another important feature that can be measured by examining the relative angular dependence (nonplanarity) of the particles that make up the massive pair is the width of the transverse momentum distribution function $G(x, \vec{k}_T^2)$ for large x. This is an important parameter that allows one to estimate the nonplanarity in large transverse momentum events. The width at large x need not be the same as measured at small x, namely $\approx$ 300 MeV/c and should be much broader if this is to explain the large nonplanarity observed at the ISR.[2]

X. CONCLUSIONS

The study of large transverse momentum processes, both exclusive and single particle inclusive, has provided considerable insight into the dynamics of possible hadronic constituents. The additional information arising from the study of reactions involving large mass production such as those described here should be equally valuable. In this paper we have attempted to provide a general framework for analyzing massive pair production. The reactions described here include the production of μ-pairs by hadron and photon beams which generalizes the treatment of Drell-Yan.[3] The production of massive hadron pairs by hadron and photon beams was also calculated by assuming that the basic interaction involved a large transverse momentum process. Fireball (a massive coherent multiquark state in our language) production and decay was not included. Finally, the simple yet important reaction involving the production of a massive hadron pair in electron-positron annihilation was described.

All the theoretical results on lepton pairs in the text are summarized in the essentially dimensional counting formula (5.1) which is further generalized in Eqs. (7.2) and (7.3). All the results on hadron pairs are summarized in Eqs. (8.8) - (8.10) and (9.4). It is important and necessary to determine experimentally what the dominant basic processes are for each physical reaction. As an example, the CIM rules would not allow a large direct quark-quark interaction, such as $qq \rightarrow qq\gamma$, and it is important to determine whether this rule is still valid in the new regime. In fact, the dominant basic processes at large transverse momentum seem to have at most two quarks or diquarks in the initial plus the final state in CIM fits to the data.

The absolute normalization of the various basic processes is difficult to compute a priori. The Drell-Yan process is particularly simple in this regard.

However, there are quite stringent bounds that can be set on the normalization of the generalized structure functions from sum rules and, more importantly, their value must be consistent with the fits to the large transverse momentum inclusive data. The normalizations also clearly depend upon assumptions of internal symmetry- color, for example.[14] Another uncertainty concerns mass corrections to ϵ. These corrections to $pp \rightarrow (\pi\pi)X$ and $p\bar{p} \rightarrow (\pi\pi)X$ are probably of a rather different form and should be important for low energies. For example, the inclusive-exclusive connection, that is the small ϵ limit, for these two reactions is quite different.

Several experimental points should be made in light of our results:

(a) The effective power analyses, the extraction of H_{eff} and N_{eff} directly from the data, should be very useful in untangling the physics of large mass production. This requires that the mass spectrum be measured at different energies so that a fixed ϵ analysis can be performed to extract the Q^2 dependence.

(b) It is important to check the counting rules by using different incident beams as well as different choices for the particles that make up the massive pair. The ϵ dependence and hence the Q^2 behavior at fixed energy should depend upon the particular type of particle pair chosen, as explained in the text.

(c) The production of massive hadron pairs in electron-positron annihilation is a particularly important test of the theory. Unlike the classic Drell-Yan process, the effects of the ψ resonances can be eliminated by choosing the initial energy appropriately. One can avoid the "contamination" that is present, for example, in the data of Christenson et al.[15] However, it is clearly very interesting to study these processes on resonance as well as off.

The general approach described here should prove helpful in correlating data from different reactions. The overall consistency of the constituent model of hadrons used here requires that there be a simple relation between the data on large transverse momentum reactions and massive pair production and this can be checked for a large number of initial beams and final detected particles.

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FIGURE CAPTIONS

1. The general decomposition of inclusive lepton pair production in hadronic collisions. The overall process $A + B \to \ell^+\ell^- X$ is written in terms of hadron-irreducible subprocesses $a + b \to \ell^+\ell^- d$.

2. Auxiliary diagram for the calculation of the spin sum and integrations over the lepton variables.

3. A sample set of subprocesses for inclusive lepton pair production, in which all of the final state quarks are free.

4. A sample set of subprocesses for inclusive lepton pair production in which the final state quarks are either bound to form a physical particle ((b)-(d)) or are required to have a fixed invariant mass ((a)-(c)).

5. A sample set of diagrams which contribute to the inclusive photoproduction of lepton pairs.

6. The general decomposition of the inclusive production of a pair of hadrons with large invariant mass in hadronic collisions. The overall process $A + B \to C + D + X$ is written in terms of the hadron-irreducible subprocesses $a + b \to c + d \to (C + r) + (D + q)$.

7. A sample set of diagrams which contribute to the production of a pair of pions with large invariant mass in pp collisions.

8. The decomposition discussed in Section IX for the production of a pair of hadrons with large invariant mass in e^+e^- annihilation.

9. A model for the production of (a) $\pi^+\pi^-$, K^+K^-, $K^+\pi^-$ and (b) π^+K^+, in e^+e^- annihilation.

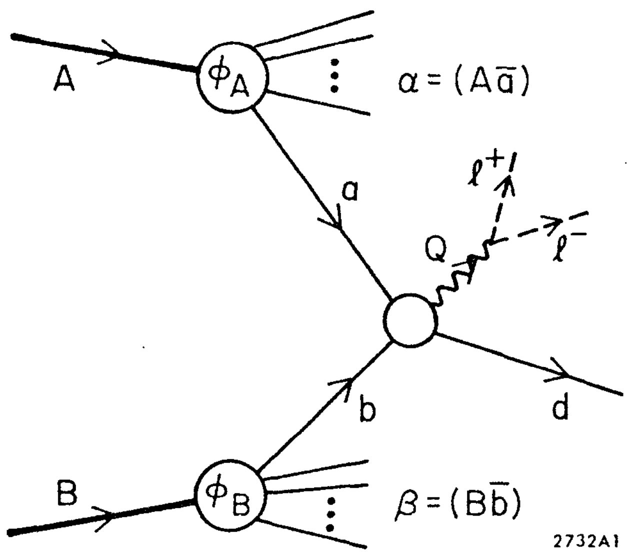


Fig. 1

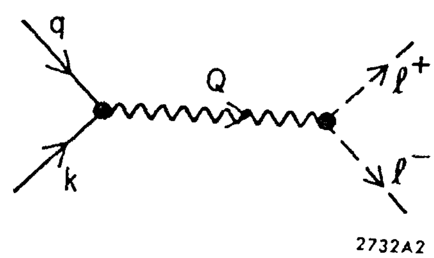


Fig. 2

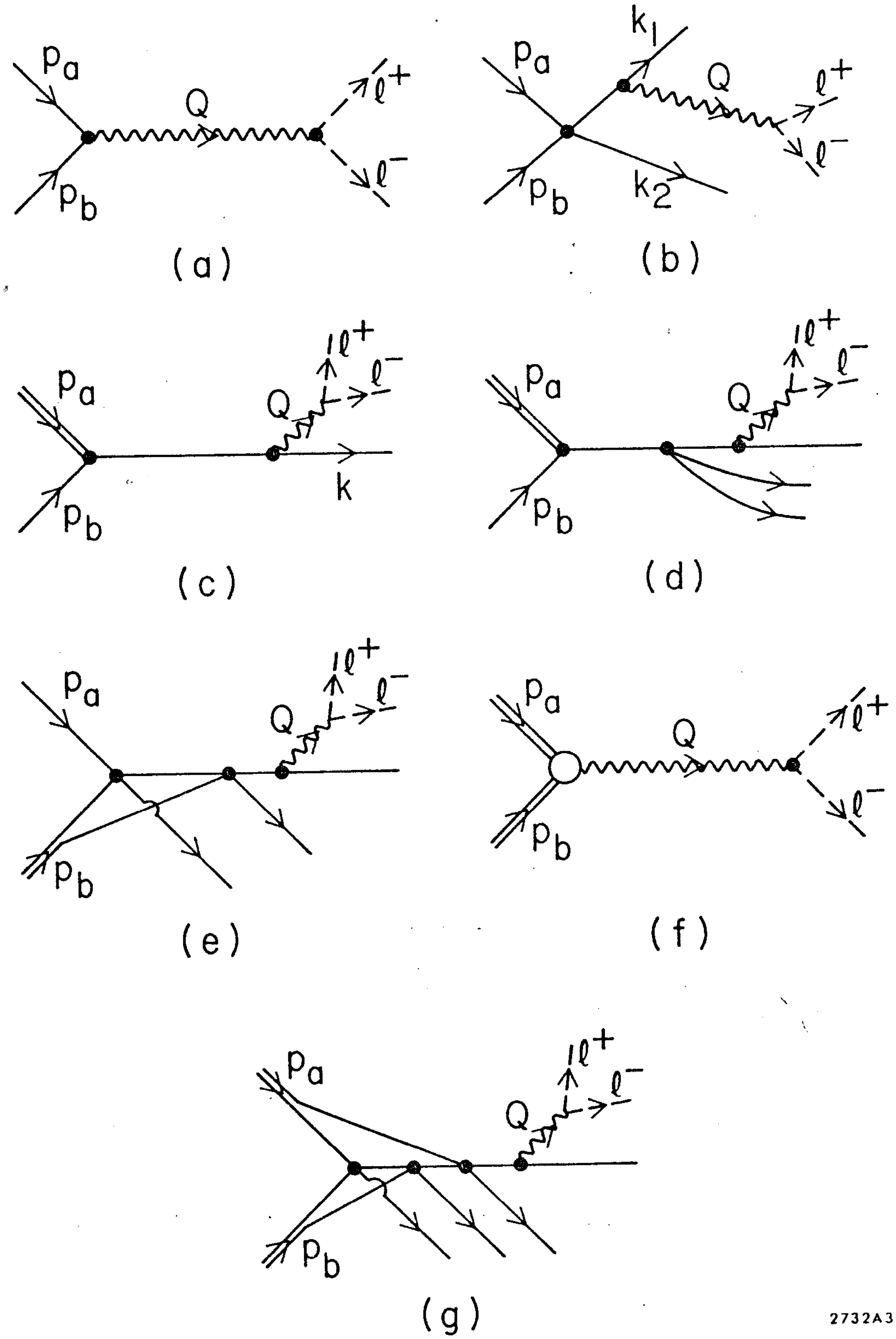


Fig. 3

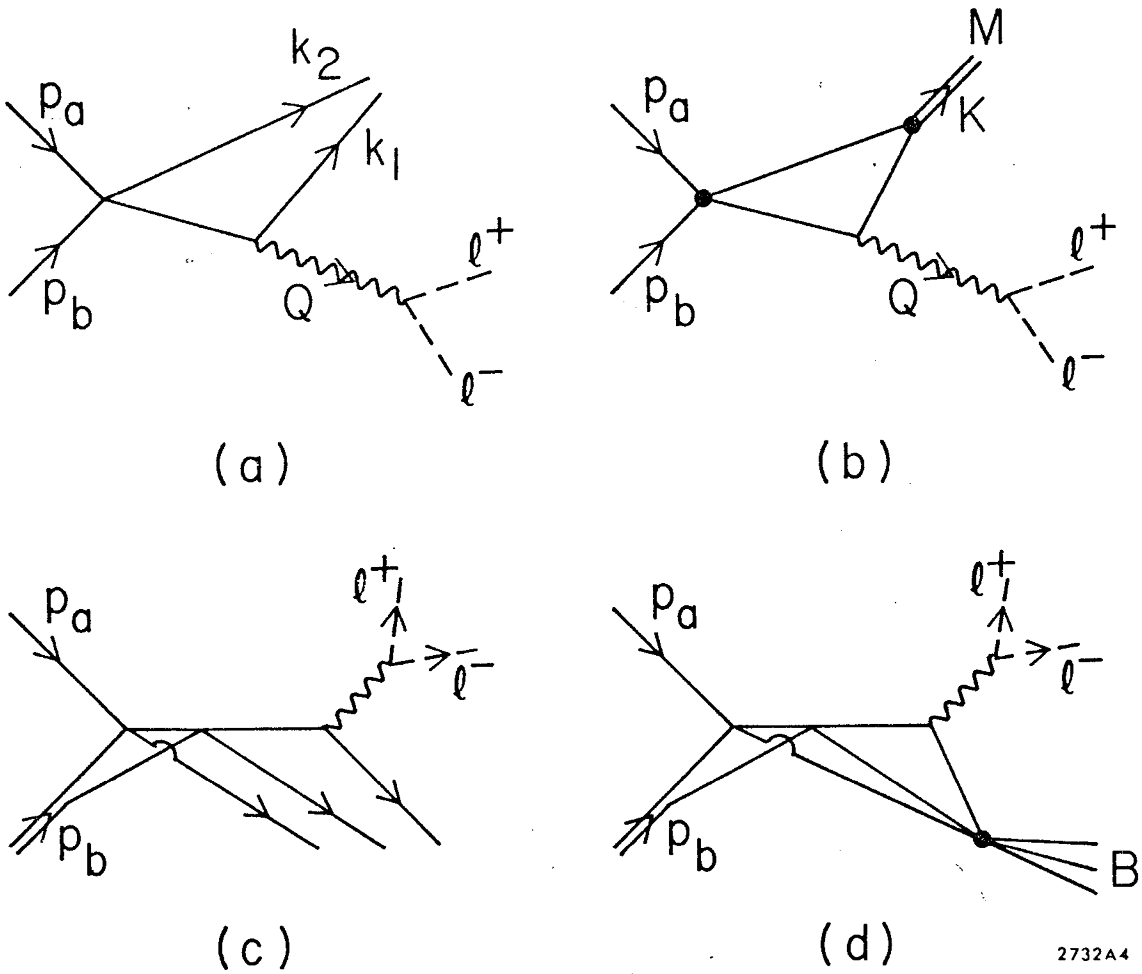
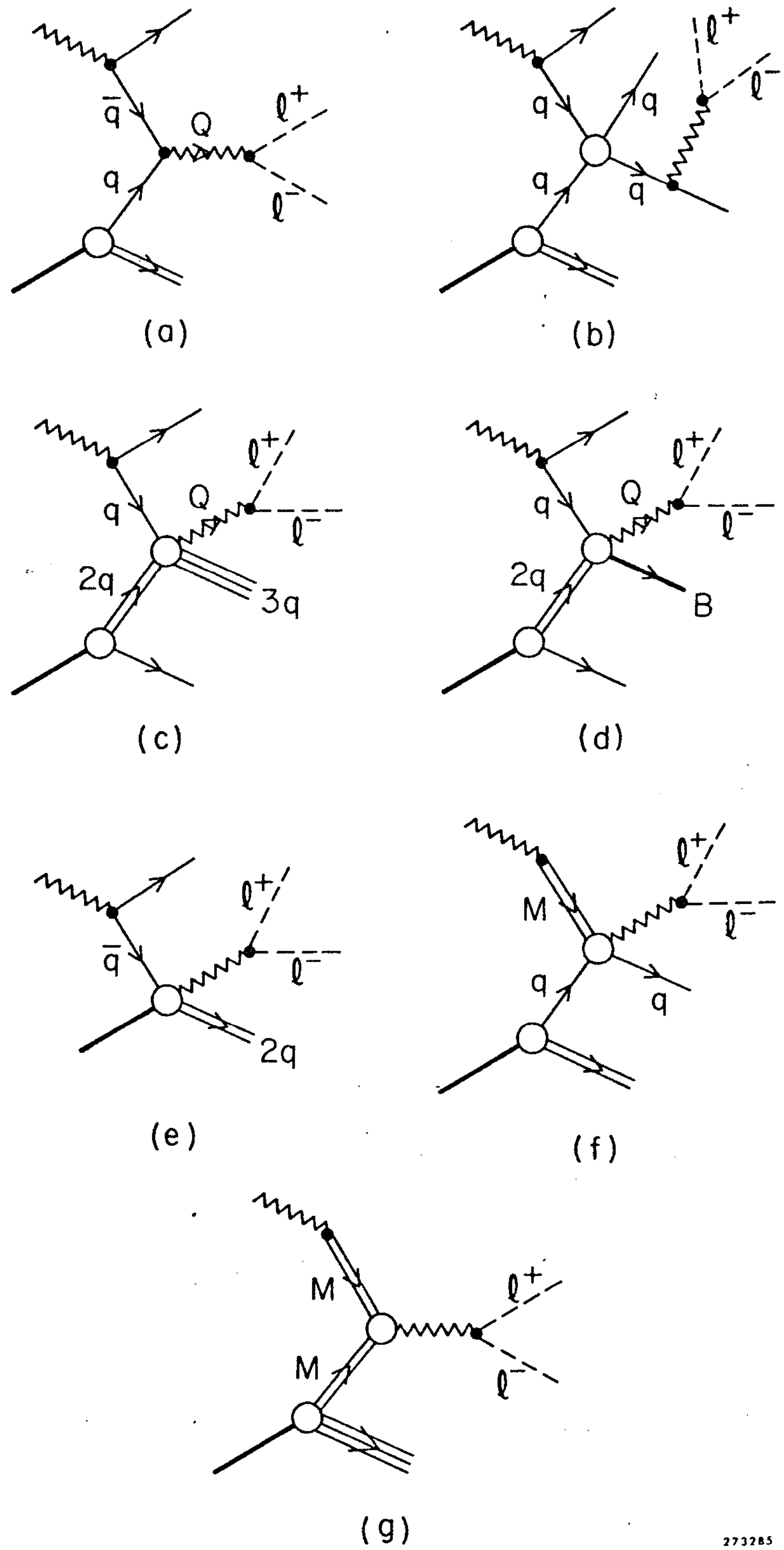


Fig. 4


q̄
q
Q
ℓ+
ℓ−
(a)
q
q
q
q
ℓ+
ℓ−
(b)
q
2q
Q
ℓ+
ℓ−
3q
(c)
q
2q
Q
ℓ+
ℓ−
B
(d)
q̄
ℓ+
ℓ−
2q
(e)
M
q
q
ℓ+
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(f)
M
M
ℓ+
ℓ−
(g)
273285

Fig. 5

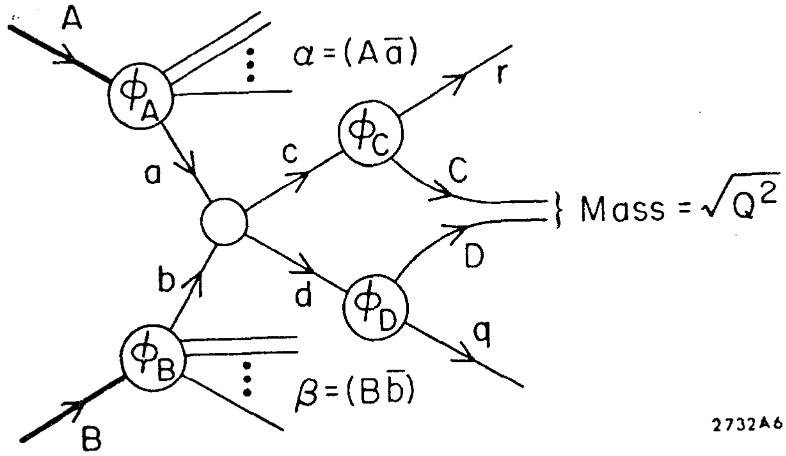

Fig. 6

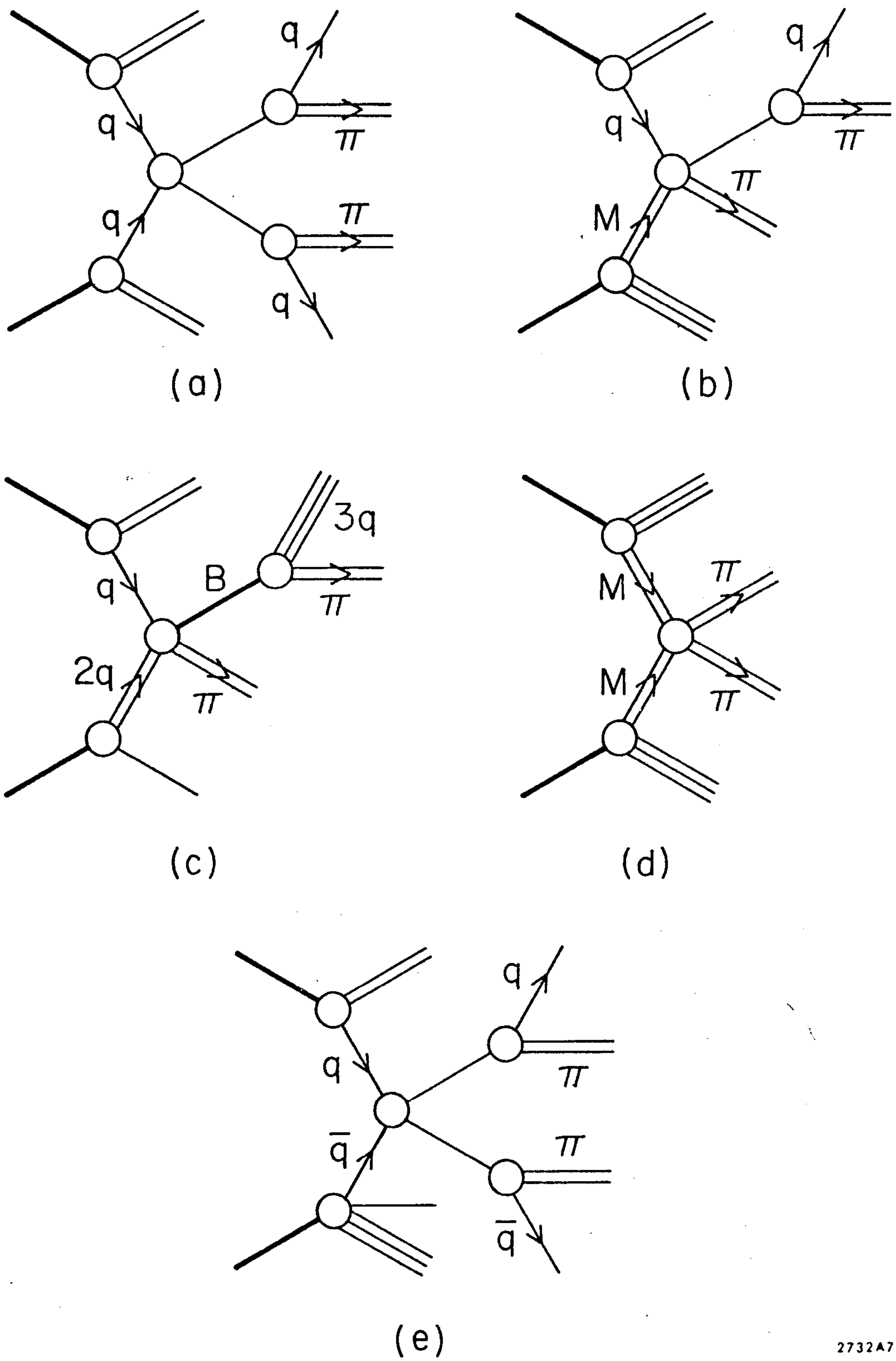

Fig. 7

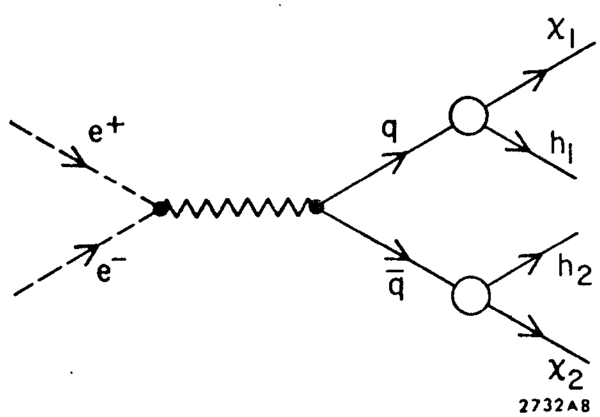

Fig. 8

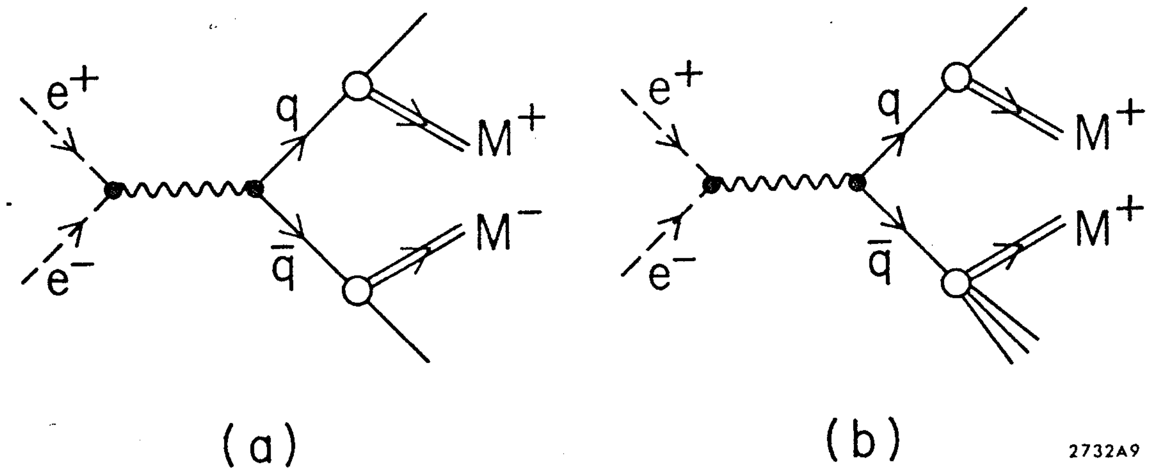

Fig. 9