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Threshold Behavior of Pion and Proton Structure Functions in the Bethe-Salpeter Description*

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Abstract The threshold behavior of the structure function νW_2 is evaluated for two- and three-body bound states described by the Blankenbecler-Sugar wave funtions. In our approach this is completely equivalent to a fully relativistic Bethe-Salpeter treatment. For spin-zero constituents and a two-body interaction which behaves as $(q^2)^{-\theta}$ at large momentum transfer, we obtain $(1-\omega)^{1+2\theta}$ and $(1-\omega)^{3+4\theta}$ for the two- and three- body bound state, respectively. For spin-½ constituents and the interaction $V = \Gamma_{(\mu)} \tilde{V} \Gamma(\mu)$ with $\tilde{V}(q,k) \underset{(q-k)\to\infty}{\simeq} ((q-k)^2)^{-1-\Delta}$ we obtain again $(1-\omega)^{1+2\Delta}$ and $(1-\omega)^{3+4\Delta}$ respectively if $\Gamma_{(\mu)} = \gamma_\mu$ but $(1-\omega)^{2+2\Delta}$ and $(1-\omega)^{5+4\Delta}$ respectively if $\Gamma_{(\mu)} = 1,\gamma_5$. Comparison with previous results on form factors show that the Drell-Yan-West relation is satisfied in all the models considered here.

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I. Introduction

In two recent papers[1,2] we have investigated the asymptotic behavior of form factors of two- and three-body bound states for both spin-zero and spin-½ constituents. Our main task was to get some ideas about the underlying structure of the pion and the nucleon (and hadrons in general) by consistently studying the pion and nucleon form factors. The latter have been proven to provide a great source of information on the nature of the constituents and their dynamics. We found that the experimental pion and nucleon form factors[3] are consistently described by assigning a quark-antiquark (two-quark) and three-quark structure to the pion and the nucleon, respectively, and the quarks interacting via a vector-gluon exchange. We consider this giving strong support to the quark picture of hadrons.

Our results may also be considered from a different point of view. Suppose we know the constituents of any hadrons (as,e.g., in the case of the deuteron) and the asymptotic behavior of its form factor. Then, we can extract the large momentum transfer (short-distance) behavior of the interaction kernel which certainly will help to improve our understanding of the dynamics of those particular constituents.[4]

A further means of probing the constituents of the hadrons (or the core region if the constituents are well established) is provided by the threshold behavior of the deep inelastic structure functions[5] $W_1(\omega)$ and $\nu W_2(\omega)$ as is also suggested by the Drell-Yan-West[6] (DYW) relation. Unfortunately, the DYW relation has not been proven for that particular kind of underlying structure we are interested in, which demands a renewed look at the structure functions being particularly appreciated with the advent of new data from SPEAR and DORIS.

In this paper, the threshold behavior of the structure functions is considered for spin-zero and spin-½ constituents. The bound states are described either by the Bethe-Salpeter (BS) equation for spin-zero constituents or, in case of spin-½ constituents, by the Blankenbecler-

Sugar (BLS) approximation. The validity of the BLS approximation in the high momentum transfer limit has been discussed in ref.2 and we like to point out that, in our approach, the two methods are completely equivalent. In fact, we need only the high momentum transfer behavior of the interaction kernel and it makes no difference starting from an ansatz of the BS or of the BLS kernel.

In case of spin-$\frac{1}{2}$ constituents we consider the exchange of scalar, pseudoscalar and of vector gluons. As in ref. 2, we find a difference between the vector interaction and the scalar and pseudoscalar interaction respectively, contrary to the dymensional counting rules.[7] The DYW relation is always satisfied in our models being in contradiction with the recent results of Ezawa.[8]

The paper is organized as follows. Section II deals with spin-zero constituents and the BS equation. Section III is devoted to the case of spin-$\frac{1}{2}$ constituents and the BLS equation. Each section is divided into two subsections a) and b) dealing with the two- and three-particle case, respectively. Table I collects all the results of refs. 1,2 and those of the present paper.

II.BS equation for spin-zero constituents.

IIa.Two-body.

In the BS model for the bound states of two spin-zero constituents, the structure functions are given by (see fig. 1):

$$W_{\mu\nu} \simeq \int d^4A\ \delta^+((p-A)^2-1)\ \delta^+((A+q)^2-1)\ \frac{1}{(A^2-1)^2}\ G_{\mu\nu} \qquad \text{(II.1)}$$

where all the masses are equal to 1, $G_{\mu\nu} = |\phi_p(k)|^2(2A_\mu+q_\mu)(2A_\nu+q_\nu)$ and the vertex function ϕ satisfies the BS equation (see fig.2)

$$\phi_p(k) = \int d^4k'\ V(k,k')\ G_1(\tfrac{1}{2}p+k')\ G_2(\tfrac{1}{2}p-k')\ \phi_p(k') \qquad \text{(II.2)}$$

In the infinite momentum frame of the "pion" defined by $p_\mu = (P+\frac{m^2}{2P}, 0_\perp, P)$, $P \to \infty$, we choose $q = (\frac{\nu}{P}, \vec{q}_\perp, 0)$ $(\nu = p\cdot q)$ and we parametrize the loop variable in the standard way writing $A = (xP+\frac{A^2+\vec{A}_\perp^2}{4xP}, \vec{A}_\perp, xP-\frac{A^2+\vec{A}_\perp^2}{4xP})$. We are interested in the limits $\vec{q}_\perp^2 \to \infty$ and $\omega = \frac{\vec{q}_\perp^2}{2\nu}$ fixed of the structure function $\nu W_2 = \frac{\nu}{P^2} W_{00} = \frac{\nu}{P^2} W_{33}$. We obtain

$$\nu W_2 \underset{P\to\infty}{\simeq} \omega^2(1-\omega) \int \frac{d^2\vec{A}_\perp}{(\vec{A}_\perp^2+\omega^2-\omega+1)^2} |\phi_p(k)|^2 , \qquad \text{(II.3)}$$

where $k^2 = \frac{\frac{1}{2}}{1-\omega}(-\vec{A}_\perp^2-\omega^2-\frac{1}{2}\omega+\frac{1}{2})$. For an interaction which behaves as $V(k,k') \underset{(k-k')^2\to\infty}{\simeq} ((k-k')^2)^{-\theta}$, $\theta > 0$, the large momentum transfer limit of the vertex function is given by[1] $\phi(k) \underset{k^2\to\infty}{\simeq} (k^2)^{-\theta}$; therefore, the threshold behavior of the structure function (II.3) reads

$$\nu W_2 \underset{\omega\to 1}{\simeq} (1-\omega)^{1+2\theta} . \qquad \text{(II.4)}$$

The same interaction gives $F(q^2) \underset{q^2\to\infty}{\simeq} (q^2)^{-1-\theta}$ for the asymptotic behavior of the form factor so that the DYW relation is satisfied.

IIb. Three-body

In the three-body case, we simply have (see fig. 3)

$$W_{\mu\nu} \simeq \iint d^4A\, d^4B\, \delta^+((A+q)^2-1)\, \delta^+(B^2-1)\, \delta^+((p-A-B)^2-1) \frac{1}{(A^2-1)^2} G_{\mu\nu} , \qquad \text{(II.5)}$$

where $G_{\mu\nu} = |\phi_p(A,B)|^2 (2A_\mu+q_\mu)(2A_\nu+q_\nu)$ and $\phi_p(A,B)$ satisfies the BS

equation for a three-body bound state (see fig.4). The definition of p,q and A is the same as in the two-body case and we take for B the natural parametrization $B = (y(1-x)P + \frac{B^2+\vec{B}_\perp^2}{4y(1-x)P}, \vec{B}_\perp, y(1-x)P - \frac{B^2+\vec{B}_\perp^2}{4y(1-x)P})$.

From Eq.(II.5) we obtain

$$\nu W_2 \simeq \omega^2(1-\omega) \int d^2\vec{A}_\perp \int_0^1 dy \int d^2\vec{B}_\perp \frac{y(1-y)|\phi_p|^2}{(\vec{A}_\perp^2(y^2(1-\omega)-y)-\vec{B}_\perp^2\omega-2\vec{A}_\perp\cdot\vec{B}_\perp\omega y-y(1-y)(1-\omega)^2-\omega)^2} \quad \text{(II.6)}$$

As in refs. 1 and 2, we assume a three-body interaction K being described by two-body interactions only, i.e., $K = V_1 + V_2 + V_3$, where V_i is the two-body interaction between particles j and k having the (previously assumed) asymptotic behavior $V_j(k,k') \underset{(k-k')^2\to\infty}{\simeq} ((k-k')^2)^{-\theta}$. This interaction and any iteration of it gives rise to disconnected kernels, so it is a difficult task to extract the asymptotic behavior of the vertex function directly from the BS equation. More convenient is to consider the (once iterated) relativistic Faddeev (RF) equation as has been fully described in ref.1 (see fig.5). We must consider the external variables on the mass-shell, as follows from Eq.(II.5), and in the limits $\nu \to \infty$, $\vec{q}_\perp^2 \to \infty$, $\omega \to 1$. In this limit (of high momentum transfer) the asymptotic behavior of the BS vertex function can be extracted from ref.1 reading

$$\phi_p(A,B) \underset{\omega\to 1}{\simeq} (1-\omega)^{1+2\theta} F(\vec{A}_\perp, y, \vec{B}_\perp), \quad \text{(II.7)}$$

where F is a function which falls off in $\vec{A}_\perp$, y, and $\vec{B}_\perp$ rapidly

enough to insure the convergence of the integrals in Eq.(II.6). Hence we obtain

$$\nu W_2 \underset{\omega \to 1}{\simeq} (1-\omega)^{3+4\theta} . \qquad \text{(II.8)}$$

For the same interaction the form factor falls off as $F(q^2) \underset{q^2 \to \infty}{\simeq} (q^2)^{-2-2\theta}$, so that the DYW relation is satisfied even in this case.

III. BLS equation for spin-½ constituents

IIIa. Two-body

As we already pointed out in a previous paper,[2] the case of two spin-½ constituents could be worked out in the framework of the BS equation, but it is a very elaborate problem to handle the spin structure in the three-particle case as the vertex function consists of 16 invariant functions. The BLS equation, on the contrary, allows a complete solution of the problem and, in addition, it looks more reliable as far as the asymptotic behavior is concerned. In fact, it overcomes the problem of the validity of the Wick rotation, and we can rigorously apply the Weinberg theorem.[9] The validity of the BLS approximation in the limit of high momentum transfer has been discussed in ref.2 (and we do not repeat here those arguments). In the case of spin-zero constituents, the BLS and the BS approaches give the same results for the asymptotic behavior of the form factors[2] and for the threshold behavior of the structure functions. The latter can be easily derived from the following

discussion of the more interesting case of spin-½ constituents.

For a bound state of two spin-½ particles, the structure functions are (still) given by Eq.(II.1) where, now,

$$G_{\mu\nu} = \mathrm{Tr}\{\phi_p(k)(\gamma\cdot A-1)^{(1)}\gamma_\mu^{(1)}(\gamma\cdot(A+q)-1)^{(1)}\gamma_\nu^{(1)}(\gamma\cdot A-1)^{(1)}\phi_p(k)$$

$$\times\ (\gamma\cdot(p-A)-1)^{(2)}\} \quad . \qquad \text{(III.1)}$$

The BS vertex function is described by Eq.(II.2), where $G_i(p)=(\gamma\cdot p-1)^{-1}$ and

$$V(k,k') = \Gamma^{(1)}_{(\mu)}\ \Gamma^{(2)(\mu)}\ \tilde{V}(k,k') \quad . \qquad \text{(III.2)}$$

Here, we shall consider $\Gamma_{(\mu)}$ being equal to 1, γ_5 or γ_μ and

$$\tilde{V}(k,k') \underset{(k-k')^2\to\infty}{\simeq} ((k-k')^2)^{-1-\Delta} \quad , \quad \Delta > 0 \quad . \qquad \text{(III.3)}$$

Introducing the vertex functions

$$\phi_p^{rs}(k) = \tilde{w}_1^r(\tfrac{1}{2}P+\vec{k})\ \tilde{w}_2^s(\tfrac{1}{2}P-\vec{k})\ \phi_p(k) \quad , \qquad \text{(III.4)}$$

we obtain for G_{00} in the infinite momentum frame

$$G_{00} \underset{P\to\infty}{\simeq} x^2P^2 \sum_{rs} |\phi_p^{rs}|^2 \quad , \qquad \text{(III.5)}$$

whose leading contribution stems from the region where both particles are on mass-shell (which justifies our definition (III.4)). Hence, we have

$$\nu W_2 \simeq \omega^2(1-\omega) \int \frac{d^2\vec{A}_\perp}{(\vec{A}_\perp^2+\omega^2-\omega+1)^2} \sum_{rs} |\phi_p^{rs}|^2 \quad . \qquad \text{(III.6)}$$

For the threshold behavior we need the high momentum transfer limit of the BS functions ϕ_p^{rs} or, equivalently,[2] the behavior of the related BLS functions χ_p^{rs} (which have been introduced in ref.2) satisfying the coupled integral equations

$$\chi_p^{rs}(\vec{q}) = 4\pi \int d^3\vec{k}\, V^{rs,r's'}(\tilde{q},\tilde{k}) \frac{(1+(\frac{1}{2}P+\vec{k})^2)^{\frac{1}{2}} + (1+(\frac{1}{2}P-\vec{k})^2)^{\frac{1}{2}}}{(1+(\frac{1}{2}P+\vec{k})^2)^{\frac{1}{2}}\,(1+(\frac{1}{2}P-\vec{k})^2)^{\frac{1}{2}}}$$

$$\times \{((1+(\tfrac{1}{2}P+\vec{k})^2)^{\frac{1}{2}} + (1+(\tfrac{1}{2}P-\vec{k})^2)^{\frac{1}{2}})^2-P^2-s\}^{-1}\, \chi_p^{r's'}(\vec{k}) \quad , \qquad \text{(III.7)}$$

where

$$V^{rs,r's'}(\tilde{q},\tilde{k}) = \tilde{w}_1^r(\tfrac{1}{2}P+\vec{q})\, \tilde{w}_2^s(\tfrac{1}{2}P-\vec{q})\, V(\tilde{q},\tilde{k})\, w_1^{r'}(\tfrac{1}{2}P+\vec{k})\, w_2^{s'}(\tfrac{1}{2}P-\vec{k}) \quad . \qquad \text{(III.8)}$$

The potential V is defined by Eq.(III.2) and $\tilde{q} = (\ \frac{1}{2}\{(1+(\frac{1}{2}P+\vec{q})^2)^{\frac{1}{2}} - (1+(\frac{1}{2}P-\vec{q})^2)^{\frac{1}{2}}\}\ ,\ \vec{q}\ )$. We write Eq.(III.7) in the infinite momentum frame parametrization, and we invert the limit $x \to 1$ and the integration over d^3k in the region of integration Λ where this procedure is allowed. For the scalar and pseudoscalar interaction we get

$$\chi^{rs}_{(1,5)}(\vec{q}_\perp,x) \underset{x\to 1}{\simeq} \frac{(1-x)^{\frac{1}{2}+\Delta}}{(\vec{q}_\perp^{\,2}+1)^{\frac{1}{2}+\Delta}} \iint_\Lambda dy\, d^2\vec{k}_\perp$$

$$\times \frac{\{y(\vec{q}_\perp-\vec{k}_\perp)^2+(1-y)(1+\vec{k}_\perp^{\,2})-y(1-y)(1+\vec{q}_\perp^{\,2})\}^{\frac{1}{2}}}{y^{\frac{1}{2}}(1-y)^{\frac{1}{2}+\Delta}(\vec{k}_\perp^{\,2}+1-sy(1-y))}\, \chi^{rs}_{(1,5)}(\vec{k}_\perp,y) + \lim_{x\to 1} \iint_{\Omega_\Lambda} \ldots\ldots \tag{III.9}$$

Ω_Λ contains the regions $y \simeq 1$ and $\vec{k}_\perp^{\,2} \simeq \infty$. This equation and a separate analysis on the high-$\vec{q}_\perp^{\,2}$ limit suggests

$$\chi^{rs}_{(1,5)}(\vec{q}_\perp,x) \underset{\substack{x\to 1\\ \vec{q}_\perp^{\,2}\to\infty}}{\simeq} \frac{(1-x)^{\frac{1}{2}+\Delta}}{(\vec{q}_\perp^{\,2})^{\frac{1}{2}+\Delta}} \tag{III.10}$$

which is confirmed by concistency, i.e., inserting Eq.(III.10) in the integral over Ω_Λ in Eq.(III.9). For the γ_μ interaction we obtain

$$\chi^{rs}_{(\mu)}(\vec{q}_\perp,x) \underset{x\to 1}{\simeq} \frac{(1-x)^{\frac{1}{2}+\Delta}}{(\vec{q}_\perp^{\,2}+1)^{\frac{1}{2}+\Delta}} \iint_\Lambda dy\, d^2\vec{k}_\perp$$

$$\times \frac{\{\vec{k}_\perp^{\,2}+(1-2y)^2\}^{\frac{1}{2}}}{y^{\frac{1}{2}}(1-y)^{3/2+\Delta}(\vec{k}_\perp^{\,2}+1-sy(1-y))}\, \chi^{rs}_{(\mu)}(\vec{k}_\perp,y) + \lim_{x\to 1} \iint_{\Omega_\Lambda} \ldots\ldots \tag{III.11}$$

giving the correct asymptotic behavior

$$\chi^{rs}_{(\mu)} \underset{\substack{x\to 1\\ \vec{q}_\perp^{\,2}\to\infty}}{\simeq} \frac{(1-x)^{\Delta}}{(\vec{q}_\perp^{\,2})^{\frac{1}{2}+\Delta}} . \tag{III.12}$$

Roughly speaking, if we insert this behavior in the integral over Ω_Λ in Eq.(III.11), we are left with the integral $\int \frac{dy}{(1-y)^{3/2}}$ over the

region $y \simeq 1$. This gives a singularity of the order $(1-x)^{-\frac{1}{2}}$ which is cancelled by a factor $(1-x)^{\frac{1}{2}+\Delta}$ inherent in the integral. From Eqs.(III.6), (III.10) and (III.12) we finally obtain

$$\nu W_2^{(1,5)} \underset{\omega \to 1}{\simeq} (1-\omega)^{2+2\Delta} \quad , \quad \nu W_2^{(\mu)} \underset{\omega \to 1}{\simeq} (1-\omega)^{1+2\Delta} \tag{III.13}$$

for the $1/\gamma_5$ and γ_μ interactions, respectively. The asymptotic behaviors (III.10) and (III.12) can also be read off from our previous analysis.[2] The corresponding form factors have the asymptotic behavior[2]

$$F^{(1,5)} \underset{q^2 \to \infty}{\simeq} (q^2)^{-3/2-\Delta} \quad , \quad F^{(\mu)} \underset{q^2 \to \infty}{\simeq} (q^2)^{-1-\Delta} \quad , \tag{III.14}$$

so that the DYW relation is satisfied for all three interactions. The recent work by Ezawa[8] agrees with our $1/\gamma_5$ result for the structure functions and with our γ_μ result for the form factors.

IIIb. Three-body

Following the line of sect. IIIa we introduce a new set ϕ^{rst} of BS functions connected with the BS vertex function ϕ by

$$\phi = \sum_{rst} \tilde{w}_1^r(\vec{A})\, \tilde{w}_2^s(\vec{B})\, \tilde{w}_3^t(\vec{C})\, \phi^{rst} \tag{III.15}$$

The structure functions $W_{\mu\nu}$ are given by Eq.(II.5), where $G_{\mu\nu}$ now takes account of the spin structure of the constituents

(cf. Eq.(III.1)). In the infinite momentum frame we obtain for G_{00} (similar as in the two-particle case)

$$G_{00} \underset{P\to\infty}{\simeq} x^2P^2 \sum_{rst} |\phi^{rst}|^2 \quad . \tag{III.16}$$

Hence, the structure function νW_2 reads

$$\nu W_2 \underset{P\to\infty}{\simeq} \omega^2(1-\omega) \int d^2\vec{A}_\perp \int_0^1 dy \int d^2\vec{B}_\perp \times \frac{y(1-y)\sum_{rst} |\phi^{rst}|^2}{\{\vec{A}_\perp^2(y^2(1-\omega)-y)-\vec{B}_\perp^2\omega-2\vec{A}_\perp\cdot\vec{B}_\perp\omega y-y(1-y)(1-\omega)^2-\omega\}^2} \quad . \tag{III.17}$$

In the limit of high momenta we are allowed[2] to substitute the BLS functions χ^{rst} for the BS functions ϕ^{rst} (cf. ref. 2). The Faddeev components $\chi^{(i)rst}$ defined by $\chi^{rst} = \chi^{(1)rst} + \chi^{(2)rst} + \chi^{(3)rst}$ satisfy the coupled equations as schematically represented in fig. 5

$$\chi^{(i)rst}_{P,M}(\vec{q}^{(i)},\vec{k}^{(i)}) = \sum_{j=1}^{3} \int\int d^3\vec{q}^{(j)\prime} d^3\vec{k}^{(j)\prime}\, V^{rst,r's't'}_{ij}(\tilde{q}^{(i)},\tilde{k}^{(i)};\tilde{q}^{(j)\prime},\tilde{k}^{(j)\prime}) \times (\vec{q}^{(j)\prime},\vec{k}^{(j)\prime})\, \chi^{(j)r's't'}_{P,M}(\vec{q}^{(j)\prime},\vec{k}^{(j)\prime}) \quad . \qquad i=1,2,3 \tag{III.18}$$

Here, M denotes the spin component and $\vec{q}^{(i)}$, $\vec{k}^{(i)}$ are the c.m. variables (used in refs. 1 and 2) defined, e.g., by $\vec{q}^{(1)} = \frac{2\vec{A}-\vec{B}-\vec{C}}{2}$, $\vec{k}^{(1)} = \frac{\vec{B}-\vec{C}}{2}$ ($\vec{C} = \vec{p}-\vec{A}-\vec{B}$). The variables $\tilde{q}^{(i)}$ and $\tilde{k}^{(i)}$ are given by

$$\tilde{q}^{(1)} = (2/3(1+\vec{A}^2)^{\frac{1}{2}} - 1/3(1+\vec{B}^2)^{\frac{1}{2}} - 1/3(1+\vec{C}^2)^{\frac{1}{2}} , \vec{q}^{(1)}) ,$$
$$\tilde{k}^{(1)} = (1/2((1+\vec{B}^2)^{\frac{1}{2}} - (1+\vec{C}^2)^{\frac{1}{2}}), \vec{k}^{(1)}) .$$

Furthermore, we have written, e.g.,

$$V_{13}^{rst,r's't'} \simeq \tilde{w}_1^r(\vec{A})\ \Gamma_{(\mu)}^{(1)}\ w_1^{r'}(\vec{A}')\ \tilde{w}_2^s(\vec{B})\ \Gamma_{(\nu)}^{(2)}\ w_2^{s'}(\vec{B}')\ \tilde{w}_3^t(\vec{C})\ \Gamma^{(3)(\nu)}$$
$$\times\ (\gamma\cdot(\tilde{B}+\tilde{C}-\tilde{B}')+1)\ \Gamma^{(3)(\mu)}\ w_3^{t'}(\vec{C}')\ ((\tilde{A}-\tilde{A}')^2)^{-1-\Delta}\ ((\tilde{B}-\tilde{B}')^2)^{-1-\Delta}$$
$$\times\ ((\tilde{B}+\tilde{C}-\tilde{B}')^2-1)^{-1} , \qquad \text{(III.18)}$$

(see ref. 2 for a better description) where $\tilde{A}$, $\tilde{B}$ etc. are defined through $\tilde{q}^{(i)}$, $\tilde{k}^{(i)}$ given before. Finally, the propagator $\mathcal{E}$ is given by

$$\mathcal{E} = \frac{(1+\vec{A}^2)^{\frac{1}{2}} + (1+\vec{B}^2)^{\frac{1}{2}} + (1+\vec{C}^2)^{\frac{1}{2}}}{(1+\vec{A}^2)^{\frac{1}{2}}\ (1+\vec{B}^2)^{\frac{1}{2}}\ (1+\vec{C}^2)^{\frac{1}{2}}}\ \{((1+\vec{A}^2)^{\frac{1}{2}}+(1+\vec{B}^2)^{\frac{1}{2}}+(1+\vec{C}^2)^{\frac{1}{2}})^2-P^2-s\}^{-1}$$

(III.19)

We consider Eq.(III.17) in the infinite momentum frame using the previous parametrization. In order to find the asymptotic behavior of the vertex function we proceed as in the two-body case by separating the integration into two regions Λ and Ω_Λ . Region Λ is understood to be that region allowing the inversion of the limit $x \to 1$ and the integration, whereas Ω_Λ contains the region $x' \simeq 1$, $\vec{A}_\perp' \simeq \infty$ and $\vec{B}_\perp' \simeq \infty$. For the scalar and the γ_5 interaction, Eq.(III.17) then reads

$$\chi^{(i)rst}_{(1,5)} \underset{x \simeq 1}{\simeq} (1-x)^{2+2\Delta} \left\{ \iint_{\Lambda} \frac{dx'}{(1-x')^{2+2\Delta}} \cdots\cdots \left(\chi^{(i)rst}_{(1,5)} + \chi^{(j)rst}_{(1,5)} \right) + \right.$$

$$\left. + \iint_{\Lambda} \frac{dx'}{(1-x')^{2+2\Delta}} \cdots\cdots \left(\chi^{(i)rst}_{(1,5)} + \chi^{(k)rst}_{(1,5)} \right) \right\} + \lim_{x \to 1} \iint_{\Omega_{\Lambda}} \cdots\cdots \quad , \tag{III.19}$$

where we have explicitely written the crucial dependence on $(1-x)$ and $(1-x')$ only. The behavior suggested by Eq.(III.19) (and being confirmed by consistency) is given by

$$\chi^{rst}_{(1,5)} \underset{x \to 1}{\simeq} (1-x)^{2+2\Delta} \quad . \tag{III.20}$$

For the vector interaction the integral equation reads

$$\chi^{(i)rst}_{(\mu)} \underset{x \simeq 1}{\simeq} (1-x)^{3+2\Delta} \left\{ \iint_{\Lambda} \frac{dx'}{(1-x')^{4+2\Delta}} \cdots\cdots \left(\chi^{(i)rst}_{(\mu)} + \chi^{(j)rst}_{(\mu)} \right) + \right.$$

$$\left. + \iint_{\Lambda} \frac{dx'}{(1-x')^{4+2\Delta}} \cdots\cdots \left(\chi^{(i)rst}_{(\mu)} + \chi^{(k)rst}_{(\mu)} \right) \right\} + \lim_{x \to 1} \iint_{\Omega_{\Lambda}} \cdots\cdots \quad . \tag{III.21}$$

In this case the correct asymptotic behavior is

$$\chi^{rst}_{(\mu)} \underset{x \to 1}{\simeq} (1-x)^{1+2\Delta} \tag{III.22}$$

following the same arguments as in the two-body case.Altogether we have

$$\nu W_2^{(1,5)} \underset{\omega \to 1}{\simeq} (1-\omega)^{5+4\Delta} \quad , \quad \nu W_2^{(\mu)} \underset{\omega \to 1}{\simeq} (1-\omega)^{3+4\Delta} \qquad \text{(III.23)}$$

for the scalar/pseudoscalar and the vector interaction, respectively. This is to be compared with the asymptotic behavior of the form factors[2]

$$F_1^{(1,5)}(q^2) \underset{q^2 \to \infty}{\simeq} (q^2)^{-3-2\Delta} \quad , \quad F_1^{(\mu)}(q^2) \underset{q^2 \to \infty}{\simeq} (q^2)^{-2-2\Delta} \; . \qquad \text{(III.24)}$$

Hence, the DYW relation is satisfied for both interactions. The three-body calculation of ref. 8 agrees with our results for both the structure function and the form factors in case of the γ_μ coupling.

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Figure Captions

Fig.1 The structure functions in the ladder approximation for a two-body bound state.

Fig.2 The BS equation for the vertex function of a two-body bound state.

Fig.3 The structure function in the ladder approximation for a three-body bound state.

Fig.4 The BS equation for the vertex function of a three-body bound state.

Fig.5 The (once iterated) relativistic Faddeev equation for the vertex function of a three-body bound state. The wavy lines represent the two-body BS T-matrix.

Table Captions

Table 1 The asymptotic behavior of form factors and structure functions for two-body (π) and three-body (p) bound states and for the different models considered in refs. 1, 2 and in the present paper.

		F_π	F_{1p}	$\nu W_2^{(\pi)}$	$\nu W_2^{(p)}$	Dimensional Counting	DYW Relation
spin-zero Constituents $V \simeq (q^2)^{-\theta}$		$(q^2)^{-1-\theta}$	$(q^2)^{-2-2\theta}$	$(1-\omega)^{1+2\theta}$	$(1-\omega)^{3+4\theta}$	YES	YES
spin-½ Constituents $\tilde{V} \simeq (q^2)^{-1-\Delta}$	γ_μ	$(q^2)^{-1-\Delta}$	$(q^2)^{-2-2\Delta}$	$(1-\omega)^{1+2\Delta}$	$(1-\omega)^{3+4\Delta}$	YES	YES
	$1, \gamma_5$	$(q^2)^{-3/2-\Delta}$	$(q^2)^{-3-2\Delta}$	$(1-\omega)^{2+2\Delta}$	$(1-\omega)^{5+4\Delta}$	NO	YES

Table I

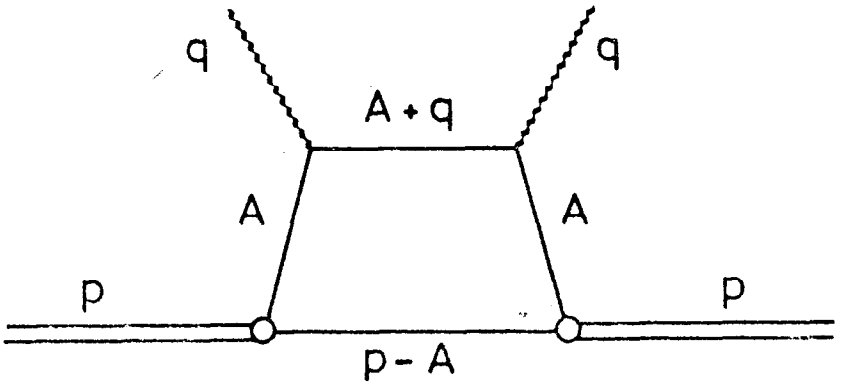

FIG. 1

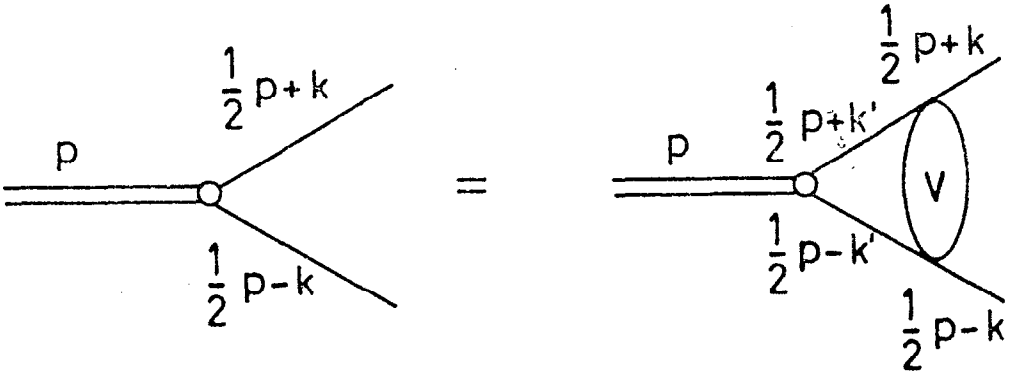

FIG. 2

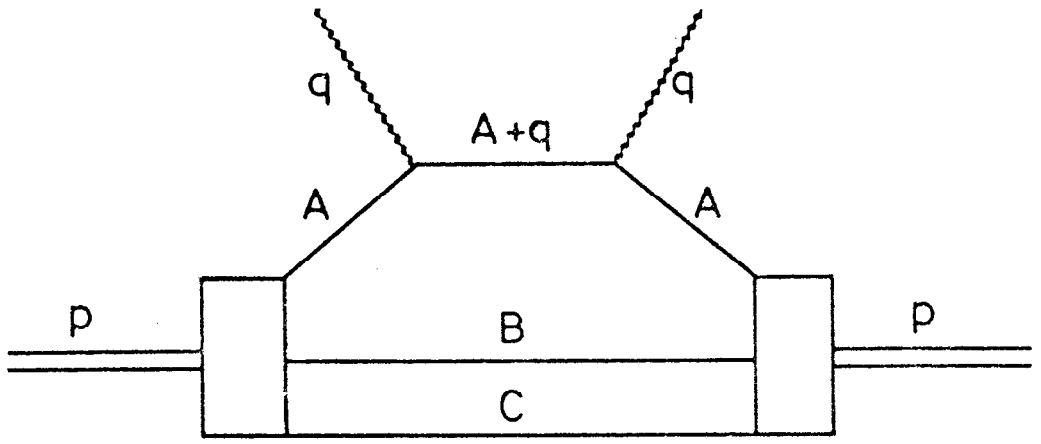

FIG.3

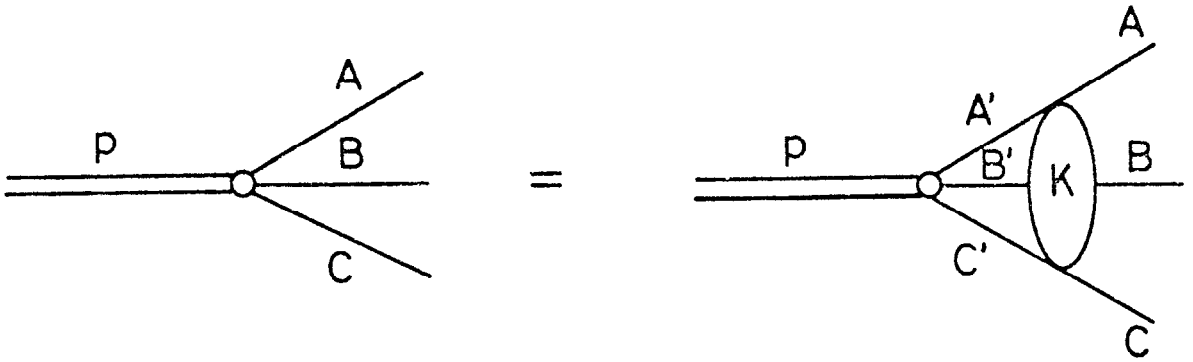

FIG.4

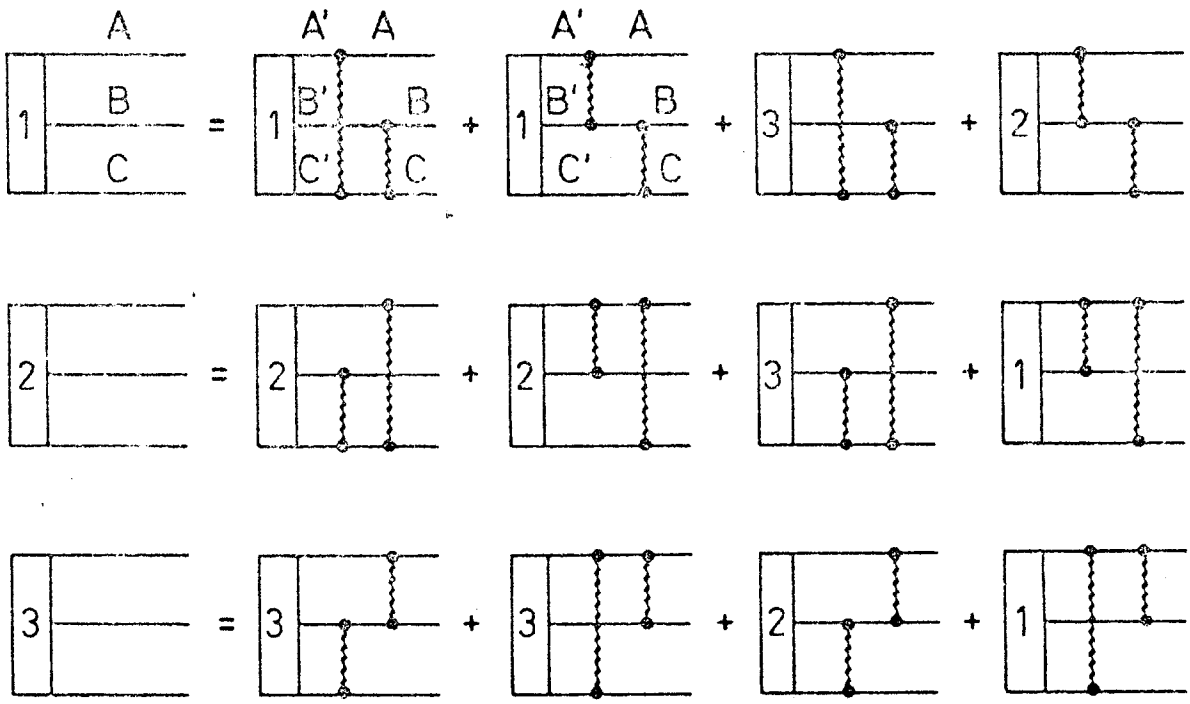

FIG. 5