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IMPLICATIONS OF HIGH BEAM CURRENTS FOR ACCELERATOR AND COMPONENT DESIGN*

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Introduction

The need for higher currents has been a primary force in design and innovation since the invention of accelerators, and a substantial amount of attention continues to be directed toward this end. In this paper, some aspects of the influence this effort has had upon the design of accelerators and storage rings are reviewed. In order to permit proper coverage and to achieve clarity, it has been necessary to limit the choice of topics for discussion and therefore to omit some of the many schemes used to achieve the present degree of success. These topics have been arbitrarily selected from the following categories: transverse interaction of a beam with its surroundings, longitudinal interaction of a beam with its surroundings, influence of vacuum on high beam currents and colliding beam-beam interactions. In the preceding paper presented at this Conference,[1] the limitation of beam performance by these phenomena has been reviewed and the reader is referred to this paper for more complete references to the original literature.

Transverse Interaction of a Beam with Its Environment

It was first observed by Kerst that the self fields of a high-current beam would reduce the transverse focusing of the beam and result in a shift of the betatron oscillation frequency, $\Delta\nu$, which is proportional to the number of particles.[2] The beam will become unstable if the betatron frequency is shifted onto a strong resonance, an effect which has certainly influenced the design of proton synchrotrons and their injectors, especially strong focusing machines. Because the maximum number of particles as limited by this tune shift increases with energy, one solution has been to increase the output energy of the injector. For example, in the Brookhaven conversion project, the linac energy output was increased to 200 MeV. At CERN, the P.S. Booster Group takes a different approach. They retain the 50 MeV linac but feed four rings, each with the sufficient number of particles to be at the $\Delta\nu$ limit, and accelerate the four beams up to 800 MeV, an energy at which they are below the $\Delta\nu$ limit when the beams are combined in the CERN proton synchrotron. At the Princeton-Penn accelerator and the Fermilab Booster, still another approach was used: to increase the pulse rate to achieve the total number of particles per second desired within the $\Delta\nu$ limit for the number of particles that could be contained in the synchrotron.

Laslett and Resegotti pointed out the important influence of the beam surroundings on the space-charge tune shift.[3] The influence of the surroundings dominates at high energies where the limiting value for the number of particles in the ring increases only linearly with energy instead of cubicly as it does at low energies. Attempts have been made to compensate the space-charge tune shift in various machines; the main difficulty with this technique arises from the fact that different particles in the beam have different tune shifts. A particularly successful method of this dynamic compensation has been worked out at the ISR where the space-charge tune shift depends upon the radial position of the particle which is strongly correlated to its momentum.[4]

High-intensity beams can also produce dynamic self fields, such as parasitic resonances in cavities, resistive-wall wake fields, etc., that will cause unstable transverse coherent oscillations. There has been a great deal of work on this subject and the preceding paper in this Conference[1] has emphasized the importance of this type of beam interaction with its environment. In electron linacs, this results in beam-breakup for high total charge in the pulses.[5] Present attempts to minimize these effects have centered on modifying the design of structures such that the parasitic modes responsible for this type of breakup resonate at different frequencies along the length of the linac, and to increase the transverse focusing by the addition of more quadrupoles.

In circular machines, there have been several approaches for controlling unstable transverse oscillations. One is to increase the Landau damping present in the beam by producing a spread in the betatron frequencies. For continuous beams like the ISR, sextupoles can be used and the betatron frequency spread can come from the variation of tune with energy.[6] For bunched beams, octupoles are used and the betatron frequency spread is derived from the tune spread with betatron amplitude. These methods have the obvious limitation in that the non-linear elements produce single-particle resonances and the necessary tune spread must not place some of the particles on a resonance.

Another approach is to use an active feedback system that senses the coherent oscillation and feeds back a signal properly phase-shifted to damp the oscillation.[7] This method has been highly successful in many cases and is most practical for damping dipole oscillations. It is difficult in any case to build feedback systems with sufficient bandwidth to stabilize all of the possible unstable modes, and the design of such a system is particularly difficult for synchrotrons with a large variation in the revolution frequency during the acceleration cycle.

The choice of the operating point for the betatron frequencies is often useful in controlling instabilities due to long-range wake fields, such as resistive-wall types, which affect each particle on its subsequent turn; this technique is used at the P.S. Booster.[8] This last point illustrates the fact that enough flexibility should be designed into machines to permit operating tunes to be varied over a large region.

The head-tail instability has been observed in many machines, as described in previous papers.[9,10] In several machines, only the "zero-head-tail mode" has been observed for which the use of sextupoles to control the sign of the chromaticity has proved successful. Indeed, this effect produces damping in such machines for proper sign of the chromaticity, and at SPEAR I, for example, the coherent oscillations excited in the stored beams by the injection kickers can be thus damped.[11] The use of sextupoles to control the chromaticity produces non-linear stop bands and care must be used in their placement in the lattice.[12] Also, because the relationship between the chromaticity and the stability of the zero-mode changes sign at transition energy, it may be necessary in proton accelerators to program the sextupole strength with the accelerating cycle.

For multi-bunch machines, the transverse oscillations of various bunches may be coupled. If the interbunch spacing is sufficiently small, it is impractical to damp each bunch separately by means of a feedback system. A solution that has been used successfully to decouple these modes has been the use of high-frequency

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quadrupoles to split the betatron frequencies of the various bunches.[13,14] Again, there is the limitation on the amount by which the betatron frequencies may be split before the tune of one of the bunches is shifted onto a damaging resonance.

Because of the difficulty of damping unstable transverse oscillations, efforts have been made to modify the environment. It was first observed at VEPP-2 that if an electrode were terminated in a matched impedance, transverse coherent oscillations would be damped.[15] In SPEAR I it was found that the damping rate of transverse oscillations, with positive chromaticity, decreased when the ferrite in the injection kickers was removed, so the ferrite, which linked the beam, must have been contributing a stabilizing influence. The group at DORIS has been able to identify parasitic modes in the rf cavities that drive transverse oscillations; they have used water-cooled probes in the cavities to damp these modes and have placed ferrite in the sections between the cavities to damp higher-frequency modes. At the ISR, the impedance of all vacuum chamber elements is determined before they are installed in the ring and hundreds of damping resistors are placed in the bottom of the chamber in order to reduce the coupling impedance between the beam and its environment below a specific limit. The approach of placing the emphasis on controlling the beam environment itself is now receiving the attention it deserves and will probably be given even more consideration in the future design of accelerators and storage rings to the degree possible. On the other hand, certain elements with intrinsically high impedances will be necessary in some machines. For example, high-voltage rf cavities are needed for the acceleration and storage of high-energy electrons.

Longitudinal Interactions of the Beam with Its Environment

The longitudinal interactions of a beam with its environment have been observed in almost all types of accelerators and storage rings. In pulsed linacs, this interaction can result in transient beam loading of the fundamental accelerating mode which varies during the pulse, producing a spread over the length of the beam pulse in the phase and energy at the output of the linac.[16,17] While this has not been a major factor limiting the maximum beam current, it does result in the degradation of the output beam quality. At Fermilab, for example, the resulting phase spread affects the performance of the debuncher used to decrease the energy spread of the beam in preparation for injection into the Booster.[18] If the proton pulse is sufficiently long, the beginning transient portion can be discarded and only the steady-state portion of the beam used. Another promising method to ameliorate this problem is to use a feed-forward system designed to sense the beam load upstream of the module to be controlled and to apply a correcting voltage.[19] For electron linacs such as SLAC, beam loading can produce an increased energy spread in the beam used for the high-energy experimental program. Here, if the beam current is sufficiently stable from pulse to pulse, the accelerating voltage may be modified by adding accelerating pulses (or decelerating pulses) of correct amplitude shape and timing.[20]

In circular accelerators and storage rings, there is a large variety of longitudinal interactions between a high-intensity beam and its environment, as is illustrated by the large number of papers on the subject presented to this Conference. The interaction of the beam with structures in the ring causes beam loading that changes the phase of the beam, resulting in a change in the beam-induced voltage which in turn further changes the phase of the beam.[21] This regenerative feedback can produce, for a single bunch, dipole or "rigid-bunch" oscillations, or, for multiple bunches, an oscillation in which all bunches are in phase. Damping can often be produced either by tuning the resonant frequency of the main rf cavity system below the driving frequency or by using a simple phase feedback system.[22] In high-harmonic accelerators such as the Fermilab main ring, it may be difficult to detune a faulty cavity sufficiently since detuning the cavity by an amount equal to the revolution frequency results in selecting the neighboring harmonic of the beam current; thus there is no detuning at all.[23] To have the bandwidth of the accelerating mode in the cavities smaller than the revolution frequency would be desirable in this case, but this is often difficult to achieve. When multiple bunches are present, it is possible to have longitudinally-coupled "rigid-bunch" motion, other than the baracentric mode, which cannot be damped by the cavity detuning. The methods used to damp this motion vary with the type of machine. For bunched beams in the ISR, a low-voltage, wide-bandwidth bunch-by-bunch feedback system is used which varies the phase of the rf voltage by a fast phase modulator.[24] In electron-positron storage rings, these methods are not practical because higher voltages are required. In DORIS, an additional cavity is driven at a frequency that is a harmonic of the revolution frequency, but not of the rf frequency, to split the synchrotron frequency of the bunches and decouple the motion.[25] Similar techniques have been applied at ADONE and at SPEAR; ADONE also uses a longitudinal feedback system that uses gated pickup signals to drive two feedback cavities at a harmonic of the revolution frequency.[26] The feedback frequency is not a harmonic of the rf frequency so that different bunches receive different feedback voltages.

In addition to longitudinal dipole oscillations, it is possible to have higher-mode longitudinal oscillations.[27] Sessler has suggested a turbulent model where the combination of many of these modes is responsible for the bunch lengthening observed in electron-positron storage rings. The quadrupole mode may be damped by a feedback system which varies the peak rf voltage, but other methods, such as Landau damping, must be used to damp higher-order modes. Landau damping may be obtained by modulating the rf voltage waveform, shrinking the rf bucket about the beam or shaking the phase of the rf accelerating system.[24,8]

One of the earliest known effects of longitudinal interaction between the beam and its environment was the spontaneous bunching of continuous beams above transition energy, an effect known as the negative mass instability, which has been extremely troublesome in the operation of electron ring accelerators.[28,29] The present methods of stabilization used are to increase the energy spread in the beam and to design the structure for a lower coupling impedance to the environment. For bunched beams passing through transition energy, this effect can produce a dilution of the longitudinal phase space of the beam. Such schemes as the triple switch or the γ_t jump have been developed to avoid this problem.[30,31] In the triple switch scheme, the rf phase is jumped three times instead of one to produce a phase oscillation that will be cancelled exactly by the longitudinal interaction. The γ_t jump is a technique of manipulating the transition energy with only minimal changes in the betatron frequencies by means of pulsed quadrupoles.

Even if all possible beam instabilities due to the interaction of the beam with its surroundings have been eliminated, there remains the problem of beam energy lost to parasitic electromagnetic modes of the surrounding structure.[32,33] This energy must be replaced by increasing the energy output from the rf accelerating system. A great deal of theoretical work has been done on this problem, and it is an important consideration in the future design of high-energy electron-positron storage rings.[34] This phenomenon has been observed at SPEAR II, and at the Fermilab Booster where it appears to arise due to the laminations of the magnets; results are reported at this Conference.[35,36] Attempts to minimize this energy loss will surely influence the design philosophy of rf accelerating cavities and other structures in future accelerators and storage rings.

Influence of Vacuum on High Beam Currents

The quality of the vacuum required for high-current beam stability is a major consideration in the design of electron-positron and proton storage rings. Of the many types of interactions the beam can have with the residual gas, perhaps the most obvious is the reduction of beam lifetime due to scattering or bremsstrahlung on the background gas.[37] The main source of gas pressure in an operating electron-positron storage ring usually stems from synchrotron radiation impinging upon the chamber walls and producing photo-electrons which in turn strike the walls and knock out gas molecules.[38] The design of vacuum chambers in electron storage rings reflects the concern with this phenomenon;[38] for example, they often have ribbed or serrated walls so that photons strike the wall normal to the surface, reducing the production of photo-electrons and hence the amount of gas desorption.

Of major importance is the material used to construct the vacuum chamber wall, since at high energies in electron storage rings the power load on the walls due to synchrotron radiation becomes critical.[39] Large electron-positron storage rings that have been proposed, such as PEP, approach the power limit that can be tolerated under present conditions. Significant improvement would require a great deal of material research and development. Aluminum chambers have been chosen for their advantages over stainless steel and copper, not only in economy but because heat transfer is better than stainless steel and the desorption coefficient is lower than copper. One of the original problems in using aluminum chambers was the failure of aluminum flanges to withstand repeated bakeouts; also the joining of aluminum to stainless steel flanges is difficult. This problem was solved in SPEAR by the explosive bonding of aluminum to stainless steel with a silver interface. Other techniques of making ultra-vacuum-tight cold welds between aluminum and stainless steel are also under study. Also it is necessary to have a large pumping speed near the place where the outgassing occurs and this has been accomplished by means of distributed ion pumps placed in every bending magnet.

Ions produced by interaction of the electron beam with the residual gas can become trapped and neutralize the beam, greatly increasing the incoherent tune shift,[3] and can also produce unstable coherent oscillations of the beam.[40] These ions may be removed by using clearing electrodes or by leaving a sufficiently large gap between bunches in the beam to permit the ions to escape to the wall. The clearing electrodes have the disadvantage of introducing additional elements into the chamber which may present undesirable electromagnetic impedances as described in the previous section.

The vacuum problems produced by high-current proton beams at the ISR have been described in detail.[41] The partial neutralization due to the production and accumulation of electrons by ionization of the background gas is largely prevented by means of clearing electrodes. In addition to minimizing the space charge tune shift, removal of the electrons is necessary to prevent coupled electron-proton instabilities that have been observed when electron removal was not fast enough. Beam-induced pressure bumps have also been observed at the ISR. The production mechanism is one in which ions produced by collisions of the beam with residual gas are driven to the wall by the electrostatic field of the beam. They liberate gas molecules from the wall which in turn are ionized by the beam, and a regenerative process can occur when the product of the beam current and the gas pressure exceed a critical value. This has been remedied by reducing the desorption coefficient by treating the surface with a glow discharge or by baking the chamber walls at high temperatures. The required high-temperature bakeout rules out the use of aluminum chambers for proton storage rings. CERN uses a stainless steel vacuum chamber and has also obtained good results with titanium materials baked at 800°C in a vacuum. Another solution used is an increase in pumping capacity. Since the problem depends upon the average current in the storage ring, there has been some discussion of achieving the desired luminosity by bunching the protons and thereby reducing the necessary average current. This solution introduces other problems due to interactions of the beam with its surroundings, as has already been discussed.

Beam-Beam Interactions

One of the goals of the colliding-beam storage rings is to achieve the maximum possible value for the luminosity, defined for bunched beams as

$$\mathscr{L} = \frac{I_1 I_2}{e^2 h f_0 A} , \qquad (1)$$

where I_1 and I_2 are the currents of the two beams, A the effective interaction area per bunch, f_0 the revolution frequency and h the number of bunches. The ultimate limit on the performance of a storage ring, referred to as the incoherent beam-beam limit, is determined by the disruptive electromagnetic force that a particle in one beam experiences as it passes through the other beam. In a review paper presented by Amman, this limit is discussed in detail.[42] At present, the exact mechanism by which this force produces a transverse blow-up of the beam is not well understood, although it is clear that it is the non-linearity of the force which is responsible for the blow-up. This force is dependent upon the beam density at the interaction point and is usually characterized by the linear tune shift, $\delta\nu$, which is experienced by a particle of small betatron oscillation amplitude as it passes through the other beam. It has been found experimentally that there is a limiting value for this tune shift above which one or both of the beams will experience a transverse blow-up. This effect places an upper limit on the useful current densities at the interaction regions, which is given by

$$\left(\frac{I}{h A}\right)_{max} = K \frac{\gamma}{\beta^*} \cdot \delta\nu_{max} , \qquad (2)$$

where γ is the relativistic energy parameter, β^* is the local betatron function at the interaction point and K a constant of proportionality. By combining the above formulae, the maximum luminosity may be written as

$$\mathscr{L}_{max} = \frac{K^2 (\delta\nu_{max})^2 \, h A \gamma^2}{e^2 f_0 (\beta^*)^2} . \qquad (3)$$

Since the interaction energy is determined by the high-energy experiments to be performed, the only design parameters that can practically be varied are the number of bunches, the effective interaction area per bunch and the betatron function at the interaction point. In many storage rings, it is possible to operate near a coupling resonance, increasing the interaction area and hence the luminosity.[43,44] The scheme proposed by Robinson and Voss[45] to utilize a lattice with extreme compression in the vertical betatron function at the interaction points (to about 1/3000 of its maximum value and 1/500 of its average value) has resulted in greatly improved luminosities over those which would have been achieved with conventional lattices.[46,47,25] In designing recent high-energy storage rings, this scheme has invariably been incorporated to arrive at the desired design luminosities.[48] The major limitation of this method is that abnormally large values of the betatron function are produced in some of the magnetic elements in the ring, making the correction of errors and chromaticity effects very difficult.

The luminosity may also be increased by simultaneously enlarging the effective interaction area and the current of the beams while remaining at the $\delta\nu$ limit. The portion of the cross-sectional area of the beam due to the energy spread is proportional to the dispersion at the interaction region and, for this reason, SPEAR was designed with a variable dispersion at the interaction region, with the result that it has been possible to improve the luminosity at low energies.[49] The natural beam emittance is determined by the interplay between the quantum fluctuation and damping due to synchrotron radiation and may be altered by modifying either the effect of the fluctuations or the damping rates. The CEA Bypass used special damping magnets to control the beam emittance by varying the transverse radiation damping.[13] Other methods of varying the damping rates have been proposed.[50] Methods that increase the quantum-fluctuation driving effects have been suggested, such as mismatching the energy-dispersion function, resonant variation of the strength of the bending field or the simpler technique of varying the betatron tune.[51,52,53]

A completely different concept devised to circumvent the beam-beam limitation is that of space-charge compensation which will soon be tested in DCI.[54] This design utilizes two rings, with both an e^+ and an e^- bunch circulating in each ring. If the currents and positions of the beams can be controlled to sufficient accuracy at the point of collision, the result should be a cancellation of the largest portion of the space-charge fields.

Conclusion

The above-mentioned schemes have all been used successfully to increase the beam current in various machines. Of course, in many cases, there are limitations that prevent any single solution from being a panacea, and, as often happens, new effects are discovered which introduce new limits on the current. The author is confident that efforts will continue to be made toward increasing the high-current limit of beams in accelerators and storage rings, and the most valuable aid for the accomplishment of this goal will remain the excellent communication that prevails among the people engaged in this work.

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