# Erratum: Upper Bound on Charmed Quarks in Nucleons from the Cross Section for $\psi(3100)$ Photo-Production (Phys. Rev. D 12, 3669 (1975)). Author: T. Goldman 

In Eqs. (4) and (8), the denominator factor $M_{\psi}^{3}$ should read $M_{\psi}$ instead, as is obvious from dimensional analysis. The misprint in the printed version occurred because of an unfortunate error in transcription from $\Gamma_{\text {had }} \Gamma_{\text {lept }} / M_{\psi}^{3}$ to $g^{2} h^{2} / M_{\psi}$, where $g$ and $h$ are dimensionless couplings. We emphasize that the numerical results are correct, as they were calculated from the correct version of the formula. In particular, the scale in Fig. (2a) is correct.

Since the completion of the paper, the experimental results have become clearer ${ }^{[1]}$ and confirm our conclusions: The experimental cross-section is an order of magnitude smaller than the theoretical calculation and shows strong evidence that most of the scattering is coherent; thus the charmed quark content of nucleons is at least an order of magnitude less than the strange quark content. Hence, this work provides an independent confirmation of the conclusions drawn from a Regge-Pomeron approach ${ }^{[2]}$ on the relative weakness of the coupling of charm to the Pomeron. (That work, however, depends on vector dominance assumptions to extract $\sigma(\psi p \rightarrow X)$ from the observed $\sigma(\gamma p \rightarrow \psi p)$.)

## References

1) B. Knapp, et al., Phys. Rev. Letters 34, 1040 (1975).
2) C. E. Carlson and P. G. D. Freund, Phys. Rev. D 11, 2453 (1975).

UPPER BOUND ON CHARM QUARKS IN NUCLEONS FROM THE CROSS SECTION FOR $\psi(3100)$ PHOTOPRODUCTION*

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## ABSTRACT

A Drell-Yan type calculation of $\psi(3100)$ photoproduction based on the recent data from SPEAR gives a surprisingly large cross section. If the results of the photoproduction experiment at FNAL are similar to naive VMD expectations, a stringent upper bound on charmed quarks in nucleons is obtained. A consequence for $\nu \mathrm{W}_{2}$ at high $\omega$ is also noted.
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[^0]The result of a Drell-Yan type ${ }^{1}$ calculation for the process $\gamma \mathrm{N} \rightarrow \psi(3100)$ +X is a surprisingly large production cross section if charmed quark-partons (c) and antipartons ( $\bar{c}$ ) are present in nucleon wave functions at a similar level to that of strange quarks. For squared-c.m. energies $s \sim 100-200 \mathrm{GeV}^{2}$, the value is in the range $150-300 \times 10^{-33} \mathrm{~cm}^{2}$, which is considerably larger than the $10-20 \times 10^{-33} \mathrm{~cm}^{2}$ suggested by simple vector meson dominance arguments. If the latter figure is verified by experiment (such as one current at $F N A L$ ) ${ }^{2}$ it follows that charmed quarks must be rarer than strange quarks in the nucleons.

The exact bound obtained depends somewhat on the parton distribution functions used, i.e., up to factors of $\sim 2-3$. Only the results obtained for parton distributions of the form employed by Chu and Gunion ${ }^{3}$ and by Farrar ${ }^{4}$ (which are supported by considerations of dimensional counting and the Constituent Interchange Models ${ }^{5}$ and are the result of detailed fits to the deep inelastic eN and $\nu \mathrm{N}$ data) are quoted here. For charmed quarks analogous in strength and origin to strange quarks, ${ }^{6,7}$ the parton distribution (probability to find a parton of momentum fraction x ) is ${ }^{3,4}$

$$
\begin{equation*}
c_{N}(x)=\bar{c}_{N}(x)=.2 x^{-1}(1-x)^{7} \tag{1}
\end{equation*}
$$

For photons, the distributions may be calculated using $Q E D^{8}$ with the result

$$
\begin{equation*}
\mathrm{c}_{\gamma}(\mathrm{y})=\overline{\mathrm{c}}_{\gamma}(\mathrm{y})=\frac{4}{9}\left(\frac{\alpha}{2 \pi}\right) \ln \left(\frac{\mathrm{s}}{\mathrm{~m}_{\mathrm{c}}^{2}}\right)\left[\mathrm{y}^{2}+(1-\mathrm{y})^{2}\right] \tag{2}
\end{equation*}
$$

where the $4 / 9$ is the square of the presumed $2 / 3$ charge of the charmed quark, $m_{c}$ is its mass, which has been taken here to be $1.6 \mathrm{GeV}^{9}$, and s is the squared total c.m. energy of the photon-nucleon collision.

Figure 1 shows the Drell-Yan process as applied here. The formula ${ }^{1}$ for computing the cross section differential in mass-squared $Q^{2}$ of the observed
final state lepton pair in the region of the $\psi$ resonance is (ignoring the possibility and effects of "color"; i.e., a factor of $1 / 3$ ):

$$
\begin{equation*}
\frac{d \sigma}{d^{2}}=\frac{\mathrm{g}^{2}{ }^{2} \int \mathrm{dx} d y \delta\left(x y-Q^{2} / \mathrm{s}\right) \mathrm{xy}\left[\mathrm{c}_{N}(\mathrm{x}) \overline{\mathrm{c}}_{\gamma}(\mathrm{y})+\overline{\mathrm{c}}_{N}(\mathrm{x}) \mathrm{c}_{\gamma}(\mathrm{y})\right]}{12 \pi\left[\left(Q^{2}-\mathrm{M}_{\psi}^{2}\right)^{2}+\mathrm{M}_{\psi}^{2} \Gamma_{\psi \operatorname{tot}}^{2}\right]} \tag{3}
\end{equation*}
$$

where h and g are the coupling constants of the $\bar{c} c \psi$ and $\bar{\ell} \ell \psi$ vertices (see Fig. 1); point vector coupling is assumed. $\mathrm{M}_{\psi}=3.1 \mathrm{GeV}$ is the mass of the $\psi$ and $\Gamma_{\psi \text { tot }} \approx 80 \mathrm{keV}$ is the inferred total width ${ }^{10}$ of the $\psi$. Since this is very narrow, the integral of Eq. (3) over the region of $Q^{2} \sim M_{\psi}^{2}$ is, to a good approximation,

$$
\begin{equation*}
\sigma(\gamma \mathrm{N} \rightarrow \mathrm{X}+(\psi \rightarrow \overline{\mathrm{l} \ell}))=\frac{\mathrm{g}^{2} \mathrm{~h}^{2}}{6 \mathrm{M}_{\psi}^{3} \Gamma_{\psi \text { tot }}} \int_{\tau}^{1} \frac{\mathrm{dx}}{\mathrm{x}} \tau\left[\mathrm{c}_{\mathrm{N}}(\mathrm{x}) \overline{\mathrm{c}}_{\gamma}\left(\frac{\tau}{\mathrm{x}}\right)+\overline{\mathrm{c}}_{\mathrm{N}}(\mathrm{x}) \mathrm{c}_{\gamma}\left(\frac{\tau}{\mathrm{x}}\right)\right] \tag{4}
\end{equation*}
$$

where $\tau=\mathrm{M}_{\psi}^{2} / \mathrm{s}$, and the off-shell heavy photon background has been ignored as it is extremely small.

The value of $g$ may be obtained from the leptonic (electrons or muons but not both, and assuming universality) decay width of the $\psi$ :

$$
\begin{equation*}
\Gamma_{\psi \rightarrow \overline{\ell \ell}}=\frac{\mathrm{g}^{2}}{12 \pi} \mathrm{M}_{\psi} \tag{5}
\end{equation*}
$$

again assuming a vector point coupling and that $\psi$ is a spin -1 meson. ${ }^{11}$ The value of $h$ has been taken to be that of a typical strong interaction: $h^{2} / 4 \pi \sim 1$. (This implies, for instance, that if $\psi$ could decay into charm nonzero mesons, then $\Gamma_{\psi \text { tot }} \sim 600 \mathrm{MeV}$; this value is calculated after the fashion of Eq. (5), as suming the two-quark final state sufficiently reflects the properties of the true final states, as is typical of quark-model calculations.) If Eq. (4) is used with the photon replaced by another nucleon, the resultant $\psi$-production cross section is in order-of-magnitude agreement with the experimental value observed
at BNL. ${ }^{12,13}$ (The corresponding calculation ${ }^{13}$ for the Drell-Yan annihilation of ordinary quarks and antiquarks into $\psi$ 's with an effective coupling strength inferred from the decay of $\psi$ into ordinary hadrons (again in a manner similar to the determination of $g$ via Eq. (5)) results in a cross section $10-30$ times smaller than observed. ${ }^{12}$ ) Since the branching ratio $B=\Gamma_{\psi \rightarrow \overline{\ell \ell}} / \Gamma_{\psi \rightarrow \text { all }}$ is known ${ }^{10}$ to be $\sim 0.07$, the production cross section for $\psi^{\prime}$ s may be obtained using Eq. (4):

$$
\begin{equation*}
\sigma(\gamma \mathrm{N} \rightarrow \psi+\mathrm{x})=\mathrm{B}^{-1} \sigma(\gamma \mathrm{~N} \rightarrow \mathrm{x}+(\psi \rightarrow \bar{\ell} \ell)) \tag{6}
\end{equation*}
$$

The results are shown as a function of $s$ in Fig. 2a. Corrections due to finite energy kinematics will be discussed in a future publication. ${ }^{13}$

The results do not depend strongly on the form in Eq. (1) as most of the cross section comes from $x \ll 1, y \sim 1$ and so depends only on the limiting value ${ }^{14}$

$$
\begin{equation*}
\lim _{x \rightarrow 0} x c(x) \approx .2 \tag{7}
\end{equation*}
$$

which should be good to a factor of 2 under our-assumptions. The longitudinal distribution of $\psi$-production $d_{\sigma} / \mathrm{d} \xi$, where $\xi=2 \mathrm{Q}_{3} / \sqrt{\mathrm{s}}$ and $\mathrm{Q}_{3}$ is the longitudinal momentum of the observed lepton pair (or $\psi$ ), is shown in Fig. 2h demonstrating that most of the cross section comes from small $x$ partons in the nucleon, ${ }^{14}$ and that the $\psi$ 's tend to be produced in the direction of and with close to the same momentum as the incident photon. Despite the fact that this is an incoherent, inelastic production model, this distribution for $\psi^{\mathrm{p}} \mathrm{s}$ is similar to what is expected from diffractive (vector dominance) production (see Fig. 2b), although the $\xi$ - and transverse-momentum distributions may be somewhat broader in this case. The formula for this cross section is

$$
\begin{equation*}
\frac{\mathrm{d}_{\sigma}}{\mathrm{d} \xi}=\frac{\mathrm{g}^{2} \mathrm{~h}^{2}}{6 \mathrm{M}_{\psi}^{3} \Gamma_{\psi t o t}} \frac{\tau}{\sqrt{\xi^{2}+4 \tau}}\left[\mathrm{c}_{\mathrm{N}}(\mathrm{x}) \bar{c}_{\gamma}(\mathrm{y})+\overline{\mathrm{c}}_{\mathrm{N}}(\mathrm{x}) \mathrm{c}_{\gamma}(\mathrm{y})\right] \tag{8}
\end{equation*}
$$

where $\mathrm{x}=\left(\xi+\sqrt{\xi^{2}+4 \tau}\right) / 2$ and $\mathrm{y}=\left(-\xi+\sqrt{\xi^{2}+4 \tau}\right) / 2$ and again an integration has been taken over the region $Q^{2} \sim M_{\psi}^{2}$.

The calculation does not depend strongly on the assumptions used except for the question as to whether or not the $\psi \overline{\mathrm{c} c}$ vertex can be approximated as a point coupling. This would be most reasonable if $\psi$ were an elementary field similar to the photon. If $\psi$ is a $\overline{c c}-b o u n d ~ s t a t e ~ h a d r o n, ~ a ~ n o n l o c a l ~ v e r t e x ~ s h o u l d ~$ be used with a momentum dependence that reflects the $\overline{\mathrm{c}} \mathrm{c}$ binding into a $\psi$. However, the corrections due to such a vertex may be small in this case for two reasons:

1) The vertex, although not pointlike, may have a small distance scale. In some recent bound state models ${ }^{9}$ of the $\psi$, it has a mean "size" (quarkantiquark separation) of $\sim 0.6 \mathrm{f}$ which is somewhat smaller than the size estimates of more ordinary hadrons ( $\sim 1.5 \mathrm{f}$ ). Note also that the observed distance scale for the onset of scaling is only $\sim 0.1-0.2 \mathrm{f}$. Nothing quantitative is obvious here, but qualitatively it seems apparent that applying a short distance or impulse approximation for $\psi$-production in this way is at least a better approximation than it would be for more ordinary hadrons (such as $\phi, \rho$, or $\omega$ ).
2) The value for the point-vertex coupling constant is approximately the same as obtained by fitting to the $\mathrm{pp} \rightarrow \psi+\mathrm{X}$ data from.BNL. ${ }^{12,13}$ In this sense, it is an effective coupling constant and its size may automatically take into account the effect of some nonlocality of the true vertex function. In this respect, it is unclear whether the value inferred from the pp data for this coupling reflects either the large mass scale of charmed quarks ${ }^{9}$ and the $\psi$ or the conversion of a larger but momentum-dependent coupling strength to a smaller, averaged constant value.

The $\psi$-production cross section suggested by naive vector dominance ${ }^{13}$ is:

$$
\begin{equation*}
\sigma(\gamma \mathrm{N} \rightarrow \psi+\mathrm{x})=\frac{40 \mathrm{nb}}{\mathrm{~b}}\left(\sigma_{\psi \mathrm{N}}^{\mathrm{tot}}\right)^{2} \tag{9}
\end{equation*}
$$

where $\sigma_{\psi \mathrm{N}}^{\text {tot }}$ is the total $\psi$-nucleon cross section in mb , which should be no less than 1 mb if $\psi$ is a hadron; ${ }^{15}$ and where b is the slope parameter in the differential cross section $\mathrm{d} \sigma(\gamma \mathrm{N} \rightarrow \psi+\mathrm{x}) / \mathrm{dt} \propto \mathrm{e}^{\mathrm{bt}}$ in $\mathrm{GeV}^{-2}$. Typically $\mathrm{b} \sim 4 \mathrm{GeV}^{-2}$ for the production of other hadronic vector mesons. The 40 nb coefficient is the combined result of choice of units and a $\psi-\gamma$ mixing strength inferred from $\Gamma_{\psi \rightarrow \bar{\ell} \ell}$ by assuming the decay sequence $\psi \rightarrow \gamma \rightarrow \overline{\ell l}$, which is consistent with experiment. ${ }^{10}$ Then, $\sigma(\gamma \mathrm{N} \rightarrow \psi+\mathrm{x}) \approx 10 \mathrm{nb}$ and this is in rough agreement with the reported value. ${ }^{2}$ Thus there must be at least a factor of 10 less charmed quark and antiquark in the nucleons than is presumed by Eq. (1). Furthermore, if the production at high energy can be experimentally shown to have a predominantly diffractive character, then the process calculated here must be a small ( $\lesssim 10 \%$ ) contribution and the charm quark content of the nucleons would have to be less than $10^{-2}$ of the strange quark content. These limits are lowered even further if one increases the value of the parameter $h$ in an attempt to recover the observed $p p \rightarrow \psi+X$ cross section ${ }^{12}$ despite the reduced charm quark content of the nucleons. ${ }^{16}$

If one clings to the theoretical prejudice that the Pomeron must ultimately be $\operatorname{SU}(4)$ symmetric, at least at $x \equiv 0$, then from the scale in Fig. $2 b$ one would conclude that the symmetric piece turns on at $x \ll 0.05$. In terms of the deep inelastic structure function, $\nu W_{2}$, this corresponds to an increase in the apparent limiting value as $\omega \rightarrow \infty$ for $\omega \gg 20$. That is, if the presence and
effects of wee strange quarks leading to an $\operatorname{SU}(3)$ symmetric Pomeron are visible as leading to an approximately constant value of $\nu \mathrm{W}_{2}$ as $\omega \rightarrow 20$, then the constant value will be larger by $\sim 2 / 3$ (since $\nu \mathrm{W}_{2}$ includes the effect of the quark charges), for $\omega>\omega_{0}$ where $\omega_{0} \gg 20$ describes the scale on which the Pomeron becomes $\operatorname{SU}(4)$ symmetric as the number of wee charmed quarks becomes comparable to all others.

I would like to acknowledge many useful conversations on this and other topics with members of the SLAC Theory Group and SLAC Theory Workshops on the $\psi$. I particularly wish to thank S. Brodsky and S. Drell for illuminating discussions on the applicability of the impulse approximation to $\psi$-production.

## FOOTNOTES AND REFERENCES

1. S. D. Drell and T.-M. Yan, Ann. Phys. (N. Y.) 66, 578 (1971).
2. As reported by T. O'Halloran, Jr., at the Symposium of the Division of Particles and Fields, Anaheim APS Meeting, February 1, 1975.
3. J. F. Gunion, Phys. Rev. D 10, 242 (1974); G. Chu and J. F. Gunion, to be published in Phys. Rev. D.
4. G. R. Farrar, Nucl. Phys. B77, 429 (1974).
5. S. J. Brodsky and R. Blankenbecler, Phys. Rev. D 10, 2973 (1974) and references therein; S. J. Brodsky and G. R. Farrar; to be published in Phys. Rev. D (1 February 1975).
6. R. P. Feynman, Photon-Hadron Interactions (Benjamin, Reading, Mass., 1972 ).
7. Just as the presence of strange quarks in nucleons is thought to reflect the wee partons (see Ref. 6) that represent part of the supposed $S U(3)$-symmetric Pomeron, so Eq. (1) follows from Refs. 4 and 5 if the Pomeron is $\mathrm{SU}(4)$-symmetric.
8. R. L. Jaffe, Phys. Rev. D 4, 1507 (1971); S. J. Brodsky (private communication); M. -S. Chen and P. Zerwas (unpublished).
9. T. W. Appelquist and H. D. Politzer, Phys. Rev. Letters 34, 43 (1975); E. Eichten et al., "The Spectrum of Charmonium," Cornell preprint, December 1974 (unpublished).
10. As reported by R. Schwitters at the Symposium of the Division of Particles and Fields, Anaheim APS Meeting, February 1, 1975, and "Report of the QED Subgroup, SLAC Workshop," SLAC-PUB-1514 (December 1974).
11. The same result is obtained if $\psi \rightarrow \bar{l} \ell$ by cascade through a heavy (off-shell) photon: $\psi \rightarrow \gamma \rightarrow \overline{\ell \ell}$.
12. J. J. Aubert et al., Phys. Rev. Letters 33, 1404 (1974), and 33, 1624. (1974).
13. R. Blankenbecler et al., SLAC Workshop Notes on Narrow States in Hadronic and Photonic Experiments, February 1975 (unpublished) and further work in progress. Similar calculations have been done by many others.
14. For $\xi \sim-1$ and $\tau \ll 1, \mathrm{x} \ll 1$ and $\mathrm{y} \sim 1$; see Eq. (8) below.
15. There are various estimates as to what is reasonable for this number. Most are based directly or indirectly on extrapolation from the $\phi-\mathrm{N}$ total cross section. See, for example, Ref. 13 above; and C. E. Carlson and P. G. O. Freund, Phys. Letters 39B, 349 (1972); R. C. Brower and J. R. Primack, "Is the $\psi$ a Ring?" U.C. -Santa Cruz preprint, unpublished.
16. Clearly this indicates that one should look to mechanisms other than DrellYan annihilation to understand the $\psi$-production cross section in purely hadronic collisions.

## FIGURE CAPTIONS

1. The Drell-Yan process for photoproduction of a massive lepton pair via the annihilation of charmed quarks and antiquarks into the $\psi$-resonance which subsequently decays: h, g, and $2 / 3$ e are coupling constants.
2. Total cross section and longitudinal distribution.
(a) Total cross section for photon + nucleon $\rightarrow \psi+$ anything (in $\mathrm{cm}^{2}$ ) as a function of squared total c.m. energy "s (in $\mathrm{GeV}^{2}$ ). Points are as computed from Eq. (6); solid curve is smooth interpolation.
(b) Longitudinal distribution (in arbitrary units) of produced $\psi$ particles $\mathrm{d} \sigma / \mathrm{d} \xi$ as a function of longitudinal momentum fraction $\xi$ of $\psi$ in the photon-nucleon c.m. frame, at two energies (solid curves). The dashed line indicates the distribution expected for diffractive (vector dominance) production.


FIG. 1


FIG. 2a


FIG. 2 b


[^0]:    * Work supported by Energy Research and Development Administration.
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