Interference Effects
Angular Distributions
How to Extract Widths
Radiative Corrections

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NOTE: The numbers quoted herein are NOT to be quoted in publication. They are meant to be illustrative, not definitive.

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This document is a working paper on the phenomenology of the $\psi$, which consists of two principal parts. It has evolved during a series of seminars on the $\psi$ with broad participation of the theoretical and experimental community at SLAC. The first part is a brief synopsis of the main tests and conclusions which can be extracted from the data, and is concerned with results. The second part is a collection of more detailed discussion of various points of the first section, and some independant discussions.

References to discussions in the second section are made in the first section by author.

## TABLE OF CONTENTS

## First Section

I. Introduction
II. Line Shapes and Partial Widths
III. Spin and Parity from Crossections and Angular Distributions

## Second Section

I. Radiative Corrections and Resonance Parameters in $\mathrm{e}^{+} \mathrm{e}^{-}$
Anihilation J. D. Jackson
II. $\psi$ Radiative Corrections
Y. Tsai
III. Interference Effects in Hadron Channels J.Bjorken, S. Brodsky
IV. Interference in Hadrons
R. Pearson
V. Front-Back Asymmetries at Resonance Peak
A. Weldon
VI. Tests for the Charge Conjugation of the $\psi$
S. Brodsky
VII. On the Effects of Nonconservation of Parity for a Resonance in the Channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$
J. D. Jackson
VIII. $P$ and $C$ Symmetries in Hadronic Final States
J. Ki hn
IX. Is $\psi\left(\psi^{\prime}\right)=\bar{\psi}\left(\bar{\psi}^{\prime}\right)$ ?
B. Ward
. X. Detailed $\psi$ Effects in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$and $\mu^{+} \mu^{-}$
R. Budny
XI. Calculations of $\mathrm{e}^{+} \mathrm{e}^{-}-\mathrm{e}^{+} \mathrm{e}^{-}$and $\mu^{+} \mu^{-}$with Beam and
Radiative Corrections
R. Giles

## Phenomenology of the $\psi$

I. Introduction

The principal channels to be investigated are

$$
\begin{aligned}
& \mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \psi \\
& \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \\
& \rightarrow \psi \rightarrow \mu^{+} \mu^{-} \\
& \rightarrow \psi \rightarrow \text { hadrons } .
\end{aligned}
$$

The energy dependence of the phenomenon across resonance is of importance: we define three regions of interest - the peak region $W=W_{\text {res }} \pm \Delta W$, with $\Delta W$ the machine resolution. For W a few MeV below the peak we have the interference region, where the resonant Breit-Wigner amplitude interferes with the electromagnetic background. The radiative tail dominates the region a few MeV above the peak.

We assume there is a single resonance in the observed peak, unless otherwise indicated.

The second section contains a discussion on the extraction of partial widths from the cross section data, and numerical estimates based on the available preliminary data. The third section is a discussion of spin and parity assignments based on the two reactions with leptonic final states.

## II. Line Shapes and Partial Widths

Ultimately, direct fitting of the shape of the resonance curves with energy should extract the most information on partial widths, but a simple procedure suffices for orientation purposes.

First, ignore radiative corrections. Then from Breit-Wigner

$$
\begin{gathered}
{[d \sigma \text {-background }]=\frac{\pi(2 J+1)}{W^{2}} \frac{\Gamma_{e}+e^{-} d \Gamma_{f}}{\left(w-w_{0}\right)^{2}+\Gamma^{2} / 4}+\text { (interference) }} \\
e^{+} e^{-} \rightarrow f
\end{gathered}
$$

Integrating across the resonance causes the interference term to cancel out

$$
\int d w[d 5-b a c k g r o u n d)=\frac{2 \pi^{2}(2 J+1)}{w^{2}} \Gamma_{e^{+}} e^{-} \frac{d \Gamma_{f}}{\Gamma_{\text {tot }}}
$$

If all final states are summed, then the area under the peak measures $\Gamma_{e^{+}} e^{-.}$ More generally,
A. Area under $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ all hadrons $=\frac{2 \pi^{2}(2 J+1)}{w^{2}} \frac{\Gamma_{e}+e^{-} \Gamma_{\text {had }}}{\Gamma_{\text {tot }}}$ which must be corrected for radiative effects.
B. Contributions of neutral modes (other than $\nu \bar{\nu}$ ) can be estimated from
the SP . 16 (HEPL) experiment.
C. The area under the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$peak $=\frac{2 \pi^{2}(2 J+1)}{W^{2}} \frac{T_{e+e^{-}} \Gamma_{\mu+\mu}}{T_{\text {tht }}}$

However, it is better to take

$$
\frac{\text { Area }\left(\mu^{+} \mu^{-}\right)}{\text {Area(hadrons) }} \text { or } \frac{(\text { Peak height }) \mu^{+} \mu^{-}}{\text {(Peak height)hadrons }}
$$

This should be independent of radiative correction and resolution. The same holds for the area under $e^{+} e^{-} \rightarrow e^{+} e^{-}$.
D. Radiative corrections: Ignoring the contribution of the interference term, the integrated area under the Breit-Wigner peak (treated as a $\delta$-function) out to an energy $\omega$ is given by a factor

times the unradiated arca (see figure).

(Fig. 1)

That is (Area) $=A_{0} \mathrm{e}^{-\delta}$ with $\sigma_{0}(\mathrm{w})=\mathrm{A}_{0} \delta\left(\mathrm{w}-\mathrm{w}_{0}\right)$.
If $W-W_{0} \gg \Delta W$, the machine resolution, this correction is independent of $\Delta W$.

The radiative tail (for $\mathrm{W} \gg \mathrm{W}_{0}$ ) is

$$
\bar{T}(w) \cong \frac{E}{\left(w-w_{0}\right)} e^{-\delta(w)} A_{0}
$$

Other corrections are expected to be small; however radiative corrections to interference effects are not negligible. Structures with rapid energy variation tend to be suppressed by a factor $e^{-\delta}$.

## E. Numerical Estimates:

Numerical integration has been caried out, using the published data, for the process $e^{+} e^{-} \rightarrow \psi 3105 \rightarrow$ charged hadrons with $W_{\text {min }}=3.10 \mathrm{GeV}$ and $W_{\text {max }}=3.12 \mathrm{GeV}$. We obtain

$$
\text { Area }=8600 \times 10^{-32} \mathrm{~cm}^{2} \mathrm{MeV}
$$

$$
\epsilon \cong \frac{4 \alpha}{\pi}\left\{\ln \frac{m_{4}}{m_{e}}-\frac{1}{2}\right\}=0.076
$$

$$
e^{\delta\left(w_{\text {max }}\right)}=e^{0.076 \ln \frac{3.105}{2 \times .015}=\frac{1}{1.415}}
$$

$$
\begin{aligned}
& \text { giving } \\
& \frac{\left.T_{e+e^{-}} T_{\text {nad charged }}\right)}{T_{\text {tot }}}=4,94 \mathrm{KeV} \frac{3}{(2 J+1)}
\end{aligned}
$$

From the ratio of two peaks we obtain

$$
\frac{T_{\text {had (charged })}}{T_{\mu+\mu-}} \cong \frac{2300}{160}=14.3
$$

If $\psi$ couples to $e^{+} \mathrm{e}^{-}$electromagnetically, then we expect

$$
T_{\text {tot }}=T_{\text {had (charged })}+T_{\text {had }} \text { (neutral) }+2 T_{e^{+} e^{-}}
$$

If it couples weakly, then perhaps one must include neutrinos to give $4 \Gamma_{\mu^{+} \mu^{-}}$ rather than 2 .

In conclusion

$$
\begin{aligned}
& T_{4 \rightarrow e^{t} e^{-}}=\frac{3}{(2 J+1)} 4.94 \mathrm{KaV}\left\{\frac{T_{\text {tot }}}{T_{\text {nad }}(\text { charged })}\right\} \\
& T_{\psi \rightarrow h a d}(\text { charged })=\frac{3}{(2 J+1)} 73 \mathrm{KeV}\left\{\frac{T \text { tot }}{T_{\text {nad (charged) }}}\right\} \\
& \text { where } \\
& \frac{\text { Tot }}{T_{\text {had (charged) }}}=1+\frac{2}{14,3}+\frac{\text { Thadcheutral) }}{\text { Thad (charged) }} \\
& \text { These numbers are intended to be illustrative only. } \\
& \text { References in the second part for this section are: } \\
& \text { Radiative corrections etc. ... J.D.Jackson and Y.Tsai } \\
& \text { Interference effects in hadrons S.Brodsky and R. Pearson }
\end{aligned}
$$

## III. Spin and Parity

This section is divided into three parts. In the first two sections the cases $J=0$ and $J>1$ are considered briefly. In the third section the case $J=1$ is considered in detail.
A. Spin 0:
(i) If $\left(\frac{\mathrm{d} \sigma^{\mu^{+} \mu^{-}}}{\mathrm{d} \Omega}-\right.$ background $)$ is not isotropic, then $\mathrm{J}_{\psi} \neq 0$.
(ii) If $\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega}\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right.$) shows an interference dip (e.g. Fig. 1), then $J_{\psi} \neq 0$ because the initial $e^{+} e^{-}$state for the background amplitude has $J_{z}= \pm 1$ and cannot interfere with the $\mathrm{J}=0$ resonant amplitude. (For $\mathrm{J}_{\psi}>1$ there can be interference in the angular distribution.)

Write the vertex as $\overline{\mathrm{u}}\left(\mathrm{g}_{\mathrm{s}}+\mathrm{ig} \mathrm{p}_{\mathrm{p}} \gamma_{5}\right) \mathrm{u}$. Then

$$
\begin{aligned}
& \frac{d \sigma}{d \Omega} \left\lvert\,=\frac{\alpha^{2}}{4 s}\left(1+\cos ^{2} \theta\right)+\frac{1}{64 \pi^{2} s}\left[\frac{s^{2}}{\left(s-m^{2}\right)^{2}+m^{2} \pi^{2}}\left(g_{s}^{2}+g_{p}^{2}\right)^{2}\right]\right. \\
& e^{t} e^{-} \rightarrow \mu^{+} \mu^{-} \\
& \frac{d \sigma}{d \Omega} \left\lvert\,=\frac{\alpha^{2}}{45}\left[\left(1+\cos ^{2} \theta\right)+2 \frac{1+\cos ^{4} \theta / 2}{\sin ^{4} \theta / 2}-4 \frac{\cos ^{4} \theta / 2}{\sin ^{2} \theta / 2}\right]\right. \\
& e^{+} e^{-} \rightarrow e^{+} e^{-} \\
& +\frac{1}{64 \pi^{2} s}\left[\frac{s^{2}}{\left(s-m^{2}\right)^{2}+m^{2} \Gamma^{2}}\left(g_{s}^{2}+g_{p}^{2}\right)^{2}\right. \\
& \left.-\frac{8 \pi \alpha s\left(s-m^{2}\right)\left(g_{s}^{2}+g_{2}^{2}\right)}{\left(\left(s-m^{2}\right)^{2}+m^{2} T^{2}\right) \sin ^{2} \theta 2}\right]
\end{aligned}
$$

B. Spin >1 (G. Ringland, D. Wright)

If the $\psi$ couplings to leptons preserves chirality then

$$
\left.\frac{d \sigma^{\text {res. }}}{d \Omega}\right|_{e^{t} e^{-} \rightarrow \mu+\mu^{-}}=A(s)\left|d_{11}^{J}(\theta)\right|^{2}+B(s)\left|d_{1,-1}^{J}(\theta)\right|^{2}
$$

If $C$ and $P$ are conserved, then $A(s)=B(s)$. The distributions are plotted for $J=2,3,9,10$. They are self explanatory. (Figs. 2-3).

The most general analysis for spin $J$ is too cumbersome to be informative.
C. Spin 1: (Budny, Cvitanovic, Giles, Pearson. . .)

If there exists no CP violation and no anomalous moment couplings, the $\psi$ contribution to the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \bar{\ell} \ell$ amplitude can be written:

$$
\bar{u}_{e}\left(p^{\prime}\right)\left[g_{v} \gamma^{\mu}+g_{A} \gamma^{\mu} \gamma_{s}\right] v_{l}(p) \frac{1}{s-m^{2}+i m \Gamma} \bar{v}_{e}(k)\left[g_{v} \gamma_{\mu}+g_{A} \gamma_{\mu} \gamma_{s}\right] u_{e}\left(k^{\prime}\right)
$$

where $g_{V}$ and $g_{A}$ are real.
This gives differential cross sections

$$
\begin{aligned}
& e^{+} e^{-} \rightarrow e^{+} e^{-} \\
& \frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2}} \frac{s}{\left(s-m^{2}\right)^{2}+m^{2} \Gamma^{2}}\left[\left(g_{V}^{2}+g_{A}^{2}\right)^{2}\left(1+\cos ^{2} \theta\right)+8 g_{V}^{2} g_{A}^{2} \cos \theta\right] \\
& +\frac{\alpha}{8 \pi} \frac{5-m^{2}}{\left(5-m^{2}\right)^{2}+m^{2} \pi^{2}}\left(g_{V}^{2}+g^{2} A\right) \frac{(1+\cos \theta)^{2}}{1-\cos \theta}
\end{aligned}
$$


(Fig. 2)

(Fig. 3)

$$
\begin{aligned}
& +\frac{\alpha^{2}}{45}\left[\left(1+\cos ^{2} \theta\right)+2 \frac{1+\cos ^{4} \theta / 2}{\sin 4 \theta / 2}-4 \frac{\cos ^{4} \theta / 2}{\sin ^{2} \theta / 2}\right] \\
& \frac{e^{t} e^{-} \rightarrow \mu^{4} \mu^{-}}{} \\
& \frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2}} \frac{5}{\left(5-m^{2}\right)^{2}+m^{2} \pi^{2}}\left[\left(g_{V}^{2}+g_{A}^{2}\right)^{2}\left(1+\cos ^{2} \theta\right)+8 g_{V}^{2} g_{A}^{2} \cos \theta\right] \\
& +\frac{\alpha}{8 \pi} \frac{5-m^{2}}{\left(5-m^{2}\right)^{2}+m^{2} \pi^{2}}\left[g_{V}^{2}\left(1+\cos ^{2} \theta\right)+2 g_{A}^{2} \cos \theta\right] \\
& +\frac{\alpha^{2}}{45}\left(1+\cos ^{2} \theta\right)
\end{aligned}
$$

where we have assumed universality of electron and muon couplings. (If not: $\mathrm{g}_{\mathrm{V}}^{2} \rightarrow \mathrm{~g}_{\mathrm{V}}(\mathrm{e}) \mathrm{g}_{\mathrm{V}}(\mu)$ and $\left.\mathrm{g}_{\mathrm{A}}^{2} \rightarrow \mathrm{~g}_{\mathrm{A}}(\mathrm{e}) \mathrm{g}_{\mathrm{A}}(\mu).\right)$

The interference terms between $\psi$ and the photon are reflected in the behavior of the total cross sections and angular distributions:

## Total Cross Sections

The interference terms cause dips in the total cross sections for both $\mu^{+} \mu^{-}$ and $\mathrm{e}^{+} \mathrm{e}^{-}$. For $\mu^{+} \mu^{-}$the dip is a near zero of the theoretical cross section on the low energy side of the peak.

$$
m-E_{d i p} \approx \frac{3}{2 \alpha}\left(\frac{g_{v}^{2}+g_{A}^{2}}{g_{r^{2}}}\right) \prod_{\psi \rightarrow e_{e}} \approx 1 M_{e} V \text { for } \psi(3105)
$$

There is a corresponding enhancement on the high energy side. After smearing by the beam resolution (fig. 4)

$$
m-E \operatorname{dip} \cong 3 \mathrm{meV} \text { for } \psi(3105), \frac{\sigma_{\text {min }}}{\sigma_{\text {background }}} \simeq 50 \%-70 \%
$$

The presence of the interference dip in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$implies that $\psi$ cannot have pure axial couplings.

The sign of the effect in $\sigma \mathrm{e}^{+} \mathrm{e}^{-}-\mathrm{e}^{+} \mathrm{e}^{-}$is reversed and markedly less pronounced. $E_{\text {dip }}>M$ because the predominant interference is with the t-channel photons cancelling the interference with the s-channel photons (Fig. 5 ). Any such dip will be buried under the radiative tail on the high energy side of the peak.

It is worth mentioning that experimentally one need not measure the cross section at the dip where event rates are low in order to see the interference terms. Any measure of the skewness of the total cross section relative to a pure BreitWigner near the peak will suffice.

(Fig. 4 )

(Fig. 5)
e.g.

changes by a factor $\sim 2$ over the region $m_{\psi}-1 . \mathrm{MeV} \rightarrow \mathrm{m}_{\psi}+1 . \mathrm{MeV}$.

## Angular Distributions

$$
\underline{\mathrm{e}}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}
$$

Any large parity violation in the $\psi$ itself $\left(g_{V} \sim g_{A}\right)$ is observable as a frontback asymmetry at the peak in $e^{+} e^{-}-\mu^{+} \mu^{-}$

$$
\begin{equation*}
\frac{d \sigma}{d \Omega} \propto(1+\cos \theta)^{2} \tag{Fig.6}
\end{equation*}
$$

For nearly pure $V$ or A the distribution at the peak is $\left(1+\cos ^{2} \theta\right)$, so one must look in the interference region to distinguish the two cases.

For pure vector the angular distribution is $1+\cos ^{2} \theta$ at all energies there is no front-back asymmetry.

For purely axial vector $\psi$, there is a front-back asymmetry

$$
A \equiv \frac{\sigma_{50^{\circ}<\theta<90^{\circ}}-\sigma_{90^{\circ}}<\theta<130^{\circ}}{\sigma_{50^{\circ}}<\theta<130^{\circ}}
$$

that is negative in the interference region and positive in the radiative tail of magnitude $\sim 35 \%$ (Fig. 6)

(Fig. 6)

## $\underline{e^{+} e^{-} \rightarrow e^{+} e^{-}}$

For the $\mathrm{e}^{+} \mathrm{e}^{-}$reaction, the information in the angular distributions is more difficult to extract. The background Bhabha process has a frontback asymmetry $A=0.66$. For $V=A$, this asymmetry is changed only slightly near the peak (Fig. 7 ). For either pure $V$ or pure A, A is decreased at the peak by the addition of a large $\left(1+\cos ^{2} \theta\right)$ term. The effects in the interference region are relatively small.
Is there more than one $\psi$ (or $\psi^{\prime}$ )?
Various theories could have more than one neutral $\psi$, conceivably degenerate in mass to an MeV . (See Barshays preprint; also colored " $\rho 0$ " degenerate with colored " $\omega$ " is another such option.) A variety of interference effects are possible, depending on whether the two lines overlap, one is broad, one narrow, ect.. The consequences for experiment are
(i) Branching ratios, distributions, ect. on the high side of the resonance may differ from those on the low side, and
(ii) The line shapes may be peculiar.

A general study for the lepton channels, assuming two spin one $\psi^{\prime} \mathbf{s}$, is given by B. Ward.

References in the second part for this section are:
Effects of P and C symmetry on angular distributions ...
A. Weldon, S. Brodsky, J. D. Jackson, and J. Kuhn

Two $\psi^{\prime}$ 's .... B. Ward
Calculations of crossections .... R. Budny, and R. Giles

(Fig. 7)


A simple coherent discussion of the problem of radiative corrections for the production of resonances in $e^{+} e^{-}$annihilation is given with emphasis on narrow resonances such as the $\psi(3105)$ and $\psi(3695)$ where the finite spreads in the energies of the beams are a significant factor in extracting partial widths from the data. Examples for the $\psi(3105)$ and $\psi(3695)$ are given.
I. Introduction

The problem of radiative corrections is familiar and well understood by workers at electron accelerators or $e^{+} e^{-}$storage rings. These notes probably contain nothing new for such experts, although the occurrence of resonances that are narrow compared to the beam energy resolution introduces aspects not normally considered. The purpose of these notes is to collect in one place the formulas relevant for the analysis of resonant line shapes and parameters in $e^{+} e^{-}$annihilation and to apply them to determination of the partial and total widths of the $\psi(3105)$ and $\psi(3695)$. Acknowledgments are due to J. D. Bjorken, G. Feldman, H. Lynch, Y. -S. Tsai, and D. R. Yennie for teaching me about various aspects of radiative corrections.

In $e^{+} e^{-}$annihilation the lowest order radiative corrections arise from the six diagrams in Fig. $1(b)$.

Fig. I(a)


Fig. 1(b)



The first two diagrams correspond to real photon emission and their sum contributes incoherently to the cross section. The other four renormalization diagrams are higher order in $\alpha$ and contribute in lowest order only by interference with the nonradiative amplitude of Fig. I(a). The calculation of the lowest order radiative corrections is done in several places. The most imnediately applicable reference is
G. Bonneau and F. Martin, Nucl. Phys. B27, 381 (1971).

Their Eq. (16) is given below as Eq. (1).
A more complete treatment of the problem of the infrared divergences associated with the vanishing of the photon mass involves the consideration of emission of arbitrary numbers of very soft photons. A basic understanding of the soft photon problem was achieved by Bloch and Nordsieck in 1937. A comprehensive modern treatment is given by
D. R. Yennie, S. C. Frautschi, and H. Suura, Annals of Phys. (N.Y.) 13, 379 (1961). See also the Brandeis 1963 Summer school lectures by Yennie:
D. R. Yennie, "Topics in Quantum Electrodynamics", in Lectures on Strong and Electromagnetic Interactions, Brandeis Summer Institute in Theoretical Physics, 1963, Vol. 1, ed. K. W. Ford, Brandeis University (1964).
The consequence of including the multiple emission of soft photons is an "exponentiation" of the lowest order logarithmic corrections into power law corrections A nice discussion of this exponentiation for soft photon emission by a classical current source can be found in
J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, McGraw-Hill, N.Y. (1965), Sect. 17.10, p. 202-207.

Radiative corrections for high-energy electron scattering by nucleons and by nuclei are treated authoritatively by
L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969)
with improvements in
Y. S. Tsai, SLAC-PUB-848 (January 1971).
II. Basic formulas
(a) Notation We consider ultrarelativistic electrons and use the following notation:
$W=$ total energy in the center of mass
$E=W / 2=$ energy of each of the electrons in the initial state,
$k=$ energy of an emitted photon
$t=\frac{2 a}{\pi}\left[\ln \left(\frac{W^{2}}{m_{e}^{2}}\right)-1\right]$


The quantity $t$ is the (classical) energy radiated per unit frequency interval at low frequencies when electron and positron in head-on collision disappear.
(b) Bonneau-Martin first order formula

Equation (16) of Bonneau and Martin for the cross section including photon emission and renormalization corrections can be written in our notation as

$$
\begin{align*}
\sigma(W)= & \sigma_{0}(W)\left\{1+\frac{2 \alpha}{\pi}\left(\frac{\pi^{2}}{6}-\frac{17}{36}\right)+\frac{13}{12} t\right. \\
& \left.+t \int_{0}^{E} \frac{d k}{k}\left[\left(1-\frac{k}{E}+\frac{k^{2}}{2 E^{2}}\right) \frac{\sigma_{0}(W-k)}{\sigma_{0}(W)}-1\right]\right\} \tag{1}
\end{align*}
$$

In the integral over $d k$ the argument ( $W-k$ ) in $\sigma_{0}$ should more correctiy be $\sqrt{W^{2}-2 W k}$, but for narrow resonances our approximation is perfectly adequate. Bonneau and Martin's upper Ilmit of integration $q_{\max }$ has been put equal to $E$, corresponding to the fact that an electron can lose all its energy in radiation. The soft-photon emission is contained in the $d k / k$ term and is just the classical result, corrected for energy conservation by the cross section $\sigma_{0}(W-k)$. It is convenient to rewrite Eq. (1) with the soft-photon part displayed separately from the "hard" photon terms:

where

$$
\begin{equation*}
\varepsilon=\frac{2 \alpha}{\pi}\left(\frac{\pi^{2}}{6}-\frac{17}{36}\right)+\frac{13}{12} t \tag{3}
\end{equation*}
$$

is a small number that changes slowly with energy. (For the $\psi(3105)$, where $t=0.076, \varepsilon=0.085$.) The last term in Eq. (2) is small compared to the first two unless the energy $W$ is far off resonance. From now on we omit this "hard" photon piece, although at appropriate points below we will come back and pick it up.

With the omission of the "hard" photon terms, our simplified version of the Bonneau-Martin formula reads

$$
\begin{equation*}
\sigma(W)=\sigma_{0}(W)[1+\varepsilon]+t \int_{0}^{E} \frac{d k}{k}\left[\sigma_{0}(W-k)-\sigma_{0}(W)\right] \tag{4}
\end{equation*}
$$

(c) Exponentiated form of the radiatively-corrected cross section

The emission of arbitrarily large numbers of soft photons with energies less than $k$ leads to the introduction of a factor

$$
\begin{equation*}
\exp [-t \ln (E / k)]=\left(\frac{k}{E}\right)^{t} \tag{5}
\end{equation*}
$$

in the integrand of the integral in Eq. (4). Then we find that the radiatively corrected cross section becomes

$$
\begin{equation*}
\sigma(W)=t \int_{0}^{E} \frac{d k}{k}\binom{k}{E}^{t} \cdot \sigma_{0}(W-k)+\varepsilon \sigma_{0}(W) \tag{6}
\end{equation*}
$$

The justification for keeping the $\varepsilon \sigma_{0}(W)$ term after exponentiation is not clear. The presence of the factor ( $k / E)^{t}$ makes the integral convergent at the lower limit. In fact,


$$
t \int_{0}^{E} \frac{d k}{k}\left(\frac{k}{E}\right)^{t}=1
$$

showing that the radiative processes redistribute the cross section in energy $W$ but do not affect the total probability. This argues for the omission of the term $\varepsilon \sigma_{0}(W)$ in (6). The first part of $\varepsilon$ is very small ( 0.0027 ) and can be viewed as some sort of "inner" correction to the width $\Gamma_{e}$ in the entrance channel. The second, energydependent, part of $E$ is larger, but is not greater than 0.1 even at PEP energies of $W=30 \mathrm{GeV}$. For simplicity, we omit the $\varepsilon \sigma_{0}(W)$ term from (6) from now on. The reader who wishes to add its contribution may do so.
(d) Folding with the energy resolution function

The incident beams in a storage ring have inherent spreads in energy coming mostly from the quantum fluctuations in the emission of synchrotron radiation. Each beam is approximately Caussian in energy and so the total energy $W$ is distributed approximately in a Gaussian fashion. If the normalized resolution function for a mean beam energy $W$ is $G\left(W-W^{\prime}\right)$, the observed cross section is

$$
\begin{equation*}
\tilde{\sigma}(w)=\int_{-\infty}^{\infty} \mathrm{d} W^{\prime} \sigma\left(W^{\prime}\right) G\left(w-W^{\prime}\right) \tag{7}
\end{equation*}
$$

The resolution function $G$ is assumed to fall off sufficiently rapidly that the limits of integration can be taken formally as $\pm \infty$ without damage to the physics. Using the radiative correction formula (6) for $\sigma\left(W^{\prime}\right)$ this becomes

$$
\begin{equation*}
\tilde{\sigma}(W)=t \int_{-\infty}^{\infty} d W^{\prime} G\left(W-W^{\prime}\right) \cdot \int_{0}^{E^{\prime}} \frac{d k}{k}\left(\frac{k}{E^{\prime}}\right)^{t} \cdot \sigma_{0}\left(W^{\prime}-k\right) \tag{8}
\end{equation*}
$$

In using Eq. (8) for relatively narrow resonances it will be convenient to make certain quite justifiable approximations. For example, with $\sigma_{0}(W)$ as a resonance whose width $\Gamma$ is small compared to its mass $M$, it is justified to replace the variable upper limit on the $k$ integration by $E=\frac{W}{2}$ and to approximate

$$
\left(-\frac{k}{E^{T}}\right)^{t}=\left(\frac{2 k}{M}\right)^{t} \cdot\left(\frac{M}{2 E^{T}}\right)^{t} \approx\left(\frac{2 k}{M}\right)^{t}
$$

(Recall that $t=0.076$ for $M=3105 \mathrm{MeV}$. )

III. High-energy radiative tail

A characteristic feature of the cross sections is a radiative tail on the high-energy side of a resonance, as shown schematically in Fig. 2. This corresponds physically to the emission of a photon by one or the other of the incident electrons causing the energy of annihilation to be in the neighborhood of the resonance mass $M$, even though the incident energy $W$ is considerably higher. For energies sufficiently above the resonance that $(W-M)$ is large compared to the larger of the line width $\Gamma$ and the width $\Delta W$ of the resolution function, Eq. (8) yields a simple result:

$$
\begin{equation*}
\tilde{\sigma}(W)=t(\text { Area })_{0} \cdot \frac{1}{W-M} \cdot\left(\frac{W-M}{E}\right)^{t} \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
(\text { Area })_{0}=\int \sigma_{0}\left(W^{\prime}\right) \mathrm{d} W^{\prime} \tag{10}
\end{equation*}
$$


is the area of the cross section without radiative corrections. If we return to Eq. (2) and pick up the "hard" photon terms, this becomes

$$
\begin{equation*}
\tilde{\sigma}(w)=t(\text { Area })_{0} \cdot\left[\left(\frac{W-M}{E}\right)^{t} \frac{1}{W-M}-\frac{1}{E}+\frac{W-M}{2 E^{2}}\right] \tag{11}
\end{equation*}
$$

The last two terms are important only very far from the resonance. The first term is basically the $1 / \Delta E$ bremsstrahlung spectrum, modified by a slowly varying factor incorporating the multiple soft-photon emission. The lowest order (Bonneau-Martin) cross section gives (11) with this factor omitted. Note that Eq. (11) is independent of $G\left(W-W^{\prime}\right)$ and depends only on $W, M$ and (Area) $O_{0}$.
IV. Cross section for a resonance whose width is large
compared to the energy resolution
For a resonance like the $\rho^{0}$ and even the $\omega^{\circ}$, its width $\Gamma$ is large compared to the beam resolution. Then the resolution function $G\left(W-W^{\prime}\right)$ can be taken as a delta function and the observed cross section $\tilde{\sigma}(W)$ is essentially equal to (6). An integration by parts gives

$$
\begin{equation*}
\sigma(W)=-\int_{0}^{E} d k\left(\frac{k}{E}\right)^{t} \frac{d}{d k} \sigma_{0}(W-k) \tag{12}
\end{equation*}
$$



In writing (12) we have dropped a term $\sigma_{0}(W / 2)$, the assumption being that, if $\sigma_{0}(W)$ is a resonant cross section peaking at $W=M$ and $W$ is not too far from resonance, such a term is negligible. Suppose that $\sigma_{0}(W)$ is a Breit-Wigner resonance with widths whose energy variation can be neglected. Then

$$
\begin{equation*}
\sigma_{0}(W)=\left(\sigma_{0}\right)_{\max } \frac{\Gamma^{2} / 4}{(M-W)^{2}+\frac{\Gamma^{2}}{4}} \tag{13}
\end{equation*}
$$

The cross section $\left(\sigma_{0}\right)_{\max }$ is the peak cross section; its value depends on the particular channel or channels being considered. With (13) inserted, Eq. (12) becomes

$$
\begin{equation*}
\sigma(w)=\left(\sigma_{0}\right)_{\max } \frac{\Gamma^{2}}{4} \int_{0}^{E}\left(\frac{k}{E}\right)^{t} \frac{2(M-w+k)}{\left[(M-w+k)^{2}+\frac{\Gamma^{2}}{4}\right]^{2}} d k \tag{14}
\end{equation*}
$$

It is probably simplest at this stage to integrate numerically in order to see what (14) gives.

For reference and orientation we eveluate (14) at $W=M$. This is essentially, but not quite, the peak cross section, the maximum being infinitesimally higher in magnitude and in position. We find

$$
\begin{equation*}
\sigma(M)=\frac{(\pi t / 2)}{\sin (\pi t / 2)}\left(\frac{\Gamma}{M}\right)^{t}\left(\sigma_{0}\right)_{\max } \tag{15}
\end{equation*}
$$

The first factor can be approximated as $(\pi t / 2) / \sin (\pi t / 2) \simeq\left(1+\pi^{2} t^{2} / 24\right)$. It is equal to unity within 0.004 or less up to PEP energies. Thus $\sigma(M) \simeq(\Gamma / M)^{t} \sigma_{\max }$. Numerically, for the $\rho^{\circ}$ meson, with $M \simeq 770 \mathrm{MeV}, \mathrm{t}=0.063, \mathrm{~T}=150 \mathrm{MeV}$, we find $(\Gamma / M)^{t} \simeq 0.90$. For the $\omega^{0}$ meson, with $\Gamma=10 \mathrm{MeV},(\Gamma / \mathrm{M})^{t} \simeq 0.76$. The reduction in peak cross section because of radiative processes is thus not negligible and is larger the smaller the width (provided the energy resolution is good--see the next section).

For completeness we note that the lowest order radiative correction gives a factor $[1-t \ln (M / \Gamma)]$ instead of $(\Gamma / M)^{t}$, corresponding to the first terms in an expansion of $(\Gamma / M)^{t}=\exp [-t \ln (M / \Gamma)]$ in powers of $t$. For the $\omega^{\circ}$ the linear radiative correction factor is 0.723 instead of 0.758 . Inclusion of $\varepsilon \sigma_{0}(M)$ from Eq. (2) or (6) adds 0.072 to the 0.723, giving 0.795 .

The high-energy radiative tail is given by Eq. (11) with

$$
\begin{equation*}
(\text { Area })_{0}=\frac{\pi}{2} \Gamma\left(\sigma_{0}\right)_{\max } \tag{16}
\end{equation*}
$$


V. Cross section for a resonance whose width is very narrow
compared to the energy resolution
Although the general case of a resonance whose width is comparable to the energy resolution can be dealt with effectively only by numerical integration, the limitopposite to that of the previous section can be discussed simply if the resolution function is known. If the resonant cross section (13) has a total width $\Gamma$ that is very small compared to the $F W H M \Delta W$ of the resolution function $G(W-W$; ) we can approximate $\sigma_{0}\left(W^{\prime}-k\right)$ in the integrand of Eq. (8) by

$$
\begin{equation*}
\sigma_{0}\left(W^{\prime}-k\right)=(\text { Area })_{0} \delta\left(W^{\prime}-M-k\right) . \tag{17}
\end{equation*}
$$

Then we obtain

$$
\tilde{\sigma}(W)=t(\text { Area })_{0} \int d W^{\prime} G\left(W-W^{\prime}\right) \frac{1}{W^{\prime}-M}\left(\frac{W^{\prime}-M}{E^{\prime}}\right)^{t} \theta\left(W^{\prime}-M\right) .
$$

This can be written as

$$
\begin{equation*}
\tilde{\sigma}(W)=t(\text { Area })_{0} \cdot \int_{0}^{\infty} \frac{d x}{x}\left(\frac{2 x}{M}\right)^{t} \cdot G(W-M-x) . \tag{18}
\end{equation*}
$$

Here we have made a very slight approximation by putting $\left(M / 2 E^{\prime}\right)^{t}=1$ in the integrand.

If the resolution function is known the integral can be evaluated (numerically) If ( $W=M$ ) is positive and large compared to the width of the resolution function, the slowly varying factors $x^{-1}(2 x / M)^{t}$ can be evaluated at $x=W-M$ and with a normalized $G$, the result (9) for the cross section in the radia:ive tail is obtained, independent of the exact shape of $G$.

For a Gaussian resolution function the peak cross section (actually the value at $W=M$ ) can be found explicitly. Writing

$$
\begin{equation*}
G(x)=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left(-x^{2} / 2 \sigma^{2}\right) \tag{19}
\end{equation*}
$$

and introducing $z=x^{2} / 2 \sigma^{2}$ in Eq. (18), we find


$$
\begin{align*}
\tilde{\sigma}(M) & =\frac{t}{2}(\text { Area })_{0} \cdot G(0) \cdot\left(\frac{2 \sqrt{2} \sigma}{M}\right)^{t} \int_{0}^{\infty} z^{\frac{t}{2}}-1 e^{-z} d z \\
& =G(0)(\text { Area })_{0}\left(\frac{2 \sigma}{M}\right)^{t} \cdot\left[2^{t / 2} \Gamma\left(1+\frac{t}{2}\right)\right] \tag{20}
\end{align*}
$$

The factor in square brackets can be approximated as

$$
\begin{aligned}
{\left[2^{t / 2} \Gamma\left(1+\frac{t}{2}\right)\right] } & \simeq 1+\frac{t}{2}(\ln 2-0.5772 \ldots) \\
& =1+0.058 t .
\end{aligned}
$$

Since $t<0.1$ even at PEP energies, this factor can be set equal to unity. The observed peak cross section for a resonance whose width is negligible compared to the energy resolution is therefore closely given by

$$
\begin{equation*}
\tilde{\sigma}(M)=\left(\frac{2 \sigma}{M}\right)^{t} \cdot G(0)(\text { Area })_{0} \tag{21}
\end{equation*}
$$

where $\sigma$ is the standard deviation of the energy resolution function and $G(0)=1 / \sigma \sqrt{2 \pi}$. The radiative processes thus decrease the peak cross section by a factor of $(2 \sigma / M)^{t}$. Typical values at the $\psi(3105)$ are $M=3105 \mathrm{MeV}, \sigma=0.78 \mathrm{MeV}$, $t=0.076$. This gives $(2 \sigma / M)^{t}=0.561$, a very significant reduction.

The corresponding calculation with the lowest order radiative correction formula gives

$$
\tilde{\sigma}(W)=(\text { Area })_{0}\left\{G(W-M)+t \int_{0}^{E} \frac{d k}{k}[G(W-M-k)-G(W-M)]\right\} .
$$

The peak cross section with the Gaussian (19) is

$$
\begin{equation*}
\tilde{\sigma}(M)=\left[1-t \ln \left(\frac{M}{2.12 \sigma}\right)\right] \cdot G(0) \cdot(\text { Area })_{0} \tag{22}
\end{equation*}
$$

This is just what one obtains by expanding Eq. (20) to first order in $t$. For the $\psi(3105)$ parameters quoted above, the factor in square brackets is 0.414 . If we add the $\varepsilon$ term from Eq. (4), this increases to 0.499 , compared to 0.561 for the exponentiated result.

VI. Area method of determining resonance widths
in the presence of radiative corrections
The area method is the most reliable one for determining resonance parameters because the details of the energy resolution are minimized to a great degree (In principle, they are eliminated entirely.). The method is well known in nuclear physics. The only new aspect here is the presence of soft-photon processes. Firstly, consider only the energy resolution. If the resonant cross section is $\sigma_{0}(W)$ then the folded cross section is

$$
\tilde{\sigma}(W)=\int G\left(W-W^{\prime}\right) \sigma_{0}\left(W^{\prime}\right) d W^{\prime}
$$

We now integrate the cross section, smeared by the resolution function, from $W_{\text {min }}$ to a varlable upper limit $W$. The lower limit $W_{\min }$ is chosen in practice to be where the resonant part of the cross section first begins to be visible above the background. The integral is

$$
A\left(W, W_{\min }\right)=\int_{W_{\min }}^{W} \tilde{\sigma}\left(W^{\prime}\right) d W^{\prime}=\int_{W_{\min }}^{W} d W^{\prime} \int d W^{\prime \prime} G\left(W^{\prime}-W^{\prime \prime}\right) \sigma_{0}\left(W^{\prime \prime}\right) .
$$

The behavior of $A\left(W, W_{m i n}\right)$ as a function of $W$ is shown schematically in Fig. $3(b)$. In Fig. $3(a)$ the cross section and the folded cross section are sketched.


Fig. 3(a)


Fig. 3(b)


At positive values of ( $W-M$ ) large compared to the observed width $\Gamma_{o b s}$ of $\tilde{\sigma}(W)$ the integral becomes constant. Its value is found by interchanging orders of integration above. Since $G$ is normalized we find

$$
\begin{equation*}
\lim _{(W-M) \gg \Gamma_{o b s}} A\left(W, W_{\min }\right)=\int d W^{\prime \prime} \sigma_{0}\left(W^{\prime}\right) \equiv(\text { Area })_{0} . \tag{23}
\end{equation*}
$$

The plateau value of $A\left(W, W_{\min }\right)$ is thus equal to the area of the original cross section, independent of the form or details of $G\left(W-W^{\prime}\right)$.

The method needs only slight modification because of the radiative corrections. We begin with the smeared cross section with radiative corrections, Eq. (8), and integrate it from $W_{\min }$ to $W$ :

$$
\dot{A}\left(W, W_{\min }\right)=\int_{W_{\min }}^{W} d W^{\prime} \int_{-\infty}^{\infty} d W^{\prime \prime} G\left(W^{\prime}-W^{\prime \prime}\right) t \int_{0}^{E^{\prime \prime}} \frac{d k}{k}\left(\frac{k}{E^{\prime \prime}}\right)^{t} \sigma_{0}\left(W^{\prime \prime}-k\right) \cdot
$$

Since we are concerned with resonances whose observed widths are small compared with $W$ it is permissible to neglect the energy variation of a factor (M/2E") ${ }^{t}$--even for the $\rho^{\circ}$ it causes an error of less than $1 \%$--and also put the upper limit on the $d k$ integration as $M / 2$. Then an integration by parts in the $d k$ integral gives

$$
A\left(W, W_{\min }\right)=\int_{W_{\min }}^{W} d W^{\prime} \int_{-\infty}^{\infty} d W^{\prime \prime} G^{\prime}\left(W^{\prime}-W^{\prime \prime}\right) \int_{0}^{M / 2} d k\left(\frac{2 k}{M}\right)^{t} \frac{d \sigma_{0}\left(W^{\prime \prime}-k\right)}{d W^{\prime \prime}} .
$$

If we now perform the $d W^{\prime \prime}$ integration and integrate by parts, we obtain

$$
A\left(W, W_{\min }\right)=\int_{W_{\min }}^{W} d W^{+} \int_{0}^{M / 2} d k\left(\frac{2 k}{M}\right)^{t} \int_{-\infty}^{\infty} d W^{\prime \prime} \sigma_{0}\left(W^{\prime \prime}-k\right) \frac{d G\left(W^{\prime}-W^{\prime \prime}\right)}{d W^{\prime}} .
$$

Now we can do the $W$ ' integration:

$$
A\left(W, W_{\min }\right)=\int_{0}^{M / 2} d k\left(\frac{2 k}{M}\right)^{t} \cdot \int_{-\infty}^{\infty} d W^{\prime \prime} \sigma_{0}\left(W^{\prime \prime}-k\right)\left[G\left(W-W^{\prime \prime}\right)-G\left(W_{\min }-W^{\prime \prime}\right)\right]
$$



Since $W=W_{\min }$ is the energy below which there is no resonant cross section, the last term $G\left(W_{m i n}-W^{\prime \prime}\right)$ can be dropped. Then a change of variables in the $d W^{\prime \prime}$ integral gives

$$
\begin{equation*}
A\left(W, W_{\min }\right)=\int_{0}^{M / 2} d k\left(\frac{2 k}{M}\right)^{t} \int_{-\infty}^{\infty} \mathrm{d}^{\prime} \sigma_{0}\left(W^{\prime}\right) G\left(W-k-W^{\prime}\right) \tag{24}
\end{equation*}
$$

To see the bahavior of $A\left(W, W_{\min }\right)$ we note that the integral over $d W$ is confined by the resonant cross section $\sigma_{0}\left(W^{\prime}\right)$ to a range of the order of $\pm \Gamma$ around $W^{\prime}=M$. The resolution function, on the other hand, is nonvanishing only for $\left|W-k-W^{\prime}\right| \leq \Delta W$. If $(W-M)$ is large compared with the larger of $\Gamma$ and $\Delta W$, the range of the $d k$ integration is confined to $k \simeq(W-M) \pm \Gamma \pm \Delta W$. Since the factor $(2 k / M)^{t}$ is slowly varying provided $k$ is not too small, it can be evaluated at $k=(W-M)$ in this limit and taken outside the integral. The remaining integrals are just as in the radiationless situation, provided $(W-M) \gg \Delta W$. We thus obtain

$$
A\left(W, W_{\min }\right) \rightarrow\left[\frac{2(W-M)}{M}\right]^{t} \cdot(\text { Area })_{0}
$$

or

$$
\begin{equation*}
(\text { Area })_{0}=\lim _{(W-M) \gg \Gamma, \Delta W}\left[\frac{M}{2(W-M)}\right]^{t} \cdot A\left(W, W_{\min }\right) . \tag{25}
\end{equation*}
$$

This is the generalization of Eq. (23) to include the effects of the radiative corrections. The integral $A\left(W, W_{m i n}\right)$ continues to increase slowly with increasing $W$ because of the radiative tail, Eq. (9), rather than levelling of $f$ to a plateau as in Fig. 3(b). The factor $[M / 2(W-M)]^{t}$, which is larger than unity but decreases with increasing energy, corrects for the rise in $A\left(W, W_{\min }\right)$. The product is larger than $A\left(W, W_{\min }\right)$, compensating for the reduced cross section near the resonance, but levels off to a well-defined plateau that is the true area of $\sigma_{0}(W)$ in the absence of soft-photon processes. In practice, one calculates $A\left(W, W_{\min }\right)$ as a function of $W$ and then begins multiplying by the correction factor for values of $W$ slightly above the resonance.
VII. Example of determination of $\Gamma_{\mathrm{e}}$ for $\psi(3105)$ and values for

## other widths

To illustrate the use of Eq. (25) we consider the integration of the cross section $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow\right.$ hadrons $)$ to determine $\Gamma_{\mathrm{e}}$ for the $\psi(3105)$. We use the data from


Augustin et al., SLAC-PUB-1504, November 11, 1974. These are not the latest or best data, but they are accessible in the literature, A summary of partial width information to date is given after the working out of the example.

The data on $e^{+} e^{-}+$hadrons near 3.1 GeV are plotted on a linear ordinate scale in Fig. 4. A background cross section of 30 nb has been subtracted. Also shown are the radiative correction factor $[M / 2(W-M)]^{t}$, the integral $A\left(W, W_{m i n}\right)$ and the product of the two.

The plateau value for the area according to Eq. (25) is

$$
(\text { Area })_{0}=11.5 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}
$$

For a narrow resonant line (13) the area is given by (16). If a $J=I^{-1}$ assignment is assumed the peak cross section for a reaction $a \rightarrow b$ is

$$
\begin{equation*}
\left(\sigma_{0}\right)_{\max }=\frac{12 \pi}{M^{2}} \frac{\Gamma_{a} \Gamma_{b}}{\Gamma^{2}} \tag{26}
\end{equation*}
$$

where $\Gamma_{a}, \Gamma_{b}$ are the partial widths for the two channels and $\Gamma$ is the total width. The product $\Gamma_{a} \Gamma_{b} / \Gamma$ is thus determined to be

$$
\begin{align*}
\frac{\Gamma_{a} \Gamma_{b}}{\Gamma} & =\frac{M^{2}}{6 \pi^{2}}\left[(\text { Area })_{0}\right]_{\mathrm{ab}} \\
& =4.34 \times 10^{-14}[\mathrm{M}(\mathrm{MeV})]^{2} \cdot[\text { Area }(\mathrm{nb} \times \mathrm{MeV})] \mathrm{MeV} \tag{27}
\end{align*}
$$

The numerical factor converts $n b$ to $(\mathrm{MeV})^{-2}$, as well as including the factor $\left(6 \pi^{2}\right)^{-1}$. For $M=3105 \mathrm{MeV}$ and (Area) $=11.5 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}$, this gives

$$
\frac{\Gamma_{e} \Gamma_{\text {had }}}{\Gamma^{-}}=4.81 \times 10^{-3} \mathrm{MeV}
$$

A rough estimate of the error in determining the area and the resonant mass indicates a generous uncertainty of 5 to $10 \%$, apart from systematic errors.

Integration according to Eq. (25) of the cross section $\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)$of Augustin et al. (called by them the cross section for 2-body, collinear events that are not pairs of electrons) gives

$$
(\text { Area })_{0}^{\mu \mu} \simeq 7.8 \times 10^{2} \mathrm{nb} \times \mathrm{MeV}
$$



Fig. 4

with an error of $20 \%$ at most, apart from systematics. Assuming this cross section is truly $e^{+} e^{-}+\mu^{+} \mu^{-}$we have

$$
\frac{\Gamma_{\mathrm{e}} \Gamma_{\mu}}{\Gamma}=3.3 \times 10^{-4} \mathrm{MeV}
$$

We thus have $\Gamma_{\mu} / \Gamma_{\text {had }}=0.068$. Assuning that $\Gamma=\Gamma_{e}+\Gamma_{\mu}+\Gamma_{\text {had }}$ and that $\Gamma_{\mu}=\Gamma_{e}$ (This is consistent with the observed resonant contribution in the $e^{+} e^{-} \rightarrow e^{+} e^{-}$ channel), we can solve for $\Gamma_{e}$, $\Gamma_{\text {had }}, \Gamma$ :


The error on $\Gamma_{\text {had }}$ and $\Gamma$ is probably less than $20 \%$, apart from systematics.
The same method of analysis has been applied to other data on the $\psi(3105)$ (SP-17 memorandum from V. Lüth, November 25, 1974) and on the $\psi(3695)$ (preprint of Abrams et al., November 23, 1974 and private commication from W. Chinowsky). A sumary of the results is as follows:
$\psi(3105)$
$\underline{e}^{+} \mathrm{e}^{-}+$hadrons $\quad$ (Area) $)_{0}=12.1 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}( \pm 5 \%)$

$$
\frac{\Gamma_{\mathrm{e}} \Gamma_{\text {had }}}{\Gamma}=5.1 \pm 0.4 \mathrm{keV}
$$

The error estimate is generous. With $\Gamma_{\text {had }} / \Gamma=0.88$ this gives

$$
\Gamma_{\mathrm{e}}=5.8 \pm 0.4 \mathrm{keV}
$$

With the same assumptions as above, $\Gamma \simeq 97 \mathrm{keV}$
$\psi(3695)$

$$
\begin{gathered}
\frac{e^{+} e^{-}}{}+\text {hadrons } \quad(\text { Area })_{0}=4.0 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}( \pm 5 \%) \\
\frac{\Gamma_{e} \Gamma_{\text {had }}}{\Gamma}=2.4 \pm 0.2 \mathrm{keV} .
\end{gathered}
$$


$\underline{e}^{+} e^{-}+\mu^{+} \mu^{-} \quad$ This channel is relatively much smaller than for the $\psi(3105)$ and it is difficult to extract a resonant cross section. A rough estimate gives $\Gamma_{\mu} / \Gamma_{\text {had }}=2 \times 10^{-2}( \pm 30 \%)$. This would imply $\Gamma_{e}=2.5 \pm 0.2 \mathrm{keV}$ and $\Gamma \sim 125 \mathrm{keV}$. As discussed in my Physics Notes JDJ/74-1, this estimate of $\Gamma$ must be viewed only as a lower limit. It is possible at the moment that the $\ell^{+} \ell^{-}$channels are contaminated with $\ell^{+} \ell^{-}+$neutrals coming from sequential decays $\psi(3695) \rightarrow \psi(3105) \pi^{\circ} \pi^{\circ}$ and $\psi(3105) n^{\circ}$. Attributing all of the observed leptonic enhancements to $\psi(3695) \rightarrow \ell^{+} \ell^{-}$thus gives an upper limit on $\Gamma_{\ell} / \Gamma_{\text {had }}$ and a lower limit to $\Gamma$. VIII. Quick and dirty method of estimating (Area)

While the area method is most reliable in determining (Area), a fair estimate can be made quickly using the radiative correction factor for the peak cross section. For a resonance seen in good resolution (Sect. IV) we found a factor $(\Gamma / M)^{t}$ and for one seen in poor resolution (Sect. V), ( $\left.2 \sigma / M\right)^{t}$ where $\sigma$ is the standard deviation of a Gaussian resolution function. Since the FWHM of a Gaussian is $\Delta W=2.35480$, a general radiative reduction factor for whatever conditions of narrowness is

$$
\begin{equation*}
\left(\sigma_{\max }\right)_{\underset{\text { with radiative }}{\text { processes }}}=\left(\frac{\Delta W_{o b s}}{M}\right)^{t} \cdot\left(\sigma_{\max }\right)_{\text {no radiative }} \tag{27}
\end{equation*}
$$

where $\Delta W_{\text {obs }}$ is the observed FWHM of the line. This interpolates between the two limits smoothly and is in error by less than $1 \%$ in comparison with $(2 \sigma / \mathrm{M})^{t}$.

If the observed line is assumed to be Gaussian in shape, its area, without radiative tail, is

$$
\begin{equation*}
(\text { Area })_{o b s}=1.0645\left(\sigma_{\max }\right)_{\text {obs }} \cdot \Delta W_{\mathrm{obs}} \tag{28}
\end{equation*}
$$

If the line shape is a Breit-Wigner, the coefficient 1.0645 becomes $\pi / 2=1.57$. We assume here that the resonance is narrow and the resolution function is Gaussian, so that the observed line shape is approximately Gaussian.

The true area can thus be estimated from Eqs. (27) and (28) to be

$$
\begin{equation*}
(\text { Area })_{0} \simeq 1.0645\left(\frac{M}{\Delta W_{o b s}}\right)^{t}\left(\sigma_{\max }\right)_{o b s} \cdot \Delta W_{o b s} \tag{29}
\end{equation*}
$$

The radiative tail produces a skewing of the symmetrical line shape, but makes only a small effect on $\Delta W_{\text {obs }}$.


As an example of the use of (29) we consider the $\psi(-3105)$ data of Augustin et al, used previously in Fig. 4. The observed peak cross section and FWHM are $\left(\sigma_{\max }\right)_{\text {obs }}=2300 \mathrm{nb}, \Delta W_{\text {obs }}=2.5 \mathrm{MeV}$. With $\mathrm{M}=3105 \mathrm{MeV}$ and $\mathrm{t}=0.076$, Eq. (29) gives

$$
(\text { Area })_{0}=1.0645 \times 1.72 \times 2300 \times 2.5=105 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}
$$

This is to be compared with the value of $11.5 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}$ found by the area method.

For the $\psi(3695)$, the data of Abrams et al. has $\left(\sigma_{\max }\right)_{o b s}=725 \mathrm{nb}$, $\Delta W_{\text {obs }}=3.0 \mathrm{MeV}$, giving (Area) $\sim 1.0645 \times 1.74 \times 725 \times 3=4.0 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}$, compared to $4.2 \times 10^{3} \mathrm{nb} \times \mathrm{MeV}$ found from the same data by the area method.

# $\psi:$ Extraction of Decay Widths and Coupling Constants. How to Deal with Radiative Corrections and Machine Energy Width 

Y. S. Tsai

## A. INTRODUCTION

The effects of radiative corrections and beam energy spread on the experiment can be divided into three categories:

1. Effects on angular distributions
2. Effects on the energy dependence of the cross section $\sigma(w)$
3. Effects on the area under a resonance $\int \sigma_{\text {resonance }} d w$.

In this report we consider only the last two.
We shall treat the radiative corrections in two steps. The first step deals with corrections necessary to obtain the decay widths and the interference effects. The second step deals with those corrections which are necessary only when one is interested in obtaining the coupling constants of $\psi$ to various particles. The first step essentially reduces the experimental cross section into the cross section represented by Fig. 1. The resultant decay widths can be represented by diagrams shown in Fig. 3, where the blobs represent all possible effects. The second step essentially takes care of the higher order electromagnetic contributions to these blobs.

There are many reasons for splitting the radiative corrections into two parts; let us list a few:

1. It is more natural to define the partial decay widths with all the effects of the blobs in Fig. 3 retained. The higher order electromagnetic contribution to the blobs can be subtracted when we evaluate the coupling constants.
2. We shall exponentiate the lowest order radiative correction in Part 1 of the radiative corrections. The blobs in Figs. 1 and 3 are not obtainable by exponentiating the lowest order electromagnetic corrections. For example, the correction due to the vacuum polarization is $1 / 11-\left.\frac{1}{2} \delta_{\text {vac }}\right|^{2}$ instead of $\exp \left(\delta_{\text {vac }}\right)$, where $\delta_{\text {vac }}$ is the lowest order correction.
3. The vacuum polarization correction is absent if $\psi$ is not coupled to a photon; for example, if $\psi$ is a neutral weak boson. Since we do not know what $\psi$ is at this moment it is best not to perform corrections which may not be justified.

## B. THE EFFECTS OF BEAM ENERGY SPREAD

AND THE SOFT PHOTON EMISSION ON THE SHAPE OF $\sigma(w)$
The energy dependence of the experimental cross sections for $e^{+}+e^{-} \rightarrow$ $\mu^{+}+\mu^{-}$and $\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \mathrm{e}^{+}+\mathrm{e}^{-}$near the resonance will reveal whether $\psi$ has the same quantum number as that of a photon. However the experimental cross section $\sigma_{\exp }{ }^{(w)} \quad$ is quite different from the theoretical cross section $\sigma_{t h}{ }^{(w)}$ because the machine energy has certain width and also the energy of the electron is degraded by the bremsstrahlung emission. Let the mean energy of the machine bew and let us denote the machine energy distribution by $G\left(w, w^{\prime}\right) d w^{\prime}$. For concreteness let us assume that $G\left(w, w^{\prime}\right)$ has a Gaussian form:

$$
\begin{equation*}
G\left(w, w^{\prime}\right)=\frac{1}{\sqrt{2 \pi} \Delta} \exp \left[-\left(w-w^{\prime}\right)^{2} /\left(2 \Delta^{2}\right)\right] \tag{1}
\end{equation*}
$$

where $\Delta$ is related to the full width at half maximum (FWHM) by

$$
\begin{equation*}
\Delta=(\text { FWHM }) / 2.3548 \approx 1 \mathrm{MeV} \tag{2}
\end{equation*}
$$

Let us denote by $B\left(w^{\prime}, w^{\prime \prime}\right) d w^{\prime \prime}$ the probability distribution of the $c . m$. energy after the bremsstrahlung emission, $w^{\prime}$ being the energy before the bremsstrahlung. The energy loss by bremsstrahlung is $w^{\prime}-w^{\prime \prime}$. Since energy loss is always positive we have

$$
\begin{equation*}
\mathrm{B}\left(\mathrm{w}^{\prime}, \mathrm{w}^{\prime \prime}\right)=0 \quad \text { if } \mathrm{w}^{\prime}-\mathrm{w}^{\prime \prime}<0 \tag{3}
\end{equation*}
$$

The expression for $B\left(w^{\prime}, w^{\prime \prime}\right)$ can be obtained from exponentiating the $\Delta E$ dependent part of the radiative corrections given by Schwinger:
where

$$
\begin{gather*}
\mathrm{e}^{\delta}=\exp \left[-\epsilon \ln \left(2 \Delta E / w^{\prime}\right)\right]=\int_{w^{\prime}-\Delta E}^{\mathrm{w}^{\prime}} \mathrm{B}\left(\mathrm{w}^{\prime}, \mathrm{w}^{\prime \prime}\right) \mathrm{d} w^{\prime \prime}  \tag{4}\\
\epsilon=\frac{2 \alpha}{\pi}\left[\ln \frac{\mathrm{w}^{\prime}}{\mathrm{m}_{\mathrm{e}}^{2}}-1\right] \tag{5}
\end{gather*}
$$

Differentiating both sides of (4) with respect to $\Delta E$, we obtain

$$
\begin{equation*}
\mathrm{B}\left(\mathrm{w}^{\prime}, \mathrm{w}^{\prime \prime}\right)=\left[\epsilon\left(\frac{2}{\mathrm{w}^{\prime}}\right)^{\epsilon} /\left(\mathrm{w}^{\prime}-\mathrm{w}^{\prime \prime}\right)^{1-\epsilon}\right] \theta\left(\mathrm{w}^{\prime}-\mathrm{w}^{\prime \prime}\right) \tag{6}
\end{equation*}
$$

The desired relation between $\sigma_{\exp }{ }^{(w)}$ and $\sigma_{\text {th }}\left(w^{\prime \prime}\right)$ is given by

$$
\begin{align*}
\sigma_{\exp }(w) & =\int_{w^{\prime \prime}}^{\infty} d w^{\prime} \int_{0}^{\infty} d w^{\prime \prime} G\left(w, w^{\prime}\right) B\left(w^{\prime}, w^{\prime \prime}\right) \sigma_{t h}\left(w^{\prime \prime}\right) \\
& \equiv \int_{0}^{\infty} d w^{\prime \prime} F\left(w, w^{\prime \prime}\right) \sigma_{t h}\left(w^{\prime \prime}\right) \tag{7}
\end{align*}
$$

The lower limit of the $d w^{\prime}$ integration comes from (3). $B\left(w^{\prime}, w^{\prime \prime}\right)$ is too singular at $w^{\prime}=w^{\prime \prime}$ for computer evaluation of (7). This singularity can be eliminated by integration by part in the following way:

$$
\begin{align*}
F\left(w, w^{\prime \prime}\right) & \equiv \int_{w^{\prime \prime}}^{\infty} d w^{\prime} G\left(w, w^{\prime}\right) B\left(w^{\prime}, w^{\prime \prime}\right) \\
& =\frac{1}{\sqrt{2 \pi} \Delta}\left(\frac{2 \sqrt{2} \Delta}{w}\right)^{\epsilon} \int_{0}^{\infty} \exp \left[-(y-z)^{2}\right] d z \epsilon \\
& =\frac{2}{\sqrt{2 \pi} \Delta}\left(\frac{2 \sqrt{2} \Delta}{w}\right)^{\epsilon} \int_{-y}^{\infty}(x+y)^{\epsilon} \mathrm{xe}^{-x^{2}} d x \tag{8}
\end{align*}
$$

where $\mathrm{y}=\left(\mathrm{w}-\mathrm{w}^{\prime \prime}\right) /(\sqrt{2} \Delta)$. There is no singularity in the integrand of (8); hence
it can be evaluated by a computer.
It should be pointed out that $\sigma_{\text {th }}$ is not the lowest order in the electromagnetic interaction. Let the lowest order in $\alpha$ cross section be $\sigma_{0}$ and write

$$
\begin{equation*}
\sigma_{\text {th }}=\left(1+5^{\prime}\right) \sigma_{0} \tag{9}
\end{equation*}
$$

$\delta^{\prime}$ contains many terms, the most obvious one being the noninfrared part of the vertex correction,

$$
\begin{equation*}
\delta_{\text {vertex }}=-\frac{\alpha}{\pi}\left[2-3 \ln \left(\mathrm{w} / \mathrm{m}_{\mathrm{e}}\right)\right] \tag{10}
\end{equation*}
$$

The vacuum polarization correction is necessary only when we know for sure that $\psi$ is coupled to leptons or hadrons electromagnetically. This will be discussed later.

## C. EXTRACTION OF PARTIAL DECAY WIDTHS

For this purpose we need to know only the area of a Breit-Wigner curve. Let us consider formation and decay of $\psi$ shown in Fig. 1, and parametrize the


Fig. 1
theoretical cross section $\sigma_{\text {th }}$ by a Breit-Wigner formula

$$
\begin{equation*}
\sigma_{t h}(\mathrm{w})=\frac{\Gamma(\psi-2 \mathrm{e}) \Gamma(\psi \rightarrow \mathrm{f})}{\left(\mathrm{w}^{2}-\mathrm{M}^{2}\right)^{2}+\Gamma_{\mathrm{t}}^{2} \mathrm{M}^{2}} \frac{16 \pi\left(2 \mathrm{~S}_{\psi}+1\right)}{\left(2 \mathrm{~S}_{\mathrm{e}}+1\right)^{2}}, \tag{11}
\end{equation*}
$$

where $S_{\psi}$ is spin of $\psi$ and $S_{e}$ is $\frac{1}{2}$. For a narrow width, the Breit-Wigner formula can be approximated by a $\delta$ function,


Fig. 2
Let us integrate the experimental curve from $w_{\min }{ }^{\text {to }} \mathrm{w}_{\max }$ as shown in Fig. 2. $\mathrm{w}_{\text {max }}$ is chosen so that

$$
\begin{equation*}
\mathrm{w}_{\max }-\mathrm{M} \gg \text { FWHM } \tag{13}
\end{equation*}
$$

Using Eqs. (7), (11), and (12), we have

$$
\begin{align*}
& \int_{\mathrm{w}_{\min }}^{\mathrm{w}} \max \left[\sigma_{\exp }(\mathrm{w})-\text { Background }\right] \mathrm{dw}=2 \pi^{2}\left(2 \mathrm{~S}_{\psi}+1\right) \\
& \quad \times \frac{\Gamma(\psi \rightarrow 2 \mathrm{e}) \Gamma(\psi \rightarrow \mathrm{f})}{\Gamma_{\mathrm{t}} \mathrm{M}^{2}} \int_{\mathrm{w}_{\min }}^{\mathrm{w}} \max d w  \tag{14}\\
& \quad \int_{M}^{\infty} G\left(\mathrm{w}, \mathrm{w}^{\prime}\right) \mathrm{B}\left(\mathrm{w}^{\prime}, \mathrm{M}\right) \mathrm{dw}^{\prime}
\end{align*}
$$

"Background" means all processes which are not represented by Fig. 1 or Eq. (11).

Under the condition (13), we expect that the value of integration becomes independent of the detail of machine width; hence $G\left(w, w^{\prime}\right)$ can be replaced by a $\delta$ function:

$$
\begin{equation*}
G\left(w, w^{\prime}\right) \rightarrow \delta\left(w^{\prime}-w\right) \tag{15}
\end{equation*}
$$

Therefore

$$
\begin{gather*}
\int_{w_{\min }}^{w_{\max }} \int_{M}^{\infty} d w^{\prime} G\left(w, w^{\prime}\right) B\left(w^{\prime}, M\right) \\
=\int_{M}^{w_{\max }} d w B(w, M)=e^{\delta} \tag{16}
\end{gather*}
$$

where

$$
\begin{equation*}
\delta=-\frac{2 \alpha}{\pi}\left[\ln \left(M^{2} / \mathrm{m}_{\mathrm{e}}^{2}\right)-1\right] \ln \left(\frac{1}{2} \mathrm{M} / \Delta \mathrm{w}\right) \tag{17}
\end{equation*}
$$

Notice that we have integrated the experimental curve from $w_{\min }$ to $w_{\text {max }}$, but $\Delta w$ is defined as $\Delta w=w_{\max }-M$, not $w_{\max }-w_{\min }$. Substituting (16) into (14) we have finally

$$
\begin{align*}
\text { area } & =\int_{w_{\min }}^{\mathrm{w}} \operatorname{\operatorname {max}}\left[\sigma_{\exp }(w)-\text { Background }\right] d w \\
& =\frac{\Gamma(\psi \rightarrow 2 e) \Gamma(\psi \rightarrow \mathrm{f})}{\Gamma_{\mathrm{t}} \mathrm{M}^{2}} 6 \pi^{2} \frac{\left(2 \mathrm{~S}_{\psi}+1\right)}{3} \mathrm{c}^{\delta}, \tag{18}
\end{align*}
$$

where $\delta$ is defined by (17).
We have done numerical integration on the left-hand side of (18) for the process ee $\rightarrow \psi_{3105} \rightarrow$ charged hadron with $w_{\min }=3.10 \mathrm{GeV}$ and $\mathrm{w}_{\max }=$ 3. 12 GeV . We obtain

$$
\begin{align*}
& \text { area }=8600 \times 10^{-33} \mathrm{~cm}^{2} \mathrm{MeV} \\
& \epsilon=\frac{2 \alpha}{\pi}\left[2 \ln \left(\mathrm{M} / \mathrm{m}_{\mathrm{e}}\right)-1\right]=0.076,  \tag{19}\\
& \mathrm{e}^{\delta}=\exp \left[-0.076 \times \ln \frac{3.105}{2 \times 0.015}\right]=\frac{1}{1.415}, \tag{20}
\end{align*}
$$

$$
-7-
$$

and

$$
\begin{equation*}
\frac{\Gamma(\psi \rightarrow 2 e) \Gamma(\psi \rightarrow \text { charged hadrons })}{\Gamma_{\mathrm{t}}}=4.94\left(\frac{3}{2 S_{\psi}+1}\right) \mathrm{keV} . \tag{21}
\end{equation*}
$$

The data for $\mathrm{e}+\mathrm{e} \rightarrow \psi \rightarrow 2 \mu$ are not very good. The ratio of two peaks yields

$$
\begin{equation*}
\frac{\sigma(\mathrm{e}+\mathrm{e} \rightarrow \psi \rightarrow \text { charged hadrons })}{\sigma(\mathrm{e}+\mathrm{e} \rightarrow \psi \rightarrow 2 \mu)}=\frac{\Gamma(\psi \rightarrow \text { charged hadrons })}{\Gamma(\psi \rightarrow 2 \mu)}=14.3 \tag{22}
\end{equation*}
$$

If $\psi$ couples to $\mathrm{e}^{+} \mathrm{e}^{-}$electromagnetically then we expect (assuming $\Gamma(\psi-2 \mu)$
$=\Gamma(\psi \rightarrow 2 e)$

$$
\begin{equation*}
\Gamma_{t}=\Gamma(\psi \rightarrow \text { charged hadrons })+2 \Gamma(\psi \rightarrow 2 \mu)+\Gamma(\psi \rightarrow \text { neutrals }) \tag{23}
\end{equation*}
$$

If $\psi$ couples to leptons directly via weak interaction, then probably

$$
\begin{equation*}
\Gamma(\psi \rightarrow 2 e)=\Gamma(\psi \rightarrow 2 \psi)=\Gamma\left(\psi \rightarrow 2 \nu_{e}\right)=\Gamma\left(\psi \rightarrow 2 \nu_{\mu}\right) . \tag{24}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\Gamma_{\mathrm{t}}=\Gamma(\psi \rightarrow \text { charged hadrons })+4 \Gamma(\psi \rightarrow 2 \mu)+\Gamma(\psi \rightarrow \text { neutral hadrons }) \tag{25}
\end{equation*}
$$

We conclude

$$
\begin{gather*}
\Gamma(\psi \rightarrow 2 \mathrm{e})=4.94 \frac{\Gamma_{\mathrm{t}}}{\Gamma(\psi \rightarrow \text { charged hadrons })} \frac{3}{\left(2 \mathrm{~s}_{\psi}+1\right)} \mathrm{keV}  \tag{26}\\
\Gamma(\psi \rightarrow \text { charged had })=14.3 \Gamma_{\psi \rightarrow 2 \mu}=14.3 \frac{1+\delta_{\text {vert }}(\mu)}{1+\delta_{\text {vert }}(\mathrm{e})} \Gamma(\psi \rightarrow 2 \mathrm{e}) \\
 \tag{27}\\
=73 \mathrm{keV} \frac{\Gamma_{\mathrm{t}}}{\Gamma(\psi \rightarrow \text { charged hadrons })} \frac{3}{(2 \mathrm{~s} \psi+1)}
\end{gather*}
$$

We note

$$
\begin{equation*}
\frac{\Gamma_{t}}{\Gamma(\psi \rightarrow \text { charged hadrons })}=1+\frac{2}{14.3}+\frac{\Gamma(\psi \rightarrow \text { neutrals })}{\Gamma(\psi \rightarrow \text { charged hadrons })} \tag{28}
\end{equation*}
$$

This factor is at least 1.14 if $\Gamma(\psi \rightarrow 2 e)$ is electromagnetic, whereas it is at least 1.28 if $\Gamma(\psi \rightarrow 2 e)$ is weak.
$\delta_{\text {vert }}(\mathrm{e})$ and $\delta_{\text {vert }}(\mu)$ are vertex corrections for electron and muon respectively and they are given in the next section.
D. DETERMINATION OF COUPLING CONSTANTS

$\Gamma(\psi \rightarrow \mathrm{f})$

Fig. 3

The widths obtained contain all the noninfrared part of the radiative corrections represented by blobs in Fig. 3. For $\Gamma_{\psi \rightarrow 2 \mu}$ and $\Gamma_{\psi \rightarrow \text { charged hadrons }}$ the final states can also contain an arbitrary number of photons because we did not do the radiative corrections to the final state. The vertex correction can be computed easily. Using (10), we obtain

$$
\begin{align*}
& \delta_{\text {vertex }}=.056 \text { for clectron }  \tag{29}\\
& \delta_{\text {vertex }}=.019 \text { for muon } \tag{30}
\end{align*}
$$

Thus in principle there is $3.7 \%$ difference between $\Gamma_{\psi \rightarrow 2 e}$ and $\Gamma_{\psi \rightarrow 2 \mu}$.

## Weak Interaction

We ask whether $\psi=\psi_{3105}$ could be a neutral weak vector boson. Let us as sume the coupling of $\psi$ to $\mathrm{e}^{+} \mathrm{e}^{-}$to be

$$
\begin{equation*}
\mathrm{H}^{\prime}=\psi^{\mu} \overline{\mathrm{u}}\left(\mathrm{~g}_{\mathrm{v}} \gamma_{\mu}+\mathrm{g}_{\mathrm{A}} \gamma_{\mu} \gamma_{5}\right) \mathrm{v} \tag{31}
\end{equation*}
$$

Then

$$
\begin{equation*}
\Gamma(\psi \rightarrow 2 e)=\frac{\mathrm{g}_{\mathrm{v}}^{2}+\mathrm{g}_{\mathrm{A}}^{2}}{12 \pi} \mathrm{M}\left[1+\delta_{\text {vertex }}(\mathrm{e})\right] \tag{32}
\end{equation*}
$$

Substituting (27) into (32), we obtain

$$
\begin{align*}
\left(\mathrm{g}_{\mathrm{v}}^{2}+\mathrm{g}_{\mathrm{A}}^{2}\right) /(4 \pi) & =4.5 \times 10^{-6}\left[1+\frac{4}{14.3}+\frac{\Gamma(\psi \rightarrow \text { neutral had })}{\Gamma(\psi \rightarrow \text { charged had })}\right] \\
& \approx 6 \times 10^{-6} \tag{33}
\end{align*}
$$

Now if $\psi \rightarrow 2$ e decay were semiweak, $\left(g_{v}^{2}+g_{A}^{2}\right) / M^{2}$ would be roughly equal to the Fermi constant $10^{-5} / \mathrm{M}_{\mathrm{p}}^{2}$. Indeed

$$
\begin{equation*}
\left(\mathrm{g}_{\mathrm{v}}^{2}+\mathrm{g}_{\mathrm{A}}^{2}\right) / \mathrm{M}^{2}=.7 \times 10^{-5} / \mathrm{M}_{\mathrm{p}}^{2} \tag{34}
\end{equation*}
$$

Therefore the decay $\psi \rightarrow 2 \mathrm{e}$ is consistent with being semiweak.

## Electromagnetic Interaction

We ask whether the decay $\psi \rightarrow \mathrm{e}^{+}+\mathrm{e}^{-}$is consistent with being electromagnetic as shown in Fig. 4.


Fig. 4

The decay width is

$$
\begin{equation*}
\Gamma(\psi \rightarrow 2 e)=\frac{\left(\mathrm{e}^{2} \mathrm{~g}_{\psi \gamma}\right)^{2}}{12 \pi} \mathrm{M}(1+\delta) \tag{35}
\end{equation*}
$$

where $\delta$ is the radiative correction. $\delta$ has three parts:

$$
\begin{equation*}
\delta=\delta_{\text {vertex }}(\mathrm{e})+\delta_{\mathrm{vac}}(\mathrm{e})+\delta_{\mathrm{vac}}(\mu)+\delta_{\mathrm{vac}}(\text { had }) \tag{36}
\end{equation*}
$$

where $\delta_{\text {vertex }}(\mathrm{e})$ is given by (30), and $\delta_{\text {vac }}(\mathrm{e}), \delta_{\text {vac }}(\mu)$ and $\delta_{\text {vac }}(\mathrm{had})$ are vacuum polarization contributions from electrons, muons, and hadrons, respectively.

$$
\begin{align*}
& \delta_{\mathrm{vac}}(\mathrm{e})=\left[\frac{2 \alpha}{\pi}-\frac{5}{9}+\frac{2}{3} \ln \frac{\mathrm{M}}{\mathrm{~m}_{\mathrm{e}}}\right]=0.024  \tag{37}\\
& \delta_{\mathrm{vac}}(\mu)=\left[\frac{2 \alpha}{\pi}-\frac{5}{9}+\frac{2}{3} \ln \frac{\mathrm{M}}{\mathrm{~m}_{\mu}}\right]=0.008  \tag{38}\\
& \delta_{\mathrm{vac}}(\mathrm{had}) \sim \delta_{\mathrm{vac}}(\mu) \tag{39}
\end{align*}
$$

Thus we obtain

$$
\begin{equation*}
\delta=.096 \tag{40}
\end{equation*}
$$

and

$$
\begin{equation*}
4 \pi \mathrm{~g}_{\psi \gamma}^{2}=\frac{3 \mathrm{\Gamma}(\psi \rightarrow 2 \mathrm{e})(137)^{2}}{\mathrm{M}(1.096)}=0.1 \tag{41}
\end{equation*}
$$

assuming $\Gamma_{\psi \rightarrow 2 \mathrm{e}} \approx 6 \mathrm{keV}$.
This is to be compared with similar calculations for the leptonic widths of $\rho, \omega$, and $\phi$ mesons

$$
\begin{aligned}
& 4 \pi g_{\omega \gamma}^{2}=0.087 \\
& 4 \pi g_{\phi \gamma}^{2}=0.067 \\
& 4 \pi g_{\rho \gamma}^{2}=0.5
\end{aligned}
$$

Thus the coupling of $\psi$ to $\gamma$ is reasonable compared with those for other vector mesons.

## Final Comments

The formalism of the radiative corrections presented in this note involves the least controversial part, the soft photon phenomena, of the radiative corrections. This is evidenced by the fact that D. R. Yennie, J. D. Bjorken, and the author all immediately wrote down the same formula (Eq. 6) after we heard about the problem. The possibility of using the area under the resonance curves to obtain the decay widths was noted immediately by many people, including David Jackson, J. D. Bjorken, and myself. Bob Pearson made the linear plot of the experimental curve which made possible the numerical integration, counting the squares on graph paper, shown by Eq. (18).

## Interference in the Hadron Channels

## J. Bjorken, S. Brodsky

The interference effects in the annihilation process $e^{+} e^{-} \rightarrow f$ for specific hadron channels $f$ and the total hadronic cross section can be a rich source of information on the hadronic couplings of the $\psi$. We assume in this section that $\psi$ has $J^{P}=1^{-}$and couples to leptons electromagnetically.

There are two amplitudes which contribute to the hadronic width for a specific decay channel $\psi \rightarrow \mathrm{f}$ :

$$
T_{f} \propto \quad\left|m_{f}^{\gamma}+m_{f}^{d}\right|^{2}
$$

i. e. :

$$
T_{f} \propto \prod_{f}^{(\gamma)} T_{f}^{(d)}+2 \Gamma_{f}^{(i n t)}
$$



Note that the decay of the $\psi$ through the photon involves the non-resonant $\psi \rightarrow \mathrm{f}$ coupling - to avoid double counting. Thus

$$
T_{\text {had }}^{(\gamma)}=\sum_{f} \Gamma_{f}^{(\gamma)}=R^{0} T_{\mu+\mu^{-}}
$$

where $R^{\circ}$ is the ratio $\sigma_{\text {had }}^{0} / \sigma_{\mu^{+} \mu^{-}}^{0}$ off resonance. For the $\sigma(3105)$ we can estimate $\Gamma_{\text {had }} \sim 75 \mathrm{keV}$ and $\Gamma_{\text {had }}^{(\gamma)} \sim 3 \times 5 \mathrm{KeV}=15 \mathrm{KeV}$

For $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow f$, there are now three contributing amplitudes

$$
\sigma_{f} \propto\left|m_{f}^{0}+\frac{m_{f}^{(x)}+m_{f}(\alpha)}{5-m^{2}+i m \Gamma}\right|^{2}
$$

In the interference region, $\left(\mathrm{s}-\mathrm{m}^{2}\right)^{2} \gg \mathrm{~m}^{2} \Gamma^{2}$

$$
\sigma_{f}=\sigma_{f}^{0}+2 \operatorname{Re} \frac{m_{f}^{i}\left[m_{f}^{(\gamma)}+m_{f}^{(d)}\right]}{5-m^{2}} \text { (Kinematic facters) }
$$

which we can compare with the $\mu^{+} \mu^{-}$channel interference

$$
\begin{aligned}
\sigma_{\mu+\mu-} & \left.=\sigma_{\mu+\mu-}^{0}+2 \operatorname{Re} \frac{m_{\mu \mu_{\mu}-m_{\mu+\mu-}^{+}}^{+_{0}}}{s-m^{2}}(k, f)\right) \\
& \equiv \sigma_{\mu+\mu-}^{\prime}(1+\in(s))
\end{aligned}
$$

Thus

$$
\sigma_{f}=\sigma_{f}^{0}\left[1+\epsilon(s)\left(1+\frac{R_{e} m_{f}^{0} m_{f}^{(d)}}{R_{Q} m_{f}^{0} m_{f}^{(\gamma)}}\right)\right]
$$

Further it is clear from the structure of the matrix elements that

$$
\frac{R_{e} m_{f}^{\circ} m_{f}^{(t)}}{R_{e} m_{f}^{0} m_{f}^{(\gamma)}}=\frac{\Gamma_{f}^{(i n t)}}{\Gamma_{f}^{(x)}}
$$

Hence

$$
\sigma_{F}=R_{f} \sigma_{\mu+\mu}^{0}\left[1+\epsilon(s)\left(1+\frac{\Gamma_{f}^{(i n t)}}{\Gamma_{f}(\gamma)}\right)\right]
$$

and

$$
\begin{aligned}
\sigma_{\text {nad }}=\sum_{f}^{\prime} \sigma_{f}= & \sigma_{0}^{f}[1+\epsilon(s)] \\
& +\epsilon(s) \sum_{f} \sigma_{0}^{f} \frac{\Gamma_{f}^{(\text {in } t)}}{\Gamma_{f}(r)}
\end{aligned}
$$

For the muon channel interference, one estimates

$$
0 \geq \in(s) \geq 0.4
$$

outside the machine resolution region. Further, since $\Gamma_{h a d} / \Gamma_{\mu^{+} \mu^{-}} \sim 20$ for the $\psi(3105)$, then $\Gamma_{\mathrm{f}}^{(\mathrm{int})} / \Gamma_{\mathrm{f}}^{(\gamma)}$ of order 3 to 5 are possible. Thus large interference effects could be seen. $\dagger$ At minimum the interference is the same as the muon channel. This is the case if the $\psi$ decays directly to hadronic states orthogonal to the states coupled to the photon. For example, if the $\psi$ is a unitary singlet, no extra interference is present. Further information is gained by examining the interference effects channel by channel. Further, as emphasized in Section I , the ratio of cross sections $\sigma_{\mathrm{f}_{1}} / \sigma_{\mathrm{f}_{2}}$ for various hadronic or leptonic channels at the same beam energy near the resonance peak can provide a sensitive test of the sign and ratio of interference effects.

Note that the above treatment is consistent with the unitarity equation

$$
\sigma_{\mathrm{had}}(s)=-4 \pi \alpha \operatorname{Im} D(s)=-\frac{4 \pi \alpha}{5} \operatorname{Im} \frac{1}{1-\pi}
$$

[^0]The contribution of a B. W. to $\pi(s)$ is equivalent to a $B$. W. in $D(s)$ with a shifted mass - provided the background is energy-independent. Thus the contribution of the resonance is

$$
\sigma_{\text {nad }}=-\frac{4 \pi \alpha}{5} \operatorname{Im} \frac{g^{2}}{5-m^{2}+i M T}
$$

However, note that $g^{2}$ is complex. The contribution $\operatorname{Img} \frac{\operatorname{Reg}}{s-m^{2}+i m T}$
corresponds to the cut

which is the $m_{f}^{0} m_{f}^{(d)}$ and $m_{f}^{0} m_{f}^{(\gamma)}$ contribution to the interference.

## Interference in Hadrons

A parable in which the four elements of the universe are represented by (no numerators allowed)

min | $1 / s$ |
| :--- |
| f |


the hadrons $\sum S Q_{i}(S)$

the leptons
$S L_{i}(S)$
and their couplings:

the hadronic component of $\psi$ is then:

$$
\begin{aligned}
& =\square+\sum_{i} \longrightarrow 4 D= \\
& =\frac{1}{s-m_{0}^{2}-s \sum g_{i}^{2} Q_{i}(s)} \\
& \equiv \frac{1}{5-m_{h a d}^{2}+i m T_{\text {had }}} \\
& \begin{array}{l}
m_{\text {had }}^{2} \cong m_{0}^{2}\left(1+2 g_{i}^{2} \operatorname{Re} Q_{i}(s)\right) \\
T_{\text {had }} \cong-m_{0} \sum g_{i}^{2} \operatorname{Im} Q_{i}(s)
\end{array}
\end{aligned}
$$

The coupling to $\gamma$ is via two means.


So one may define $\lambda=\lambda_{0}+\sum_{\mathrm{i}} \mathrm{g}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}} \mathrm{Q}_{\mathrm{i}}(\mathrm{s})$. Additional beasties appearing in the $\gamma$ are and So one has the full photon propagator


$$
\equiv \frac{1}{5} \frac{1}{1-\pi(s)}
$$

with

$$
\pi\left(c_{)}\right)=\left[e^{2} L\left(s_{j}\right)+\sum q_{i}^{2} Q\left(\mathcal{Q}_{i}\right)+\frac{\lambda^{2} 5}{s-m_{m d}^{2}+i m \Gamma_{\operatorname{mad}}}\right.
$$

keeping only the leading corrections in $\mathrm{e}^{2}$ one obtains:

$$
\left|\frac{1}{1-\Pi(s)}\right|^{2} \cong \frac{\left(s-m_{n \times x}^{2}\right)^{2}+m_{1}^{2} \Gamma_{\text {nad }}^{2}}{\left(s-m_{4}^{2}\right)^{2}+m^{2} \Gamma_{\text {tot }}^{2}} \quad m_{4} \cong m_{n}^{2}\left(1+\operatorname{Re}\left(\lambda^{2}\right)\right)
$$

If one assumes scaling for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ had. background, this gives $\operatorname{Im} Q_{i}(s)=\operatorname{ImL}(s)=$. $-1 / 12 \pi$.which allows a simple form for $\operatorname{Im} \pi$, viz.

$$
\begin{aligned}
& \operatorname{Im} \pi^{\text {(hand) }}=\frac{-1}{12 \pi} \frac{1}{\left(5-n_{\text {mil }}^{2}\right)^{2}+m^{2} \Gamma_{\text {mad }}^{2}}\left(\sum_{i}\left(R_{2} \lambda g_{c} S+q_{i}\left(S-m_{\text {nad }}^{2}\right)\right)^{2}\right. \\
& \left.+\frac{s^{2} \Sigma g_{i}^{2}}{144 \pi^{2}}\left(\left(\sum q_{i}^{2}\right)\left(\sum q_{i}^{2}\right)-\left(\sum g_{i} q_{i}\right)^{2}\right)\right) \\
& \text { then one has for } \sigma^{\text {had }}=\frac{4 \pi \alpha}{5}-\operatorname{Im} \frac{\pi}{i 1-\frac{\pi}{2}} \\
& \text { where we shall let } \operatorname{Re} \lambda-\lambda, \Sigma g_{i}^{2}-g^{2}, \sum q_{i}^{2}=Q^{2}, \sum q_{i} q_{i}=q^{2} \mathbb{Q} \\
& \sigma^{\text {had }}=\frac{\alpha}{35} \frac{\left(\lambda j g+\left(c-m_{\text {had }}^{2}\right) Q\right)^{2}+s^{2} g^{2}\left(Q^{2} g^{2}-(q \cdot Q)^{2}\right) / 144 \pi^{2}}{\left(s-m_{4}^{2}\right)^{2}+n 1^{2} \Gamma_{\text {tot }}^{2}}
\end{aligned}
$$

or if it pleases one, one may define a factor of e out of $\lambda$ and $Q$ to give

$$
\sigma^{\text {had }}=\frac{4 \pi \alpha^{2}}{35}\left\{0 \cdots \text { or } \sigma_{0}\{a .0\}\right.
$$

i. e.

$$
\sigma^{\text {had }}=\sigma_{0} \frac{\left[\lambda^{2} s^{2} g^{2}\right]+\left[Q^{2}\left(\left(s-m_{n a d}^{2}\right)^{2}+s^{2} g^{4} / 144 \pi^{2}\right)\right]+\left[\lambda s\left(s-m_{\text {had }}^{2}\right) g Q-s^{2} g^{2}\left(g Q^{2} / 44 \pi^{2}\right]\right.}{\left(s-m_{4}^{2}\right)^{2}+m^{2} \Gamma_{\text {tot }}^{2}}
$$

the first term represents direct hadron production from the $\psi$.
The second term can be rewritten as

$$
\sigma_{0} Q^{2} \frac{\left(5-m_{h n d}^{2}\right)^{2}+m^{2} T_{\text {nad }}^{2}}{\left(s_{j}-m_{4}^{2}\right)^{2}+m^{2} T_{t+t}^{2}}=Q^{2} \sigma^{t} e^{2} \rightarrow \mu^{+} \mu-
$$

where $Q^{2}=\Sigma q^{i} \quad$ is often called $R$.
The third term represents the interferrence between direct hadron production and production through the photon, and is highly model dependent because of the factor $q \cdot Q$. If to take a simpleminded example $g$ measured baryon number $\mathrm{g} \cdot \mathrm{Q} \equiv 0$.

The experimental observation of such an effect is moreover quite difficult as the second piece has precisely the same energy dependence of the direct term.

## A. Weldon

## Front-Back Asymmetry at Resonance Peak

(Parity)

$$
\begin{aligned}
&\left.\frac{d \sigma}{d \Omega}\right|_{\text {pot. }}=\frac{1}{P^{2}} \sum_{J} \sum_{J^{\prime}}\left(J+Y_{l}\right)\left(J^{\prime}+y_{2}\right)(-)^{\lambda-\mu}\left\langle\lambda_{c} \lambda_{d}\right| T^{J}\left|\lambda_{a} \lambda_{b}\right\rangle^{*}\left\langle\lambda_{c} \lambda_{d}\right| T^{J}\left|\lambda_{a} \lambda_{b}\right\rangle \\
& \times \sum_{l}\left(J^{\prime} \ell ; \lambda_{1}-\lambda\right) \subset\left(J J^{\prime} \ell ; \mu_{1}-\mu\right) P_{l}(c \circlearrowleft \theta)
\end{aligned}
$$

Look at resonance, i.e. $J=J^{\prime}=$ fixed. Take forward, backward asymmetry:

$$
\begin{aligned}
& \left.\frac{d \sigma}{d \Omega}(z)\right|_{\text {pol }}-\left.\frac{d \sigma}{d \Omega} \cdot(-z)\right|_{p d} \\
& \left.\quad=\frac{\left(J+y_{2}\right)^{2}}{P^{2}}\left|\left\langle\lambda_{c} \lambda_{d}\right| T^{I}\right| \lambda_{3} \lambda_{b}\right\rangle\left.\right|^{2}(-)^{\lambda-\mu} \sum_{\ell_{\text {odd }}} C\left(I J l ; \lambda_{1}-\lambda\right)\left(I J J \ell ; \mu_{1}-\mu\right) \quad 2 P_{l}(\omega \infty \theta)
\end{aligned}
$$

Use $\mathrm{C}(\mathrm{JJ} \ell ; \mathrm{m}-\mathrm{m})=0$ when $\ell$ odd $+\mathrm{m}=0$. Thus only $\lambda= \pm 1$ and $\mu= \pm 1$ contribute to front-back asymmetry. Use C(JJ $\ell ;-1,1)=-\mathrm{C}(\mathrm{JJ} \ell ; 1,-1)$ for $\ell$ odd. Sum polarizations:

$$
\begin{aligned}
\left.\frac{d \sigma}{d \Omega}(z)\right|_{p e e}-\left.\frac{d \sigma}{d \Omega}(-z)\right|_{p o l}=\frac{\left(J+y_{2}\right)^{2}}{4 p^{2}}\{ & \left.\left.\left\{\left|\left\langle y_{2}-y_{2}\right| J^{J}\right| y_{2}-y_{2}\right\rangle\right|^{2}-\left|\left\langle y_{2}-y_{2}\right| T^{J}\right|-y_{2} y_{2}\right\rangle\left.\right|^{2} \\
& \left.\left.\left.+k y_{2} y_{2}\left|T^{J}\right|-y_{2} y_{2}\right\rangle\left.\right|^{2}-\left|\left\langle-y_{2} y_{2}\right| T^{J}\right| y_{2}-y_{2}\right\rangle\left.\right|^{2}\right\} \\
& \times \sum_{\text {odd }} \mid\left(\left.(J J R ; 1,-1)\right|^{2} 2 P_{\ell}(\cos \theta)\right.
\end{aligned}
$$

## Construct eigenstates of $P$ and $C$ :

$$
\begin{array}{lll}
|A\rangle=\frac{\left|y_{2}-\gamma_{2}\right\rangle-\left|-y_{2} y_{2}\right\rangle}{\sqrt{2}} & \underline{p} & + \\
|B\rangle=\frac{(-)^{J}}{r_{2}} & + \\
|C\rangle=\frac{\left|y_{2}-y_{2}\right\rangle+\left|-y_{2} y_{2}\right\rangle}{} & (-)^{J-1} & - \\
\left.|D\rangle=-1-y_{2}-y_{2}\right\rangle & (-)^{J} & - \\
r_{2} & (-)^{J-1} & -
\end{array}
$$

In terms of these states,

$$
\begin{aligned}
\frac{d \sigma}{d \Omega}(z)-\frac{d \sigma}{d \Omega}(-z)=\frac{\left(I+y_{2}\right)^{2}}{m^{2}} & \left\{R_{l}\left(\langle A| T^{\top}|A\rangle\langle B| T^{\top}|B\rangle\right)+\operatorname{Re}\left(\langle A| T^{\top}|B\rangle\langle B| T^{\top}|A\rangle^{x}\right)\right\} \\
& \times \sum_{\text {axd }}|(I J J \ell ; 1,-1)|^{2}+P_{\ell}(\cos \theta)
\end{aligned}
$$

Thus if $|\psi\rangle$ is an eigenstate of parity (either $|\mathrm{A}\rangle$ or $|\mathrm{B}\rangle$ ), no asymmetry is possible.

NOTE: If $|\psi\rangle$ were degenerate (i.e., two particles with opposite parity), an asymmetry would be possible from the first term above.

## S. Brodsky

One of the intriguing possibilities concerning the $\psi^{\prime} s$ - especially if they have anything to do with the weak interactions - is that one or more of these particles has positive charge conjugation. There are two main tests pertinent to $\mathrm{e}^{+} \mathrm{e}^{-}$colliding beams
(a) Direct decay into $2 \gamma$ or other $\mathrm{C}=+$ final state
(b) A front-back charge asymmetry off resonance due to interference with the normal one $-\gamma$ processes. We shall discuss this test in some detail below.

It should be emphasized that even if the angular distribution for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$ at each of the $\psi$ peaks shows a $1+\cos ^{2} \theta$ distribution, it is still possible that one or more $\psi^{\text {i }}$ s could be an axial vector, $\mathrm{J}^{\mathrm{PC}}=1^{++}$. If $\psi$ is a neutral carrier of the weak current, it could well be an axial vector. If there were degenerate vector and axial vector $\psi^{\prime} s$, then a linear $\cos \theta$ term would occur at the resonance peak in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$and other channels. This is equivalent to the parity violation tests, discussed elsewhere.

The main characteristic of a state with even $C$ is ihat it couples with equal magnitude and sign to particles and antiparticles. The interference of the (real) one photon amplitude for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$with the real part of the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \psi \rightarrow \mu^{+} \mu^{-}$ with $\mathrm{C}(\psi)=+$, then changes $\operatorname{sign}$ (odd in $\cos \theta$ ) when the $\mu^{+}$is detected along the positron versus the electron direction. Thus:

$$
\Delta(\theta)=\frac{N^{+}(\theta)-N^{-}(\theta)}{N^{+}(\theta)+N^{-}(\theta)}=\frac{2 R_{e} m_{\gamma}^{+} m_{\psi}}{\left|M_{\gamma}\right|^{2}+\left|m_{\psi}\right|^{2}} \neq 0
$$

where $\mathrm{N}^{ \pm}(\theta)$ is the rate $\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega}$ when $\mu^{ \pm}$is detected at an angle $\theta$ to the direction of the incident positron beam.

As an example, we suppose the $\psi$ has simple axial vector $\gamma_{\mu} \gamma_{5}$ coupling to the electron and muon current, with coupling constant $\mathrm{g}_{\mathrm{A}}^{(\mathrm{e})}$ and $\mathrm{g}_{\mathrm{A}}^{(\mu)}$, respectively. Then, using Budny's paper, we find

$$
\Delta(\theta)=4 \frac{\cos \theta}{1+\cos ^{2} \theta} \frac{B_{5}}{B_{4}}
$$

where

$$
\begin{aligned}
B_{5} & =\frac{g_{A}^{(e)} g_{A}^{(M)}}{e^{2}} R_{e} \frac{S}{S-M^{2}+i M \Gamma} \\
& =\frac{g_{A}^{(e)} g_{A}^{(M)}}{e^{2}} \frac{\left(S-M^{2}\right) S}{\left(S-M^{2}\right)^{2}+M^{2} \Gamma^{2}}
\end{aligned}
$$

and

$$
B_{4}=1+\left(\frac{g_{A}^{(e)} g_{A}^{(r)}}{e^{2}}\right)^{2} \frac{S^{2}}{\left(S-\mu^{2}\right)^{2}+\mu^{2} \Gamma^{2}}
$$

Thus $\Delta$ is negative below resonance if $\mathrm{g}_{\mathrm{A}}^{(\mathrm{e})} \mathrm{g}_{\mathrm{A}}^{(\mu)}>0$. Note also

$$
\frac{\left|g_{A}^{(e)} g_{A}^{(r)}\right|}{12 \pi}=\frac{\prod_{4 \rightarrow e^{+} e^{-}}^{\frac{1}{2}} \Gamma_{4 \rightarrow M+M}^{\frac{1}{2}}}{M_{4}}
$$

so approximately for the $\psi(3105)$
where at $\quad S=M^{2}-n M \Gamma, \quad E_{c m}=M_{\psi}-n \Gamma / 2$,

$$
\begin{aligned}
& B_{5}=-7 \times 10^{-4} \frac{M}{\Gamma} \frac{n}{1+n^{2}} \sim 24 \frac{n}{1+n^{2}} \\
& B_{4}=1+\frac{1}{1+n^{2}}\left[7 \times 10^{-4} \frac{M}{\Gamma}\right]^{2} \sim 1+\frac{(24)^{2}}{1+n^{2}}
\end{aligned}
$$

For

$$
\begin{gathered}
n=2 \theta, \quad E \mathrm{~cm}=M+-.85 \mathrm{meV} \\
\Delta \sim-2 \frac{\cos \theta}{1+\cos ^{2} \theta}
\end{gathered}
$$

which is a sizeable interf erence. For $60^{\circ}$ to $90^{\circ}$ detection

$$
\int\left(N+-N-\cos \theta / \int(N++N-) \cos \theta=3 / 13 \cdot 4 \frac{B_{5}}{B_{4}}\right.
$$

gives a $50 \%$ asymmetry at $\mathrm{n}=20$.
We also expect a similar interference for any hadron channel; es. a front-back asymmetry of $\pi^{+}$vs. $\pi^{-}, \mathrm{K}^{+}$vs. $\mathrm{K}^{-}, \mathrm{p}$ vs. $\overline{\mathrm{p}}$, etc., in inclusive or exclusive processes. The effect is larger than the muon case due to the increased ratio

$$
\frac{\Gamma_{4 \rightarrow \pi \pm \mathbb{Z}}^{1 / 2}}{\Gamma_{4 \rightarrow \mu^{+} \mu^{-}}^{1 / 2} / \frac{\Gamma_{\gamma \rightarrow \pi \pm x}^{1 / 2}}{\Gamma_{\gamma \rightarrow \mu^{1 / 2}}^{1-} \mu^{-}} \text {. }}
$$

for inclusive $\pi^{+}$. For the $\psi(3105)$, this ratio is 2 , and gives a $50 \%$ asymmetry at $n-10$. However, the asymmetry is presumably smeared and reduced by the quark-jet effects, as well as machine resolution. The above discussion is meant to be representative of the magnitude of the effect due to a positive charge conjugation particle. Radiative corrections, which induce a background charge asymmetry at the $1 \%$ level, are relatively unimportant.


The observable consequences in the angular distribution and in the longitudinal polarization of the muons of a parity-violating spin 1 resonant state are summarized. Particular attention is focused on the possibility of lack of universality between electrons and muons.

## I. Introduction

While there is increasing evidence that the $\psi(3105)$ and $\psi(3695)$ are exotic hadrons of some sort rather than intermediate vector bosons, the situation is sufficiently uncertain that the consequences of other assignments should be explored. In this note we consider the channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$and examine the observable effects in the angular distribution and the muon longitudinal polarization of a spin 1 resonant state that couples to $e^{+} e^{-}$and $\mu^{+} \mu^{-}$with interactions that may not conserve parity. In the angular distribution we include interference with the s-channel photon pole amplitude, but for simplicity ignore such interference for the muon polarization. The channel $e^{+} e^{-} \rightarrow e^{+} e^{-}$can be treated similarly, but is complicated by the additional t-channel photon exchange. It seems probable that the channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$ is more useful in establishing the presence or absence of a small effect than $e^{+} e^{-} \rightarrow e^{+} e^{-}$since the latter process has an asymmetric angular distribution from the t-channel contribution.

There are discussions in the Russian literature on the effects of an intermediate vector boson on these reactions and R. V. Budny (Oxford, now SLAC) has made detailed calculations. Here, however, we focus on the possibility of different couplings of electrons and muons to the intermediate vector particle. After all, we must keep searching for ways in which the muon can be distinguished from the electron: II. Basis of the calculation and notation

The computations are elementary lowest order Feynman diagram calculations. We give no details, but merely summarize the notation. The two diagrams considered are given below


4L-2187

| LAWRENCE RADIATION LABORATORY - UNIVERSITY OF CALIFORNIA PHYSICS NOTES | MEMO NO. PAGE <br> JDJ/74-2 2 |
| :---: | :---: |
| On the effects of nonconservation of parity for a resonance in the channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$ | $\frac{\text { NAME }}{\text { J. D. Jackson }}$ |

The lepton-resonance couplings are $\gamma_{\alpha}\left(g_{V}+g_{A} \gamma_{5}\right)$ for $e^{+} e^{-}$and $\gamma_{\alpha}\left(g_{V}^{\prime}+g_{A}^{\prime} \gamma_{5}\right)$ for $\mu^{+} \mu^{-}$, the convention on the Dirac matrices being the Pauli choice in which the standard ( $V-A$ ) coupling is $\gamma_{\alpha}\left(1+\gamma_{5}\right)$. Time-reversal invariance is assumed; the $g$ 's are all taken to be real. With the $\psi(3105)$ and $\psi(3695)$ in mind, we neglect the lepton masses throughout. For simplicity we average over initial spins. If the storage ring beams are transversely polarized, the results are applicable to averages over azimuth. In the formulas it is convenient to have two sets of symbols. We define

$$
\begin{equation*}
B=\frac{2 g_{V} g_{A}}{g_{V}^{2}+g_{A}^{2}}, \quad \dot{B}^{\prime}=\frac{2 g_{V}^{\prime} g_{A}^{\prime}}{g_{V}^{\prime 2}+g_{A}^{\prime 2}} \tag{I}
\end{equation*}
$$

and alternatively,

$$
\begin{array}{ll}
\gamma_{V}=\frac{g_{V}}{\sqrt{g_{V}^{2}+g_{A}^{2}}}, & \gamma_{V}^{\prime}=\frac{g_{V}^{\prime}}{\sqrt{g_{V}^{\prime 2}+g_{A}^{\prime 2}}} \\
\gamma_{A}=\frac{g_{A}}{\sqrt{g_{V}{ }^{2}+g_{A}^{2}}}, & \gamma_{A}^{\prime}=\frac{g_{A}^{\prime}}{\sqrt{g_{V}^{\prime 2}+g_{A}^{\prime 2}}} . \tag{3}
\end{array}
$$

We thus have $\beta=2 \gamma_{V} \gamma_{A}, B^{\prime}=2 \gamma_{V}^{\prime} \gamma_{A}^{\prime}$ if conversion is necessary. The sums of the squares of the coupling constants can be expressed in terms of the partial widths for the decay of the resonance of mass $M$,

$$
\begin{align*}
g_{V}^{2}+g_{A}^{2} & =\frac{12 \pi \Gamma_{e}}{M}  \tag{4}\\
g_{V}^{\prime 2}+g_{A}^{\prime 2} & =\frac{12 \pi \Gamma_{\mu}}{M}
\end{align*}
$$

The differential cross sections are expressed in units of the standard asymptotic QED cross section for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$:

$$
\begin{equation*}
\left(\sigma_{\mu \mu}\right)_{Q E D} \equiv \frac{4 \pi}{3} \frac{a^{2}}{W^{2}} \tag{5}
\end{equation*}
$$

where $\alpha=1 / 137$ and $W$ is the total c.m.s. energy. It is useful to have symbols for

| LAWRENCE RADIATION LABORATORY - UNIVERSITY OF CALIFORNIA PHYSICS NOTES | MEMO NO. PAGE <br> JDJ/74-2 3 |
| :---: | :---: |
| On the effects of nonconservation of parity for a resonance in the channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$ |  |

the resonant and interference terms in the cross section. We thus define

$$
\begin{equation*}
\left|A_{R}\right|^{2}=\frac{9}{4 \alpha^{2}} \frac{\Gamma_{e} \Gamma_{\mu}}{\left[(M-W)^{2}+\frac{\Gamma^{2}}{4}\right]} \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\operatorname{Re} A_{R}=\frac{3}{2 \alpha} \frac{\sqrt{\Gamma_{e} \Gamma_{\mu}}(M-W)}{\left[(M-W)^{2}+\frac{\Gamma^{2}}{4}\right]} \tag{7}
\end{equation*}
$$

III. Differential cross section

The c.m.s. differential cross section, averaged over initial spins and summed over final spins, is

$$
\frac{d \sigma}{d \Omega}=\left(\sigma_{\mu \mu}\right)_{Q E D} \cdot \frac{3}{16 \pi}\left\{1+\cos ^{2} \theta+\left|A_{R}\right|^{2}\left(1+\cos ^{2} \theta+2 \beta B^{\prime} \cos \theta\right) \quad\left\{\begin{align*}
& \\
& -2 \operatorname{Re} A_{R}\left[\gamma_{V} \gamma_{V}^{\prime}\left(1+\cos ^{2} \theta\right)+2 \gamma_{A} \gamma_{A}^{\prime} \cos \theta\right] \tag{8}
\end{align*}\right\}\right.
$$

The angle $\theta$ is the angle between the directions of the momenta of leptons of the same charge initially and finally. The first term is the s-channel photon contribution, the second the resonant term, and the third the interference. The negative sign in the interference term is a consequence of the assumption of a normal resonance with a counter-clockwise rotation through resonance on an Argand diagram. This term can contribute positively, of course, with suitable relative signs of the couplings.

Ten specific examples are listed in the table below. The couplings are cited in a stylized notation, with $\mathrm{V}, \pm \mathrm{V}$ meaning $g_{A}=g_{\mathrm{A}}^{\prime}=0$ and $g_{\gamma} g_{V}^{\prime} \gtrless 0$; $\mathrm{V} \pm \mathrm{A}$, $V \pm A$ meaning $g_{V}=g_{A}, g_{V}^{\prime}=g_{A}^{\prime}$ and $g_{V} g_{V}^{\prime}>0$; and so on.


The table contains some interesting if not surprising results. The first three entries (with upper signs on the first two) correspond to the universality of the electron and muon couplings. The first two are normal parity-conserving situstions. The resonant amplitude squared has the familiar $\left(1+\cos ^{2} \theta\right)$ behavior, while the interference term reflects whether the resonant and background amplitudes represent states with the same parities or not. The third entry is the classic (V $\pm \mathrm{A}$ ) parityviolating situation. Both terms show the strong forward peaking of $(1+\cos \theta)^{2}$, independent of whether the universal coupling is ( $V-A$ ) or $(V+A)$. For both the $\psi(3105)$ and $\psi(3695)$ such a strong asymmetry is excluded by the data.

The last seven entries correspond to different couplings for muons and electrons. Entries 5 and 6 show that with ( $V-A$ ) at one vertex and (V + A) at the other the angular distribution peaks strongly backward instead of forward. Perhaps the most interesting circumstances are contained in the last four entries in which one of the leptonic couplings conserves parity while the other does not. Inspection shows that these situations are distinguished from the parity-conserving cases (1 or 2) only in the magnitude of the interference term. This is easy to understand. With parity conser:ed at one vertex, only the corresponding part of the coupling at the other vertex is operative in the interference term. The angular behavior is as if parity were conserved throughout, but the magnitude is reduced because the coupling at one vertex is not fully operative.
 is difficult because the effect is washed out by the energy spread in the beams. For this reason it may be possible to establish the existence of an interference minimum and to distinguish a $J=1^{+}$assignment from a $J=1^{-}$assuming parity conservation, but it is unlikely that parity-violating effects of the sort given in entries 7, 8, 9, 10 can be distinguished from parity conservation throughout, except perhaps in the extreme of purely $V$ coupling at one vertex and purely $A$ coupling at the other. IV. Longitudinal polarization of the muons

We have seen that there is a possibility with different couplings of the mons and electrons of a parity violation being manifest only weakly in the angular distributions. It is natural then to seek evidence of nonconservation of parity in the longitudinal polarization of the muons. For simplicity we neglect the interference between resonant and background amplitudes. We have in mind the $\psi(3105)$ where the resonance is observed to be approximately 15 to 20 times the background. Furthermore, for reasons of counting rate in any experiment, data would probably be taken at the resonant peak where interference effects are negligible.

The c.m.s. differential cross section for observation of a $\mu^{-}$at angle $\theta$ with helicity $\lambda$ is

$$
\begin{aligned}
\left(\frac{d \sigma}{d \delta}\right)_{\lambda}= & \frac{1}{2}\left(\sigma_{\mu \mu}\right)_{Q E D}\left|A_{R}\right|^{2} \cdot \frac{3}{16 \pi}\left\{1+\cos ^{2} \theta+2 \beta B^{\prime} \cos \theta\right. \\
& \left.-2 \lambda\left[\beta^{\prime}\left(1+\cos ^{2} \theta\right)+2 \beta \cos \theta\right]\right\}
\end{aligned}
$$

The sum of this cross section over $\lambda= \pm 1 / 2$ gives the resonant term in Eq. (8). The longitudinal polarization is
$P_{\text {long }}=\frac{\left(\frac{d \sigma}{d \Omega}\right)_{+}-\left(\frac{d \sigma}{d \Omega}\right)_{-}}{\left(\frac{d \sigma}{d \Omega}\right)_{+}+\left(\frac{d \sigma}{d \Omega}\right)_{-}}=-\left[\frac{\beta^{\prime}\left(1+\cos ^{2} \theta\right)+2 \beta \cos \theta}{1+\cos ^{2} \theta+2 \beta \beta^{\prime} \cos \theta}\right]$
wtere $g$ and $B^{\prime}$ are defined in Eq. (1). For positive muons the sign is opposite (and the angle $\theta$ is suitably redefined relative to the incaming positron).


We note in Eq. (10) that even if $\beta=0$ or $\beta^{\prime}=0$ (so that the angular distribution shows no asymmetry) there is still longitudinal polarization provided parity is not conserved at one of the vertices. Observation of the longitudinal polarization therefore allows differentiation among the possibilities $7,8,9,10$ of the table given above, even between type 7 and type 8 , for example, with their identical angular distributions. For $\beta=\beta^{\prime}$ or $\beta=0, \beta^{\prime} \neq 0$ the longitudinal polarization is equal to $-\beta^{\prime}$, independent of angle. For $\beta=0, \beta^{\prime} \neq 0$ this is easily understood as a result of the production in a parity-conserving interaction of a particle that subsequently decays via a parity-violating interaction, e.g., $\Lambda \rightarrow p \pi^{-}$. When $\beta^{\prime}=0, \quad \beta \neq 0$ the longitudinal polarization changes sign for $\theta<\frac{\pi}{2}$ and $\theta>\frac{\pi}{2}$. The integrated cross section (9) is the same for both helicities. The absence of net longitudinal polarization reflects the parity conservation in the decay of the resonant state.

## V. Summary

The effects of nonconservation of parity with a $J=1$ resonance in the channel $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$are explored. With the same couplings for electrons and muons, parity-violating effects show up directly in the angular dependence of the differential cross section. If the electronic and muonic couplings to the resonance are different, however, the angular distributions are less definitive. In particular, if either coupling conserves parity, the angular variation of the differential cross section may be very difficult to distinguish from the completely parity-conserving situation. Even in these circumstances the longitudinal polarization of the muons provides a means of establishing parity violation if it exists and of determining which leptonic coupling is responsible.

## J. Kuhn

## $P$ and C symmetries in hadronic final states

Consider the differential, semiinclusive cross section $\mathrm{d} \sigma\left(\mathrm{E}_{\pi^{+}}, \mathrm{E}_{\pi^{-}}, \theta^{+}, \theta^{-}, \theta^{+-}\right)$ for the reaction $\mathrm{e}^{+}+\mathrm{e}^{-} \rightarrow \pi^{+}+\pi^{-}+\mathrm{X} . \quad\left(\theta^{+}, \theta^{-}\right.$are the angles between $\vec{\pi}^{+}, \overrightarrow{\mathrm{e}}^{+}$and $\vec{\pi}^{-}, \overrightarrow{\mathrm{e}}^{+} ; 1 \theta^{+-} \mid$the angle between $\vec{\pi}^{+}$and $\vec{\pi}^{-} ; \theta^{+-\geqslant}<0$ if $\left(\vec{\pi}^{+} \times \vec{\pi}^{-}\right) \cdot \overrightarrow{\mathrm{e}}^{+} \geqslant 0$ )


Fig. 1
$P$ invariance $\Rightarrow \mathrm{d} \sigma\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \theta^{+}, \theta^{-}, \theta^{+-}\right)=\mathrm{d} \sigma\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \theta^{+}, \theta^{-},-\theta^{+-}\right)$
(P-transformation and rotation of Fig. 1 yields Fig. 2)


Fig. 2

Tests for $P$ invariance are also possible with integrated cross sections, egg.,

$$
\int \mathrm{dE}^{+} \mathrm{dE}^{-} \frac{\mathrm{d} \sigma}{\mathrm{dE}} \mathrm{dE}^{-}\left(\ldots, \theta^{+-}\right)=\int \mathrm{dE}^{+} \mathrm{dE}^{-} \frac{\mathrm{d} \sigma}{\mathrm{dE} E^{+} \mathrm{dE}^{-}}\left(\ldots,-\theta^{+-}\right)
$$

C invariance $\Rightarrow \mathrm{d} \sigma\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \theta^{+}, \theta^{-}, \theta^{+-}\right)=\mathrm{d} \sigma\left(\mathrm{E}^{-}, \mathrm{E}^{+}, \pi-\theta^{-}, \pi-\theta^{+}, \theta^{+}\right)$


Fig. 3
$C P$ invariance $\Rightarrow d \sigma\left(E^{+}, E^{-}, \theta^{+}, \theta^{-}, \theta^{+-}\right)=d \sigma\left(E^{-}, E^{+}, \pi-\theta^{+},-\theta^{+-}\right)$


## If we assume $\psi$ couples to leptons via one photon, we get simple tests of

## C symmetry! (a la Pais and Treiman)

Define W by

$$
\mathrm{d} \sigma\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \theta^{+}, \theta^{-}, \theta^{+-}\right)=\mathrm{W}\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \mathrm{z}^{+}, \mathrm{z}^{-}, \mathrm{z}^{+-}\right) \Delta^{-\frac{1}{2}} \mathrm{~d} \mathrm{E}^{+} \mathrm{dE}^{-} \mathrm{dz}{ }^{+} \mathrm{dz}{ }^{-} \mathrm{dz}{ }^{+-}
$$

where $z=\cos \theta$

$$
\Delta=1-\left(\mathrm{z}^{+}\right)^{2}-\left(\mathrm{z}^{-}\right)^{2}-\left(\mathrm{z}^{+-}\right)^{2}+2 \mathrm{z}^{+} \mathrm{z}^{-} \mathrm{z}^{+-}
$$

One $\gamma$-exchange yields a parity symmetric W :

$$
\mathrm{W}=\mathrm{A}_{1}+\mathrm{A}_{2} \mathrm{z}^{+} \mathrm{z}^{-}+\mathrm{A}_{3}\left(\mathrm{z}^{+2}+\mathrm{z}^{-2}\right)+\mathrm{A}_{4}\left(\mathrm{z}^{+2}-\mathrm{z}^{-2}\right)
$$

$A$ are functions of $E^{+}, E^{-}, z^{+-}$,
$C$ invariance implies

$$
\begin{gathered}
\mathrm{A}_{1,2,3}\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \mathrm{z}^{+-}\right)=\mathrm{A}_{1,2,3}\left(\mathrm{E}^{-}, \mathrm{E}^{+}, \mathrm{z}^{+-}\right) \\
\mathrm{A}_{4}\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \mathrm{z}^{+-}\right)=-\mathrm{A}_{4}\left(\mathrm{E}^{-}, \mathrm{E}^{+}, \mathrm{z}^{+-}\right)
\end{gathered}
$$

Integration over $\mathrm{z}^{+}$and $\mathrm{z}^{-}$yields test for C symmetry, e.g.,
(i) $\frac{1}{4} \int_{-1}^{1} \mathrm{dz}^{+} \mathrm{dz}^{-} \mathrm{W}=\mathrm{A}_{1}+\frac{1}{3} \mathrm{~A}_{3}$ should be symmetric in $\mathrm{E}^{+}, \mathrm{E}^{-}$;
(ii) $\int_{0}^{1} \mathrm{~d} z^{+} \int_{0}^{1} \mathrm{~d} z^{-} \mathrm{W}\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \mathrm{z}^{+-}, \mathrm{z}^{+}, \mathrm{z}^{-}\right)-\mathrm{W}\left(\mathrm{E}^{+}, \mathrm{E}^{-}, \mathrm{z}^{+-}, \mathrm{z}^{+},-\mathrm{z}^{-}\right)$

$$
=\int_{0}^{1} d z^{+} \int_{0}^{1} d z^{-} 2 A_{2} z^{+} z^{-}=2 A_{2}
$$

should be symmetric in $\mathrm{E}^{+}, \mathrm{E}^{-}$
etc.

We consider now in more detail the exclusive channel $\mathrm{e}^{+}+\mathrm{e}^{-} \longrightarrow \pi^{+}+\pi^{-}+\mathrm{N}$, where N is any neutral particle. The most general matrix element is ( $\psi$ is assumed to have spin 1):


$$
\partial \eta^{*}=\bar{v}\left(e^{*}\right) \gamma_{\mu}^{(-i)}\left(g_{V}+g_{A} \gamma_{5}^{-}\right) \mu\left(e^{-}\right)
$$

$$
\frac{(-i)}{9^{2}-M^{2}} J^{\mu}
$$


$g_{V}$ and $g_{A}$ are arbitrary complex numbers. $A^{ \pm}, B, C$ are functions of the invariants $\mathrm{q}^{2}, \mathrm{q} \pi^{+}, \mathrm{q} \pi^{-}$. We define

$$
\begin{array}{ll}
\overrightarrow{\mathrm{n}}^{ \pm}:=\vec{\pi}^{ \pm} /\left|\vec{\pi}^{ \pm}\right| ; & \vec{n}_{\perp}:=\vec{\varepsilon} /\left(2 E \left|\vec{\eta}+||\vec{\pi}|)=\vec{n}^{-} \times \vec{n}^{+},\right.\right. \\
\mathrm{a}^{ \pm}:=A^{ \pm} \mid \vec{\pi}^{ \pm} ; & \theta_{0}, c:=B, C \cdot\left(2 E\left|\vec{\pi}^{+}\right| \vec{\pi} \mid\right),
\end{array}
$$

After performing the usual calculations, we arrive at the following conclusions:

1) If $\psi$ couples to leptons electromagnetically, the angular distribution is proportional to $\left(n_{\perp}^{2}-n_{\perp 3}^{2}\right)$, where $n_{\perp}$ is normal to the production plane.
No parity violation is possible. Any C violation can only show up in different energy distributions of $\pi^{+}$and $\pi^{-}$. There is no change of the angular distribution in the interference region.
$\sum_{\text {spins }} \frac{m^{2}}{E^{2}}|\gamma \eta|^{2}=2\left(\overrightarrow{i n}^{2}-m_{1}^{2}\right)\left(\frac{Q^{2}}{q^{2}-M^{2}} b+\left.\frac{e}{q^{2}} C\right|^{2} ;\right.$
2) If this simple form fails to describe the angular distribution, we have to apply the following general formulas:

Direct term:

$$
\begin{aligned}
& \frac{m^{2}}{4 E^{2}} \sum_{\text {spins }}\left|30 z^{4}\right|^{2}= \\
& \left(\left.\lg _{A}\right|^{2}+\left|g_{V}\right|^{2}\right) / 2\left[\left(\vec{n}^{+2}-n_{3}^{+2}\right)\left|a^{+}\right|^{2}+\left(\vec{n}^{-2}-n_{3}^{-2}\right)\left|a^{-}\right|^{2}+\left(\vec{n}_{1}^{2}-n_{23}^{2}\right) \mid \theta^{2}\right.
\end{aligned}
$$

## Interference term:

$$
\begin{aligned}
& \frac{m^{2}}{4 E^{2}} \sum_{\text {spins }} 2 \operatorname{Re} \partial q^{4} d \partial c^{c^{*}}= \\
& =\frac{e}{q^{2}} \operatorname{Re} \frac{1}{a^{2}-M^{2}}\left\{\left(\vec{n}_{L}^{2}-n_{13}^{2}\right) g_{L} e c^{*}+i g_{A}\left[\left(\vec{n}^{*} \times \vec{n}_{1}\right)_{3} a^{+} c^{*}+\left(\vec{n} \times \vec{n}_{1}\right)_{3} a^{-} c^{*}\right]\right.
\end{aligned}
$$

We should mention, that one can have parity violation, if $A^{ \pm}$and $B$ are different from zero, even if the coupling of $\psi$ to leptons is pure vector.

$$
\text { Is } \psi\left(\psi^{t}\right)=\bar{\psi}(\bar{\psi}) ?
$$

A number of models (not to be elaborated upon here) would suggest that $\bar{\psi} \neq \psi$ and, perhaps, $\bar{\psi}^{\prime} \neq \psi^{\prime}$. In view of this possibility, we have decided to investigate, to somc dcgrce, the extent to which it can be checked by the kinematical aspects of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$and $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$in gross.

Of course, the most general situation (arbitrary spin, arbitrary interaction vertices, etc.) would require an immense amount of effort. Here, in view of the present general trend of the data, we shall focus on a spin 1 assignment with the following form for the effective interaction density:

$$
\begin{align*}
& \mathcal{L}_{I}^{e f f}=-e\left[\bar{e}\left(g_{V}-q_{A} \partial_{5}\right) \gamma_{A} e \psi^{\beta}+\bar{e}\left(g_{V}^{*}-g_{A}^{*} \gamma_{5}^{\prime}\right) \gamma_{B} e \psi^{\beta}\right. \\
& +\bar{\mu}\left(\bar{a}_{v}-\bar{g}_{A} \gamma_{5}\right) 犭_{\beta} \mu \psi^{\beta}+\underset{\sim}{\left.\left(\bar{q}_{v}^{*}-\bar{G}_{A}^{*} \frac{y}{5}\right) \partial_{\beta}^{\prime} \mu^{\beta}\right]} \tag{1}
\end{align*}
$$

(For convenience, we are writing $g_{V, A}, \bar{g}_{V, A}$ in units of e.) We shall also, for simplicity, take $m_{\psi}=m_{\bar{\psi}}$. A more general analysis appears in the appendix. Further, we shall presume $\bar{g}_{V, A}=g_{V, A}$ and point out where appropriate the change in our results when this assumption is relaxed.

The relevant kinematics is summarized by Fig. 1.




Figure 1

We define, conventionally,

$$
\begin{equation*}
s=(p+q)^{2}=\left(p^{\prime}+q^{\prime}\right)^{2} \quad, \quad t=\left(p^{\prime}-p\right)^{2}=\left(q^{\prime}-q\right)^{2} \tag{2}
\end{equation*}
$$

When we ignore lepton masses, we have

$$
\begin{equation*}
t \simeq-\frac{S}{2}(1-\cos \theta) \tag{3}
\end{equation*}
$$

where $\theta$, as usual, is the center of momentum scattering angle of the positron in the Bhabha process, for example.

In Fig. 1, the propagation functions for $\psi, \bar{\psi}$ are to be understood to have their denominators modified to

$$
\begin{equation*}
k^{2}-m_{a}^{2}+i \Gamma_{a} m_{a} \tag{4}
\end{equation*}
$$

in view of the manifest instability of these objects. The parameters $\Gamma_{a}$ then characterize this instability in the standard fashion. Having laid down our conventions, let us now get on with the implications of Fig. 1.

A quantity of direct experimental significance is the differential cross section. We shall only consider effects not explicitly dependent on lepton masses. Furthermore, in view of the present state of the data, we shall not consider the various polarized cross sections in the present discussion. These may appear elsewhere. Thus, here, we shall record only the respective unpolarized differential cross sections, hoping to pinpoint some gross feature which would be a signal for (1). We consider first $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$.

Upon effecting the standard, trivial, manipulations we find (we are assuming $\left.m_{\psi}=m_{\bar{\psi}}, \Gamma_{\dot{\psi}}=\Gamma_{\bar{\psi}}\right)$

$$
\begin{align*}
& \left.\frac{d \sigma^{\mu \bar{\mu}}}{d \Omega}=\frac{\alpha^{2}}{4 s}\left[\left(1+\operatorname{coc}^{2} \theta\right)\left(1+\frac{4 s\left(s-m_{4}^{2}\right)[(\operatorname{Reg}}{v}\right)^{2}-\left(I m g_{v}\right)^{2}\right]\right) \\
& +\frac{2 s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\left\{\left(\left|g_{Y}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}+\right. \\
& \left.\left[(\operatorname{Reg})^{2}\right)^{2}-\left(I \operatorname{Img}_{V}\right)^{2}+\left(\operatorname{Reg} g_{A}\right)^{2}-\left(I m g_{A}\right)^{2}\right]^{2} \\
& \left.\left.-4\left[\operatorname{Reg}_{V} \operatorname{Img}_{V}+\operatorname{Reg} \operatorname{Im}_{A}\right]^{2}\right\}\right) \\
& +8 \operatorname{coct} \theta\left(\frac{s\left(s-m_{4}^{2}\right)\left[\left(\operatorname{Keg}_{A}\right)^{2}-\left(I_{m} g_{A}\right)^{2}\right]}{\left[\left(S-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right. \tag{5}
\end{align*}
$$

$$
\begin{aligned}
& +\frac{2 s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} \cdot m_{4}^{2}\right]}\left\{\left(\operatorname{Reg}_{A} \operatorname{Reg}_{V}+\operatorname{Im} g_{A} \operatorname{Im} g_{V}\right)^{2}\right. \\
& \left.\left.+\left(\operatorname{Reg} \operatorname{Reg}_{A}-\operatorname{Im} g_{V} \operatorname{Im} g_{A}\right)^{2}-\left(\operatorname{Reg}_{V} \operatorname{Im} g_{A}+\operatorname{Reg} \operatorname{Im}_{A} g^{2}\right\}\right)\right]
\end{aligned}
$$

The most glaring gross feature of this last result is the dependence of the "dip" terms on coupling constant . Within our framework we see that the occurrence of a dip or rise before the peak is entirely dependent on the phases of $g_{V, A^{*}}$. The dip completely reverses to become a rise as $g_{V, A}$ are varied from pure real to pure imaginary simultaneously. All dip terms vanish if

$$
\begin{equation*}
\left|\operatorname{Reg}_{i}\right|=|\operatorname{Img}| \quad, i=h, V . \tag{6}
\end{equation*}
$$

For $g_{i} \neq \bar{g}_{i}$, the last condition is clearly changed to

$$
\operatorname{Reg}_{i} \operatorname{ke}_{i} \bar{g}_{i}=\operatorname{Im}_{i} \operatorname{Im}_{i} \bar{g}_{i} \quad, i=A, V
$$

It would, therefore, be very interesting if a definitive statement could be made about the lack or presence of a dip in the $\mu$-pair cross section.

As we would have expected, the difference between (1) and the more simple situation where $\psi=\bar{\psi}$ is simply in the energy dependence, as the angular structure for $\bar{\mu} \bar{\mu}$ can only be that of s-channel vector and axial vector, namely

$$
\begin{equation*}
1+\cos ^{2} \theta \quad \text { and } \quad \operatorname{coc} \theta \tag{7}
\end{equation*}
$$

This, thus, continues to be true if we let $m_{\psi} \neq m_{\bar{\psi}}, \Gamma_{\psi} \neq \Gamma_{\bar{\psi}}$. But, of course, in this last situation, the energy dependence becomes slightly more complicated. To repeat, we have recorded this case in the appendix. Finally, let us remark that the only effect of taking $\bar{g}_{i} \neq g_{i}$, ist to make the replacement $g_{i}^{2} \rightarrow g_{i} \bar{g}_{i}$ for terms
quadratic in $g_{i}$ and to replace terms like

$$
\left(\left.\lg _{A}\right|^{2}+\left|g_{V}\right|^{2}\right)^{2} \quad \text { by } \quad\left(\left|g_{A}\right|^{2}+\left.\lg _{V}\right|^{2}\right)\left(\left.\lg _{A}\right|^{2}+\left|\bar{g}_{V}\right|^{2}\right), \text { etc. }
$$

for terms quartic in $\mathrm{g}_{\mathrm{i}}$.
We turn next to the Bhabha process. Again, by standard methods

$$
\begin{aligned}
& \frac{d \sigma^{c \pi}}{d \Omega}=\frac{\alpha^{2}}{4 s}\left[( 1 + \operatorname { c o s } ^ { 2 } \theta ) \left(1+\frac{2 s\left(s-m_{4}^{2}\right)\left(g_{\gamma}^{2}+g_{v}^{*^{2}}\right)}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right.\right. \\
& \left.+\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{\psi}^{2}\right]}\left\{2\left(\left|g_{V}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}+\left(g_{V}^{2}+g_{A}^{2}\right)^{2}+\left(g_{V}^{*^{2}}+g_{A}^{m^{2}}\right)^{2}\right\}\right) \\
& -\frac{4 \cos ^{4} \theta / 2}{\sin ^{2} \theta / 2}\left(1+\frac{s\left(s-m_{4}^{2}\right)\left(g_{V}^{2}+g_{V}^{4^{2}}+g_{A}^{2}+g_{A}^{x^{2}}\right)}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right) \\
& +\frac{2\left(1+\cos ^{4} \theta 12\right)}{\sin ^{4} \theta 12} \\
& +\cos \theta\left(\frac{4 s\left(s-m_{4}^{2}\right)\left(g_{A}^{2}+g_{A}^{*^{2}}\right)}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}+\right.
\end{aligned}
$$

$$
\left.\frac{8 . s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\left\{2\left(\operatorname{Re}\left(g_{A}^{*} g_{v}\right)\right)^{2}+g_{v}^{2} g_{A}^{2}+g_{v}^{*^{2}} g_{A}^{*^{2}}\right\}\right)
$$

$$
\begin{aligned}
-4 \cos ^{4} \theta / 2( & \frac{s^{2}\left[\left(s-m_{4}^{2}\right)\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)-\Gamma_{4}^{2} m_{4}^{2}\right]}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]} \\
& \times\left\{2\left(\left|g_{V}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}+8\left(R \in\left(g_{A}^{*} g_{V}\right)\right)^{2}+\left(g_{A}^{2}+g_{V}^{2}\right)^{2}\right. \\
& +4 g_{V}^{2} g_{A}^{2}+\left(g_{A}^{*^{2}}+g_{V}^{* 2}\right)^{2}+4 g_{V}^{\left.*_{V}^{2} g_{A}^{*}\right\}} \\
& +\frac{s\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)\left(g_{V}^{2}+g_{V}^{*^{2}}+g_{A}^{2}+g_{A}^{* 2}\right)}{\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{4\left(1+\cos ^{4} \theta / 2\right)}{\sin ^{2} \theta / 2}\left(\frac{s\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)\left(g_{V}^{2}+g_{v}^{*^{2}}\right)}{\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+r_{4}^{2} m_{4}^{2}\right]}\right) \\
& -\frac{4\left(1-\cos ^{4} \theta / 2\right)}{\sin ^{2} \theta / 2}\left(\frac{s\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)\left(g_{A}^{2}+g_{A}^{* 2}\right)}{\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+r_{4}^{2} m_{4}^{2}\right]}\right)
\end{aligned}
$$

$$
\begin{aligned}
& +2\left(1+\cos ^{4} \theta / 2\right)\left(\frac{s^{2}\left\{2\left(\left.1 g_{V}\right|^{2}+\left.g_{A}\right|^{2}\right)^{2}+\left(g_{V}^{2}+g_{A}^{2}\right)^{2}+\left(g_{\psi}^{\left.\left.*^{2}+g_{A}^{*}\right)^{2}\right\}}\right.\right.}{\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right) \\
& -2\left(1-\cos ^{4} \theta / 2\right)\left(\frac{\left.s^{2}\left\{8\left(\operatorname{Re}\left(g_{A}^{*} g_{V}\right)\right)^{2}+4 g_{V}^{2} g_{A}^{2}+4 g_{V}^{*^{2}} g_{A}^{* 2}\right\}\right)-}{\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right)-
\end{aligned}
$$

Again, the dip phenomenon is the most glaring feature, but the analoga of the remarks made above for $\mu \bar{\mu}$ do not strictly apply. For, even if the analogue of (6) does pertain, there still may appear a "dip" due to the term

$$
\begin{align*}
-4 \cos \theta / 2( & \frac{s^{2}\left[\left(s-m_{4}^{2}\right)\left(s \sin ^{2} \theta 12+m_{4}^{2}\right)-\Gamma_{4}^{2} m_{4}^{2}\right]}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s \sin ^{2} \theta^{\prime} 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]} \\
& \times\left\{a^{2}\left(\left.\lg _{V}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}+8\left(\operatorname{Re}\left(g_{A}^{*} g_{V}\right)\right)^{2}+\left(g_{A}^{2}+g_{V}^{2}\right)^{2}\right.  \tag{9}\\
& \left.+4 g_{A}^{2} g_{V}^{2}+\left(g_{A}^{*^{2}}+g_{V}^{* 2}\right)^{2}+4 g_{A}^{A^{2} g_{V}^{*}}\right\}
\end{align*}
$$

although admittedly it would be down from the terms respecting (6) by $\mathrm{O}\left(\mathrm{g}^{2}\right)$ and may thus escape observation.

Furthermore, the appearance of terms $\propto(1-\cos \theta)^{-1}$ in the inference between t-channel $\gamma$ and s-channel $\psi, \bar{\psi}$ means that the "dip" phenomena can be enhanced by considering data at angles away from $\cos \theta=0$, presumably. Additionally, these terms will respect the analogue of (6), since they arise from schannel $\psi, \bar{\psi}$. Thus, a clever use of s-dependence and angular dependence taken together is probably the best way to investigate the dip phenomena, as one would have guessed. To repcat, a very suggestive gross signal for (1) would be the absence of a dip in (5). The occurrence of a rise would also be very interesting, although it would not be as suggestive because of the possibility of negative metric. When polarized cross section data becomes available, perhaps a clearer test will be the apparent $T$-violating terms in the respective cross sections. To repeat, the analysis relevant to this latter situation may appear elsewhere.

Appendix

In the unlikely event that experiment permits probing of the respective resonances, we shall record in this appendix the differential cross sections for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$and $e^{+} e^{-}-e^{+} e^{-}$when $m_{\psi} \neq m_{\bar{\psi}}, \Gamma_{\psi} \neq \Gamma_{\bar{\psi}}$, still taking, however, $g_{i}=\bar{g}_{i}$. As we remarked above, the effect of relaxing this last assumption is trivial $-g_{i}^{2} \rightarrow g_{i} \bar{g}_{i}$, etc. (see page 5 ).

For the $\overline{\mu \mu}$ case, we find (again by the standard methods)

$$
\begin{aligned}
& \frac{d \sigma^{\mu \mu}}{d \Omega}=\frac{\alpha^{2}}{4 S}\left[\left(1+\cos ^{2} \theta\right) / 1+\frac{s\left[\left(s-m_{4}^{2}\right)\left(g_{v}^{2}+g_{v}^{*^{2}}\right)+i \Gamma_{4} m_{4}\left(g_{v}^{\left.\left.*^{2}-g_{v}^{2}\right)\right]}\right.\right.}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right. \\
& +\quad \frac{s\left[\left(s-m_{\frac{1}{4}}^{2}\right)\left(g_{v}^{2}+g_{v}^{*^{2}}\right)-i \Gamma_{\frac{1}{4}} m_{4}\left(g_{v}^{*^{2}}-g_{v}^{2}\right)\right]}{\left[\left(s-m_{\frac{1}{4}}^{2}\right)^{2}+\Gamma_{\frac{1}{4}}^{2} m_{\frac{1}{4}}^{2}\right]}
\end{aligned}
$$

$$
+\left\{\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}+\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{\frac{4}{4}}^{2} m_{\frac{2}{4}}^{2}\right]}\right\}\left(\left|g_{V}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}
$$

$$
+s^{2}\left\{[ ( s - m _ { 4 } ^ { 2 } ) ( s - m ^ { 2 } \frac { 1 } { 4 } ) + \Gamma _ { 4 } m _ { 4 } \Gamma _ { 4 } m _ { \overline { 4 } } ] \left(\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}\right)^{2}+\left(g_{V}^{2}+g_{A}^{2}\right)^{2}\right.\right.
$$

$$
)+i\left[\Gamma_{4} m_{4}\left(s-m_{4}^{2}\right)-\Gamma_{\frac{1}{4}} m_{\overline{4}}\left(s-m_{4}^{2}\right)\right]\left(\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}\right)^{2}-\left(g_{V}^{2}+g_{A}^{2}\right)^{2}\right.
$$

$$
\left.\int\right\}\left(\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{\frac{4}{4}}^{2}\right]\right)
$$

$$
\begin{aligned}
& +2 \cos \theta\left(\frac{s\left[\left(s-m_{4}^{2}\right)\left(g_{A}^{2}+g_{A}^{+2}\right)+i \Gamma_{4} m_{4}\left(g_{A}^{* 2}-g_{A}^{2}\right)\right]}{\left[\left(S-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right. \\
& +\frac{s\left[\left(s-m_{4}^{2}\right)\left(g_{A}^{2}+g_{A}^{* 2}\right)+i \Gamma_{\overline{4}} m_{\overline{4}}\left(g_{A}^{2}-g_{A}^{*^{2}}\right)\right]}{\left[\left(s-m_{\overline{4}}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}\right]} \\
& +4\left\{\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}+\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right\}\left(\operatorname{Re}_{A}^{*} g_{Y}\right)^{2} \\
& +4 s^{2}\left\{\left[\left(s-m_{4}^{2}\right)\left(s-m_{4}^{2}\right)+\Gamma_{4} m_{4} \Gamma_{4} m_{4}\right]\left(g_{V}^{*^{2}} g_{A}^{*^{2}}+g_{v}^{2} g_{A}^{2}\right)\right. \\
& +i\left[\Gamma_{4} m_{4}\left(s-m_{\frac{2}{2}}^{2}\right)-\Gamma_{4} m_{\overline{4}}\left(s-m_{4}^{2}\right)\right]\left(g_{V}^{*^{2}} g_{A}^{*^{2}}-g_{V}^{2} g_{A}^{2}\right) \\
& \left.3 /\left(\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\right)\right]
\end{aligned}
$$

For the Bhabha process, we find

$$
\begin{aligned}
& \frac{d \sigma^{e \bar{e}}}{d \Omega}=\frac{\alpha^{2}}{4 s}\left[( 1 + \operatorname { c o s } ^ { 2 } \theta ) \left(1+\frac{s\left[\left(s-m_{4}^{2}\right)\left(q_{v}^{2}+q_{v}^{*}\right)+i \Gamma_{4}^{2} m_{4}\left(g_{v}^{*}-g_{v}^{2}\right)\right]}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right.\right. \\
& +\frac{s\left[\left(s-m_{4}^{2}\right)\left(g_{v}^{2}+q_{V}^{* 2}\right)+i \Gamma_{4} m_{\overline{4}}\left(g_{v}^{2}-g_{V}^{* 2}\right)\right]}{\left[\left(s-m_{\frac{2}{4}}^{2}\right)^{2}+\Gamma_{\frac{\Gamma_{4}}{2}}^{2} m_{\frac{2}{4}}^{2}\right]} \\
& +\left\{\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}+\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{\frac{T_{4}^{2}}{2}}^{2} m_{\frac{2}{4}}^{2}\right]}\right\} \\
& \times\left(\left.\lg _{r}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2} \\
& +s^{2}\left\{[ ( s - m _ { 4 } ^ { 2 } ) ( s - m _ { \frac { 2 } { 4 } } ^ { 2 } ) + \Gamma _ { 4 } m _ { 4 } \Gamma _ { 4 } m _ { 4 } ] \left(\left(g_{4}^{*}+g_{A}^{*}\right)^{2}\right.\right. \\
& +\left(g_{v}^{2}+g_{A}^{2}\right)^{2}+i\left[\left(s-m_{4}^{2}\right) \Gamma_{4} m_{4}-\left(s-m_{4}^{2}\right) \Gamma_{4} m_{4}\right] \\
& \left.\times\left(\left(g_{V}^{*}+q_{A}^{*^{2}}\right)^{2}-\left(g_{V}^{2}+g_{A}^{2}\right)^{2}\right)\right\} /\left(\left[\left(s-m_{4}^{2}\right)^{2}+r_{4}^{2} m_{4}^{2}\right]\right. \\
& \left.\left.\times\left[\left(s-m_{\frac{2}{4}}^{2}\right)^{2}+\Gamma_{4}^{2} m_{\frac{2}{4}}^{2}\right]\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{4 \cos ^{4} \theta / 2}{\sin ^{2} \theta / 2}\left(1+\frac{1}{2} S\left[\left(S-m_{4}^{2}\right)\left(g_{V}^{2}+g_{A}^{2}+g_{V}^{+^{2}}+g_{A}^{A^{2}}\right)-i \Gamma_{4} m_{4}( \right.\right. \\
& g_{V}^{2}+g_{A}^{2}-g_{V}^{\left.\left.H^{2}-g_{A}^{*^{2}}\right)\right] /\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]} \\
& +\frac{1}{2} s\left[\left(s-m_{\overline{4}}^{2}\right)\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}+g_{V}^{2}+g_{A}^{2}\right)-i \Gamma_{\overline{4}} m_{-}\left(g_{V}^{*}+g_{A}^{*^{2}}-g_{V}^{2}\right.\right. \\
& \left.\left.\left.-g_{A}^{2}\right)\right] /\left[\left(s-m_{\frac{2}{4}}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}\right]\right) \\
& +\frac{2\left(1+\cos ^{4} \theta / 2\right)}{\sin ^{4} \theta / 2} \\
& +2 \cos \theta\left(\frac{s\left[\left(s-m_{4}^{2}\right)\left(g_{A}^{2}+g_{A}^{* 2}\right)+i \Gamma_{4} m_{4}\left(g_{A}^{* 2}-g_{A}^{2}\right)\right]}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right. \\
& +\frac{s\left[\left(s-m_{\frac{2}{4}}^{2}\right)\left(g_{A}^{2}+g_{A}^{2^{2}}\right)+i \Gamma_{\overline{4}} m_{\overline{4}}\left(g_{A}^{2}-g_{A}^{* 2}\right)\right]}{\left[\left(s-m_{\frac{2}{4}}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}\right]}
\end{aligned}
$$

$$
\begin{aligned}
& +4\left\{\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}+\frac{s^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{\frac{\pi}{4}}^{2} m_{4}^{2}\right]}\right\}\left(R_{e} g_{A}^{*} g_{\gamma}\right)^{2} \\
& +4 s^{2}\left\{\left[\left(s-m_{4}^{2}\right)\left(s-m_{4}^{2}\right)+\Gamma_{4} m_{4} \Gamma_{\overline{4}} m_{4}\right]\left(g_{V}^{*} g_{A}^{*}+g_{V}^{2} g_{A}^{2}\right)\right. \\
& +i\left[\left(s-m_{\frac{2}{4}}^{2}\right) \Gamma_{4} m_{4}-\left(s-m_{\frac{2}{4}}^{2}\right) \Gamma_{\frac{\pi}{4}} m_{\overline{4}}\right]\left(g_{r}^{\left.\left.*^{2} g_{A}^{* 2}-g_{V}^{2} g_{A}^{2}\right)\right\} / ~}\right. \\
& \left.\left(\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s-m_{\frac{2}{4}}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\right)\right) \\
& -4 \cos ^{4} \theta / 2\left(\frac { 1 } { 2 } S \left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)\left(g_{V}^{2}+g_{A}^{2}+g_{V}^{* 2}+g_{A}^{* 2}\right)\right.\right. \\
& \left.+i \Gamma_{4} m_{4}\left(g_{V}^{2}+g_{A}^{2}-g_{V}^{*^{2}}-g_{A}^{*^{2}}\right)\right] /\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right. \text {. } \\
& +\frac{1}{2} S\left[\left(\operatorname{sen}^{2} \theta / 2+m_{\frac{2}{4}}^{2}\right)\left(g_{V}^{2}+g_{A}^{2}+g_{V}^{*^{2}}+g_{A}^{*^{2}}\right)\right. \\
& \left.+i \Gamma_{\overline{4}} m_{\overline{4}}\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}-g_{V}^{2}-g_{A}^{2}\right)\right] /\left[\left(\sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{\frac{\pi}{4}}^{2} m_{\frac{2}{4}}^{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& +s^{2}\left[\frac{\left(s-m_{4}^{2}\right)\left(s \sin _{i}^{2} \theta / 2+m_{4}^{2}\right)-\Gamma_{4}^{2} m_{4}^{2}}{\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right. \\
& \left.+\frac{\left(s-m_{\frac{2}{4}}^{2}\right)\left(s \sin ^{2} \theta / 2+m_{\overline{4}}^{2}\right)-\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}}{\left[\left(s-m_{\frac{4}{4}}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}\right]\left[\left(s \sin ^{2} \theta / 2+m_{\overline{4}}^{2}\right)^{2}+\Gamma_{\frac{\Gamma}{4}}^{2} m_{\overline{4}}^{2}\right]}\right] \\
& x\left\{\left(\left|g_{r}\right|^{2}+\left|g_{A}\right|^{2}\right)^{2}+4\left(\operatorname{Re} g_{A}^{*} g_{V}\right)^{2}\right\} \\
& +\frac{1}{2} s^{2}\left\{[ ( s - m _ { 4 } ^ { 2 } ) ( s \operatorname { s i n } ^ { 2 } \theta / 2 + m _ { 4 } ^ { 2 } ) - \Gamma _ { 4 } m _ { 4 } \Gamma _ { \overline { 4 } } m _ { \overline { 4 } } ] \left\{\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}\right)^{2}+4 g_{v}^{*^{2}} g_{A}^{*^{2}}\right.\right. \\
& \left.+\left(g_{\gamma}^{2}+g_{A}^{2}\right)^{2}+4 g_{r}^{2} g_{A}^{2}\right\}+i\left[\left(s-m_{\psi}^{2}\right) \Gamma_{4} m_{\overline{4}}+\left(\sin ^{2} \theta / 2\right.\right. \\
& \left.\left.+m_{4}^{2}\right) \Gamma_{4} m_{4}\right]\left\{\left(g_{V}^{* 2}+g_{A}^{* 2}\right)^{2}-\left(g_{V}^{2}+g_{A}^{2}\right)^{2}+4\left(g_{V}^{*^{2}} g_{A}^{* 2}-g_{V}^{2} g_{A}^{2}\right)\right\} \\
& -\iint\left(\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(\sin ^{2} \theta / 2+m_{\frac{1}{4}}^{2}\right)^{2}+\Gamma_{\frac{4}{4}}^{2} m_{\frac{2}{4}}^{2}\right]\right) \\
& +\frac{1}{2} s^{2}\left\{[ ( s - m _ { \frac { 2 } { 4 } } ^ { 2 } ) ( \operatorname { s i n } ^ { 2 } \theta / 2 + m _ { 4 } ^ { 2 } ) - \Gamma _ { \frac { 1 } { 4 } } m _ { 4 } \Gamma _ { 4 } m _ { 4 } ] \left\{\left(g_{V}^{2}+g_{A}^{2}\right)^{2}+4 g_{V}^{2} g_{A}^{2}\right.\right. \\
& \left.+\left(g_{y}^{* 2}+g_{A}^{+^{2}}\right)^{2}+4 g_{y}^{* 2} g_{A}^{* 2}\right\}+i\left[\left(s-m_{4}^{2}\right) \Gamma_{4} m_{4}+\right. \\
& \left.\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right) \Gamma_{4} m_{4}\right]\left\{\left(g_{V}^{2}+g_{A}^{2}\right)^{2}-\left(g_{V}^{*^{2}}+g_{A}^{*^{2}}\right)^{2}+4\left(g_{V}^{2} g_{A}^{2}-g_{V}^{k^{2}} g_{A}^{*}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \} /\left(\left[\left(s-m_{4}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{4}^{2}\right]\left[\left(\sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\right) \\
& +
\end{aligned}
$$

$$
\begin{aligned}
& \left.\left.\left.-\Gamma_{4} m_{4}\left(s \sin ^{2} \theta_{2}+m_{4}^{2}\right)\right\}\left[\left(g_{v}^{*^{2}}+g_{A}^{*^{2}}\right)^{2}-\left(g_{v}^{2}+g_{A}^{2}\right)^{2}\right]\right)\right] \\
& \left.\left(\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\left[\left(s \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]\right)\right) \\
& -\infty\left(1-\cos ^{4} \theta / 2\right)\left(4 \left\{\frac{s^{2}}{\left[\left(\sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+\Gamma_{4}^{2} m_{4}^{2}\right]}\right.\right. \\
& \left.+\frac{s^{2}}{\left[\left(s \sin ^{2} \theta / 2+m^{2}\right)^{2}+\Gamma_{4}^{2} m_{\frac{2}{4}}^{2}\right]}\right\}\left(\operatorname{Reg}_{A}^{*} g_{Y}\right)^{2} \\
& +2.5^{2}\left(\left\{\left(5 \sin ^{2} \theta / 2+m_{4}^{2}\right)\left(5 \sin ^{2} \theta / 2+m_{\frac{2}{4}}^{2}\right)+\Gamma_{4} m_{4} \Gamma_{\frac{\pi}{4}} m_{\overline{4}}\right\}\right. \\
& \times\left[g_{Y}^{*^{2}} g_{A}^{*^{2}}+y_{Y}^{2} g_{A}^{2}\right]+i\left\{\Gamma_{\overline{4}} m_{\overline{4}}\left(s i^{2} i^{2} \in / 2+m_{4}^{2}\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left(\left[\left(\sin \sin ^{2} \theta / 2+m_{4}^{2}\right)^{2}+m_{4}^{2} \Gamma_{4}^{2}\right]\left[\left(\sin \sin ^{2} \theta / 2+m_{\frac{1}{4}}^{2}\right)^{2}+\Gamma_{\overline{4}}^{2} m_{\overline{4}}^{2}\right]\right) \\
& )]
\end{aligned}
$$

Detailed $\psi$ Effects in $\mathrm{e}^{+} \mathrm{e}^{-}-\mathrm{e}^{+} \mathrm{e}^{-}$and $\mu^{+} \mu^{-}$

The crossection will be calculated from

assuming $\gamma_{\mu}\left(g_{V}+g_{A} \gamma_{5}\right)$ with real $g_{V}$ and $g_{A}$ and complex $M_{0}$. For generality, we will assume the beam is transversely polarized, and that the final polarizations are not summed in case the $\mu^{+}$or $\mu^{-}$helicity is observed, or storage rings with longitudinally polarized beams are built. Our notation is

$$
\begin{aligned}
& s=4 E^{2}, \quad t=-s(1-\cos \theta) / 2 \\
& Q=t /\left(e^{2}\left(t-M_{0}^{2}\right)\right), \quad R=s /\left(e^{2}\left(s-M_{0}^{2}\right)\right) \\
& e^{2}=4 \pi \alpha
\end{aligned}
$$

$\zeta_{-}$and $\zeta_{+}$are the initial transverse polarizations
$\phi$ is the angle between this direction and the scattering plane
$h_{-}$and $h_{+}$are the final helicities.
The differential crossection for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$is

$$
\begin{aligned}
& \frac{325}{\alpha^{2}} \frac{d \sigma}{d \Omega}=\left(1+h_{-} h_{+}\right) 4 B_{1} \\
& +\left(1-h_{-} h_{+}\right)\left\{(1-\cos \theta)^{2} B_{2}+(1+\cos \theta)^{2} B_{3}+S_{1} \xi_{2} \sin ^{2} \theta\right. \\
& \left.\left[\cos 2 \varphi B_{4}+\sin 2 \varphi B_{5}\right]\right\}+\left(h_{-}-h_{+}\right)\left\{(1+\cos \theta)^{2} B_{6}\right. \\
& \left.+\zeta_{1} \zeta_{2} \sin ^{2} \theta\left[\cos 2 \varphi B_{7}+\sin 2 \varphi B_{8}\right]\right\}
\end{aligned}
$$

where

$$
\begin{array}{ll}
B_{1}=\left|C_{1}\right|^{2} & B_{2}=\mid C_{2} 1^{2} \\
B_{3}=\frac{1}{2}\left\{\left|C_{3}+C_{4}\right|^{2}+\left|C_{5}+C_{6}\right|^{2}\right\} & B_{6}=\frac{1}{2}\left\{\left|C_{3}+C_{4}\right|^{2}-\left|C_{5}+C_{6}\right|^{2}\right\} \\
B_{4}=R_{e} C_{2}^{*}\left(C_{3}+C_{4}+C_{5}+C_{6}\right) & B_{7}=R_{e} C_{2}^{*}\left(C_{3}+C_{4}-C_{5}-C_{6}\right) \\
B_{5}=\operatorname{lm} C_{2}^{*}\left(C_{3}+C_{4}-C_{5}-C_{6}\right) & B_{2}=\operatorname{lm}_{2} C_{2}^{*}\left(C_{3}-C_{5}\right) \\
C_{1}=\frac{5}{t}\left[1+\left(g_{v}^{2}-g_{A}^{2}\right) Q_{1}\right] & C_{2}=1+\left(g_{v}^{2}-g_{A}^{2}\right) R \\
C_{3}=\frac{5}{t}\left[1+\left(g_{r}+g_{A}\right)^{2} Q\right] & C_{4}=1+\left(g_{v}+g_{A}\right)^{2} R \\
C_{5}=\frac{5}{t}\left[1+\left(g_{v}-g_{A}\right)^{2} Q\right] & C_{6}=1+\left(g_{v}-g_{A}\right)^{2} R
\end{array}
$$

The differential crossection for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$is given by this expression with $\mathrm{s} / \mathrm{t}$ and Q set to zero. In the present experiment the beam does not live long enough to become polarized, and final helicities are not measured, thus the above expression reduces to

$$
\begin{aligned}
& \frac{85}{\alpha^{2}} \frac{d \sigma}{d \Omega}=4 B_{1}+(1-\cos \theta)^{2} B_{2}+(1+\cos \theta)^{2} B_{3} \\
& B_{1}=\left(\frac{s}{+}\right)^{2}\left[1+\left(g_{v}^{2}-g_{A}^{2}\right) 2 \operatorname{Re} Q+\left(g_{v}^{2}-g_{A}^{2}\right)^{2}|Q|^{2}\right] \\
& B_{2}=\left[1+\left(g_{v}^{2}-g_{A}^{2}\right) 2 \operatorname{Re} R+\left(g_{v}^{2}-g_{A}^{2}\right)^{2}|R|^{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
B_{3}= & \left(\frac{s}{t}+1\right)^{2}+\left(g_{v}^{2}+g_{A}^{2}\right)\left(\frac{S}{t}+1\right) \geq R_{e}\left[\frac{3}{t} Q+R\right] \\
& +\frac{1}{2}\left[\left(q_{v}+g_{A}\right)^{2}+\left(g_{v}-g_{A}\right)^{2}\right]\left|\frac{S}{t} Q+R\right|^{2}
\end{aligned}
$$

The values of $g_{V}, g_{A}$, and $\Gamma$ are sufficiently small that $Q$ can be neglected in $B_{1}-B_{3}$, but the $R e R$ and $|R|^{2}$ terms can become large. In these,

$$
\begin{aligned}
& \operatorname{Re} R=\frac{\Sigma\left(S-M_{0}^{2}\right)}{e^{2}\left(\left(S-M_{0}^{2}\right)^{2}+M_{0}^{2} \Gamma^{2}\right)} \\
& |\mathbf{R}|^{2}=\frac{S^{2}}{e^{4}\left(\left(S-M_{0}^{2}\right)^{2}+M_{0}^{2} \Gamma^{2}\right)}
\end{aligned}
$$

If transverse polarization can be achieved, an interesting term to measure is $B_{4}$. This can be large near the resonance.
(ed. note: For further details see preprint by R. Budny "Detailed $W^{0}$ Effects in $\mathrm{e}^{+} \mathrm{e}^{-}-\mathrm{e}^{+} \mathrm{e}^{-}$" and "Effects of Neutral Weak Currents in Annihilation",

Physics Letters 45B, No. 4, 340 (1973).

Some examples of crossections averaged over a gaussian beam $\frac{1}{\sqrt{2 \pi} \sigma} e^{-\left(w-w_{0}\right)^{2} / 25^{2}} \quad$ with $\quad \sigma=18026 \mathrm{MeV}$
are given in the following ( total crossections are integrated over
$\left.50^{\circ}<\theta<130^{\circ}, \Gamma_{\text {tot }}=75 \mathrm{keV}\right)$.




$$
\begin{array}{ll}
\text { I } & \sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right) \\
\text {II } & \sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-}\right)
\end{array}
$$




III $\Delta^{\mathrm{T}} \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-}-\mu^{+} \mu^{-}\right)$from $\mathrm{B}_{4}$ if $\xi_{-}^{5}+=-(.924)^{2}$ integrated
IV $\Delta^{T} \sigma\left(e^{+} e^{-} \rightarrow e^{+} e^{-}\right) \quad\left\{-\frac{\pi}{4}<\varphi<\frac{\pi}{4}\right\}+\left\{\frac{5 \pi}{4}<\varphi<\frac{5 \pi}{4}\right\}-\left\{\frac{\pi}{4}<\varphi<\frac{3 \pi}{4}\right\}-\left\{\frac{5 \pi}{4}<\varphi<\frac{7 \pi}{4}\right\}$









XIII $\mathrm{d} \sigma / \mathrm{d} \cos \theta\left(\mathrm{e}^{+} \mathrm{e}^{-}\right)$at $\mathrm{W}=3.102 \mathrm{GeV}$
XIV same at $W=3.103 \mathrm{GeV}$





## Calculations by R. Giles and R. Pearson

The results plotted include exponentiated radiative corrections and folding with a Gaussian beam distribution: $\sigma\left(\mathrm{E}_{\mathrm{cms}}\right)=1.2 \mathrm{MeV}$ (standard deviation)

## Resonance parameters:

$\Gamma_{\text {TOTAL }}=92 \mathrm{keV}$
$\Gamma_{\psi \rightarrow \mathrm{e}^{+} e^{-}}=\Gamma_{\psi \rightarrow \mu^{+} \mu^{-}}=5.5 \mathrm{keV}$
$\psi$ assumed to be spin 1 with no CP violation

$$
\langle\ell \bar{l} \mid \psi\rangle=\bar{u}_{\ell}\left(\mathrm{g}_{\mathrm{v}} \gamma^{\mu}+\mathrm{g}_{\mathrm{A}} \gamma^{\mu} \gamma_{5}\right) \mathrm{v}_{\ell} \epsilon_{\mu}^{(\psi)} \quad\left(\mathrm{g}_{\mathrm{v}}, \mathrm{~g}_{\mathrm{A}} \text { real }\right) .
$$

PURE V

$\sigma_{\mathrm{e}^{+}} \mathrm{e}^{-} \rightarrow \mu^{+}{ }^{-}$
(radiative corrected and smeared over beam)

$\sigma_{e^{+}} e^{-} \rightarrow e^{+} e^{-}(n, b)$
(radiative corrected and smeared over beam)
$\mathrm{V}=\mathrm{A}$

same

same

PURE A

same

same




[^0]:    $\dagger$ However, These effects can be obscured by the presence of a large contribution from direct hadron production spread by machine resolution.

