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## SURVEY OF THEORETICAL SPECULATIONS ON THE NATURE OF $\psi(3105)$ AND $\psi(3695)^{*}$.

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#### Abstract

We discuss various theoretical schemes for the recently discovered massive long-lived particles $\psi(3105)$ and $\psi(3695)$.

We concentrate, in particular, on the merits and difficulties of supposing that these particles are colored vector mesons, charm-anticharm hadron states or intermediate weak bosons. Various critical experimental tests are suggested.


[^0]I. Introduction

The discovery of massive particles with extremely narrow widths at SLAC $^{1,2}$ and Brookhaven ${ }^{3}$ has resulted in a myriad of theoretical speculations on what their true nature may be. The purpose of this report is to discuss, in as cogent and orderly a fashion as possible, various theoretical schemes which may account for the existence of these particles. We shall divide the theoretical models to be discussed into three broad classes. To wit, models in which these particles are:

1. colored vector mesons
2. charm-anticharm hadron states
3. intermediate weak bosons.

It is possible that the $\psi(3105)$ and the $\psi(3695)$ belong to two different categories of the above. We shall not consider such a possibility further here. (It is also possible that they are something else - but what?)

As we shall see, none of the theoretical explanations for the $\psi(3105)$ and $\psi(3695)$ will be truly successful by themselves. Hence, it becomes imperative to obtain other experimental confirmation for each of the theoretical schemes considered. Several possible critical tests of the various models will be suggested.

## II. Experimental Information

We list below various relevant physical parameters of the $\psi(3105)$ and $\psi(3695)$ which have been deduced from the data of Refs. 1 and 3. The extraction of the natural widths of these particles and of their partial widths into hadrons and leptons is discussed in more detail in the report of the QED Subgroup of the SLAC Workshop. ${ }^{4}$ The spin parity of the $\psi^{\prime}$ 's is not yet known, although it probably is $1^{-}$.
$\psi(3105)$

$$
\begin{aligned}
& \mathrm{M}=3.105 \mathrm{GeV} \\
& \Gamma_{\mathrm{h}} \cong 90 \mathrm{keV} \\
& \Gamma_{\mathrm{e}} \cong 5.5 \mathrm{keV}
\end{aligned}
$$

$\Psi^{r}(3695)$

$$
\begin{aligned}
& \mathrm{M}=3.695 \mathrm{GeV} \\
& \Gamma_{\mathrm{h}} \gtrsim 200 \mathrm{keV} \\
& \Gamma_{\mathrm{e}} \cong 2.5 \mathrm{keV}
\end{aligned}
$$

The $\psi(3105)$ has also been seen ${ }^{3}$ in the process $p+B e \rightarrow e^{+}+e^{-}+x$. The cross section on Be, assuming a production cross section $\frac{d \sigma}{d p} \sim e^{-6 \mathrm{p}} \perp$ for 28.5 $\mathrm{GeV} / \mathrm{c}$ incident photons, is quoted to be approximately $10^{-34} \frac{1}{\mathrm{~cm}}$ / nucleon. No evidence seems to have been found in this experiment for the $\psi(3695)$. The smaller branching ratio of $\Gamma_{\mathrm{e}} / \Gamma_{\mathrm{h}}$ for the $\psi(3695)$ together with threshold factors limiting the phase space and presumably suppressing its production in this process permit a natural explanation for this fact.

There exists an experimental bound from SLAC on the photoproduction of the $\psi(3105)$ :

$$
\sigma_{\gamma \mathrm{N}} \rightarrow \psi \mathbb{N} \leq 29 \mathrm{nb}
$$

Another important piece of information is that the decay of $\psi(3695)$ into $\psi(3105)$
plus two charged pions has been seen with a branching ratio of perhaps $40 \%$.

## III. The Color Hypothesis

The narrow widths of the $\psi(3105)$ and $\psi(3695)$ suggest that some conservation law is at work. A popular possibility is that these particles are the first observed "colored" vector mesons. Their decay iṣ suppressed because "color" is conserved or almost conserved.

We recall that the idea of color was originally introduced to avoid having the lowest baryon state be described by a totally symmetric quark wave function. The relevant invariance group of the strong interaction is enlarged from $\operatorname{SU}(3)$ (or $\operatorname{SU}(4)$ if one believes in charm) to $\mathrm{SU}(3) \times \mathrm{G}_{\text {color }}$. The choice for $G_{\text {color }}$ is largely arbitrary but the most reasonable choice appears to be again $\mathrm{SU}(3)$. Color models where the invariance group is $\mathrm{SU}(3) \times \mathrm{SU}(3)$ ' include the red, white and blue fractionally charged quark model of Gell-Mann and Zweig and the integrally charged Han-Nambu model. Other models are certainly possible, but not as attractive. For example, one can consider an $\mathrm{SU}(3) \times \mathrm{O}(3)$ model.

An important distinction between different types of color models is whether or not colored hadrons exist. In the red, white and blue model, all hadrons and the electromagnetic current are color singlets. In the Han-Nambu model, colored hadrons exist and the electromagnetic current itself carries color. Clearly only models of this latter type are of interest here.

From now on, for definitiveness, we shall restrict ourselves to the $S U(3) \times \operatorname{SU}(3)^{\prime}$ Han-Nambu model. In this model the electric charge is given by

$$
\mathrm{Q}=\mathrm{I}_{3}+\frac{1}{2} \mathrm{Y}+\mathrm{I}_{3}^{\prime}+\frac{1}{2} \mathrm{Y}^{\prime}
$$

where $\overrightarrow{\mathrm{I}}$ and $\mathrm{Y}^{\prime}$ are the "isospin" and the "hypercharge" of the color $\mathrm{SU}(3)^{\prime}$ group. It is clear from the above that the electromagnetic current transforms in this scheme according to the $(1,8)+(8,1)$ representation of $\mathrm{SU}(3) \times \mathrm{SU}(3)^{\text {' }}$.

If we let the quarks transform according to a $(3, \overline{3})$ representation of $\mathrm{SU}(3) \times \mathrm{SU}(3)^{\prime}$ then they will have integral charges. Explicitly one has a charge matrix

$$
\left[\begin{array}{lll}
\rho_{1} & \mathrm{p}_{2} & \mathrm{p}_{3} \\
\mathrm{n}_{1} & \mathrm{n}_{2} & \mathrm{n}_{3} \\
\lambda_{1} & \lambda_{2} & \lambda_{3}
\end{array}\right]=\left[\begin{array}{rrr}
0 & 1 & 1 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{array}\right]
$$

where the quarks with subscripts $i=(1,2,3)$ have color quantum numbers $\mathrm{I}_{3}^{\prime}=-\frac{1}{2}, \mathrm{Y}^{\prime}=-\frac{1}{3} ; \mathrm{I}_{3}^{\prime}=\frac{1}{2}, \mathrm{Y}^{\mathbf{y}}=-\frac{1}{3} ;$ and $\mathrm{I}_{3}^{\prime}=0, \mathrm{Y}^{\mathbf{\prime}}=\frac{2}{3}$ respectively. The $\mathrm{SU}(3)$ quantum numbers for ( $\mathrm{p}, \mathrm{n}, \lambda$ ) are $\mathrm{I}_{3}=\frac{1}{2}, \mathrm{Y}=\frac{1}{3} ; \mathrm{I}_{3}=-\frac{1}{2}, \mathrm{Y}=\frac{1}{3}$; and $\mathrm{I}_{3}=0$, $Y=-\frac{2}{3}$ respectively.

By assumption the lowest baryon and meson states are color singlets.
For example one writes in the quark mnemonic

$$
\pi^{+}=\frac{1}{\sqrt{3}}\left(p_{1} \overline{\mathrm{n}}_{1}+\mathrm{p}_{2} \overline{\mathrm{n}}_{2}+\mathrm{p}_{3} \overline{\mathrm{n}}_{3}\right)
$$

Besides these color singlet states the Han-Nambu scheme allows for color nonsinglet states. These states are presumed to have a higher mass than the singlet states because of dynamical properties of the quark interaction. Below we shall discuss the hypothesis that the $\psi(3105)$ and the $\psi(3695)$ are indeed colored mesons.

If the $\psi$ 's are colored hadrons their coupling to leptons is indirect and it occurs because of mixing with the photon (see Fig. 1).


Fig. 1

Since the photon belongs to the $(1,8)+(8,1)$ representation of $\mathrm{SU}(3) \times \mathrm{SU}(3)^{\prime}$ it follows that the $\psi^{\prime}$ s must transform (at least in part) according to the $(1,8)$ representation. That is, they are color octets. Various possibilities are now open. Among these we shall consider that:

1. $\psi(3105)$ and $\psi(3695)$ belong to the same color octet $(1,8)$ and thus color is not an exact symmetry of the strong interactions.
2. $\psi(3105)$ and $\psi(3695)$ belong to different color octets $(1,8)$ which are ordinary $\mathrm{SU}(3)$ singlets. Thus, color is not broken.
3. $\psi(3105)$ and $\psi(3695)$ are members of color octets but are not pure singlet $\mathrm{SU}(3)$ states - they are a mixture of octet and singlet in ordinary $\mathrm{SU}(3)$ (that is, $(1,8)+(8,8))$. This last possibility is the most natural from the point of view of $\operatorname{SU}(6) \times$ color, and color is not necessarily broken.
(1) Let us consider the first case. If color is broken the photon can couple to two states of each color octet, the ones with $I_{3}^{\prime}=0, Y^{\prime}=0$ and either $\overrightarrow{\mathrm{I}^{\prime}}=1$ or $\overrightarrow{\mathrm{I}^{\prime}}=0$ (analogous to the $\rho^{o}$ and $\omega_{8}$ in $\mathrm{SU}(3)$ ). From this viewpoint the presence of both $\psi^{\prime}$ 's is natural. However the amount of color breaking is sizable. One has

$$
\frac{\Delta \mathrm{M}^{2}}{\mathrm{M}^{2}}=\frac{(3.7)^{2}-(3.1)^{2}}{(3.7)^{2}} \approx 30 \%
$$

It is difficult to reconcile such large breaking of color in the masses with the extremely narrow widths of these particles. (Remember however that, in usual $\mathrm{SU}(3)$, the mass breaking is a lot larger than the coupling breaking.) One could suppress the decay of one of the particles by $\overrightarrow{I^{\prime}}$ conservation in addition to color symmetry. However, decay of the other particle is not suppressed since it has $\overrightarrow{I^{\prime}}=0$. Furthermore the decay

$$
\psi(3695) \rightarrow \psi(3105)+\pi^{+}+\pi^{-}
$$

violates $I^{\prime}$. Thus it becomes difficult to understand both $\psi^{\prime}$ 's within this first model.

We should note for later purposes that experimentally the above decay is suppressed relative to a typical strong transition such as $\rho^{\prime}(1600) \rightarrow \rho+\pi^{+}+\pi^{-}$ by one to two orders of magnitude. Taking the suppression factor to be $10^{-2}$ and the known value of the $\rho^{\prime} \rho \pi \pi$ coupling constant, we then obtain a decay width $\Gamma_{\psi^{\prime} \rightarrow \psi \pi \pi}$ of the correct order of magnitude

$$
\begin{aligned}
\Gamma_{\psi^{\prime} \rightarrow \psi \pi \pi} & =(0.01) \Gamma_{\rho^{\prime} \rightarrow \rho \pi^{+} \pi^{-}} \frac{(\text { phase space }) \psi^{\prime}}{(\text { phase space }) \rho^{\prime}} \\
& \sim 0.01 \times 200 \times 0.1 \mathrm{MeV} \\
& \sim 200 \mathrm{keV}
\end{aligned}
$$

(2) The second possibility avoids all difficulties associated with color symmetry breaking by assuming that $\psi(3695)$ and $\psi(3105)$ are in different color multiplets. With this assignment it is natural to suppose color is not broken, or broken only slightly. Note however that electromagnetism would give at least $1-2 \%$ splitting, which is of the order of $30-60 \mathrm{MeV}$. If there is no other splitting we would expect only the $\left(\frac{\sqrt{3}}{2} \mathrm{~V}_{3}+\frac{1}{2} \mathrm{~V}_{8}\right)$ member of the octet to mix with the photon and couple to leptons. The other members do not couple to leptons electromagnetically, but they may couple weakly. The color octet to which the $\psi(3695)$ belongs is presumably a radial or orbital excitation of the $\psi(3105)$ color octet. This is perhaps not so unnatural if one recalls the $\rho, \rho^{\prime}$ example.

A direct prediction of this scheme is that one should find charged partners to both the (3105) and (3695). These charged states should have masses at most 100 MeV away from the neutral mass values. Electromagnetism will probably split the $\overrightarrow{I^{\prime}}=1, Y^{\prime}=0$ states and the $\overrightarrow{I^{\prime}}=\frac{1}{2}, Y^{\prime}= \pm 1$ states differently so that
one should see doublets of positively and negatively charged states.
(3) The third color scheme which we want to consider makes use of ordinary $\operatorname{SU}(3)$ singlet and octet mixing. The ordinary $\phi$ and $\omega_{\text {a }}$ are not pure $\operatorname{SU}(3)$ states. Rather one has a mixing

$$
\begin{aligned}
\omega & =\cos \theta \phi_{8}+\sin \theta \phi_{1} \\
\phi & =-\sin \theta \phi_{8}+\cos \theta \phi_{1}
\end{aligned}
$$

This follows from the (35) representation of $\mathrm{SU}(6)$. In quark models the mixing angle is such that $\phi=\lambda \bar{\lambda}$ and $\omega=\frac{1}{\sqrt{2}}(\mathrm{p} \overline{\mathrm{p}}+\mathrm{n} \overline{\mathrm{n}})$. The idea is that $\psi(3695)$ and $\psi(3105)$ are also mixtures of ordinary $\mathrm{SU}(3)$ singlets and octets while being colored octets. That is, they are mixtures of $(1,8)$ and $(8,8)$ representations. This would follow again if we classify them in a $(35,8)$ representation of $\mathrm{SU}(6) \times$ color. Therefore, a natural hypothesis is that they are the same mixture as the $\omega$ and the $\phi$. Then

$$
\begin{aligned}
\psi^{a}(3105) & =\cos \theta \psi_{(8,8)}^{a}+\sin \theta \psi_{(1,8)}^{a} \quad a=1, \ldots, 8 . \\
\psi^{a}(3695) & =-\sin \theta \psi_{(8,8)}^{a}+\cos \theta \psi_{(1,8)}^{a} \\
\tan \theta & =\sqrt{2}
\end{aligned}
$$

We could use the suggestive notation (both usual isospin $I=0$ and $G=-1$ )

$$
\psi^{a}(3105) \equiv \omega^{a} \quad \psi^{a}(3695) \equiv \phi^{a} \quad a=1, \ldots, 8
$$

The other unmixed members of the $(8,8)$ are written then as

$$
\mathrm{I}=1: \vec{\rho}^{\mathrm{a}} ; \quad \mathrm{I}=\frac{1}{2}: \mathrm{K}^{*^{\mathrm{a}}}, \overline{\mathrm{~K}}^{*^{\mathrm{a}}} \quad \mathrm{a}=1, \ldots, 8
$$

The mixing to the photon comes in the $(1,8) u^{\prime}-$ spin singlet combination

$$
\frac{\mathrm{em}^{2}}{\mathrm{f}} \mathrm{~A}^{\mu}\left(\psi_{\mu(1,8)}^{3}+\frac{1}{\sqrt{3}} \psi_{\mu(1,8)}^{8}\right)=\frac{\mathrm{em}^{2}}{\mathrm{f}} \mathrm{~A}^{\mu}\left[\frac{2 \sqrt{2}}{3}\left(\frac{\sqrt{3}}{2} \omega_{\mu}^{3}+\frac{1}{2} \omega_{\mu}^{8}\right)+\frac{2}{3}\left(\frac{\sqrt{3}}{2} \phi_{\mu}^{3}+\frac{1}{2} \phi_{\mu}^{8}\right)\right]
$$

Note that $\vec{\rho}^{\mathrm{a}}$ cannot mix with the photon.

If $\mathrm{SU}(3)^{\prime}$ is broken only electromagnetically or if there is further breaking only in the photon direction $\left({ }^{-2} 1_{1}\right)$, then the $u^{\prime}$-spin singlet states $\left(\frac{\sqrt{3}}{2} \omega^{3}+\frac{1}{2} \omega^{8}\right)$ and $\left(\frac{\sqrt{3}}{2} \phi^{3}+\frac{1}{2} \phi^{8}\right)$ are automatically eigenstates of the mass matrix and should be directly identified with the observed $\psi(3105)$ and $\psi(3695)$ respectively. The other member of the $\omega^{\mathrm{a}}$ or $\phi^{\text {a }}$ color octets cannot mix with the photon, and thus cannot couple to leptons electromagnetically. Therefore, the other $\omega^{\mathrm{a}}$ and $\phi^{\mathrm{a}}$ cannot be seen in the $\mathrm{e}^{+} \mathrm{e}^{-}$channel, the SPEAR experiments, or in the MIT-Brookhaven experiment. On the other hand, if $\mathrm{SU}(3)^{\text { }}$ is broken appreciably in any other direction, say $\left({ }^{1} 1_{-2}\right)$, then $\omega^{3}, \omega^{8}, \phi^{3}, \phi^{8}$ are separately eigenstates of the mass matrix, and we would then expect to see these four states in the SPEAR experiments. For the moment we leave this possibility open, but concentrate on a breaking of $S U(3)^{\prime}$ in the photon direction ( ${ }^{-2} 1_{1}$ ) only, since only two $\psi^{\prime}$ s have been seen so far. We will discuss later the possibility for $S U(3)^{\prime}$ to be broken.

This scheme provides a natural explanation for the two states without having to put one of the particles in an excited radial or orbital representation. Furthermore, it makes direct predictions for the ratio of the leptonic widths of the $\psi$ 's. Since the photon couples to $\phi_{8}$ but not to $\phi_{1}$, and to $\psi_{1,8}$ but not to $\psi_{8,8}$, one predicts

$$
\frac{\Gamma\left(\psi(3695) \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)}{\Gamma\left(\psi(3105) \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)}=\frac{\Gamma\left(\omega \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)}{\Gamma\left(\phi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)} \cdot\left[\frac{\mathrm{M}_{\psi(3695)} \cdot \mathrm{M}_{\phi}}{\mathrm{M}_{\psi(3105)} \cdot \mathrm{M}_{\omega}}\right]
$$

This is in rough agreement with the data if one includes the phase space corrections in the square brackets. If these corrections are dropped the agreement is excellent. (Note that the problem of whether one includes broken mass effects in such vector dominance calculations is a traditionally unsolved problem.)

Similarly, it is possible to calculate the ratio of the coupling of $\rho, \omega, \phi$, $\psi(3105), \psi(3695)$ to photons in a quark model. Writing

$$
\langle 0| \mathrm{J}_{\mu}^{\mathrm{E} . \mathrm{M} .}|\mathrm{V}\rangle=\frac{\mathrm{m}_{\mathrm{V}}^{2}}{\mathrm{f}_{\mathrm{V}}} \epsilon_{\mu}
$$

one finds

$$
\mathrm{f}_{\rho}^{-2}: \mathrm{f}_{\omega}^{-2}: \mathrm{f}_{\phi}^{-2}: \mathrm{f}_{\psi(3105)}^{-2}: \mathrm{f}_{\psi(3695)}^{-2}=9: 1: 2: 8: 4
$$

In this scheme the $\psi(3695) \sim \phi^{\alpha}$ is made up of $\lambda \bar{\lambda}$ quarks while $\psi(3105) \sim \omega^{\alpha}$ is made up of $p$ and $n$ quarks in the $u^{\prime}$-spin singlets combination. The decay $\psi(3695) \rightarrow \psi(3105)+\pi \pi$ is then suppressed (say, relative to $\left.\rho^{\prime} \rightarrow \rho \pi \pi\right)$ in a quark model by the so-called Zweig rule, which states that quark diagrams with disconnected quark lines are suppressed. This suppression factor in the decay rates, or coupling constants squared, is of the order of 50-100 in similar situations. With such a suppression factor, as noted earlier, the decay width $\Gamma_{\psi^{\prime} \rightarrow \psi \pi \pi}$ is of the correct order of magnitude $\approx 200 \mathrm{keV}$.

We note also that one expects more kaons in the decay products of $\phi$ than in that of $\omega$. Hence, from the identification of $\psi(3105)$ as the $\omega$ type state in usual $\operatorname{SU}(3)$ space and the $\psi^{\prime}(3695)$ as the $\phi$ type state in usual $\operatorname{SU}(3)$ space, this color scheme predicts that more kaons will be seen in the direct decay products of the $\psi^{\prime}$ (i.e., not via the $\psi$ cascade) than those of $\psi$. A naive estimate from the quark model gives a factor of 2 to 3 . As noted before, if $\operatorname{SU}(3)^{\prime}$ is broken even slightly, in a direction other than the photon, $\omega^{3}, \omega^{8}, \phi^{3}, \phi^{8}$ can all become physical particles, just like $\rho, \omega, \phi$ in the color singlet case. Then the ratio of their coupling to photons is
$\mathrm{f}_{\rho}^{\mathbf{- 2}}: \mathrm{f}_{\omega}^{\mathbf{- 2}}: \mathrm{f}_{\phi}^{\mathbf{- 2}}: \mathrm{f}_{\omega^{3}}^{-2}: \mathrm{f}_{\phi}^{-2}: \mathrm{f}_{\omega}^{-2}{ }^{8}: \mathrm{f}_{\phi}^{-2}=9: 1: 2: 2: 1: 6: 3$

Identifying $\omega^{3}$ and $\phi^{3}$ with $\psi$ and $\psi^{\prime}$, the widths $\Gamma_{\psi \rightarrow \text { ee }}$ and $\Gamma_{\psi^{\prime} \rightarrow \text { ee }}$ come out much closer to data (within $10 \%$ ). They are a factor of 4 down from the exact SU(3)' case. This scheme also predicts two more narrow width resonances in the $\mathrm{e}^{+} \mathrm{e}^{-}$scattering whose decay widths to $\mathrm{e}^{+} \mathrm{e}^{-}$are approximately three times larger than those of $\psi$ and $\psi^{\prime}$.

Within the color scheme, the absence of more sharp resonances implies an exact $\operatorname{SU}(3)^{\prime}$ symmetry except for a possible breaking in the photon direction (not due to electromagnetism). In the latter case, we still have only $\psi$ and $\psi^{\prime}$. The main effect of this breaking is the enhancement of hadronic decay relative to the radiative decay modes of both $\psi$ and $\psi^{\prime}$. Note also that if we allow for such a small $\mathrm{SU}_{3}^{\prime} \mathrm{u}^{\prime}$-spin singlet breaking the decay to final noncolor hadrons which will go through this interaction will obey G-parity selection rules since usual isospin I is still a good quantum number. $\psi$ and $\psi^{\prime}$ being of $\omega$ or $\phi$ type in usual $\mathrm{SU}_{3}$ space have $\mathrm{I}=0$ and therefore can decay only to odd number of pions. Experimentally one finds indications that the $\pi^{+} \pi^{-}$and $\pi^{+} \pi^{-} \pi^{+} \pi^{-}$decay mode for $\psi$ are absent.

This scheme (and the other two before it) runs into difficulty when one tries to understand the decay widths of the $\psi^{\prime} s$. The process

$$
\psi \rightarrow \gamma+\text { ordinary hadrons }
$$

is allowed by $\mathrm{SU}(3)^{\prime}$ since the photon carries away the color degree of freedom. To get a feeling for the numbers involved we remark that a typical radiative width, like $\omega \rightarrow \pi \gamma$ is around $\frac{1}{2}-1 \mathrm{MeV}$ (while $\phi \rightarrow \eta \gamma$ is $\sim 0.1 \mathrm{MeV}$ ). This value is already above the total widths of the $\psi(3105)$ and $\psi(3695)$. Furthermore, to make matters worse the phase space available to these particles is $3-4$ times that of ordinary vector mesons and the number of possible decay channels is also much greater. Hence to explain the size of the $\psi(3105)$ width, assuming
that it is mostly radiative, one has to understand how to suppress it by a factor of more than an order of magnitude from what is normally expected in strong interactions.

At the present stage, this is an outstanding question for any color scheme. We will attempt here a possible resolution of this puzzle in the context of a vector meson dominance model. In that case the relevant diagram is shown in Fig. 2.


Fig. 2
Denoting the transition form factor from a colored vector meson to the photon by $\gamma_{\psi}\left(9^{2}\right)$, the width is then proportional to

$$
\Gamma_{\psi \rightarrow \gamma+\text { hadrons }} \propto \gamma_{\psi}^{2}(0)
$$

On the other hand, the decay of $\psi$ into lepton pairs involves $\gamma_{\psi}^{2}\left(q^{2}=m_{\psi}^{2}\right)$. A priori we have no reason to believe that $\gamma_{\psi}\left(q^{2}\right)$ is slowly varying when such a large extrapolation is involved from $q^{2}=0$ to $q^{2} \approx 10-14$. Assuming that $\gamma_{\psi}(0) \ll$ $\gamma_{\psi}\left(\mathrm{q}^{2}=10\right)$ by a factor of approximately 0.1 , we obtain a suppression factor of $10^{-2}$ for $\psi \rightarrow \gamma+$ hadrons.

This assumption has direct implications in photoproduction since again $\gamma_{\psi}(0)$ appears. For example we can estimate photoproduction of $\psi$ and $\psi^{\prime}$ relative to $\rho$ and $\phi$ photoproduction.

$$
\frac{\sigma(\gamma \mathrm{N} \rightarrow \psi \mathbb{N})}{\sigma(\gamma \mathrm{N} \rightarrow \rho \mathrm{~N})}=\frac{\gamma_{\psi}^{2}(0)}{\gamma_{\rho}^{2}(0)}\left[\frac{\sigma_{\mathrm{Tot}}(\psi \mathrm{~N})}{\sigma_{\mathrm{Tot}}(\sigma \mathrm{~N})}\right]^{2} \times\left(\mathrm{t}_{\min } \text { effects }\right) .
$$

$$
\rightarrow \frac{\sigma\left(\gamma \mathrm{N} \rightarrow \psi^{\prime} \mathrm{N}\right)}{\sigma(\gamma \mathrm{N} \rightarrow \phi \mathrm{~N})}=\frac{\gamma_{\psi^{\prime}}^{2}(0)}{\gamma_{\phi}^{2}(0)}\left[\frac{\sigma_{\operatorname{Tot}}\left(\psi^{\prime} \mathrm{N}\right)}{\sigma_{\operatorname{Tot}}{ }^{(\phi N)}}\right]^{2} \times\left(\mathrm{t}_{\min } \text { effects }\right)
$$

The suppression factors can be estimated

$$
\begin{aligned}
& \frac{\gamma_{\psi^{\prime}}(0)}{\gamma_{\rho}^{2}(0)}=\frac{\Gamma_{\psi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}}{\Gamma} \frac{\mathrm{m}}{\rho \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}} \frac{\gamma_{\psi}(0)}{\mathrm{m}_{\psi}}\left(\frac{\gamma_{\psi^{\prime}}(10)}{\gamma^{2}}\right)^{2} \approx 2 \times 10^{-3} \\
& \frac{\gamma_{\psi^{\prime}}^{2}(0)}{\gamma_{\phi}^{2}(0)}=\frac{\Gamma}{\psi^{\prime} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}} \frac{\mathrm{m}_{\phi}}{\Gamma_{\phi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}^{\mathrm{m}_{\psi^{\prime}}}\left(\frac{\gamma_{\psi^{\prime}}(0)}{\gamma_{\psi^{(14)}}^{(14)}}\right)^{2} \approx 5 \times 10^{-3}}
\end{aligned}
$$

Further, one could assume that $\sigma_{\operatorname{Tot}}(\psi N) \approx \sigma_{\operatorname{Tot}}(\rho N)$ and $\sigma_{\text {Tot }}\left(\psi^{\prime} N\right) \approx \sigma_{T o t}(\phi N)$ since they have the same quark content respectively. The $t_{\text {min }}$ effects are negligible at NAL energies. Thus one obtains the prediction (valid within onc order of magnitude)

$$
\begin{aligned}
& \sigma(\gamma \mathrm{N} \rightarrow \psi \mathrm{~N}) \approx 2 \times 10^{-3} \sigma(\gamma \mathrm{~N} \rightarrow \rho \mathrm{~N}) \\
& \sigma\left(\gamma \mathrm{N} \rightarrow \psi^{\prime} \mathrm{N}\right) \approx 5 \times 10^{-3} \sigma(\gamma \mathrm{~N} \rightarrow \phi \mathrm{~N}) .
\end{aligned}
$$

The spectroscopy of this model is very rich. Because one is mixing an $(8,8)$ with a $(1,8)$ one has a total of 72 vector states. The breaking pattern occurs in two directions. The first is usual $\operatorname{SU}(3)$ breaking which gives $\phi$-like, $\omega$-like, $\rho$ like, and $\mathrm{K}^{*}$-like states which are octets in color, as mentioned before. The second breaking is the breaking of color by electromagnetism and perhaps additional small $\mathrm{SU}(3)^{\prime}$ breaking. As mentioned before, as long as the breaking is only in the $u^{\prime}$-spin singlet direction, there are only two $\psi$ 's that couple to the photon. As a result all states fall into $u^{\prime}-$ spin multiplets with $u^{\rho}=0, u^{\prime}=\frac{1}{2}$, and $u^{\prime \prime}=1$. Furthermore, $u^{\prime}-$ spin remains conserved up to weak interactions.

If one assumes standard $\operatorname{SU}(3)$ octet breaking and validity of first order calculation, one expects that the $\rho$-like states are degenerate with the $\omega$-like states ( $\mathrm{m} \sim 3105 \mathrm{MeV}$ ), while the $\mathrm{K}^{*}$-like states are located approximately at

$$
\mathrm{M}_{\mathrm{K}_{\mathrm{a}}^{*}}^{2}=\frac{1}{2}\left[\mathrm{M}_{\psi_{3105}}^{2}+\mathrm{M}_{\psi_{3695}}^{2}\right] \cong 11.6 \mathrm{GeV}^{2}
$$

In this case one expects that $\psi(3695)$ can decay into the $u^{\prime}$-singlet $\rho$-like state $\psi_{\rho}$ plus a pion, conserving color. These decays (which are also suppressed by Zweig's rule as $\psi(3695) \rightarrow \psi(3105)+\pi \pi)$ are apparently not seen experimentally. Within the context of the model this could mean a number of things.
(1) First order $\operatorname{SU}(3)$ breaking calculations for these high mass states are not reliable, up to $10-20 \%$. Indeed it could be that the $\rho$-like states are above or near 3550 MeV .
(2) There may be additional $\operatorname{SU}(3)$ breaking, say in the (27)direction, which is more effective for these states than for ordinary hadrons.
(3) There may be a dynamical reason which suppresses isospin-changing transitions among color states.
(4) The model is wrong.

We should comment that if $\psi_{\rho}$ is above or near 3550 MeV then, when produced, it decays strongly, $\psi_{\rho} \rightarrow \psi_{\omega}(3105)+\pi$, conserving color and isospin and not suppressed by Zweig's rulc.

Some of the vector states predicted by this model are doubly charged. The presence of such doubly charged states is a definite prediction of this color model, and their observation will provide a strong support for the model. Not all states predicted by the model are expected to be sharp resonances, since strong color transitions may occur whenever kinematically possible. An example of such a transition has been mentioned in the previous paragraph.

If $\psi(3105)$ and $\psi(3695)$ are indeed colored vector mesons, this will have important consequences for deep inelastic scattering and $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. In deep inelastic scattering soon after it is energetically possible to produce color states one should begin to feel the effect of the "color thaw". This would give (temporary) scaling violations. A (naive) way of estimating the effect of the color thaw is to use the quark-parton model. Below the color threshold, effectively one has color singlets, so that for the proton scaling function one has

$$
f_{p}(x)=\frac{4}{9}(p(x)+\bar{p}(x))+\frac{1}{9}(n(x)+\bar{n}(x))+\frac{1}{9}(\lambda(x)+\bar{\lambda}(x))
$$

since the effective proton (neutron, lambda) quark charge is $2 / 3(-1 / 3,-1 / 3)$;
for the neutron one has, via isospin,

$$
\mathrm{f}_{\mathrm{n}}(\mathrm{x})=\frac{1}{9}(\mathrm{p}(\mathrm{x})+\overline{\mathrm{p}}(\mathrm{x}))+\frac{4}{9}(\mathrm{n}(\mathrm{x})+\overline{\mathrm{n}}(\mathrm{x}))+\frac{1}{9}(\lambda(\mathrm{x})+\bar{\lambda}(\mathrm{x}))
$$

where of course $p(x)$ is the distribution of proton quarks in the proton, etc.
Hence

$$
\mathrm{f}_{\mathrm{p}}(\mathrm{x})+\mathrm{f}_{\mathrm{n}}(\mathrm{x})=\frac{5}{9}(\mathrm{p}(\mathrm{x})+\overline{\mathrm{p}}(\mathrm{x})+\mathrm{n}(\mathrm{x})+\overline{\mathrm{n}}(\mathrm{x}))+\frac{2}{9}(\lambda(\mathrm{x})+\bar{\lambda}(\mathrm{x}))
$$

Above the color threshold one is probing the distributions of partons with charge. These are $\mathrm{p}_{2}, \mathrm{p}_{3}, \mathrm{n}_{1}, \lambda_{1}$ and one has

$$
\begin{aligned}
f_{p}(x)+f_{n}(x) & =\left[p_{1}(x)+p_{2}(x)+p_{3}(x)+\bar{p}_{1}(x)+\bar{p}_{2}(x)+\bar{p}_{3}(x)\right. \\
& \left.+n_{1}(x)+n_{2}(x)+n_{3}(x)+\bar{n}_{1}(x)+\bar{n}_{2}(x)+\bar{n}_{3}(x)\right]+2\left[\lambda_{1}(x)+\bar{\lambda}_{1}(x)\right]
\end{aligned}
$$

Since protons are color singlets we expect

$$
p_{i}(x)=\frac{1}{3} p(x) ; n_{i}(x)=\frac{1}{3} n(x) ; \quad \lambda_{i}(x)=\frac{1}{3} \lambda(x)
$$

Hence, above color threshold one has

$$
f_{p}(x)+f_{n}(x)=[p(x)+n(x)+\bar{p}(x)+\bar{n}(x)]+\frac{2}{3}[\lambda(x)+\bar{\lambda}(x)]
$$

If we neglect the strange quark contribution we find that

$$
\frac{\left[\mathrm{f}_{\mathrm{p}}(\mathrm{x})+\mathrm{f}_{\mathrm{n}}(\mathrm{x})\right]_{\text {ABOVE }}}{\left[\mathrm{f}_{\mathrm{p}}(\mathrm{x})+\mathrm{f}_{\mathrm{n}}(\mathrm{x})\right]_{\text {BELOW }}} \approx \frac{9}{5}
$$

This should be noticeable in the FNAL $\mu$-scattering experiment. A similar test involves the bound, valid in Han-Nambu models,

$$
\frac{1}{2} \leq \frac{f_{n}}{f_{p}} \leq 2
$$

However, violation of this bound may occur in unfavored kinematical configurations ( $x \rightarrow 1$ ) where indeed color production may be suppressed.

Finally let us mention that in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons one expects that after the color thaw the cross section should eventually settle down to the Han-Nambu value:

$$
\mathbf{R}=\frac{\sigma_{\mathbf{h}}}{\sigma_{\mu \mu}}=4
$$

As a final remark let us ask where we can find partners of the $\psi^{\prime}$ s. A possible place is photoproduction at sufficiently high energies. Also, associated production either at SPEAR or with hadronic targets at high energies should be very interesting.

## IV. The Charm Hypothesis

Another possibility for the $\psi(3105)$ and $\psi(3695)$ that has received considerable attention is to identify these particles as bound states of charm-anticharm quarks. Charm was introduced as far back as $1964{ }^{5}$ as a possible new quantum number, which like strangeness should be conserved in strong and electromagnetic interactions but violated by weak interactions. The relevant invariance group for the strong interaction is then $\operatorname{SU}(4)$, not $\operatorname{SU}(3)$. However, the symmetry is supposed to be badly broken so that charmed hadrons have much bigger masses than non-charmed hadrons. In terms of quarks, SU(4) introduces a fourth quark " C " with charge $+2 / 3$, isospin 0 , hypercharge 0 , and charm 1. All other quarks have charm equal to 0 .

The idea of charm was revived in 1970 in order to resolve a dilemma of weak interaction theory. It was observed that the experimental upper limits on $\Delta S \neq 0, \Delta Q=0$ weak transitions require that the current-current theory of weak interactions must fail on an energy scale of $5-20 \mathrm{GeV}$, well below the 300 GeV unitarity limit. It was argued then that by adding to the Cabbibo current a $\Delta C=1$ part, the magnitude of these transitions could be understood without modifying the current-current structure. In the quark mnemonic the weak current of hadrons is represented by

$$
J=\bar{p}(n \cos \theta+\lambda \sin \theta)+\bar{c}(\lambda \cos \theta-n \sin \theta)
$$

where $\theta$ is the Cabibbo angle. It is important to add that this explanation of the $\Delta S \neq 0, \Delta Q=0$ transitions requires that the charmed quark be no heavier than a few GeV.

In $\mathrm{SU}(4)$ the familiar $\mathrm{SU}(3)$ vector meson nonet is joined by seven new states. Six of these new vector mesons carry charm, $C \neq 0$, but the seventh
has $\mathrm{C}=0$. In the $\mathrm{SU}(3)$ quark model the $\rho, \omega$, and $\phi$ are successfully described as

$$
\begin{aligned}
& \rho^{0}=\frac{1}{\sqrt{2}}(\mathrm{p} \overline{\mathrm{p}}-\mathrm{n} \overline{\mathrm{n}}) \\
& \omega=\frac{1}{\sqrt{2}}(\overline{\mathrm{p}}+\mathrm{n} \overline{\mathrm{n}}) \\
& \phi=\lambda \bar{\lambda}
\end{aligned}
$$

so that in the extension to $\mathrm{SU}(4)$ the new $\mathrm{C}=0$ meson is expected to be a fairly pure $\bar{c} c$. In analogy to the purity of the $\phi$ as $\lambda \bar{\lambda}$ it is sometimes referred to as the "charmed" $\phi$

$$
\phi_{\mathrm{c}}=\mathrm{c} \overline{\mathrm{c}}
$$

The charmed $\phi, \phi_{c}$, has the correct quantum numbers to be produced in $e^{+} e^{-}$collisions. Because of the upper limit on the charmed quark mass one expects $\mathrm{m}_{\phi_{\mathbf{c}}} \lesssim 10 \mathrm{GeV}$. Hence it could well be that $\psi(3105)$ and $\psi(3695)$ are charm-anticharm vector mesons of the $\phi_{c}$ variety. There is no ready explanation in $\operatorname{SU}(4)$ for the appearance of two states. One must assume, perhaps, that the higher state is a radial or orbital excitation of the lower state. Within specific models, ${ }^{6}$ to be discussed below, this explanation is perhaps natural.

Assuming for the moment that the $\psi$ 's are $\phi$ 's we can ask why is the width of these states so small. Clearly since $\phi_{c}$ would like to decay preferentially into a pair of charmed mesons, it must be below the threshold to do so. It can decay into non-charmed states only by breaking Zweig's rule. We can estimate this width ${ }^{7}$ by correcting the $\phi \rightarrow 3 \pi$ width by phase space

$$
\Gamma_{\phi_{\mathrm{c}}} \sim 600 \mathrm{keV} \times \frac{\mathrm{M}_{\phi_{\mathrm{c}}}}{\mathrm{M}_{\phi}} \approx 2 \mathrm{MeV}
$$

This number is clearly too big. Perhaps it should be even made bigger by taking into account the increased number of channels available for $\phi_{c}$ decay. If we want to retain the hypothesis that the $\psi$ 's are charm-anticharm mesons we must understand why the above estimate is wrong. One possible explanation, in the context of asymptotically free field theories, has been proposed recently ${ }^{6}$ and will be discussed below.

The SU(4) quark model makes numerous predictions for the interactions of $\phi_{c}$ relative to the other vector mesons $\rho, \omega$, and $\phi$. These predictions have been nicely discussed in the review article of Gaillard, Lee, and Rosner. ${ }^{7}$ Of particular interest is the coupling of the vector mesons to the photon

$$
\langle 0| \mathrm{J}_{\mu}^{\mathrm{e} . \mathrm{m}} \cdot|\mathrm{~V}\rangle=\frac{\mathrm{m}_{\mathrm{V}}^{2}}{\mathrm{f}_{\mathrm{V}}} \epsilon_{\mu} ;
$$

one has the relations

$$
\mathrm{f}_{\rho}^{-2}: \mathrm{f}_{\omega}^{-2}: \mathrm{f}_{\phi}^{-2}: \mathrm{f}_{\phi}^{-2}=9: 1: 2: 8
$$

From these relations we obtain $\Gamma_{\psi} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \approx 15 \mathrm{keV}$ to be compared with the experimental value $\approx 5 \mathrm{keV}$. In view of the ambiguities due to the very large $\mathrm{SU}(4)$ breaking, this is not necessarily an unreasonable result.

The $\phi_{c}$ decays into hadrons by strong annihilation of the $c \bar{c}$ pairs into states made of $p, n$, and $\lambda$ quarks. We expect strong quark-quark interaction to be mediated by forces which are singlets in ordinary $\mathrm{SU}(3)$ so that $1 / 3$ of the final states will contain $\lambda$ quarks. In contrast, only $1 / 6$ of photon-induced final states will contain $\lambda$ quarks. Therefore in the $\phi_{c}$ decay peak we expect $1 / 3$ of the events to contain $K$ mesons, compared to approximately $1 / 6$ of the events away from the peak. However $\psi$ and $\psi^{\prime}$ decays to kaons with approximately the same rate, in contrast to the color scheme prediction.

In the case of charm, as in the case of color before, the most crucial test of the hypothesis $\psi=\phi_{c}$ will be the success or failure of searches for other $\operatorname{SU}(4)$ particles. Identifying $\psi=\phi_{\mathrm{C}}$ we learn the approximate magnitude of the $\mathrm{SU}(4)$ breaking and rough predictions of the masses of other particles can be made. This is done systematically in Ref. 7, and will not be repeated here.

Following Ref. 7, we estimate that the charmed pseudoscalar $\overline{\mathrm{c}} \mathrm{n}$ has a mass on the order of $\approx 2.15 \mathrm{GeV}$. Therefore at $\sqrt{\mathrm{s}}=4.3 \mathrm{GeV}$ in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation we would expect a threshold corresponding to the associated production of charmed particles. In the Gell-Mann-Zweig quark-parton model, we expect that beyond this threshold $\mathrm{R}=\sigma_{\mathrm{h}} / \sigma_{\mu \mu}=3 \frac{1}{3}$. Of the hadronic events, $10 \%$ arise from creation of $\lambda \bar{\lambda}$ quarks and $40 \%$ from $\overline{\mathrm{c}} \mathrm{c}$ pairs. Since the charmed states decay primarily into strange particles, a dramatic signal accompanying the crossing of this threshold is the increase in the function of events with K's from $\sim 1 / 6$ to $\sim 1 / 2$. Other features of the decay products of $C \neq 0$ states are discussed in Ref. 7.

Without corresponding experimental evidence, the hypothesis of the $\psi(3105)$ and $\psi(3695)$ as charm-anticharm vector mesons is in theoretical trouble because naive theoretical estimates do not give a sufficiently narrow width for their decay. A theoretical scheme which may avoid this difficulty has been proposed by Appelquist and Politzer ${ }^{6}$ in the context of asymptotically free field theories of the strong interactions. They assume that the underlying field theory is $\mathrm{SU}(4)$ $\times \mathrm{SU}(3)_{\text {color }}$, where the color symmetry is a hidden exact symmetry along the lines of the red, white and blue model. Hadrons are always color singlets. The 12 quarks of the theory interact by exchanging eight massless color gluons; both quarks and gluons are "confined" and not observable states. In such a theory,
the effective strong interaction coupling constant $\alpha_{S}$ is small, $\alpha_{S}<1$, at large momenta (say above 2 GeV ) so that Bjorken scaling holds. For low momenta $\alpha_{s}>1$ which accounts for the strong binding of ordinary hadrons and also (perhaps! ) provides the mechanism for quark and gluon confinement. For large momenta perturbation theory can be used since the coupling constant is small. Appelquist and Politzer argue that in $c \bar{c}$ and $\lambda \bar{\lambda}$ bound states the scale of momenta is determined by the quark masses $m_{c}$ and $m_{\lambda}$. For $\phi_{c}=\bar{c} c$ with $m_{c}$ $\gtrsim 1.5 \mathrm{GeV}$ the scale of momenta is large enough so that $\alpha_{\mathrm{s}}$ is in the perturbation theory region, while for $\phi=\lambda \bar{\lambda}, \alpha_{S}$ is in the strong coupling region. Hence our estimate for $\Gamma_{\phi_{c}}$ from $\Gamma_{\phi}$ is likely to be wrong. Appelquist and Politzer find $\Gamma_{\phi_{c}} \propto\left(\alpha_{s}\right)^{6}$ and hence $\Gamma_{\phi_{C_{+}}} \ll \Gamma_{\phi}$ may be a plausible consequence. By estimating $\Gamma_{\phi_{C}}$ (total) and $\Gamma_{\phi_{c}} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$from the decay of the $\psi(3105)$ they arrive at a value $\alpha_{S}=0.25$. This value justifies their calculation of the decay of orthocharmonium the same way as one calculates the decay of orthopositronium in QED. (That is where $\Gamma_{\propto}\left(\alpha_{S}\right)^{6}$ comes from.) A Balmer series of excited states of charmonium would also be expected on the basis of their calculation.

The consistency of this interpretation is brought into question by the existence of the $\psi(3695)$. If one attempts to interpret $\psi(3695)$ as an excited state in a relativistic Balmer series for charmonium, then the $\psi(3105)$ must be regarded as strongly bound and it is not surprising that one finds $\alpha_{s} \approx 1$. Such a value is not consistent with Appelquist and Politzer's estimate and it means that their perturbation theory calculations cannot be taken seriously. Hence it is necessary for the proponents of this theory to find some other explanation for the $\psi(3695)$.

Two observations would provide strong support for the charmonium scheme of things :

1) Production of charmed mesons ( $\mathrm{J}^{\mathrm{P}}=0^{-}$, charged and with mass $\sim 2.1$ GeV ) presumably pair produced in $\mathrm{e}^{+} \mathrm{e}^{-}$collisions and observed to decay predominantly to $\mathrm{K} \mathrm{s}-\mathrm{i} . \mathrm{e} ., \tan ^{2} \theta_{\mathrm{c}} \sim \# \pi^{\mathbf{s}} \mathrm{s} / \# \mathrm{~K}^{\mathbf{s}} \mathrm{s}$
2) Magnetic dipole transitions from the ${ }^{3} S_{1} \psi$ and $\psi^{\prime}$ states to their paracharmonium partners with partial decay rates $\sim 1 \mathrm{keV}$. Thus an allowed M1 transition between the $\psi(3105)$ and its $1_{S_{0}}$ paracharmonium partner has a rate

$$
\begin{equation*}
\Gamma \simeq\left(\frac{\mathrm{Q}}{\mathrm{e}}\right)^{2} \frac{16 \alpha}{3} \mathrm{k}^{3} / \mathrm{M}_{\mathrm{Q}_{\mathrm{c}}}^{2} \tag{.2}
\end{equation*}
$$

where k is their energy difference and $\mathrm{M}_{Q_{c}}$ is the mass of the charmed quark. Using

$$
M_{\rho}^{2}-M_{\pi}^{2} \approx M_{K}^{2}-M_{K}^{2} \approx \frac{1}{2} \operatorname{GeV}^{2} \approx M_{\psi^{\prime}}^{2}-M_{\psi}^{2}
$$

$\mathrm{k}-\mathrm{M}_{\psi^{\prime}}-\mathrm{M}_{\psi} \sim 80 \mathrm{keV}, \mathrm{M}_{\mathrm{Q}_{\mathrm{c}}} \sim 2 \mathrm{GeV}, \Gamma \sim 3 \mathrm{keV}$. If the $\psi^{\mathbf{r}}(3695)$ is interpreted as a radially excited charmonium state its decay to the $\psi(3025)$ is a forbidden dipole mode due to orthogonality of the spatial wave functions and in this case

$$
\Gamma^{\prime} \sim\left(\frac{\mathrm{Q}}{\mathrm{e}}\right)^{2} \frac{16 \alpha}{3} \frac{\mathrm{k}^{\mathbf{t}^{3}}}{\mathrm{M}_{\mathrm{Q}_{\mathrm{c}}}^{2}}\left(\frac{\mathrm{k}^{2} \mathrm{R}^{2}}{6}\right)^{2}
$$

and depends on the radius of the $\mathrm{c} \overline{\mathrm{c}}$ state. For $\mathrm{k}^{\prime} \sim 670 \mathrm{MeV}$ and
$\mathrm{R}^{2} \sim \frac{1}{\mathrm{M}_{\mathrm{Q}_{\mathrm{c}}} \cdot \epsilon_{\mathrm{B}}} \sim\left(\frac{1}{4} \mathrm{f}\right)^{2}, \Gamma^{\prime}$ may be as large as $\sim 50 \mathrm{keV}$ and easily seen.

It is also possible for $\psi$ and $\psi^{\prime}$ to be states in a $\operatorname{SU}(4) \times \operatorname{SU}(3)^{\prime}$ group (i.e., charm plus Han-Nambu color). This predicts $R=\frac{\left.\text { d( } e^{+} e^{-} \rightarrow \text { hadron }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}=6$ at asymptotic energies and more resonances in the $\mathrm{e}^{+} \mathrm{e}^{-}$channel. However it does not help to explain the problems we already have in either the charm or the color scheme.

## V. Intermediate Weak Boson Hypothesis

The widths of the $\psi(3105)$ and $\psi(3695)$ decays into leptons imply that these particles have an effective coupling to leptons of the order of the Fermi constant. To be more specific, if we assume that the coupling of the $\psi^{\prime} s$ to $\mathrm{e}^{+} \mathrm{e}^{-}$ pairs is of the form

$$
\mathscr{L}_{\text {int }}=\mathrm{g} \psi_{\mu} \overline{\mathrm{e}} \gamma_{\mu}\left(\mathrm{c}_{\mathrm{V}}-\gamma_{5} \mathrm{c}_{\mathrm{A}}\right) \mathrm{e}
$$

then the width $\Gamma_{\psi}$ is given by

$$
\Gamma_{\psi}=\frac{\mathrm{g}^{2}}{12 \pi}\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right) \mathrm{m}_{\psi}
$$

which gives

$$
\frac{\mathrm{g}^{2}}{\mathrm{~m}_{\psi}^{2}}\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)=\lambda \mathrm{G}_{\mathrm{F}} / \sqrt{2}
$$

with $\lambda \approx 0.8$ for the $\psi(3105)$ and $\lambda \approx 0.3$ for the $\psi(3695)$. These numbers encourage one to assume that the $\psi$ 's may be neutral weak bosons. It must be remarked, however, that ordinary vector mesons have leptonic widths of the same order of magnitude as the $\psi^{\prime}$ s. Hence, leptonic widths by themselves can be misleading.

The reported presence of the decay $\psi(3695) \rightarrow \psi(3105)+\pi^{+}+\pi^{-}$with a width of around $100-200 \mathrm{keV}$, would seem to us to put the hypothesis of the $\psi \mathrm{s}$ as weak bosons in grave peril. The effective coupling for this decay, when one takes into account the phase space limitations as described earlier, is of order of $1 / 3$ to $1 / 10$ of the strong $\rho^{\prime} \rightarrow \rho \pi \pi$ decay. This is difficult to reconcile with most simple models of weak interactions. However, there may be ways out. ${ }^{5}$ Another "experimental" difficulty, not unrelated perhaps to the above, is the large coupling that the $\psi^{\prime}$ 's have to hadrons vis-a-vis leptons. Naive quark
model estimates, using universality, would predict rates differing at most by an order of magnitude although multiquark schemes can boost the coupling ratio even further. However a weak IVB scheme would suggest that leptons should appear at a comparable rate to the pions in the $\psi^{\prime}(3695) \rightarrow \psi(3105)$ decay process.

With these caveats in mind, let us further test the hypothesis that these particles are neutral weak bosons. If this were the case we can ask in what theoretical framework do the $\psi^{\prime}$ 's naturally fall. Three possibilities suggest themselves immediately:
a) They are part of a unified renormalizable gauge theory of weak and electromagnetic interactions.
b) They belong in a renormalizable, but not unified, theory.
c) They are weak bosons in a nonrenormalizable theory. The first possibility, which is the most attractive, unfortunately is not viable. The other two alternatives, although possible theoretically, do not seem to be especially desirable. We shall discuss all three below.
a) It is clear that the $\psi$ 's cannot be the weak neutral vector boson in the familiar Weinberg-Salam $S U(2) \times U(1)$ theory. For one thing there is only one massive neutral boson in this theory with a mass $m_{z} \gg m_{\psi}$. However, the $\psi s$ cannot also be neutral bosons in any other unified gauge theory because of the observed weakness of their dimensionless coupling to leptons, g<<e. A fundamental property of gauge theories that include electromagnetism is that the universal gauge coupling constants g are necossarily larger than the electric charge e (up to Clebsch-Gordan coefficients). This is familiar in the $\operatorname{SU}(2) \times$ $\mathrm{U}(1)$ model where

$$
e=\frac{\mathrm{gg}^{\prime}}{\sqrt{g^{2}+\mathrm{g}^{\prime 2}}}
$$

In Appendix A we show that if the true photon is an orthogonal combination of $n$ gauge bosons with couplings $g_{1}, g_{2}, \ldots, g_{n}$, then the electric charge is given by

$$
\frac{1}{e^{2}}=\frac{1}{g_{1}^{2}}+\ldots+\frac{1}{g_{n}^{2}}
$$

which proves our assertion that $g_{i} \geq e$. Therefore, even if one were successful in building a unified theory with low mass intermediate bosons, the couplings in that theory would be too large to identify the bosons with $\psi$.

The re are some ways out of the above argument. Normally all gauge particles in unified theories are taken to couple to all leptons and quarks. If $\psi(3105)$ and $\psi(3695)$ were gauge particles then their coupling to leptons and hadrons would be at least as large as e implying leptonic widths many times larger than observed. However, one could suppose that the unified theory contains the $\psi$ 's as a subgroup which did not have any direct couplings to leptons. The $\psi$ 's could couple to other matter (Higgs particles, other fermions, etc.) with $g_{\psi} \geq e$. Through mixing with the photon and/or other gauge particles they could acquire an effective coupling to leptons which could be arranged to be small. This can be done through a generalization of Berkcley type models, ${ }^{8}$ where the $\psi^{\prime}$ s would not directly couple to hadrons or leptons. Another suggestion, somewhat different, would introduce heavy leptons which are mixed with the electron and muon. Then, although the photon coupling to all leptons is e, one could suppress the $\psi$ 's couplings by picking an appropriate coupling scheme and mixing angle. We see no compelling reasons, at this stage, for trying to invoke such epicycles.
b) In the second class of models, mentioned above, the $\psi$ 's are not unified with electromagnetism. The simplest possibility is to associate the $\psi^{\prime}$ s with $U(1)$ groups that completely commute with the group of unified weak and electromagnetic interactions. In this case there is no restriction on their couplings. All gauge theories written down so far have such freedom. Thus the existence of such $\psi^{\prime}$ s would not ruin the possibility of having a renormalizable theory of the weak interactions.

An example of a theory that could accommodate the $\psi^{\prime} \mathrm{S}$ would be $\mathrm{SU}(2) \times$ $U(1) \times U(1) \times U(1)$ where the last two $U(1)$ 's have $\psi(3105)$ and $\psi(3695)$ as their neutral weak bosons. One could suppose for instance that $\psi(3105)$ coupled to baryon number (B) plus lepton number ( L ) in the combination ( $\mathrm{B}+\mathrm{L}$ ), as Sakurai ${ }^{9}$ suggested recently, and that $\psi(3695)$ coupled to $(B-L)$ Other, more exotic, possibilities are also clearly possible. Theoretically at least one unappealing feature of such theories is that the presence of the $\psi$ 's gives little restriction on the overall structure of the scheme. There is basically no a priori reason for their existence. Putting it another way, although the $\psi^{\prime}$ s may account for some (or all) of the neutral current weak effects, they do not tell us anything about the charged currents.
c) The last possible scheme for the $\psi$ 's as weak bosons supposes that they are part of some nonrenormalizable theory of weak interactions. An example of theories of this kind is one in which the $\psi(3105)$ and $\psi(3695)$ are neutral members of an $S U(2) \times U(1)$ or $\operatorname{SU}(3)$ gauge group of massive weak vector bosons. Such schemes would in general imply that the charged vector bosons have masses in the neighborhood of 3-5 GeV. Hence they could already be in trouble with the existing NAL data, ${ }^{10}$ if this data is taken at face value. From a theoretical standpoint, models of this sort are objectionable because they are
rendered nonrenormalizable when one attempts to introduce electromagnetism in the scheme. Hence they appear to us to be a step backward.

Irrespective of theoretical prejudices, it is still useful to suppose that the $\psi$ 's are neutral weak bosons and try to see what they predict for neutral current processes. This in general requires one to make additional assumptions on how they couple to neutrinos and quarks.

Our motivation for considering the $\psi$ as a candidate for an intermediate vector boson is suggested by its small decay width $\Gamma_{\psi \rightarrow \text { ee }} \approx 5 \mathrm{keV}$ (as well as $\Gamma_{\psi \rightarrow \text { hadr }} \approx 80 \mathrm{keV}$ ) which implies an effective coupling to leptons and quarks of the order of the Fermi coupling.

To be precise, let us begin by assuming the coupling of the $\psi$ to the $\mathrm{e}^{+} \mathrm{e}^{-}$ pairs has the form $\mathrm{g} \psi_{\mu} \overline{\mathrm{e}} \gamma_{\mu}\left(\mathrm{c}_{\mathrm{V}}-\gamma_{5}{ }^{\mathrm{c}} \mathrm{A}\right)$ e. The decay width $\Gamma_{\psi \rightarrow \text { ee }}$ is then given by

$$
\Gamma_{\psi \rightarrow e e}=\frac{\mathrm{g}^{2}}{12 \pi}\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right) \mathrm{m}_{\psi}
$$

so that

$$
\frac{\mathrm{g}^{2}}{4 \pi}\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)=5 \times 10^{-6}
$$

Consequently

$$
\frac{\mathrm{g}^{2}\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)}{\mathrm{m}_{\psi}^{2}}=\lambda \frac{\mathrm{G}_{\mathrm{F}}}{\sqrt{2}} \text { with } \lambda \cong 0.8
$$

Note that the mass of $\psi$ is relatively "light" as compared with the mass of the intermediate weak vector bosons in unified gauge theories. However, in the latter case, the coupling is of order e (electromagnetic coupling) while here $g$ is of order $e^{2}$.

Having fixed the parameter $\lambda$, we now proceed to examine the neutrino experiments.

## A. Neutrino-electron scattering

Assuming $\psi$ couples to neutrinos in the standard V-A form with the same coupling g as with electron, then the cross sections for $\nu_{\mu} \mathrm{e} \rightarrow \nu_{\mu} \mathrm{e}$ and $\bar{\nu}_{\mu} \mathrm{e} \rightarrow$ $\bar{\nu}_{\mu}$ e are given by

$$
\begin{aligned}
& \sigma\left(\nu_{\mu} \mathrm{e}\right)=\lambda^{2} \frac{\mathrm{G}^{2} \mathrm{~s}}{4 \pi} \frac{\mathrm{~m}_{\psi}^{2}}{\mathrm{~m}_{\psi}^{2}+\mathrm{s}} \frac{\left(\mathrm{c}_{\mathrm{V}}+\mathrm{c}_{\mathrm{A}}\right)^{2}+\frac{1}{3}\left(\mathrm{c}_{\mathrm{V}}-\mathrm{c}_{\mathrm{A}}\right)^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \\
& \sigma\left(\bar{\nu}_{\mu} \mathrm{e}\right)=\lambda^{2} \frac{\mathrm{G}^{2} \mathrm{~s}}{4 \pi} \frac{\mathrm{~m}_{\psi}^{2}}{\mathrm{~m}_{\psi}^{2}+\mathrm{s}} \frac{\left(\mathrm{c}_{\mathrm{V}}-\mathrm{c}_{\mathrm{A}}\right)^{2}+\frac{1}{3}\left(\mathrm{c}_{\mathrm{V}}+\mathrm{c}_{\mathrm{A}}\right)^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}}
\end{aligned}
$$

We remark that because of the $\psi$ propagator, the cross sections behave asymptotically as a constant. In Fermi theory, they rise linearly with energy. However, in practice $s=2 m_{e} E_{\nu}$ is smaller thaq $m_{\psi}^{2}$, hence the propagator effect is difficult to be detected and one has $\frac{\mathrm{m}_{\psi}^{2}}{\mathrm{~m}_{\psi}^{2}+\mathrm{s}} \approx 1$.

$$
\text { For } c_{V}=c_{A}=1 \text { one has }
$$

$$
\begin{aligned}
& \sigma\left(\nu_{\mu} \mathrm{c}\right)=3 \times 10^{-42} \mathrm{~cm}^{2} \cdot(\mathrm{E} / \mathrm{GeV}) \\
& \sigma\left(\bar{\nu}_{\mu} \mathrm{e}\right)=10^{-42} \mathrm{~cm}^{2} \cdot(\mathrm{E} / \mathrm{GeV})
\end{aligned}
$$

If either $\mathrm{c}_{\mathrm{V}}$ or $\mathrm{c}_{\mathrm{A}}$ is equal to zero (purely vector or axial coupling),

$$
\sigma\left(\nu_{\mu} \mathrm{e}\right)=\sigma\left(\bar{\nu}_{\mu} \mathrm{e}\right)=4 \times 10^{-42} \mathrm{~cm}^{2} \cdot(\mathrm{E} / \mathrm{GeV})
$$

The Gargamelle data are quoted as follows:

$$
\sigma\left(\nu_{\mu} \mathrm{e}\right) \leq 0.26 \times 10^{-41} \mathrm{~cm}^{2}(\mathrm{E} / \mathrm{GeV})
$$

$$
3 \times 10^{-43} \mathrm{~cm}^{2}\left(\frac{\mathrm{E}}{\mathrm{GeV}}\right)<\sigma\left(\bar{v}_{\mu} \mathrm{e}\right)<3 \times 10^{-42} \mathrm{~cm}^{2}\left(\frac{\mathrm{E}}{\mathrm{GeV}}\right)
$$

B. Deep inelastic neutrino-nucleon scattering

In terms of the variable $\mathrm{x}=\frac{\mathrm{Q}^{2}}{2 \mathrm{~m} \nu}$ and $\mathrm{y}=\frac{2 \mathrm{~m} \nu}{\mathrm{~S}}$ the differential cross sections for neutrino and anti-neutrino are written in the usual form:

$$
\begin{aligned}
\frac{d^{2} \nu, \bar{\nu}}{d x d y}=\frac{\lambda^{2}}{\left(c_{V}^{2}+c_{A}^{2}\right)^{2}} \frac{G^{2} s}{2 \pi} \frac{1}{\left(1+x y \frac{s}{m_{\psi}^{2}}\right)^{2}}\{ & \left(1-y-x y \frac{m^{2}}{s}\right) F_{2}(x)+x^{2} F_{1}(x) \\
& \left.\mp x y\left(1-\frac{y}{2}\right) F_{3}(x)\right\}
\end{aligned}
$$

Assuming Bjorken scaling, spin $1 / 2$ constituents and maximal V-A interference $2 \mathrm{xF}_{1}(\mathrm{x})=\mathrm{F}_{2}(\mathrm{x})=-\mathrm{x} \mathrm{F}_{3}(\mathrm{x})$, the differential cross sections take a simple form:

$$
\begin{aligned}
& \frac{\mathrm{d}^{2} \sigma(\nu \mathrm{~N} \rightarrow \nu+\ldots)}{\mathrm{dxdy}}=\frac{\lambda^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \frac{\mathrm{G}^{2} \mathrm{~s}}{2 \pi} \frac{1}{\left(1+\mathrm{xy} \frac{\mathrm{~s}}{\mathrm{~m}_{\psi}^{2}}\right)^{2}} \mathrm{~F}_{2}(\mathrm{x}) \\
& \frac{\mathrm{d}^{2} \sigma(\bar{\nu} \mathrm{~N} \rightarrow \bar{\nu}+\ldots)}{\mathrm{dx} \mathrm{dy}}=\frac{\lambda^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \frac{\mathrm{G}^{2} \mathrm{~s}}{2 \pi} \frac{1}{\left(1+\mathrm{xy} \frac{\mathrm{~s}}{\mathrm{~m}_{\psi}^{2}}\right)^{2}}(1-\mathrm{y})^{2} \mathrm{~F}_{2}(\mathrm{x})
\end{aligned}
$$

Let us parametrize $\mathrm{F}_{2}(\mathrm{x})=\mathrm{a}+\mathrm{b} \sqrt{\mathrm{x}}$ (Regge parametrization). The threshold behavior near $x \approx 1$ for $F_{2}(x)$ can be taken into account by introducing a multiplicative factor $(1-\mathrm{x})^{\alpha}$. Integrating over x and y , one obtains:
$\sigma(\nu \mathrm{N} \rightarrow \nu+\ldots)=\frac{\lambda^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \frac{\mathrm{c}^{2} \mathrm{~m}_{\psi}^{2}}{2 \pi}\left\{\mathrm{a} \ln \left(1+\frac{\mathrm{s}}{\mathrm{m}_{\psi}^{2}}\right)+2 \mathrm{~b}\left(1-\frac{\arctan \sqrt{\mathrm{s} / \mathrm{m}_{\psi}^{2}}}{\sqrt{\mathrm{~s} / \mathrm{m}_{\psi}^{2}}}\right)\right\}$

$$
\begin{aligned}
\sigma(\bar{\nu} \mathrm{N} \rightarrow \bar{\nu}+\ldots)= & \frac{\lambda^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \frac{\mathrm{G}^{2} \mathrm{~m}_{\psi}^{2}}{2 \pi}\left\{\mathrm{a}\left[\left(1+\frac{\mathrm{m}_{\psi}^{2}}{\mathrm{~s}}\right) \ln \left(1+\frac{\mathrm{s}}{\mathrm{~m}_{\psi}^{2}}\right)-\frac{3}{2}-\frac{\mathrm{m}_{\psi}^{2}}{\mathrm{~s}}\right]\right. \\
& \left.+\mathrm{b}\left[4 \frac{\mathrm{~m}_{\psi}^{2}}{\mathrm{~s}}\left(1+\frac{1}{3} \frac{\mathrm{~m}_{\psi}^{2}}{\mathrm{~s}}\right) \ln \left(1+\frac{\mathrm{s}}{\mathrm{~m}_{\psi}^{2}}\right)+2-\frac{4}{3} \frac{\mathrm{~m}_{\psi}^{2}}{\mathrm{~s}}-\frac{16}{3} \frac{\arctan \sqrt{\mathrm{~s} / \mathrm{m}_{\psi}^{2}}}{\sqrt{\mathrm{~s} / \mathrm{m}_{\psi}^{2}}}\right]\right\}
\end{aligned}
$$

For $\mathrm{s} \ll \mathrm{m}_{\psi}^{2}$ the cross sections are linear in s , and

$$
\begin{aligned}
\sigma(\nu \mathrm{N} \rightarrow \nu+\ldots) & =3(\bar{\nu} \mathrm{~N} \rightarrow \bar{\nu}+\ldots)=\frac{\lambda^{2}}{\left(\mathrm{c}_{\mathrm{V}}^{2}+\mathrm{c}_{\mathrm{A}}^{2}\right)^{2}} \frac{\mathrm{G}^{2}}{2 \pi}\left(\mathrm{a}+\frac{2 \mathrm{~b}}{3}\right) \mathrm{s} \\
& =2.5\left(\mathrm{a}+\frac{2 \mathrm{~b}}{3}\right) 10^{-39} \mathrm{~cm}^{2}(\mathrm{E} / \mathrm{GeV})
\end{aligned}
$$

Here we take $c_{V}=c_{A}=1$. The parameters a and $b$ can be determined by fitting the above equation with low energies Gargamelle data $\sigma\left(\nu \mathrm{N} \rightarrow \mu^{-}+\ldots\right)=$ $0.7810^{-38} \mathrm{~cm}^{2}(\mathrm{E} / \mathrm{GeV})$ and from the ratio $\mathrm{R}=\frac{\sigma(\nu \mathrm{N} \rightarrow \nu+\ldots)}{\sigma\left(\nu \mathrm{N} \rightarrow \mu^{-+} \ldots\right)}=0.22 \pm 0.03$. We extract $\sigma(\nu \mathrm{N} \rightarrow \nu+\ldots)=0.1710^{-38} \mathrm{~cm}^{2}(\mathrm{E} / \mathrm{GeV})$ from which we have

$$
a+\frac{2 b}{3}=0.7
$$

In Fig. 3 we plot $\sigma(\nu \mathrm{N} \rightarrow \nu+\ldots)$ and $\sigma(\bar{\nu} \mathrm{N} \rightarrow \bar{\nu}+\ldots)$ for two extreme values of $a$ and $b(a=0.7, b=0)$ and $(a=0, b=1.05)$. For comparison, on the same figure are plotted the cross sections when the mass of the intermediate boson is very large. If we fit the parameters a and $b$ to the area $\int_{1}^{10} \sigma(\nu) d E_{\nu}$ of Gargamelle instead of to the cross sections at $\mathrm{E}_{\nu} \cong 1 \mathrm{GeV}$, then $\sigma(\nu), \sigma(\bar{\nu})$ will be raised somewhat. We estimate that $\sigma(\nu)$ and $\sigma(\bar{\nu})$ at $\mathrm{E} \approx 100 \mathrm{GeV}$ will be at most a factor of two larger than shown in Fig. 3. This is still substantially smaller than the linear curve.

Our phenomenological analysis shares some features with models recently discussed (Adler and Tuan, Sakurai). However here the parameters are fixed


Fig. 3


Fig. 4
by SPEAR data, while in previous works the parameters are extracted from neutrino experiments. Moreover, because of the "light" mass of $\psi$, the cross sections $\sigma(\nu)$ and $\sigma(\bar{\nu})$ as plotted in Fig. 3 soon deviate from the straight lines which characterize models with high mass intermediate boson. This can be tested in the near future.

We conclude with a few comments :

1. All the calculations given above are done only for $\psi$ The coming of the second $\psi^{\prime}(3695)$ makes the weak neutral boson interpretation unnatural. One can of course stick to this interpretation (see, however, objections later on): in this case the presence of $\psi(3695)$ whose coupling to leptons is about $\sqrt{1 / 3}$ that of $\psi(3105)$ will increase $\sigma(\nu)$ and $\sigma(\bar{\nu})$ by about $25 \%$ (see Fig. 4).
2. If $\psi$ and $\psi^{\prime}$ are both weak neutral bosons, the decay width of $\psi^{\prime} \rightarrow \psi \pi \pi$ can be estimated to be at most of the order of a few eV.
$\left(\Gamma_{\psi^{\prime} \rightarrow \psi \pi \pi}=\frac{\mathrm{g}^{2}}{256 \pi^{3}}\left(1-\frac{\mathrm{m}}{\mathrm{M}}\right)^{3}(\mathrm{M}+\mathrm{m})\right)$ where m and M are mass. of $\psi$ and $\psi^{\prime}$.
Hence this puts the neutral current interpretation in a very difficult position. One possible way out is to allow a strong coupling for $\psi, \psi^{\prime}$ when they couple in pairs (e.g., $\psi^{\upharpoonright} \psi \pi \pi$ vertex). This will enhance both $\sigma(\nu \mathrm{p} \rightarrow \nu \psi \mathrm{x})$ and $\sigma(e p \rightarrow e \psi x)$.

Another possible way out is to consider $\psi$ as the weak boson and $\psi^{1}$ as a color or charm state. In this case, one has to invoke some unknown suppression factor in the cross section ( $\nu \mathrm{p} \rightarrow \nu \mathrm{x}$ ) via $\psi$ exchange to fit FNAL data. 3. It will be interesting to search for the charged partners $\mathrm{W}^{+} \mathrm{W}^{-}$whose mass may not be very far from 4 GeV . Its effect on $\nu \rightarrow \mu^{-}$and $\bar{\nu} \rightarrow \mu^{+}$scattering, namely the x and y distributions, as well as the total cross sections,
will be different from that predicted by models with high $W^{ \pm}$mass. Experimentally, $\sigma\left(\nu \rightarrow \mu^{-}\right)$and $\sigma\left(\bar{\nu} \rightarrow \mu^{+}\right)$rise approximately linearly up to NAL energies. However, a simple estimate of the charged-boson production cross sections $\sigma\left(\nu \mathrm{N} \rightarrow \mu^{-} \mathrm{W}^{+}+\ldots\right)$ shows that it has the same order of magnitude as the deep inelastic $\sigma\left(\nu \mathrm{N} \rightarrow \mu^{-}+\ldots\right)$. Any lower bound on the mass of $\mathrm{W}^{ \pm}$should take into account the presence of $\mathrm{W}^{ \pm}$production cross section or the possibility of the presence of more than one $W^{ \pm}$.
4. The contribution of $\psi$ to deep inelastic electroproduction can be estimated to be approximately $10^{-4} \frac{\mathrm{Q}^{2}}{\mathrm{~m}^{2}}$ (the interference term of $\psi$ with photon $\gamma$ ).
5. Assuming the charged partners $\mathrm{W}^{ \pm}$of $\psi$ have approximately the $\psi$ mass (i.e., within a factor of 2) and the same coupling $g\left(\frac{g^{2}}{4 \pi} \simeq 10^{-6}\right)$ then the amplitudes for $\mathrm{K}_{\mathrm{L}} \rightarrow \mu^{+} \mu^{-}$and $\mathrm{K}_{\mathrm{L}} \rightarrow 2 \gamma$ are suppressed respectively to $\mathrm{G} \alpha^{2}$ and $\mathrm{G} \alpha$. This means the introduction of charm is not needed to suppress the $\mathrm{K}_{\mathrm{L}} \rightarrow 2 \mu$ decay.
6. It should be noted that in the quark-parton model $\frac{\Gamma_{\gamma} \rightarrow \text { hadron }}{\Gamma_{\gamma \rightarrow e^{+}} e^{-}}=\sum_{i=1}^{N} Q_{i}^{2}$ where $Q_{i}$ is the charge of the $i^{\prime}$ th quark (in units of e). The sum goes over N quarks (12 in the color-charm quark model). On the other hand, if $\psi$ is weak $U(1)$ neutral boson, then $\frac{\Gamma_{\psi} \rightarrow \text { hadron }}{\Gamma_{\psi} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}=\mathrm{N}$. Experimentally one finds

$$
\frac{\Gamma_{\psi \rightarrow \text { hadron }}}{\Gamma_{\psi \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}}} \cong 14
$$

in the $\psi(3105)$. However, as far as $\psi(3695)$ is concerned, this ratio is above a hundred, which is another indication that the interpretation of this particle as a weak neutral boson is in trouble.

Let us summarize the situation on the weak boson hypothesis. First of all, theoretically none of the schemes is attractive. Further, the schemes do not in any natural way explain why the branching ratios of $\psi^{\prime}, \Gamma_{h}^{\prime} / \Gamma_{\mathrm{e}}^{\prime}>\Gamma_{\mathrm{h}} / \Gamma_{\mathrm{e}}$, or why
the decay $\psi(3695) \rightarrow \psi(3105)+\pi^{+} \pi^{-}$occurs with a strong coupling constant. On the other hand, the neutral current experiments may be "explained" using the $\psi^{\prime}$ 's as the weak bosons. Clearly more experimental tests are desirable. Evidence of parity violation in the ee $\rightarrow$ ee and $\mu \mu$ channels and the appearance of leptons in the $\psi^{\prime}(3695) \rightarrow \psi(3105)$ decays would be suggestive for such schemes. Finding many partners of the $\psi^{\prime} \mathrm{s}$ (both charged or neutral) would be probably difficult to reconcile with the weak boson hypothesis. Seeing diffractive photoproduction of the $\psi^{\prime}$ s, over a large energy range, would in all probability dispel all doubts that these particles are not hadrons.

## Appendix A

The covariant derivative for a gauge model is

$$
\nabla_{\mu}=\partial_{\mu}-i g_{1} A_{1}^{\alpha} F_{1}^{\alpha}-i g_{2} A_{2}^{\alpha} F_{2}^{\alpha}-\ldots-i g_{n} A_{n}^{\alpha} F_{n}^{\alpha}
$$

where $F_{i}^{\alpha}$ are generators (matrices). The charge operator is

$$
Q=Q_{1}+Q_{2}+\ldots+Q_{n}
$$

Rewrite

$$
\begin{aligned}
g_{1} A_{1} Q_{1} & +g_{2} A_{2} Q_{2}+\ldots+g_{n} A_{n} Q_{n}= \\
& =e\left(Q_{1}+Q_{2}+\ldots+Q_{n}\right) A+\sum \text { (operator) } \cdot \text { (orthogonal fields) } \\
A & =T_{\gamma i} A_{i}=\text { (photon) } ; T=\text { orthogonal matrix } .
\end{aligned}
$$

From BHY, ${ }^{8}$ do it for the first two:

$$
\begin{aligned}
g_{1} A_{1} Q_{1}+g_{2} A_{2} Q_{2} & =g_{1} \sin \phi_{2}\left(Q_{1}+Q_{2}\right)\left(\sin \phi_{2} A_{1}+\cos \phi_{2} A_{2}\right) \\
+\frac{g_{1}}{\cos \phi_{2}} & {\left[\cos ^{2} \phi_{2} Q_{1}-\sin ^{2} \phi_{2} Q_{2}\right]\left(\cos \phi_{2} A_{1}-\sin \phi_{2} A_{2}\right) } \\
\tan \phi_{2} & =g_{2} / g_{1} \\
e_{1} & =g_{1} \\
e_{2} & =g_{1} \sin \phi_{2}
\end{aligned}
$$

Now form the recursion

$$
\begin{aligned}
e_{1} & =g_{1} \\
e_{2} & =e_{1} \sin \phi_{2} \\
e_{3} & =e_{2} \sin \phi_{3} \\
& \cdot \\
& \cdot \\
e & =e_{n}=e_{n-1} \sin \phi_{n}
\end{aligned}
$$

where $\tan \phi_{\mathrm{k}}=\mathrm{g}_{\mathrm{k}} / \mathrm{e}_{\mathrm{k}-1}$, which gives $\sin \phi_{\mathrm{k}}=\frac{\mathrm{g}_{\mathrm{k}}}{\sqrt{\mathrm{e}_{\mathrm{k}-1}^{2}+\mathrm{g}_{\mathrm{k}}^{2}}}$.
Therefore,

$$
\frac{1}{e_{k}^{2}}=\frac{1}{e_{k-1}^{2} \sin ^{2} \phi_{k}}=\frac{1}{\mathrm{~g}_{\mathrm{k}}^{2}}+\frac{1}{\mathrm{e}_{\mathrm{k}-1}^{2}}
$$

Clearly, we now have

$$
\frac{1}{e^{2}} \equiv \frac{1}{e_{n}^{2}}=\frac{1}{g_{n}^{2}}+\frac{1}{g_{n-1}^{2}}+\ldots+\frac{1}{g_{1}^{2}}
$$

which proves our assertion.

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