SLAC WORKSHOP NOTES ON NARROW STATES
IN HADRONIC AND PHOTONIC EXPERIMENTS*
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#### Abstract

We discuss the production of the $\psi(3105)$ and $\psi(3695)$ in hadronhadron and photon-hadron collisions. We investigate whether we can define experimental tests which will definitively support or rule out current theoretical models for the $\psi^{\prime} s$.


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## I. INTRODUCTION •

The recent discovery of the $\psi(3105)$ and $\psi(3695)$ at SPEAR ${ }^{1,2}$ and the simultaneous observation of what appears to be the lower mass resonance at the Brookhaven $\mathrm{AGS}^{3}$ are currently forcing a major reexamination of the theoretical foundations of our knowledge of particle physics. It may be that these striking new observations will furnish the modicum of information necessary to demonstrate the validity of some particular model. However, it is more consistent with the history of physics to suggest that many new experiments are necessary before theorists have enough information to assimilate these particles into a comprehensive view of the world.

One of the major questions which has arisen is whether the new states are hadrons or nonhadrons. If they are the former, then we should expect the discovery of whole spectroscopic families of similar states. We then expect an interesting theoretical pursuit of new symmetries incorporating and systematizing the new particles. This is a relatively conservative viewpoint. There are difficulties in thinking of the new states as hadrons: the widths (inverse lifetimes) are several orders of magnitude smaller than those of familiar hadronic states. The necessity for new quantum numbers and powerful selection rules is obvious. Various forms of these selection rules have been reviewed by Harari in a parallel document. ${ }^{4}$

If the $\psi$ 's are not hadrons then there is little reason to expect large families of states but there is a chance that fundamental new theoretical understanding will result from careful study of the presently known particles and their interactions. This rather unconservative possibility may take one of many forms, but the $\psi^{\prime}$ 's must fit into a picture which is consistent with our present understanding of the weak and electromagnetic interactions. The possibility that the
$\psi^{\prime} s$ may be the carriers of the (neutral) weak interaction is evident. It will be examined separately. A more general possibility is that the new states may be vector gauge particles which couple to conserved quantum numbers other than charge (some combination of baryon number and lepton number, for example). In all of these cases we have no difficulty in explaining the narrow decay widths.

Our purpose here is to examine the hadronic and photonic production of the new states with particular emphasis upon the means by which one might distinguish hadronic from nonhadronic character. .Certain insight into this question comes from asking under what circumstances the production of $\psi^{\prime}$ s in hadronic collisions (at BNL, NAL, and elsewhere) may be made consistent with the SPEAR data. The general aspects of hadronic production are thus the subject of Section II, where we consider associated production, effective coupling strengths, and models for production. The latter allow us to make some predictions for rates and energy dependences for differing hadronic beams. In Sec. III we analyze the bearing of photoproduction experiments upon the question of hadronic versus nonhadronic character and show that observation of this process does not eliminate the possibility of nonhadronic character for the $\psi^{\prime}$ s. In Section $I V$, the occurrence of $\psi^{\prime}$ 's as resonances in proton-antiproton annihilation is studied, and Section $V$ is devoted to conclusions.

This document is intended as an informal record of activity of several SLAC theorists in the month following the new discoveries. This document has several companions, the results of efforts by other workshop groups. They should be consulted. We thank S. Drell and J. D. Bjorken for organizing this effort and for their leadership in staying close to the data.

## II. THE PRODUCTION OF $\psi$ 's IN HADRONIC COLLISIONS

It seems evident that the $\psi(3105)$ is observed as an enhancement in the reaction $p+B e \rightarrow e^{+} e^{-}+$anything at an energy of $P_{l a b}=28.5 \mathrm{GeV}$. The higher mass state, the $\psi(3695)$, is not detected. Its production cross section times its branching ratio into lepton pairs is below the level of $1 \%$ (to $90 \%$ confidence) ${ }^{5}$ of that for the $\psi(3105)$. If the enhancement is indeed the same state observed at SPEAR, then there are opportunities for studying the properties of the $\psi$ 's in hadronic collisions by varying production reactions (projectile and target), energies and other experimental parameters.

In this section we explore first the possibility that the $\psi^{\text {t }}$ s are hadrons produced in rather conventional fashion. This will be Part A. In Part B we examine the possibility that the $\psi^{\prime}$ s are produced in fashion similar to the production of photons (though the $\psi^{\prime}$ s may couple directly to different quantum numbers). In Part C we consider several concrete models for hadronic production of the $\psi$ 's; two mechanisms are consistent with either hadronic or nonhadronic character for the $\psi$ while the third would indicate fundamental nonhadronic character. The possibility that different production mechanisms may play different relative roles as a function of energy is explored.

Before turning to these alternate possibilities we develop a bit of simple formalism. Let the cross section for inclusive production of the i-th resonance in a given hadronic collision be $\sigma_{\mathrm{i}}\left(\mathrm{s}, \mathrm{M}^{2}\right)$ where s is the total c.m. (energy) ${ }^{2}$ and M is the mass detected (usually as a lepton pair). These production cross sections will be sensitive not only to the production matrix elements but to kinematic limitations (due to energy availability, momentum transfer damping, and so forth). We believe these kinematic limitations to be relatively severe at

BNL energies and particularly so for the higher mass state. This question will be explored in greater detail in the appropriate dynamical contexts below.

Since we now know of a major cascade decay mode of the $\psi(3695)$ into the $\psi(3105)+2$ pions, it is evident that an inclusive measurement of lepton pairs in the lower mass region will actually reflect not only the hadronic production of the lower mass state, but also that of the higher one. If we denote the branching ratio for all cascade decay modes of the $\psi(3695)$ as $b_{2 \rightarrow 1}$ and the branching ratios for leptonic decays of the two states $a s b_{i \rightarrow l}$ then we can define two useful ratios:

$$
\begin{align*}
r_{1} & =\frac{\sigma_{2}\left(\mathrm{~s}, \mathrm{M}_{2}^{2}\right) \times \mathrm{b}_{2} \rightarrow 1 \times \mathrm{b}_{1} \rightarrow \overline{\ell \bar{l}}}{\sigma_{1}\left(\mathrm{~s}, \mathrm{M}_{1}^{2}\right) \times \mathrm{b}_{1} \rightarrow \overline{\ell \ell}} \\
& =\frac{\sigma_{2}\left(\mathrm{~s}, \mathrm{M}_{2}^{2}\right)}{\sigma_{1}\left(\mathrm{~s}, \mathrm{M}_{1}^{2}\right)} \times \mathrm{b}_{2 \rightarrow 1}  \tag{2.1}\\
\mathrm{r}_{2} & =\frac{\sigma_{1}\left(\mathrm{~s}, \mathrm{M}_{1}^{2}\right) \times \mathrm{b}_{1} \rightarrow \overline{\ell \bar{l}}+\sigma_{2}\left(\mathrm{~s}, \mathrm{M}_{2}^{2}\right) \times \mathrm{b}_{2} \rightarrow 1 \times \mathrm{b}_{1} \rightarrow \overline{\ell \ell}}{\sigma_{2}\left(\mathrm{~s}, \mathrm{M}_{2}^{2}\right) \times \mathrm{b}_{2} \rightarrow \overline{\ell \ell}} \\
& =\frac{\left(\sigma_{1}+\sigma_{2} \mathrm{~b}_{2} \rightarrow 1\right) \times \mathrm{b}_{1} \rightarrow \overline{\ell \ell}}{\sigma_{2} \mathrm{~b}_{2} \rightarrow \overline{\ell \ell}} \tag{2.2}
\end{align*}
$$

The first of these ratios $\left(r_{1}\right)$ is the fraction of lepton pairs detected in the $\psi(3105)$ region which get there by the cascade mechanism. If there is kinematic suppression of the production of the higher mass state, for example, then $\sigma_{2}$ and this ratio will be small. The second ratio $\left(r_{2}\right)$ is that of the number of lepton pairs seen in the 3105 mass region (due both to direct production of the $\psi(3105)$ and cascade decay of the $\psi(3695)$ ) to the number of lepton pairs in the 3695 mass region. This ratio will be large if there is kinematic suppression or
if the branching ratio into lepton pairs of the $\psi(3695)$ is smaller than that of the $\psi(3105)$, and it will be very large if both statements are true. This appears the case experimentally, and on the basis of models consulted below. Indeed, if this ratio is not large it will be evidence that the amplitude for hadronic production of the $\psi(3695)$ is much larger than that for the $\psi(3105)$.

There are other mechanisms for producing large mass lepton-antilepton pairs in hadronic collisions and, until the recent discoveries of giant peaks in the data, these mechanisms were the subject of considerable study. It is with little justice that we now refer to these processes as the background. We believe the background to be relatively smooth and to be dominated by off-shell photons produced in the hadron collision (the shoulder in the old Lederman BNL data ${ }^{6}$ is now explained by the resonance peaks). The precise mechanisms responsible for this background will be important to us if there is close dynamical connection between the hadronic production of photons and that of $\psi$ particles. This possibility is carefully explored below. We will denote the hadronic production cross section for a timelike photon as $\sigma_{\gamma}\left(\mathrm{s}, \mathrm{M}^{2}\right)$. The resultant photon converts to hadrons or to a lepton pair. The ratio for the latter two processes is just the parameter $R$ measured in the SPEAR experiment (under the peaks and their radiative tails).
A. Associated Production

The kinematics of the production process make it unlikely but not impossible that the $\psi(3105)$ and the $\psi(3695)$ carry a nonadditive quantum number, such as color, which would require a form of associated production. To be explicit, let us assume that the enhancement observed by the BNL-MIT experiment is a member of an octet in a new $\operatorname{SU}(3)$. Then, because the initial state is a singlet in this quantum number there must be another member of an octet in the final
state. The kinematics for production at $P_{l a b}=28.5 \mathrm{GeV} / \mathrm{c}$ are as follows:

$$
\begin{array}{ll}
\mathrm{pp} \rightarrow \mathrm{pC}_{\mathrm{B}} \psi & \mathrm{Q}=3.39-\mathrm{M}_{\mathrm{C}_{\mathrm{B}}} \mathrm{GeV}  \tag{2.3}\\
\mathrm{pp} \rightarrow \mathrm{pp} \psi \psi & \mathrm{Q}=-0.65 \mathrm{GeV}
\end{array}
$$

The production of a pair of $\psi$ 's is absolutely ruled out except to the extent that the Fermi motion in Beryllium smears the effective $\sqrt{5}$ over a range of order 0.8 GeV . To the extent we can rule out a baryon carrying this new quantum number with mass less than 3.4 GeV , the same sort of thing can be said about the channel with a colored baryon. The possibility that there might be some sort of threshold effect operating might be inferred from the fact that the MITBNL collaboration see no enhancement at the location of the $\psi(3695)$, although this production could also be suppressed by a small branching ratio into leptons.

The expected ratio of the $\psi(3105)$ to $\psi(3695)$ in the lepton pair channel is $r_{2}$ in Eq. (2.2). We can input some experimental numbers. The number quoted for $r_{2}$ by the MIT-BNL collaboration

$$
r_{2} \gtrsim 100 \quad \text { (with } 90 \% \text { confidence) }
$$

The ratio $b_{1} \rightarrow e^{+} e^{-/ b} b_{2} \rightarrow e^{+} e^{- \text {from analysis of the SPEAR data }{ }^{7}}$

$$
\begin{equation*}
10 \lesssim b_{1} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-/ b_{2}} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-\lesssim 37} \tag{2.4}
\end{equation*}
$$

and we get

$$
\begin{equation*}
\frac{\left(\sigma_{1}+\sigma_{2} b_{2} \rightarrow 1\right)}{\sigma_{2}} \geq 2.7 \tag{2.5}
\end{equation*}
$$

This is not unreasonable in view of the expected kinematic suppression which could have a form something like

$$
\begin{equation*}
\frac{\sigma_{1}}{\sigma_{2}} \propto\left[\frac{\mathrm{Q}(3105)}{\mathrm{Q}(3695)}\right]^{\mathrm{n}}=\left(\frac{2.45}{1.87}\right)^{\mathrm{n}} \tag{2.6}
\end{equation*}
$$

where n is some large power and $\mathrm{Q}(\mathrm{x})$ is the Q -value of the reaction $\mathrm{pp} \rightarrow \mathrm{pp} \psi(\mathrm{x})$. If $n$ is 3 this is 2.25 ; if $n$ is $6,5.06$, and if $n$ is 10 this is 14.9 . We will discuss the threshold behavior in terms of specific models later.

One type of associated production which cannot be ruled out is the reaction

$$
\begin{equation*}
\mathrm{pp} \rightarrow \gamma \psi \mathrm{pp} \tag{2.7}
\end{equation*}
$$

where we have a (possibly soft) photon in the final state. Due to the formation of the $\psi$ in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation we must consider the photon to have a component which transforms like an octet in the new $\operatorname{SU}(3)$.

Taking the enhancement of Christenson et al. ${ }^{6}$ in $\mathrm{pp} \rightarrow \mu^{+} \mu^{-} \mathrm{X}$ to be some combination of the $\psi$ and $\psi^{\prime}$ we can estimate the energy dependence of the production cross section as shown in Fig. 1. This estimate is quite crude. In principle the best way to decide the question of associated production is to look more carefully at the final state for evidence of the necessary particles. A careful study of the energy dependence near threshold, however, can be valuable and this is being done at Brookhaven.

Actually, the rates for producing the $\psi(3105)$ and the $\psi(3695)$ at BNL are not small considering the $Q$-value of the production mechanism at this low energy assuming no extra quantum number suppression. Fig. 2 shows these rates are not inconsistent with the rates for production of other heavy hadronic states ( $(\mathrm{p} \overline{\mathrm{p}})$ pairs, $\phi^{\prime} \mathrm{s}$, and deuterium and large $\mathrm{p}_{\mathrm{T}}$ pions). There does not seem to be any need for more quantum number suppression in order to understand the absence of the $\psi(3695)$. These simple indications also suggest that the production of the $\psi(3105)$ and $\psi(3695)$ will rise substantially through NAL and ISR energies. For more detailed calculations of the cross sections we have to make some assumptions and use models.


FIG. 1--Energy dependence for $\psi$ production.


FIG. 2--The solid lines represent interpolations of differential cross sections for the process pp $\rightarrow \pi+$ anything at different values of the $p_{T}$ of the pion. On the same graph are differential cross sections at $\mathrm{p}_{\mathrm{T}}=0$ for $\mathrm{pp} \rightarrow \overline{\mathrm{p}}, \mathrm{pp} \rightarrow \phi(1019), \mathrm{pp} \rightarrow \psi, \mathrm{pp} \rightarrow \psi^{\Downarrow}$, and $\mathrm{pp} \rightarrow \overline{\mathrm{d}}$.

## Other Hadronic Reactions

If we assume the c $\bar{c} \operatorname{explanation~for~the~} \psi^{\prime}$ s, it obviously is important to pursue the analogy of the $\phi$ and the $\psi$. As well as invoking Zweig's rule to explain the lack of decay modes for a c $\vec{c}$ state we can understand approximately the hadronic $\phi$ production in terms of a quark model for the exchange process. Experimentally, ${ }^{8}$ the ratio

$$
\begin{equation*}
\frac{\sigma\left(\pi^{-} \mathrm{p} \rightarrow \omega \mathrm{n}\right)}{\sigma\left(\pi^{-} \mathrm{p} \rightarrow \phi \mathrm{n}\right)} \approx 285 \pm 50 \tag{2.8}
\end{equation*}
$$

is observed around $5 \mathrm{GeV} / \mathrm{c}$. Since both processes proceed through rho exchange this suppression can be thought to quantitatively represent the lack of nonstrange quarks in the $\phi$, and/or strange quarks in $\pi^{-}$and $p$. Assuming another factor of $5-10$ for the possible extra purity of a state made from charmed quarks we can estimate the cross section for $\pi^{-} p \rightarrow(3105) \mathrm{n}$. Shortly above threshold near $P_{l a b} \simeq 9 \mathrm{GeV} / \mathrm{c}$ the cross section can be expected to be on the order of

$$
\begin{align*}
\sigma\left(\pi^{-} \mathrm{p} \rightarrow \psi \mathrm{n}\right) & \simeq .02-.05 \mu \mathrm{~b}  \tag{2.9}\\
& =20-50 \mathrm{nb}
\end{align*}
$$

This is shown in Fig. 3.
The search for this reaction by detecting the $\psi$ in the leptonic decay mode is certainly within reason and the observation of a simple exclusive reaction would definitely rule out models needing associated production.


FIG. 3--Expected range of cross section for $\pi^{-} p \rightarrow \psi(3105) \mathrm{n}$ as explained in text.

## B. Photon-like Production

In the next few sections we wish to examine the hypothesis that the same hadronic processes responsible for the "photon" background in the BNL-MIT experiment are also responsible for $\psi$ production. Various models for hadronic production of $\gamma^{\prime}$ s exist, including the bremsstrahlung model ${ }^{9}$ and the Drell-Yan mechanism, ${ }^{10}$ and these models may be easily modified to include $\psi$ production. The only ambiguity arises in the value of the coupling constant of the $\psi$ to hadrons, although some information about this vertex in the various models is available from the SPEAR experiment. In order to be satisfactory, these models must account for both the magnitude of the cross section at 3.1 and 3.7 GeV for $\psi$ and $\psi$ ' production, respectively, and the magnitude of the background due to one-photon processes.

We can circumvent many of the problems involved in computing absolute cross sections by first concentrating on the ratio of peak ( $\psi$-production) to background ( $\gamma$-production). If the $\psi$ and $\gamma$ are produced in roughly the same way, much of the production dynamics cancels out, apart from relative coupling constants and leptonic branching ratios. In the remainder of this section, we discuss the ratio of peak to background cross sections in some detail, while in the following sections we consider both the Drell-Y an and bremsstrahlung models.

We assume that the amplitudes may be written as

$$
\begin{gather*}
M_{i}^{\gamma}=e \beta_{i \gamma} M_{\gamma 0}\left(s, Q^{2}, \ldots\right) \frac{1}{Q^{2}}  \tag{2.10a}\\
M_{i}^{\psi}=c \beta_{i \psi} M_{\psi 0}\left(s, Q^{2}, \ldots\right)\left[\left(Q^{2}-M_{\psi}^{2}\right)+i M_{\psi} \Gamma\right]^{-1} \tag{2.10b}
\end{gather*}
$$

with the notation as shown in Fig. 4. In general, there are many amplitudes for each process and we must decide later whether we must add them coherently or incoherently. This will depend upon kinematic and other factors. Let us assume



FIG. 4--Notation for the bremsstrahlung production of $\gamma$ and $\psi$.
that this question is settled and write the cross sections as

$$
\begin{align*}
\frac{\mathrm{d} \sigma^{\gamma}}{\mathrm{d} Q^{2}} & =\mathrm{e}^{2} \beta_{\gamma}^{2} \sigma_{0 \gamma}\left(\mathrm{~s}, \mathrm{Q}^{2}\right) \frac{1}{\mathrm{Q}^{4}} \\
\frac{\mathrm{~d} \sigma^{\psi}}{\mathrm{dQ}} & =\mathrm{c}^{2} \beta_{\psi}^{2} \sigma_{0 \psi}\left(\mathrm{~s}, \mathrm{Q}^{2}\right)\left[\left(\mathrm{Q}^{2}-\mathrm{m}_{\psi}^{2}\right)^{2}+\mathrm{m}_{\psi}^{2} \Gamma^{2}\right]^{-1}
\end{align*}
$$

To compare with experiment it is useful to integrate these theoretical cross sections over the $\psi$ mass region, assuming that the "bare" cross sections
$\sigma_{0}^{\gamma}\left(\mathrm{s}, \mathrm{Q}^{2}\right)$ and $\sigma_{0}^{\psi}\left(\mathrm{s}, \mathrm{Q}^{2}\right)$ are not rapidly varying and may simply be evaluated at the $\psi$ mass $Q=3.105 \mathrm{GeV}$. To evaluate the $\psi$ contribution, the extreme narrowness of the resonance makes a delta-function approximation appropriate:

$$
\begin{equation*}
\sigma_{\psi} \equiv \int \mathrm{dQ}^{2} \frac{\mathrm{~d} \sigma_{\psi}}{\mathrm{dQ}^{2}} \cong \mathrm{c}^{2} \beta_{\psi}^{2} \sigma_{0 \psi}\left(\mathrm{~s}, \mathrm{~m}_{\psi}^{2}\right) \frac{\pi}{\mathrm{m}_{\psi} \Gamma} \tag{2.12}
\end{equation*}
$$

The one-photon contribution per unit energy interval is:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma_{\gamma}}{\mathrm{dQ}} \approx \mathrm{e}^{2} \beta_{\gamma}^{2} \sigma_{0 \gamma}\left(\mathrm{~s}, \mathrm{~m}_{\psi}^{2}\right) \frac{2 \mathrm{~m}_{\psi}}{\mathrm{m}_{\psi}^{4}} \tag{2.13}
\end{equation*}
$$

The ratio of these theoretical quantities is

$$
\begin{equation*}
\mathrm{r}_{\text {theory }}=\frac{\sigma_{\psi}}{\mathrm{d} \sigma_{\gamma} / \mathrm{dQ}}=\frac{\beta_{\psi}^{2}}{\beta_{\gamma}^{2}} \frac{\mathrm{c}^{2}}{\mathrm{e}^{2}}\left(\frac{\sigma_{0 \psi}^{-}\left(\mathrm{s}, \mathrm{~m}_{\psi}^{2}\right)}{\sigma_{0 \gamma}\left(\mathrm{~s}, \mathrm{~m}_{\psi}^{2}\right)}\right) \frac{\pi \mathrm{m}_{\psi}^{2}}{2 \Gamma_{\text {tot }}} \tag{2.14}
\end{equation*}
$$

This quantity is particularly easy to compare with experiment: one simply counts the number of events in the peak at 3105 and divides by the number of background events per unit of energy. The energy bin size chosen then cancels out when the experimental and theoretical values of $r$ are equated. Then solving for the unknown dynamical parameters in the theoretical expression yields

$$
\begin{equation*}
\frac{\beta_{\psi}^{2}}{\beta_{\gamma}^{2}} \cdot\left(\frac{\sigma_{0 \psi}}{\sigma_{0 \gamma}}\right)=\mathrm{r}_{\exp } \frac{\mathrm{e}^{2}}{c^{2}} \frac{2 \Gamma_{\mathrm{tot}}}{\pi \mathrm{~m}_{\psi}^{2}} \tag{2.15}
\end{equation*}
$$

Everything on the right-hand side is measurable, and in fact rough numbers are presently known.

A crucial quantity on the left-hand side of Eq. (2.15) is the ratio of "bare" cross sections for production of photons and $\psi$ particles. The coupling strengths have been extracted. If both processes are topologically similar, that is, if the photon and the $\psi$ are produced in the same fashion within the hadronic blob, then
we would expect the ratio of bare cross sections to be close to unity. For example, if $\psi$ and $\gamma$ both couple to quarks internally and in the same fashion, then the ratio will be one. If there is an additional mechanism, however, for production of $\psi^{\prime} s$, then the ratio will be larger than one. An example is the possible coupling of $\psi^{\prime}$ s to neutral gluons, or other neutral objects, within the blob. The parameter $c$ in the expression above may be extracted from knowledge of the partial width of the decay $\psi(3105) \rightarrow \ell^{+} \ell^{-}$obtained at $\operatorname{SPEAR}\left(\Gamma_{\psi \rightarrow \ell^{+} \ell^{-}} \approx\right.$ 6 keV ) and we find $\frac{\mathrm{c}^{2}}{4 \pi}=5.8 \times 10^{-6}$. The total width has been extracted by a number of theorists as approximately 90 keV . With a rough estimate of the value of $r_{\text {exp }}$ from the BNL experiment (3 background events per 25 MeV and a peak of 242 events) we find $r_{\exp }=2.0 \mathrm{GeV}$. With these values, the right-hand side of Eq. (2.15) has the value $1.5 \times 10^{-2}$. By comparison, the ratio of the square of the effective coupling of $\psi(3105)$ to leptons to that of photons to leptons is $\frac{\mathrm{c}^{2}}{4 \pi} / \frac{\mathrm{e}^{2}}{4 \pi}=7.9 \times 10^{-4}$.

We now consider how the $\psi$ and the $\gamma$ may couple within the hadronic blob and ask whether the production amplitudes for different internal topologies are to be taken coherently or incoherently. With the small inelasticity at lower energies, we should use a coherent sum for the bremsstrahlung mechanism. The Drell-Yan process, on the other hand, is always an incoherent sum. A simple assumption for bremsstrahlung is that the $\psi$ and the $\gamma$ couple only to the six valence quarks within the two-nucleon system. For the Drell-Yan process we must go to the sea of antiquarks and we reserve treatment of this mechanism to a later section. The coherent (incoherent) sum over electromagnetic charges of the (Gell-Mann - Zweig) valence quarks in a pp system gives $4 \mathrm{e}^{2}\left(2 \mathrm{e}^{2}\right)$ while for pn it. is $\mathrm{e}^{2}\left(5 / 3 \mathrm{e}^{2}\right)$. The average coherent and incoherent values for Beryllium are thus $\beta_{\gamma}^{2}=\frac{5}{2} \mathrm{e}^{2}\left(\frac{11}{6} \mathrm{e}^{2}\right)$.

In general we do not yet know what quantum numbers are involved in the $\psi$ coupling to hadrons. Likely candidates are the electromagnetic and baryonic charges - there isn't much else. For coupling to baryon number, the coherent sum gives $\beta_{\psi}^{2}=4 \mathrm{~b}^{2}$ (the incoherent gives $2 / 3 \mathrm{~b}^{2}$ ) where $\mathrm{b} / 3$ is the coupling of a $\psi$ to a quark of baryon number $1 / 3$. Utilizing the parameters given above we can then determine $b$ :

$$
\left(\frac{\sigma_{0 \psi}}{\sigma_{0 \gamma}}\right) \frac{b^{2}}{4 \pi}- \begin{cases}6.82 \times 10^{-5}  \tag{2,16}\\ 3.01 \times 10^{-4} & \text { coherence assumption } \\ \text { incoherence assumption }\end{cases}
$$

It is particularly interesting to compare the coupling of psi to a quark (coupling $=\mathrm{b} / 3=9.7 \times 10^{-3} \times\left(\sigma_{0 \gamma} / \sigma_{0 \psi}\right)^{\frac{1}{2}}$ ), with effective coupling of psi to a lepton (coupling $=\mathrm{c}=8.5 \times 10^{-3}$ ). These couplings, determined by assumed coherence, are thus the same within a factor of $1.14 \times\left(\sigma_{0 \gamma} / \sigma_{\psi 0}\right)^{\frac{1}{2}}$. The coupling of $\psi$ to quarks derived assuming incoherence is larger than its coupling to leptons by another factor of 2.1 .

The fact that the quark coupling is larger is vital if we are to explain the branching ratio of $\psi$ to hadrons versus leptons at SPEAR with a reasonably small number of quarks. Indeed, if we assume that the cross sections for the two processes in question are incoherent sums over amplitudes as in Fig. 5,

the branching ratio, which has an experimental value of about 15 , gives

$$
\mathrm{N}=15 \times \frac{9 \mathrm{c}^{2}}{\mathrm{~b}^{2}}=\left\{\begin{array}{ll}
11.5  \tag{2.18}\\
2.6
\end{array} \times\left(\frac{\sigma_{\psi 0}}{\sigma_{\gamma 0}}\right)^{\frac{1}{2}} \quad \text { "coherent" b } \quad\right. \text { "incoherent" b }
$$

Mixing coherent production with incoherent, when we extract " $b$ ", would easily allow us to have effectively 3 or 4 or 9 quarks at SPEAR.

As a further check we can compute the ratio ( $\psi \rightarrow$ hadrons/one photon background). The theoretical value is

$$
\begin{equation*}
r_{\text {th }}=\frac{\sigma_{\psi}}{d \sigma_{\gamma} / \mathrm{dQ}}=\mathrm{N} \frac{\mathrm{c}^{2} \mathrm{~b}^{2}}{9 \mathrm{e}^{4}}\left(\frac{\pi \mathrm{~m}_{\psi}^{2}}{2 \Gamma_{\text {tot }}}\right) \tag{2.19}
\end{equation*}
$$

where $N$ is the number of quarks. For $\Gamma_{\text {tot }}=90 \mathrm{keV}$ and using the value of b obtained by assuming coherence in the hadronic production of $\psi$, we obtain

$$
\begin{equation*}
r_{\mathrm{th}}=\mathrm{N} \times(140 \cdot \mathrm{MeV}) \tag{2.20}
\end{equation*}
$$

Experimentally this ratio has a val ue of about $\frac{12,000 \mathrm{nb} \cdot \mathrm{MeV}}{8 \mathrm{nb}} \approx 1500 \mathrm{MeV}$. Consistency is achieved for $N \approx 11$ quarks. For incoherent coupling we found a value of $b^{2}$ which is larger by a factor of 4.4 , giving $N \approx 3$.

Thus we see that assuming a photon-like production for the $\psi$ is not immediately incompatible with the BNL-MIT peak to background ratio or with the SPEAR data. There is, however, a large degree of uncertainty in the exact magnitude of the background. If the true photon background is $50 \%$ larger than we have assumed, then it would be possible to have a universal coupling of the $\psi$ to leptons and quarks and still retain a reasonable value for $N$ at SPEAR. If, on the contrary, the observed background is dominated by accidentals, the effective value of $r$ would rise and $N$ would fall below unity, making photon-like production unlikely.

We may also conclude from the relationship (2.19) that the consistency of

SPEAR and BNL data cannot tolerate excessive deviation of the ratio $\sigma_{0 \psi} / \sigma_{0 \gamma}$ from unity. That is, if the minimal assumptions made above are correct, photons and $\psi$ particles must be produced in roughly the same fashion, albeit with different couplings. This raises the very interesting question of whether the $\psi^{\prime}$ s couple through the photon to leptons or to hadrons, or to both. The latter possibility can be eliminated at once since the ratio of leptonic to hadronic processes at SPEAR is very different on and off "resonance". The other two possibilities are both a priori viable. Differing character of the hadronic final states on and off "resonance" would eliminate the possibility that the $\psi$ couples to hadrons via the photon. This leaves the possibility that the $\psi$ couples directly to hadrons but indirectly through the photon to leptons. If this is true (a direct coupling to both leptons and hadrons is also possible), then we can extract the effective $\gamma-\psi$ coupling from the leptonic process. The net coupling of $\psi(3105)$ to leptons is $\mathrm{c}=8.5 \times 10^{-3} \dot{\mathrm{~m}}_{2}^{2}$ In standard vector dominance notation, where the
$\gamma-\psi$ vertex is written as $\frac{\mathrm{e}_{\psi}}{\mathrm{h}_{\gamma \psi}}$, we have

$$
\begin{array}{cc}
\mathrm{h}_{\gamma \psi}=\frac{\mathrm{e}^{2}}{\mathrm{c}}=10.8 \\
\frac{\mathrm{~h}_{\psi \gamma}^{2}}{4 \pi}=9.3 & \frac{\mathrm{e}}{\mathrm{~h}_{\gamma \psi}} \mathrm{m}_{\psi}^{2}=2.8 \times 10^{-2} \mathrm{~m}_{\psi}^{2}
\end{array}
$$

This value is comparable to the familiar vector dominance couplings of $\rho, \omega$, and $\phi$. A discussion of how $\psi$ 's with these couplings might fit into a charmed SU(4) is given in Section III. A.

NOTE: In the preceding analysis, we have neglected the contribution to lepton pairs at 3.1 GeV due to cascade decay of the $\psi^{\prime} \rightarrow \psi+$ hadrons. If we let
$\mathrm{r}_{1}=\frac{\# \mathrm{e}^{+} \mathrm{e}^{-} \text {at } \mathrm{M} \mathrm{e}^{+} \mathrm{e}^{-}=3.105 \text { due to cascade of } \psi_{2} \rightarrow \psi_{1}+\text { hadrons }}{\# \mathrm{e}^{+} \mathrm{e}^{-} \text {at } \mathrm{M}_{\mathrm{e}^{+} \mathrm{e}^{-}}=3.105 \text { due to direct production of } \psi_{1}}$
then
$r_{1}=\frac{\sigma_{2}\left(\mathrm{~s}, \mathrm{M}_{2}^{2}\right)}{\sigma_{1}\left(\mathrm{~s}, \mathrm{M}_{1}^{2}\right)} \mathrm{b}_{\psi_{2}} \rightarrow \psi_{1}+$ hadrons.
where $\sigma_{1}, \sigma_{2}$ are defined in Section A. Since $\sigma_{1} / \sigma_{2}>3$ in most models and $\mathrm{b}_{2-1} \sim \frac{1}{2}$ the cascade decays are at most $1 / 6$ of the total number of events at 3.1 GeV and thus can be neglected at this stage.
C. The Drell-Yan Mechanism

In this section we present the results of detailed calculations in the DrellYan model for the production cross section in hadronic experiments of lepton pairs with mass $Q$, evaluated at the masses of the $\psi(3105)$ and $\psi(3695)$, expressed as a function of $s$, the squared total c.m. energy of the collision. Both the $\psi$ and $\gamma$ contributions have been calculated for $\mathrm{pp}, \mathrm{np}, \pi \mathrm{p}$, and $\pi \mathrm{ncol}-$ lisions. The parton model diagrams for the two contributions are given in Fig. 6. We have used the point vector coupling vertices shown in Figs. 7a and 7d, which may or may not be reasonable approximations to the more detailed possibilities indicated in Figs. 7b and 7c. We have also used the parton distribution functions listed in Table I. These are very similar to the fairly conservative distributions that Farrar, and Chu and Gunion have shown to be consistent with available deep inelastic eN and $\nu \mathrm{N}$ scattering data. (We have not yet considered the modified Khuti-Weisskopf distributions of Tuan et al., which are more difficult to reconcile with the inelastic data, although they should produce somewhat larger lepton pair production cross sections than calculated here,)

(a) VIA PHOTON

(b) VIA $\psi, \psi^{\prime}$

2636A5
Fig. 6--Parton model diagrams for hadrons $h_{1} h_{2}$ scattering to produce a massive lepton pair via quark-antiquark annihilation into (a) a photon, (b) a $\psi$ or $\psi^{\prime}$.

In the approximation used here, the Drell-Yan process cannot distinguish hadronic $\psi$ and $\psi^{\prime}$ which are ce bound states of charmed quarks from $\psi$ and $\psi^{\prime}$ which are the particles of fundamental fields (as are the photon and neutral vector gluons) of arbitrary (hadronic or other) character.

The formulae involved in the calculation are as follows:

$$
\begin{equation*}
\frac{d \sigma}{d Q}=\left(\frac{4 \pi \alpha^{2}}{3}\right) \frac{2}{Q^{3}}\left\{\tau \int_{\tau}^{1} \frac{d x}{x} \sum_{i} e_{i}^{2}\left[f_{i}(x) \bar{f}_{i}\left(\frac{\tau}{x}\right)+\left(x \rightarrow \frac{\tau}{x}\right)\right]\right\} \tag{2.22}
\end{equation*}
$$



FIG. 7--Parton model diagrams for $\psi$-fermion couplings. (a) Point vector coupling of $\psi$ to quarks; (b) Charmonium model -3 vector gluons couple ordinary quarks to the ce bound state identified as $\psi$;
(c) Photon mediated (GVMD) coupling of $\psi$ to charged leptons;
(d) Direct point vector coupling of $\psi$ to leptons.

TABLE I

## Parton Distribution Functions


Neutron: $u_{n}(x)=d_{p}(x) ; d_{n}(x)=u_{p}(x) ;$ all others $=\bar{u}_{p}(x)$

Photon: $q_{\gamma}(x)=\bar{q}_{\gamma}(x) \approx 7.6 \mathrm{e}_{\mathrm{q}}^{2}\left(\frac{\alpha}{2 \pi}\right)\left(\mathrm{x}^{2}+(1-\mathrm{x})^{2}\right)$

$$
\begin{equation*}
\frac{\mathrm{d} \sigma^{\psi}}{\mathrm{dQ}}=\frac{8 \pi \Gamma_{\psi} \rightarrow \mathrm{e}^{-\Gamma_{\psi} \rightarrow \text { hadrons }}}{\left[\left(\mathrm{Q}^{2}-\mathrm{M}_{\psi}^{2}\right)^{2}+\mathrm{M}_{\psi}^{2} \Gamma_{\psi \text { tot }}^{2}\right] \mathrm{M}_{\psi}}\left\{\tau \int_{\tau}^{1} \frac{\mathrm{dx}}{\mathrm{x}}\left[\mathrm{f}_{\mathrm{i}}(\mathrm{x}) \overline{\mathrm{f}}_{\mathrm{i}}\left(\frac{\tau}{\mathrm{x}}\right)+\left(\mathrm{x} \rightarrow \frac{\tau}{\mathrm{x}}\right)\right]\right\} \tag{2.23}
\end{equation*}
$$

and similarly for $\mathrm{d}_{\sigma} \psi^{\prime} / \mathrm{dQ}$, where $\tau=\mathrm{Q}^{2} /(\mathrm{s})$ and we have assumed that $\psi$ and $\psi^{\prime}$ have a common coupling strength $\left(h_{j}^{2}=\frac{1}{3} \sum_{i} h_{i}^{2}\right)$ to all ordinary quarks. This is what one expects if the vertex (Fig. 7a) is a point approximation of Fig. 7b as the vector gluons should not distinguish between $u$, $d$, and $s$ quarks (at least, not strongly). An alternative scheme, for which, however, we know of no justification, is to choose $h_{i} \propto Q_{i}$ the quark charges. This makes the u-quark contribution dominant. The effect is approximately to multiply all our results using ordinary quarks by a factor of three (3). The relation between couplings and widths is

$$
\begin{equation*}
\Gamma_{\psi \rightarrow \mathrm{e}}=\frac{\mathrm{g}_{\psi-\mathrm{e}}^{2} \mathrm{M}_{\psi}}{12 \pi}, \Gamma_{\psi \rightarrow \text { hadr }}=\frac{\mathrm{g}_{\psi-\mathrm{e}}^{2} \sum_{i} \mathrm{~h}_{\mathrm{i}}^{2} \mathrm{M}_{\psi}}{12 \pi} \tag{2.24}
\end{equation*}
$$

and similarly for $\psi^{\prime}$. (The use of $\mathrm{h}_{\mathrm{i}} \mathrm{g}_{\psi_{\mathrm{e}}}$ for the $\psi \overline{\mathrm{q}}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}}$ coupling is intended to follow as an analogy to the $\gamma \bar{q}_{i} q_{i}$ couplings $e e_{i}$. ) We have used the following values for these parameters:

$$
\begin{array}{ll}
\Gamma_{\psi \rightarrow e \mathrm{e}}=6 \mathrm{keV} & \Gamma_{\psi_{\mathrm{tot}}} \simeq \Gamma_{\psi \rightarrow \mathrm{hadr}}=90 \mathrm{keV} \\
\Gamma_{\psi^{\prime} \rightarrow e^{-e}}=2 \mathrm{keV} & \Gamma_{\psi_{\text {tot }}^{\prime}}
\end{array}
$$

Note the last two values especially, as the calculated cross section for $\psi^{\prime}$ depends linearly on the branching ratio $\Gamma_{\psi^{\prime} \rightarrow \text { had }} / \Gamma_{\psi^{\prime}} \quad$ which we have taken to be $10 \%$. This value leads to a ratio of $\psi^{\prime}$-signal to $\psi$-signal which is consistent with experimental results at BNL; however, we have not chosen it to fit the data. Rather, this branching ratio is what one would expect if, in addition to $\psi^{\mathrm{r}} \rightarrow \psi \pi \pi$ having a large ( $\sim 40 \%$ ) fraction of $\psi^{\text {r }}$ decays, there is a similar large
fraction taken up by $\psi^{\prime} \rightarrow \chi+\gamma$ where $\chi$ is a ce pseudoscalar meson (of mass $\sim 3 \mathrm{GeV}$ in any $\mathrm{SU}(4)$ picture $)$. Both of these decays must be excluded from $\Gamma_{\psi^{\prime}} \rightarrow$ had ${ }^{\text {as this width is supposed to be a measure of the effective coupling of }}$ $\psi^{\boldsymbol{r}}$ to ordinary hadrons (which $\psi$ and $\chi$ are not). The $\psi^{\gamma}$ total width is suggested by other analyses in this work, but in any event lies comfortably between the experimental lower and upper limits of .3 and 2.7 MeV .

Our results are plotted in Fig. 8 for $p$ p and Fig. 9 for $\pi \bar{p} p$ as a function of s. These are the cross sections integrated over the peak of the Breit-Wigner by multiplying by $\pi \Gamma_{\text {tot }}$; i.e., treating the Breit-Wigner as a delta-function approximation. For the photon contribution, the integration was achieved by multiplying by 25 MeV taken as a typical bin width for these experiments.

For purposes of comparison, we have also calculated:

1) The cross section assuming $\psi$ is a $c \bar{c}$ bound state of charmed quarks that couples strongly only to such quarks and that charmed quarks are found in ordinary hadrons at about the same level as strange quarks (this must at least be valid for $\mathrm{x} \leq$ some $\mathrm{x}_{\mathrm{o}}$ if the Pomeron is to be $\mathrm{SU}(4)$ symmetric). See Fig. 10.
2) The photoproduction cross section using the Drell-Yan mechanism, both with and without charmed quarks in the nucleon (see Fig. 11).

In the latter case, the Drell-Yan cross section is negligible compared to experiment and to estimates using vector meson dominance with $\sigma_{\tau}(\psi N) \approx 1 \mathrm{mb}$. In (1) we assumed a not quite symmetric Pomeron, i.e.,

$$
\begin{equation*}
\bar{c}_{p}(x)=c_{p}(x) \approx 0.1(1-x)^{7} / x \approx \frac{1}{2} s_{1}(x) \tag{2.25}
\end{equation*}
$$

and used the experimental value from BNL to normalize the coupling of $\psi$ to charmed quarks:

$$
\begin{equation*}
\left(\mathrm{h}_{\mathrm{c}} \mathrm{~g}_{\psi \mathrm{ee}}\right)^{2} / 4 \pi \approx 0.4 \tag{2.26}
\end{equation*}
$$



FIG. 8--The total cross sections for $\mathrm{pp} \rightarrow \ell^{+} \ell^{-}+X$ as a function of total $\mathrm{c} . \mathrm{m}$. energy squared s for $\ell^{+} l^{-}$pair masses of 3.1 and 3.7 GeV via the mechanisms of Fig. 6 (a) and 6 (b).


FIG. 9--The total cross sections for $\pi^{-} \mathrm{p} \rightarrow \ell^{+} \ell^{-}+X$ as a function of total $\mathrm{c} . \mathrm{m}$. energy squared $s$ for $l^{+} l^{-}$pair masses of 3.1 and 3.7 GeV via the mechanisms of Fig. 6 (a) and 6 (b).


FIG. 10 --The total cross sections for pp and $\pi^{-} \mathrm{p} \rightarrow \psi\left(\rightarrow \ell^{+} \ell^{-}\right)+\mathrm{X}$ as a function of $s$ assuming that Fig. $6(b)$ is dominated by charmed quarks annihirating into $\psi$ with a large effective coupling constant.


FIG. 11--The total cross section for photoproduction of $\psi$ 's off of nucleons, as a function of $s$, both with and without significant amounts of charmed quarks in nucleons.

This is about the strength suggested by Appelquist and Politzer ${ }^{11}$ for $\alpha_{S}$, the gluon-charmed quark coupling, although it at first scems on the small side for a strong coupling constant. We also used these results in the calculation of (2.22) with charmed quarks.

We have not included the additional factor of $1 / 3$ suppression that occurs if quarks carry color quantum numbers. Our conclusions are:

1) The Drell-Yan process does not explain the magnitude of signal seen at BNL (unless the parton distributions are quite different from those inferred from hadronic data).
2) The calculated peak to background ratio at the 3.1 and ratio of 3.7 signal to 3.1 signal are consistent with the experimental data.
3) $\gamma N$ and especially $\pi N$ experiments benefit from relatively large $\psi$ and $\psi^{\prime}$ production cross sections even at modest energies.
4) The predicted $\gamma \mathrm{N} \rightarrow \psi \mathrm{X}$ cross section is sufficiently large that when combined with accurate experimental results it may restrict the level of charmed quarks in nucleons.
5) The $\psi$ and $\psi^{\prime}$ provide a "strong signal" method for testing Drell-Yan scaling: If an experiment is run at $s$ and $s^{\prime}$ such that $\frac{(3.1)^{2}}{s}=\tau=\frac{(3.7)^{2}}{s^{1}}$ then there should be no relative effect if the Drell-Yan scaling function depends on $\tau$ only, and we should have:
$\sigma_{\psi \rightarrow \ell^{+} \ell^{-}}(\mathrm{S}) \approx \frac{\Gamma_{\dot{\psi} \rightarrow \ell \ell}}{\Gamma_{\psi^{\prime} \rightarrow \ell \ell}}\left(\frac{3.7}{3.1}\right)^{3}\left(\frac{\Gamma_{\psi^{\prime} \text { tot }}}{\Gamma_{\psi^{\prime} \rightarrow \text { had }}}\right) \sigma_{\psi^{\prime} \rightarrow \ell^{+} \ell^{-}}\left(\mathrm{s}^{\prime}\right)$
We have continued in these calculations despite (1) because:
(a) This is a definite model with no completely free adjustable parameters.
(b) There exist models (e.g., the CIM) which include many other processes
apart from the Drell-Yan. We are at present involved in a systematic study of these processes and cannot see any reason why at present energies they should not be comparable or larger than the Drell-Yan process. This is particularly true of $p p$ reactions where there are few $\bar{q}$ 's in the initial state to drive the Drell-Yan process. However the $\tau$ distribution in these processes is not dissimilar from Drell-Yan and so the shapes of the final curves may be very similar to those presented here, and conclusions 2-4 may yet be important and interesting.
D. The Bremsstrahlung Mechanism

We can make a rough estimate of the bremsstrahlung contribution by utilizing the calculation of Berman, Levy, and Neff. ${ }^{9}$ This computation assumed an effective charge in the hadronic production amplitude of $e$. For consistency with Section II. B we rewrite their results (which were computed for $p_{\ell}=28.5$ $\mathrm{GeV} / \mathrm{c}$ ) for bremsstrahlung production of photons in the form

$$
\begin{equation*}
\frac{\mathrm{d} \sigma^{\gamma}}{\mathrm{dQ}^{2}}=3.3 \times 10^{-35} \beta_{\gamma_{\mathrm{coh}}}^{2} \mathrm{~cm}^{2} / \mathrm{GeV}^{2} \text { at } \sigma^{2}=\mathrm{m}_{\psi}^{2}=9.6 \mathrm{GeV}^{2} \tag{2.28}
\end{equation*}
$$

For proton-proton scattering $\beta_{\gamma}^{2}=4 \mathrm{e}^{2}$ while for $\mathrm{p}-\mathrm{n}$ scattering $\beta_{\gamma}=\mathrm{e}^{2}$. Thus for $\psi$ production we have
$\frac{\mathrm{d}^{\psi}}{\mathrm{dQ}^{2}}=3.3 \times 10^{-35} \frac{\mathrm{c}^{2}}{\mathrm{e}^{2}} \beta_{\psi}^{2} \frac{\mathrm{Q}^{4}}{\left(\mathrm{Q}^{2}-\mathrm{m}_{\psi}^{2}\right)^{2}+\Gamma^{2} \mathrm{~m}^{2}} \mathrm{~cm}^{2} / \mathrm{GeV}^{2}$
where $\beta_{\psi}^{2}=4 b^{2}$ for both pp and pn collisions. For production on a Be target the average electromagnetic coupling is then $\beta_{\gamma_{a v}}^{2}=\frac{5}{2} \mathrm{e}^{2}$ and the baryonic coupling is $4 \mathrm{~b}^{2}$.

The $\gamma$-background eross section per nucleon is then

$$
\begin{equation*}
\frac{\mathrm{d} \sigma^{\gamma}}{\mathrm{dQ}^{2}}=7.6 \times 10^{-36} \mathrm{~cm}^{2} / \mathrm{GeV}^{2} \text { at } \mathrm{Q}^{2}=\mathrm{m}_{\psi}^{2} \tag{2.30}
\end{equation*}
$$

while the integrated contribution of the $\psi$ is

$$
\begin{align*}
\sigma_{\psi} & =3.3 \times 10^{-35} \frac{\mathrm{c}^{2}}{\mathrm{e}^{2}} \beta_{\psi}^{2} \frac{\pi \mathrm{~m}_{\psi}^{3}}{\Gamma_{\text {tot }}} \mathrm{cm}^{2} / \mathrm{GeV}^{2} \\
& =9.3 \times 10^{-35} \mathrm{~cm}^{2} \tag{2.31}
\end{align*}
$$

This is in excessively good agreement with the preliminary BNL/MIT results.
We might well argue that Bremsstrahlung is the dominant mechanism at these relatively low energies since the (incoherent) Drell-Yan mechanism yields a cross section which is an order of magnitude smaller. We further believe that this situation may well be inverted at higher energies. Loss of coherence (over the nucleon) tends to reduce the bremsstrahlung contribution, compensating for kinematic enhancement as we go to higher energies. We know that the Drell-Yan mechanism undergoes a rapid kinematic liberation as the total energy is increased. The latter is due to the fact that wee antiquarks (small x ) carry larger amounts of energy at larger values of $\mathrm{E}_{\mathrm{cm}}$. The $\psi$ is very near or above threshold for most quark-antiquark annihilations at BNL energies.

A further question of interest is the rate which a similar bremsstrahlung calculation gives for production of the $\psi(3695)$ at BNL energies. Assuming that the $\psi^{\prime}$ couples with the same strength as the $\psi$ (there are indications that this is not precisely the case) and using a kinematic suppression in production of a factor of three, we obtain

Experimentally, this ratio is quoted as $>100$ (at $90 \%$ confidence). According to Ref. 9 , the ratio of photon cross sections is about 3 . This ratio is computed in a high energy approximation and may not fully reflect the kinematic limitations of BNL energies. The branching ratio for $\psi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$is about $1 / 15$ while that for $\psi^{\prime}$ involves $\Gamma_{\psi^{\prime}} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$, which is known to be about 3 keV , and $\Gamma_{\psi^{\prime}}$ tot which is known only very roughly ( $300 \mathrm{keV}<\Gamma_{\psi^{\mathrm{t}}}^{\text {tot }}<2.7 \mathrm{MeV}$ ). To the extent that our several assumptions are correct, we can obtain a lower bound on $\Gamma_{\psi^{\prime}}^{\text {tot }}$ :
$\frac{\Gamma_{\psi^{\prime}}^{\mathrm{tot}}}{\Gamma_{\psi^{\prime} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}} \geq \frac{100}{\mathrm{~b}_{\psi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}} \cdot \frac{\frac{\mathrm{d} \sigma}{\gamma}\left(\mathrm{Q}^{2}=\mathrm{m}_{\psi^{\prime}}^{2}\right)}{\frac{\mathrm{d} \sigma^{\gamma}}{\mathrm{dQ}^{2}}\left(\mathrm{Q}^{2}=\mathrm{m}_{\psi}^{2}\right)}\left(\frac{\mathrm{m}_{\psi^{\prime}}}{\mathrm{m}_{\psi}}\right)^{2}\left(\frac{\mathrm{~b}^{\prime}}{\mathrm{b}}\right)^{2} \geq 708\left(\frac{\mathrm{~b}^{\prime}}{\mathrm{b}}\right)^{2}$
or, with $b=b^{\prime}$ (equal effective coupling of $\psi$ and $\psi^{\prime}$ to hadrons) and $\Gamma \psi^{\prime} \rightarrow e^{+} e^{-}$ $\approx 3 \mathrm{keV}$,

$$
\begin{equation*}
\Gamma_{\psi^{\prime}}^{\mathrm{tot}} \approx 2.1 \mathrm{MeV} \tag{2.34}
\end{equation*}
$$

This estimate is undoubtedly large, though within experimental bounds. This is most likely due to underestimation of the kinematic suppression of $\psi^{\prime}$ production (a kinematic suppression of a factor of 10 would give $\Gamma_{\psi^{\prime}}^{\text {tot }}>640 \mathrm{keV}$ ). The lower bound on $\Gamma_{\psi^{\prime}}^{\text {tot }}$ would also be made smaller by a downward revision in the reported experimental cross section ratio $\left(\sigma_{\psi} / \sigma_{\psi^{r}}\right)$, by a reduction of the partial width $\Gamma_{\psi^{\prime} \rightarrow e^{+} e^{-}}$, or by a smaller effective coupling of $\psi^{\prime}$ to hadrons $\left(\frac{b^{\prime}}{b}<1\right)$. Without the last it would be difficult to argue for a total width less than a few hundred keV. If the total width is quite large, so that the branching
ratio $\mathrm{b}_{\psi^{\prime} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \text {is very small, then the cascade effect makes the experimen- }}$ tally measured cross section ratio larger than the actual value. As estimated above, however, this is only about a $10 \%$ effect.
III. PHOTOPRODUCTION EXPERIMENTS AND THE NATURE OF THE $\psi$ 's Photoproduction is potentially of great interest in elucidating the interactions of the $\psi$ particles with photons and hadrons. However, as we shall discover, the observation of $\psi$ production by photon beams incident on hadrons will not necessarily settle the question of hadronic versus nonhadronic character of these particles.

If the $\psi^{\prime}$ 's are hadrons and if they are $1^{--}$states then it is possible to produce them by vector dominance of the incident photon followed by diffractive hadronic scattering into the final state. Moreover, a consistent view of the $\psi^{\prime}$ s as hadrons would then also require that the diffractive scattering be of characteristic hadronic magnitude. This possibility is discussed in Section A, where it becomes clear that one must again invoke special pleading to justify hadronic character for the $\psi$ 's. It is possible to make such arguments along the same lines as those which might justify the narrow widths. If the $\psi^{\prime}$ s are composite hadrons then it is likely that there will be larger spectroscopic families and that these families will contain narrow pseudoscalar states. In Part B we consider the possible Primakoff photoproduction of these states.

The alternate possibility which has been the theme of earlier sections is that the $\psi$ 's are not hadrons. It is then unlikely that vector dominance is relevant and the question of photoproduction takes on new dynamical meaning. Indeed, the effective coupling estimates of Section II indicate that even if the $\psi$ 's do mix with the photon, the $\psi$-hadron cross section will be too small to support experimental observation, via vector dominance, at even the nanobarn level. The nonhadronic character would mean that if photoproduction occurs it does not proceed through elastic or quasi-elastic scattering of $\psi$ 's from hadrons. The interesting possibility then is that the photon turns into ordinary hadrons (rhos,
$\phi^{\prime} s$, etc.) which then produce the $\psi^{\prime}$ s in a fashion entirely analogous to their observed production in the BNL experiment. This discussion is the subject of Part C.

## A. Vector Dominance and Diffractive Photoproduction

Under the assumption that the $\psi$ and $\psi^{\prime}$ are hadrons and that vector dominance is applicable we can make some very strong predictions about diffractive photoproduction. We can define the differential cross section for $\psi \mathrm{N}$ scattering

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\gamma \mathrm{~N} \rightarrow \psi \mathrm{~N})=\frac{\mathrm{e}^{2}}{\mathrm{~h}_{\psi}^{2}} \frac{\mathrm{~d} \sigma}{\mathrm{dt}}(\psi \mathrm{~N} \rightarrow \psi \mathrm{~N}) \tag{3.1}
\end{equation*}
$$

where the width into $e^{+} e^{-}$is given by

$$
\begin{equation*}
\Gamma_{\left(\psi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)}=\frac{\alpha^{2}}{3} \frac{4 \pi}{\mathrm{~h}_{\psi}^{2}} \mathrm{M}_{\psi} \tag{3.2}
\end{equation*}
$$

For the $\psi(3105)$ and the $\psi(3695)$ we can use the leptonic widths around $5 . a \mathrm{keV}$ and 2.5 keV respectively to obtain

$$
\begin{align*}
& \mathrm{h}_{\psi}^{2} / 4 \pi(3105)=11.0 \pm 2  \tag{3.3a}\\
& \mathrm{~h}_{\psi}^{2} / 4 \pi(3695)=26.2 \pm 5 \tag{3.3b}
\end{align*}
$$

It is important to note that these coupling constants are out of line for the simple SU(4) quark model where the $\psi$ and $\psi^{\prime}$ are made out of charmed quarks. Under these assumptions the ratios of the vector meson couplings

$$
\begin{equation*}
\frac{h^{2}}{4 \pi}: \frac{h_{\omega}^{2}}{4 \pi}: \frac{h_{\phi}^{2}}{4 \pi}: \frac{h_{\psi}^{2}}{4 \pi}: \frac{h_{\psi}^{2}}{4 \pi} \tag{3.4a}
\end{equation*}
$$

are predicted to be

$$
\begin{equation*}
\frac{1}{9}: 1: \frac{1}{2}: \frac{1}{8}: \frac{1}{8} \tag{3.4b}
\end{equation*}
$$

Experimental values from Orsay give

$$
\begin{array}{ll}
h_{p}^{2}  \tag{3.4c}\\
\frac{p}{4 \pi} & =2.56 \pm .2
\end{array} \quad \frac{h_{\omega}^{2}}{4 \pi}=19.2 \pm .2 \quad \frac{h_{\phi}^{2}}{4 \pi}=11.3 \pm .3
$$

so that the ratios can be written

$$
\begin{equation*}
.133 \pm .015: 1.0: .585 \pm .07: .57 \pm .1: 1.36 \pm .3 \tag{3.4~d}
\end{equation*}
$$

The fact that the last two coupling constants are wrong does not necessarily rule out the charmed quark model for the $\psi^{\prime} s$ in any way. It simply means that there may be substantial $\mathrm{SU}(4)$ breaking due to the large effective mass of the charmed constituents or that our understanding of vector dominance is not as complete as it might be.

Since the $e^{+} e^{-}$width of the $\psi$ is approximately a factor of 5 smaller than would be predicted by $\operatorname{SU}(4)$ we might consider estimating the mass of the charmed quarks by making a simple phase space correction

$$
\begin{equation*}
\Gamma_{\mathrm{obs}}^{\mathrm{e}^{+} \mathrm{e}^{-}}=\Gamma_{\mathrm{SU}_{4}}^{\mathrm{e}^{+} \mathrm{e}^{-}}\left(1-\frac{4 \mathrm{~m}_{\mathrm{c}}^{2}}{\mathrm{M}_{\psi}^{2}}\right)^{3 / 2} \tag{3.5}
\end{equation*}
$$

derived by noting that the coupling of the photon depends on the velocity of the quarks in the bound state:

$$
\begin{equation*}
\left(1-\frac{4 m_{c}^{2}}{M_{\psi}^{2}}\right)^{3 / 2} \simeq 1 / 5 \Rightarrow m_{c} \cong 1.26 \tag{3.6}
\end{equation*}
$$

We get a relatively large value for the effective mass of the charmed quark but one which is significantly smaller than the mass used in the charmonium model for the $\psi^{\prime} s$.

If we forget about $\operatorname{SU}(4)$ and use the observed coupling constants we can plunge ahead and calculate the photoproduction cross section (4.1). In order to normalize things conveniently we might write

$$
\begin{equation*}
\frac{\frac{d \sigma}{d t}(\gamma \mathrm{~N} \rightarrow \psi \mathrm{~N})}{\frac{d \sigma}{d t}(\gamma \mathrm{~N} \rightarrow \phi \mathrm{~N})} \simeq \frac{11.0}{11.3} \frac{\frac{\mathrm{~d} \sigma}{\mathrm{dt}}(\psi \mathrm{~N} \rightarrow \psi \mathrm{~N})}{\frac{d \sigma}{d t}(\phi \mathrm{~N} \rightarrow \phi \mathrm{~N})} \tag{3.7}
\end{equation*}
$$

An approximate fit to $\phi$ photoproduction can be obtained from Cornell data ${ }^{12}$

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\gamma \mathrm{~N} \rightarrow \phi \mathrm{~N})=(2.85 \pm 0.2) \mathrm{e}^{(5.4 \pm 0.3) \mathrm{t}}\left(\mu \mathrm{~b} / \mathrm{GeV}^{2}\right) \tag{3.8}
\end{equation*}
$$

which integrates to give around 500 nb . In order to estimate how big the ratio of differential cross sections can be in order that the $\psi$ can be considered a hadron we need to do some further estimation. A large cross section would certainly indicate strong interactions but there may be several ways consistent with the hadronic hypothesis that we can have small cross sections. One suggestion discussed by Carlson and Freund ${ }^{13}$ is

$$
\begin{equation*}
\lim _{s \rightarrow \infty} \frac{\sigma_{\text {tot }}^{\mathrm{AP}}}{\sigma_{\text {tot }}^{\mathrm{BP}}}=\left(\frac{\mathrm{m}_{\mathrm{B}}^{2}}{\mathrm{~m}_{\mathrm{A}}^{2}}\right) \tag{3.9}
\end{equation*}
$$

If we neglect the real part of the forward amplitude this should give approximately

$$
\begin{equation*}
\frac{\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{Ap} \rightarrow \mathrm{Ap})}{\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{Bp} \rightarrow \mathrm{Bp})}=\frac{\mathrm{m}_{\mathrm{B}}^{4}}{\mathrm{~m}_{\mathrm{A}}^{4}} \tag{3.10}
\end{equation*}
$$

This means that we would expect the total $\psi \mathrm{N}$ cross section to be on the order of a millibarn and (assuming identical diffractive slopes)

$$
\begin{equation*}
\sigma(\gamma \mathrm{N} \rightarrow \psi \mathrm{~N}) \approx 500\left(\frac{1.019}{3.105}\right)^{4} \mathrm{nb} \approx 5.8 \mathrm{nb} \tag{3.11}
\end{equation*}
$$

An independent estimate of the $\psi$ hadronic cross section which yields a small cross section uses the charmonium model of Appelquist and Politzer ${ }^{11}$ and assumes that the classical Bohr radius gives an estimate of the size of the state. Using

$$
\begin{equation*}
\mathrm{R}_{\text {Bohr }}=\frac{\mathrm{n}^{2}}{\left(4 / 3 \alpha_{\mathrm{s}}\right)\left(\mathrm{m}_{\mathrm{c}} / 2\right)} \tag{3.12}
\end{equation*}
$$

with $\alpha_{S}$ the effective coupling constant in the range $(1 / 3,1)$ we get

$$
\begin{equation*}
\mathrm{R}_{\text {Bohr }}=(0.14-0.48) \mathrm{n}^{2} \mathrm{fm} \tag{3.13}
\end{equation*}
$$

Assuming the $\phi$ is about 1 fm across and $\sigma_{\text {tot }} \propto r^{2}$ then yields the estimate

$$
\begin{equation*}
\sigma(\gamma \mathrm{N} \rightarrow \psi \mathrm{~N})=0.2-26 \mathrm{nb} . \tag{3.14}
\end{equation*}
$$

Within certain contexts we therefore see that it is quite plausible to have small diffractive cross sections and still consider the $\psi$ a hadron.

There is another argument for small hadronic cross sections which has been advanced in the case in which the $\psi^{\prime} s$ are $c \bar{c}$ bound states. This conjecture is that the diffractive scattering of a $\psi$ should be small below inelastic thresholds. The latter must involve charmed particles. This suggestion seems to follow from any picture in which diffractive scattering is the shadow of inelastic channels. The consequence of this argument is that diffractive scattering should be of ordinary hadronic magnitude above these thresholds but far smaller below them. From the unitarity equation we may write, at $t=0$,

$$
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\gamma \mathrm{~N} \rightarrow \psi \mathrm{~N}) \propto\left[\sigma_{\operatorname{tot}}(\gamma \mathrm{N})\right]^{2}+16 \pi\left|\operatorname{Re} \mathrm{M}_{\mathrm{e} \ell}(\psi \mathrm{~N})\right|^{2}
$$

Above inelastic thresholds the imaginary part is large and the real part small. Below threshold, the imaginary part (which is primarily the sum over inelastic channels) will be small. If the real part is also small there then the elastic differential cross section will be small and the simple argument above will be correct. However, there is no guarantee that this is indeed the case. In general, a nondiffractive production process will increase the cross section both above and below its threshold, but the magnitude of the effect below threshold and its dependence upon dynamical parameters are open questions. It is the real part which is responsible for the cross section below threshold and we must ask what determines its magnitude and energy dependence.

We may examine this problem by using a simple (and probably unrealistic) example of an invariant amplitude. Consider an amplitude of the form

$$
\operatorname{Im} M=f_{0} \sqrt{s-s_{c}}\left(s-s_{c}+\Delta\right)^{-1} \theta\left(s-s_{c}\right)
$$

which achieves its maximum value at $s=s_{c}+\Delta$. At general $s$ one finds (neglecting the elastic cut contribution)

$$
\mathrm{R}(\mathrm{~s}) \equiv\left|\frac{\mathrm{M}(\mathrm{~s})}{\operatorname{Im} \mathrm{M}\left(\mathrm{~s}_{\mathrm{c}}+\Delta\right)}\right|=\frac{2 \sqrt{\Delta}}{\sqrt{\Delta}+\sqrt{\mathrm{s}_{\mathrm{c}}-\mathrm{s}}}
$$

Now observe that $R\left(s_{c}+3 \Delta\right)=R\left(s_{c}-\Delta\right)$, and that $R\left(s_{c}\right)=2 \sqrt{\Delta}\left(\sqrt{\Delta}+\sqrt{s_{c}}\right)^{-1}$. Therefore $\operatorname{Re} \mathrm{M}(\mathrm{s})$ is of the same order as $(\mathrm{Im} M)_{\text {max }}$ in the region $\mathrm{s}_{\mathrm{c}}-\Delta \lesssim \mathrm{s} \lesssim \mathrm{s}_{\mathrm{c}}$.

The extent of the region over which $\operatorname{Re} M$ is comparable to ( $\operatorname{Im} M$ ) max is determined by the parameter $\Delta$, which must be specified on dynamical grounds. If the production matrix element is slowly varying, then $\operatorname{Im} M$ will tend to increase until all particles are relativistic, suggesting that $\Delta \sim s_{c}$. While this argument is certainly not definitive, it suggests, except for $t_{\text {min }}$ effects, that $\mathrm{d} \sigma / \mathrm{dt}$ is comparable to $\left[\sigma_{\text {tot }}\left(\mathbf{s}_{\mathbf{c}}+\Delta\right)\right]^{2}$ for all $\mathbf{s} \gtrsim \mathbf{s}_{\mathbf{c}}-\Delta$. That is, that $\mathrm{d} \sigma / \mathrm{dt}$ is large for some distance below threshold. In models where the $\psi$ is a quarkantiquark bound state, the parameter $\Delta$ may be related to the quark mass or to the exchange mass characterizing the production of a charmed pair.

The clear implication of this argument is that if $\Delta$ is large, then the hadronic cross section of $\psi^{\prime} \mathrm{s}$ on other hadrons might be large far below charmed meson production thresholds. This would mean that one may not invoke the threshold argument to justify a small photoproduction cross section.
B. Narrow Pseudoscalar States and Primakoff Photoproduction

If the $\psi(3105)$ and $\psi(3695)$ are hadrons all our experience with hadronic spectroscopy suggests that there should exist pseudoscalar particles related to the vector states. To be explicit we adopt the terminology of charm and consider the existence of a pseudoscalar meson, $\chi$, made from cec just like the $\psi$ 's. One way of looking for such a state or states would be to look for evidence of the electromagnetic transitions

$$
\begin{align*}
& \psi(3105) \rightarrow \chi+\gamma  \tag{3.15}\\
& \psi(3695) \rightarrow \chi+\gamma
\end{align*}
$$

by searching for nearly monochromatic photons emerging from $\psi$ decays. If, however, there are substantial other one-photon decays of the $\psi$ 's it may not be possible to get sufficient resolution on $\gamma$ energies in order to detect the $\chi$.

What we would like to investigate here is the possible Primakoff photoproduction of a $\chi$ detected by its subsequent decay into $2 \gamma^{\prime}$ s. We will briefly comment on the Primakoff photoproduction of other heavy states with decays of the form $\mathrm{A} \rightarrow$ hadron $+\gamma$.

Define

$$
\begin{equation*}
\Delta_{0}=\frac{\mathrm{M}_{\psi}^{2}}{2 \mathrm{~K}} \tag{3.16}
\end{equation*}
$$

which is the minimum momentum transfer for the reaction $\gamma \mathrm{N} \rightarrow \chi \mathrm{N}$. The formula for Primakoff photoproduction at high energy off a nucleus with charge Z i

$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Delta^{2}}=\mathrm{z}^{2} \frac{\Delta_{\mathrm{T}}^{2}}{\left(\Delta_{0}^{2}+\Delta_{\mathrm{T}}^{2}\right)^{2}} 8 \pi \frac{\Gamma_{\chi} \rightarrow 2 \gamma}{\mathrm{~m}_{\chi}^{3}}\left|\Gamma\left(\Delta^{2}\right)\right|^{2}
$$

where $\Delta^{2}=\Delta_{T}^{2}+\Delta_{L}^{2}, \Delta_{T} \cong K \theta, \Delta_{L} \cong \Delta_{0}$ are momentum transfers. To evaluat this formula we need some estimate of the range of masses considered likely fc
$\chi$. This obviously depends on some theoretical model. Using $\operatorname{SU}(4)$ or charmonium ideas we get a range of masses, $m_{\chi} \in(2.2-3.1)$. This translates into the values of $\Delta_{0}$ plotted in Fig. 12. The next thing which needs to be estimated is the width $\Gamma_{\chi \rightarrow 2 \gamma^{\prime}}$. We have these estimates from

$$
\begin{align*}
& \Gamma_{\pi^{o} \rightarrow 2 \gamma}=7.8 \pm .9 \mathrm{eV} \\
& \Gamma_{\eta \rightarrow 2 \gamma}=374 \pm 60 \mathrm{eV}  \tag{3.18}\\
& \Gamma_{\eta^{\prime} \rightarrow 2 \gamma} \leq 19 \pm 3 \mathrm{keV}
\end{align*}
$$

Tentative guesses are that the width should scale as $\mathrm{m}^{3}$ and one finds

$$
\begin{equation*}
\Gamma_{\chi \rightarrow 2 \gamma} \epsilon(24-340) \mathrm{keV} \tag{3.19}
\end{equation*}
$$

For a rough estimate of the size of the cross section we can use in Eq. (3.17) the form factor for a nucleus of radius $R$ with uniform density

$$
\begin{equation*}
\Gamma(\Delta)=\frac{3(\sin \Delta R-\Delta R \cos \Delta R)}{(\Delta R)^{3}} \tag{3.20}
\end{equation*}
$$

Assuming a 50 keV width into $\gamma \gamma$, the Primakoff production off Pb is estimated in Figure 13. With a substantial branching ratio into detectable decay mode $2 \gamma$ the production of $\chi^{\prime}$ s should be marginally detectable at SLAC energies and result in a clear signal at NAL energies. Some rough estimates of the $\pi^{\circ}$ $\pi^{\circ}$ background needs to be made but this appears to be an interesting signal.

In color schemes there should be a substantial number of narrow states with a $\pi^{ \pm} \gamma$ decay mode. These can be produced with sizable cross sections via the Primakoff effect in high energy $\pi$ beams. The detection of these resonances depends sensitively on the background effects and will not be examined here in detail.


FIG. 12--Minimum momentum transfer in photoproduction of $\eta$ 's and $\chi$ 's.


FIG. 13--Primakoff photoproduction of pseudoscalar $\chi$ off Pb .
C. Photoproduction Without $\psi$ Vector Dominance

If the $\psi$ particles do not couple directly to photons, then the diffractive photoproduction mechanism discussed above does not exist. If the $\psi^{\prime}$ s are not hadrons then their cross sections on ordinary hadrons will be small on the usual scale. In either case, observation of $\psi$ production by photons incident on hadrons requires another explanation.

The situation in which the $\psi^{\prime}$ s do not have appreciable hadronic interaction is of particular interest. Indeed, it is the implication of Section II that the total $\psi$-hadron cross section may be small. Using the effective couplings derived there we can relate the total $\psi$-proton cross section to the total photon-proton cross section:


Since $\sigma_{\gamma p}^{\text {tot }}=10^{-28} \mathrm{~cm}^{2}$, the total cross section for a $\psi$ on a proton is about

$$
\sigma_{\psi \mathrm{p}}^{\mathrm{tot}} \approx 1 \mu \mathrm{~b}
$$

The elastic $\psi$-proton cross section is even smaller. Assuming that the elastic scattering slopes for photons and $\psi^{\prime}$ s are comparable, we can compute the ratio of elastic cross sections from unitarity:

$$
\begin{equation*}
\frac{\sigma_{\psi p}^{\mathrm{el}}}{\sigma_{\gamma \mathrm{p}}^{\mathrm{el}}} \approx\left(\frac{\sigma_{\psi \mathrm{p}}^{\mathrm{tot}}}{\sigma_{\gamma \mathrm{p}}^{\mathrm{tot}}}\right)^{2} \approx 2 \times 10^{-4} \tag{3.22}
\end{equation*}
$$

Since $\sigma_{\gamma \mathrm{p}}^{\mathrm{el}}$ is about 0.1 microbarn, we then have a $\psi$-p elastic cross section of about 0.02 nb . The inclusive cross section $\psi+\mathrm{p} \rightarrow \psi+\mathrm{X}$ is presumably somewhere between the elastic and the total cross section. Inserting the coupling obtained in Section II, assuming that the photon couples to the $\psi$, then gives us a diffractive photoproduction cross section. If the total cross section sets the scale, the photoproduction cross section is about 1 nb . If the elastic $\psi$-p cross section sets the scale then the photoproduction cross section is only about $10^{-5} \mathrm{nb}$. Observation of appreciable photoproduction then demands that the $\psi$ be produced in some other way.

One evident possibility is that the photon turns into an ordinary vector meson (or a quark-antiquark pair) which then interacts with the hadron target to produce the $\psi$ in a fashion entirely analogous to its production in protonproton (-neutron) collisions. Such a process is shown in Fig. 14


We can estimate the cross section by our now familiar technique of taking ratios:


We then obtain

$$
\sigma_{\gamma \mathrm{p} \rightarrow \psi \mathrm{p}}^{\mathrm{VMD}} \approx\left(10^{-3}\right) \times \sigma_{\gamma \mathrm{p} \rightarrow \phi \mathrm{p}}^{\mathrm{VMD}} \approx 0.5 \mathrm{nb} .
$$

Summing over inelastic final states will give a larger inclusive $\psi$ photoproductic cross section. This process is thus potentially far larger than the diffractive photoproduction process. The evident conclusion is that we should not assume that photoproduction is diffractive and then, on the basis of this assumption, assert that the $\psi$-proton cross section must be of hadronic magnitude. The data may be easily explainable even if the coupling of the photon to the $\psi$ is identicall: zero.
IV. THE $\psi^{\prime}$ (3.7) AND $\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{p} \overline{\mathrm{p}}$ AND $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}$. A POSSIBLE LINK.

During the London conference data was presented for $\mathrm{p} \overline{\mathrm{p}}$ elastic scattering at fixed angle and $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}$at various angles, at beam momenta of 5 and 6.2 GeV . A striking feature is that at $90^{\circ}$ the differential cross section $\mathrm{d} \sigma / \mathrm{dt}$ for the elastic scattering is the same for both values of beam momenta, in contradiction to a simple extrapolation of the $\overline{\mathrm{p}} \mathrm{p}$ data from lower energies, and the theoretical expectations of the CIM and the dimensional counting laws of Brodsky and Farrar ${ }^{14}$ which seem to work for other reactions. These rules predict a $s^{-10}$ behavior of the cross section, which in this case would mean a drop of around an order of magnitude as one goes from the lower to the higher energy. A lab momentum of 6.2 GeV is equivalent to $\sqrt{\mathrm{s}}=3.68$ and hence within the experimental resolution they may have been right on the $\psi^{\prime}$, which would thus be responsible for the anomalously high cross section. Below we take this to be the case under the assumption that the $\psi^{\prime}$ is a spin 1 particle with a pure vector coupling to the $\mathrm{p}-\overline{\mathrm{p}}$. This enables us to estimate the necessary branching ratio of $\psi^{\prime}$ in $p \bar{p}$. Similar assumptions for $p \bar{p} \rightarrow \pi^{+} \pi^{-}$enable us to estimate the branching ratio of $\psi^{\prime}$ into $\pi^{+} \pi^{-}$, but in this case there is not data around $90^{\circ}$ so we have to make extrapolations which may be quite wrong. We have to await better data in order to draw any final conclusions. However, we expect our estimate to be at least of the correct order of magnitude for this reaction.

In any case, we shall see that the rapid fall-off of hadronic (strong) interactions at large angles and large energies can allow interesting upper bounds to be placed on the rather weak couplings of particles such as the $\psi^{+}(3.7)$, and perhaps heavier ones. In the present case, bounds can be obtained (if data is known at the right energy) for particles of any quantum numbers that can couple to a $\mathrm{p} \overline{\mathrm{p}}$ system.

NOTE:
It may be interesting to explore $\mathrm{K}^{-} \mathrm{p}$ scattering, as well, to extract properties of possible strange baryonic resonances, since cross sections fall rapidly beyond $90^{\circ}$ with a very small backward hadronic peak just as in $\overline{\mathrm{p} p}$.

Our model is then that $\mathrm{p} \overrightarrow{\mathrm{p}}$ elastic scattering takes place via a $\psi^{\prime}$ in the s-channel


We, of course, assume that interference effects between hadronic and resonance mechanisms are negligible, and try to derive an upper bound to $\Gamma_{p \bar{p}}$. We then have

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}=\frac{1}{64 \pi \mathrm{sp}^{2}} \times \frac{1}{4} \times \frac{1}{\left[\left(\mathrm{~s}-\mathrm{M}^{2}\right)^{2}+\mathrm{M}^{2} \Gamma^{2}\right]} \mathrm{g}_{\mathrm{p}}^{4} \mathrm{I}^{\mu \nu} \mathrm{F}_{\mu \nu} \tag{4.1}
\end{equation*}
$$

where

$$
\mathrm{I}^{\mu \nu}=\operatorname{Tr}\left[\left(p_{1}+\mathrm{m}\right) \gamma^{\mu}\left(\phi_{2}-\mathrm{m}\right) \gamma^{\nu}\right]
$$

and

$$
\mathrm{F}_{\mu \nu}=\operatorname{Tr}\left[\left(\mathfrak{q}_{1}+\mathrm{m}\right) \gamma_{\mu}\left(q_{2}-\mathrm{m}\right) \gamma_{\nu}\right]
$$

$M$ is the mass of the $\psi^{\prime}, m$ the proton mass and $p$ is the center of mass 3momentum. An evaluation of $I^{\mu \nu} \mathrm{F}_{\mu \nu}$ gives

$$
\begin{equation*}
\mathrm{I}^{\mu \nu} \mathrm{F}_{\mu \nu}=8\left[\mathrm{t}^{2}+\mathrm{u}^{2}+8 \mathrm{~m}^{2} \mathrm{M}^{2}-8 \mathrm{~m}^{4}\right] \tag{4.2}
\end{equation*}
$$

The coupling constants $\mathrm{g}_{\mathrm{p}}$ can be translated into the width for $\psi^{\prime} \rightarrow \mathrm{p} \overline{\mathrm{p}}$ via the relation $\Gamma_{p}=\frac{\mathrm{Mg}^{2}}{12 \pi}$ (which is good to $2 \%$ even allowing for the nonzero mass of the proton), and the Breit-Wigner denominator can be eliminated by integrating both sides of Eq. (4.1) over the resolution of the machine, which is quoted as 20 MeV in mass (or $1 / 7 \mathrm{GeV}^{2}$ in s ). The differential cross section $\mathrm{d} \sigma / \mathrm{dt}$ at $90^{\circ} \sim 1 \times 10^{-32} \mathrm{~cm}^{2} / \mathrm{GeV}^{2}$. This gives the bound

$$
\begin{equation*}
\frac{1}{7} \times \frac{d \sigma}{d t}\left(90^{\circ}\right) \geq \frac{36 \pi^{2}}{\mathrm{p}^{2}}\left[\mathrm{p}^{4}+\mathrm{m}^{2} \mathrm{M}^{2}-\mathrm{m}^{4}\right] \frac{\Gamma_{\mathrm{p} \overline{\mathrm{p}}}^{2}}{\Gamma_{\text {tot }}} \frac{1}{\mathrm{M}^{5}} \tag{4.3}
\end{equation*}
$$

Evaluating the numerics gives .

$$
\begin{equation*}
\frac{\Gamma_{\mathrm{p} \overline{\mathrm{p}}}^{2}}{\Gamma_{\mathrm{T}}} \leq 1.03 \times 10^{-6} \mathrm{GeV} \tag{4.4}
\end{equation*}
$$

Taking $\Gamma_{\mathrm{T}}=1 \mathrm{MeV}$ gives for the branching ratio

$$
\begin{equation*}
\frac{\Gamma_{\mathrm{p} \bar{p}}}{\Gamma_{\mathrm{T}}} \lesssim 3.2 \% \tag{4.5}
\end{equation*}
$$

With an axial vector coupling this quantity is about $1 \%$.
Allowing for a different width, experimental cross section, or resolution, we can write
$\frac{\Gamma_{\mathrm{p}}}{\Gamma_{\mathrm{T}}} \lesssim 3.2 \times 10^{-2} \times \sqrt{\frac{1 \mathrm{MeV}}{\Gamma_{\mathrm{T}}}} \times \sqrt{\frac{\mathrm{d} \sigma}{\mathrm{dt}} / 10^{-32}} \mathrm{~cm}^{2} / \mathrm{GeV}^{2} \times \sqrt{\frac{7 \Delta s}{1 \mathrm{GeV}^{2}}}$
We do not expect the numbers in the square roots to vary very much from the values given above so that $3.2 \%$ is a good estimate of the branching ratio if the $\psi^{\prime}$ is to be responsible for this cross section in the manner outlined above.

We now turn to $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}$. We can write

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}\right)=\frac{1}{64 \pi \mathrm{sp}^{2}} \frac{1}{4} \frac{\mathrm{~g}_{\mathrm{p}}^{2} \mathrm{~g}_{\pi}^{2}}{\left(\mathrm{~s}-\mathrm{M}^{2}\right)^{2}+\mathrm{M}^{2} \Gamma^{2}} \mathrm{I}^{\mu \nu} \mathrm{G}_{\mu \nu} \tag{4.7}
\end{equation*}
$$

where

$$
\mathrm{G}_{\mu \nu}=\left(\mathrm{q}_{1}-\mathrm{q}_{2}\right)_{\mu}\left(\mathrm{q}_{1}-\mathrm{q}_{2}\right)_{\nu}
$$

and

$$
\bar{I}^{\mu \nu} G_{\mu \nu}=2 M^{4}-8 p^{2} \mathrm{~s} \cos ^{2} \theta
$$

This means that

$$
\begin{equation*}
\frac{\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}\right)}{\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{p} \overline{\mathrm{p}})}=\frac{\mathrm{g}_{\pi}^{2}}{\mathrm{~g}_{\mathrm{p}}^{2}} \frac{\mathrm{I}^{\mu \nu} \mathrm{G}_{\mu \nu}}{\mathrm{I}^{\mu \nu} \mathrm{F}_{\mu \nu}} \tag{4.8}
\end{equation*}
$$

The coupling constants may again be rewritten in terms of widths, and the rest of the right hand side can be evaluated numerically to give

$$
\begin{equation*}
\frac{\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\pi^{+} \pi^{-}\right)}{\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{p} \overline{\mathrm{p}})}=1.3 \frac{\Gamma_{\pi}}{\Gamma_{\mathrm{p}}} \frac{1-0.74 \cos ^{2} \theta}{1+0.38 \cos ^{2} \theta} \tag{4.9}
\end{equation*}
$$

As stated above the $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{+} \pi^{-}$cross section has not been measured experimentally at $90^{\circ}$ but extrapolations from smaller angles would indicate that it is comparable, perhaps slightly larger than the elastic cross section. This would indicate that the branching ratio into $\pi^{+} \pi^{-}$is similar to that into $\mathrm{p} \overline{\mathrm{p}}$; i.e., of the order of a few percent. However better data is required before the ratio $\Gamma_{\pi} / \Gamma_{p}$ can be determined accurately from Eq. (4.9).

Even if the branching ratio into $p \bar{p}$ is measured to be much smaller and the $\psi^{\prime}$ has nothing to do with the above processes, it is still possible that higher mass particles would contribute significantly in $\bar{p} \bar{p}$ elastic scattering because of the rapid fall with $s$ of the usual strong hadronic contribution at large angles.

The branching ratio of the $\psi(3695)$ into $\bar{p} \bar{p}$ estimated here is quite large. One reason to believe that this might be possible would be to associate the $\psi^{\prime} s$ with the exotic states envisioned by Rosner ${ }^{16}$ which couple to $B \bar{B}$ channels. The
$\psi(3105)$ could couple strongly to, for example, $\Delta \boldsymbol{J}$ and the $\psi(3695)$ to $\overline{\mathrm{p}}$. . It is important to remember, however, that there are other resonances expected in the $p \bar{p}$ channel around this energy and the identification of the entire effect with the $\psi(3695)$ is highly speculative and should be taken as a bound only.

Another possibility is to assume that the anomalously large $\mathrm{p} \overline{\mathrm{p}}$ cross section is not due to the $\psi^{\prime}$, but to its related $\mathrm{O}^{-}$state. Models in which the $\left(1^{-}\right) \psi^{\prime}$ is a $c \bar{c}$ bound state have its little brother $\mathrm{O}^{-}$mass a little smaller than the $1^{-}$ ( 55 MeV in the model of Appelquist and Politzer). Recalculating the elastic scattering as above under the same assumptions about resolution as before, we obtain

$$
\begin{equation*}
\frac{\Gamma_{\mathrm{p} \overline{\mathrm{p}}}^{2}}{\Gamma_{\text {tot }}}=2.6 \times 10^{-3} \mathrm{MeV} \tag{4.10}
\end{equation*}
$$

which gives

$$
\begin{equation*}
\frac{\Gamma_{\mathrm{p}} \bar{p}}{\Gamma_{\text {tot }}}=5.1 \times 10^{-2} \times\left[\Gamma_{\text {tot }} / 1 \mathrm{MeV}\right]^{-1 / 2} \tag{4.11}
\end{equation*}
$$

Hence if the width of the $\mathrm{O}^{-}$is comparable to that of the $\psi^{\prime}$ the branching ratio into $\mathrm{p} \overline{\mathrm{p}}$ will also be of the order of a few percent.

## V. DISCUSSION

The hope of the authors when starting this workshop was that the various theoretical models presented to explain the $\psi$ and $\psi^{\prime}$ could be distinguished on the basis of simple experiments which could be quickly performed and analyzed. This hope did not stand up well during the course of the analysis of the models done in the workshop. Although we do not see a definitive test of all of the models, we can say some things.

Our analysis of the BNL-MIT experiment has shown that $\psi$ production is compatible with a photon-like production through mechanisms such as bremsstrahlung, Drell-Yan, or perhaps some sum of it and other constituent interchange model processes. These alternatives could most easily be distinguished by an analysis of the energy dependence of the production cross section. On the other hand, truly hadronic associated production has not been ruled out either. Here too the energy dependence would be important, as well as an identification of the observed final states. In a c $\bar{c}$ model for the $\psi$, we expect the cross section to change markedly as we pass the threshold for associated production of charmed-anticharmed states, and detailed knowledge here would strongly restrict models of the production matrix element and hence also models of the $\psi^{\prime} \mathrm{s}$.

One original assumption of ours which did not stand up to careful examination was the idea that photoproduction of the $\psi$ would be able to answer definitively whether or not it was a hadron. We found, within the context of vector dominance, various reasons, consistent with the hypothesis that the $\psi$ 's are hadrons, why the cross section $\sigma_{\text {tot }}(\psi$ p) could be below a millibarn. This translates into an expected photoproduction cross section below 5 nanobarns. At this level of
cross section we also found several mechanisms which could photoproduce the $\psi$ even though it does not interact strongly.

The idea that the $\psi^{\prime}$ s are c $\bar{c}$ bound states is currently attractive but will only be fully viable when charmed vector mesons are found. The search for charmed particles is probably one of the most interesting paths to pursue experimentally at this point. Various signals have already been discussed by Lee, Gaillard, and Rosner ${ }^{15}$ and we have very little to add to that here. The discovery of charm will probably be dramatic. It becomes a very difficult dynamical question, however, to decide at what point the absence of the experimental evidence for charmed particles will begin to rule out the cex explanation for the $\psi^{\prime} \mathrm{s}$.

In any hadronic model for the $\psi^{\prime}$ 's we would expect neutral pseudoscalar mesons with a fairly substantial branching ratio into $\gamma \gamma$. This would imply that the meson can be produced via Primakoff photoproduction at high energy. At some level, the observation of narrow pseudoscalars is absolutely essential for the interpretation of the $\psi^{\prime}$ s as hadrons. There are many roles outside of the strong interactions that vector or axial vector particles can play in various forms of gauge theories while one does not expect a priori any pseudoscalars that do not interact strongly.

Since $\psi$ 's that carry a new quantum number such as color can only be produced in associated production with a photon or another colored hadron, one way to examine this option is to look carefully at the final state or to look at exclusive channels. Furthermore, the reaction $\pi^{-} p \rightarrow \psi(3105) \mathrm{n}$ has been discussed in detail. Observation of this reaction at a reasonable rate would certainly support the analogy of the $\psi$ and the $\phi$ and rule out any sort of new conserved quantum number carried by the $\psi$.

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