SLAC-PUB-14956



CERN-PH-TH/2012-105, SLAC-PUB-14956, IPPP/12/17, DCPT/12/34, LPN12-045

# Systematic improvement of QCD parton showers

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**Abstract** In this contribution, we will give a brief overview of the progress that has been achieved in the field of combining matrix elements and parton showers. We exemplify this by focusing on the case of electron–positron collisions and by reporting on recent developments as accomplished within the SHERPA event generation framework.

## 1 Monte Carlo event generation at a glance

Event generators are widely used to model the multi-hadron final states of high-energy particle collisions. For a very comprehensive review, we refer the interested reader to Ref. [1]. The underlying principle for organizing the computer simulation of events is factorization, i.e. to factorize the evolution of each event into several phases ordered according to their energy domains. We broadly distinguish two major phases governed by two different physics regimes: we can apply short-distance/perturbative methods to describe the physics at the harder energy

[1]Presented at Linear Collider 2011: Understanding QCD at Linear Colliders in searching for old and new physics, 12–16 September 2011, ECT*, Trento, Italy



Work supported in part by US Department of Energy contract DE-AC02-76SF00515.

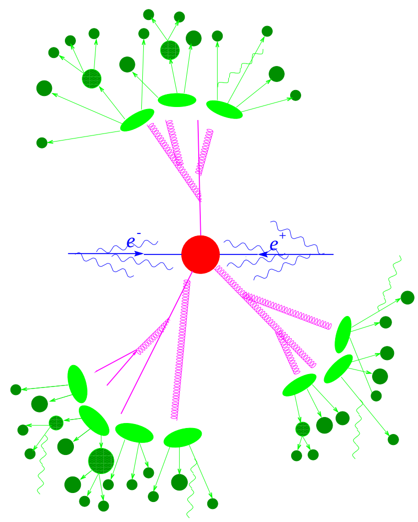

Figure 1: The various phases of Monte Carlo event generation, illustrated for lepton–lepton collisions. The outer, circular part visualizes the event evolution driven by non-perturbative dynamics (depicted by the green blobs) while the inner part shows the phases related to short-distance phenomena (depicted by the red and blue objects).

scales while for the description of soft effects, we have to rely on phenomenological models encoding our observations regarding the confinement of the collision products, a mechanism for which a rigorous understanding has not been developed yet. The separation is mainly driven by the nature of QCD where the strong coupling becomes small at large scales, such that the theory becomes asymptotically free and can be formulated in terms of partons. Contrary at scales of $\mathcal{O}(1)$ GeV, the coupling strength has increased substantially and non-perturbative dynamics dictates the evolution of the events. An extremely important property of QCD is the formation of jets, which manifest themselves as sprays of particles leaving localized energy deposits in the detectors. Correspondingly, the phases of the event generation can also be described in terms of jet production and (intra-jet) evolution, cf. e.g. Ref. [2].

Fig. 1 gives the details by showing not only the main but all phases, which we consider in Monte Carlo event generation. The phases where physics can be mastered with perturbative methods are visualized in the inner part of the figure. In blue we show the effects of initial-state radiation off the incoming leptons, which commonly are encoded in an inclusive manner by electron structure functions. The red objects visualize the hard interaction (shown by the big red blob in the middle representing the process $e^-e^+ \to q\bar{q}g$) producing energetic parton jets that give rise to subsequent QCD bremsstrahlung (shown by the branching pattern in magenta). The physics of the hard process is best described by relying on exact matrix-element expressions – with the current frontier given by $n$-leg tree-level ($n \sim 10$) and QCD virtual ($n \sim 5$) amplitudes – whereas all bremsstrahlung effects are described by parton showering based on matrix-element approximations that are correct in the singular phase-space regions of QCD. The phases of non-perturbative dynamics are represented by the green-coloured blobs in the outer sphere of the figure. They depict the transition of the coloured partons into primary, unstable hadrons and their subsequent decays into stable, detectable hadrons, which can be described by phase-space or effective models. The parton–hadron conversion is "parametrized" by hadronization models, such as the renowned models of Lund string fragmentation [3] or cluster hadronization [4].

A full-fledged Monte Carlo event generator incorporates physics implementations according to all phases of event evolution, from the evaluation of scattering matrix elements to the description of hadron decays. The Monte Carlo approach is inherent to all phases: cross sections are physical objects and, hence, a probabilistic picture can be identified for each phase. We can draw events from the resulting probability densities by generating random numbers. Pythia [5], Herwig [6, 7] and Sherpa [8, 9] are examples for (well) established event generators in the LHC era. Common to them is the generation of hadron-level predictions, which can be compared directly to experimental data, once the data are corrected for detector effects.

## 2 Parton shower basics and modern formalisms

The final states of the hard interactions often produce partons that are still sufficiently energetic to induce further radiation, because there is enough time for them to interact perturbatively before hadronization sets in. Owing to the singularity structure intrinsic to QCD, these emissions preferably populate the collinear and soft regions of phase space, and very conveniently it is in these limits that QCD amplitudes factorize. This can be taken further, i.e. be promoted to a factorization at the cross-section level:

$$d\sigma_{n+1} \;=\; d\sigma_n\,\frac{\alpha_s(t)}{2\pi}\,\frac{dt}{t}\,dz\;P_{a\to bc}(z)\;. \tag{1}$$

Here $\alpha_s$, $t \equiv p_a^2$ and $z$ respectively denote the strong coupling constant, the propagator and the momentum-fraction variable used in the splitting process. The function $P_{a\to bc}(z)$ characterizes the parton splitting $a \to bc$ (e.g. $q \to qg$) in detail, encoding the functional dependence on $z$, and possibly the splitting angle. For example, if one considers the leading collinear region, i.e. small-angle radiation off outgoing partons, the Altarelli–Parisi (or DGLAP) splitting functions are obtained; a nice introduction to the subject can be found in [10]. Eq. (1) expresses more than factorization of the multi-parton cross section, it ultimately forms the basis for a recursive definition of multiple emissions ordered in $t$. As a result collinear/soft partons can be added in an iterative procedure, and we arrive at an emission pattern as shown in Fig. 1 where the initially energetic $q\bar{q}g$ partonic ensemble has evolved down to a scale (magnitude of the ordering variable $t$) of the order of $t^{1/2} \sim 1$ GeV. This (i) regulates the (collinear) divergences and (ii) sets a scale conveniently close to the onset of hadronization. Emissions below this cut-off are said to be unresolvable. The iterative scheme ensures that all kinematically enhanced contributions are taken into account, which from a more formal point of view means that the leading logarithmic (LL) terms are summed up to all orders. The enhancements are manifest in the intra-jet evolution and in the rapid particle multiplicity growth, both of which being well described by the parton shower approximation.

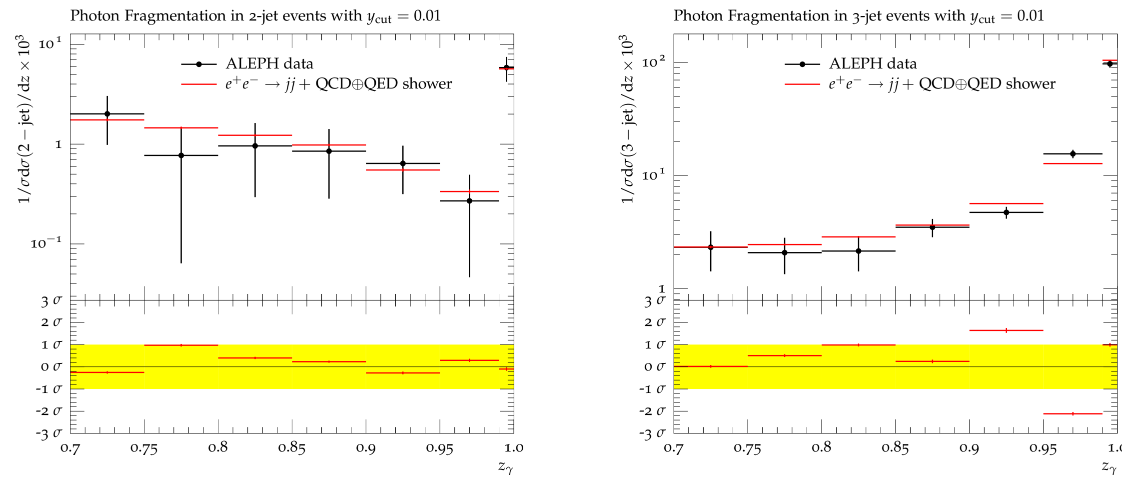


Figure 2: The $z_\gamma$ distribution as measured by ALEPH in hadronic $Z^0$ decays at LEP1 [37] and predicted by SHERPA's QCD+QED CSSHOWER evolution added to $e^-e^+ \to q\bar{q}$ hard scatterings.

Over the last decade the activities in the field of parton shower modelling have been seen to be intensified for several reasons; there was a push for designing new Monte Carlo programs for the LHC era resulting in a careful revision of existing programs.[2] There was also a strong demand to adjust parton showers to work well with input from (multi-leg and loop) higher-order matrix elements, and furthermore to interconnect them with models for multiple parton interactions and the underlying event. These efforts led to a number of refinements in shower algorithms and, moreover, the construction of new parton showers [14–29]. We want to illustrate this very briefly by presenting two selected results obtained from dipole-like shower schemes developed within the SHERPA collaboration.

## 2.1 Example – SHERPA's CSSHOWER

The CSSHOWER was derived from the dipole subtraction formalism used in next-to-leading order (NLO) calculations where CS stands for the names of the pioneers of this formalism, Catani and Seymour. To construct the shower algorithm, in particular its corresponding splitting functions, one exploits the dipole factorization of the real-emission matrix elements; the various CS dipole functions are translated into shower kernels by working in 4 dimensions, the large $N_C$ limit, and averaging over spins. This was originally described in [16] and worked out in detail, as well as implemented, in Refs. [20, 21]; furthermore, dipole showers were verified to reproduce the DGLAP equation [30–32]. The CSSHOWER entails nice properties such as its Lorentz-invariant formulation, on-shell splitting kinematics with rather local recoil compensation by spectator partons, exact/complete phase-space mapping of emissions and an inherent inclusion of soft colour coherence. Nevertheless, for the production of vector bosons in hadronic collisions, one (rather minor) shortcoming of the initially proposed NLO-like recoil strategy particularly was

[2] The next-generation programs PYTHIA8 [11] and HERWIG++ [12, 13] emerged from this initiative.

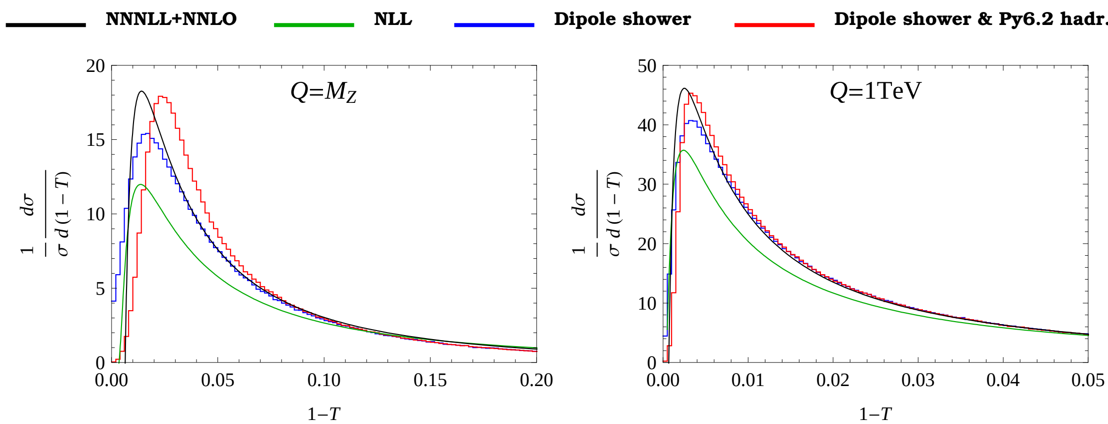


Figure 3: The all-particle one-minus-thrust event shapes in electron–positron annihilation at LEP1 and TeV energies. The comparison is between analytical results at N$^3$LL+NNLO and NLL accuracy (black and green curves), and numerical results obtained from SHERPA's colour dipole model neglecting/including hadronization corrections (blue/red histograms).

discussed in the literature [33]. Unlike in $b$-space exponentiation this recoil scheme does not generate the vector boson $p_T$ spectra continuously through each emission, but finally resolutions were put forward as in [34–36].

The CSSHOWER allows for the straightforward inclusion of QED effects; technically there is almost no difference between a $q \to q\gamma$ and $q \to qg$ splitting apart from the spectator concept (all oppositely charged particles in QED versus the colour-linked parton in large $N_\mathrm{C}$ QCD). The respective emission probabilities factorize trivially allowing a democratic treatment of photon and QCD parton radiation. This has been discussed in [35]. As an example we show in Fig. 2 results of a crucial benchmark for the combined QCD+QED CSSHOWER model, which is to reproduce the scale-dependent photon fragmentation function $D_\gamma(z_\gamma, y_\mathrm{cut})$ as measured by the ALEPH collaboration in hadronic $Z^0$ decays at LEP1 [37]. The events are classified by $n$-jet topologies and resolution measures $y_\mathrm{cut}$, and are required to have at least one reconstructed jet containing a photon with energy fraction $z_\gamma > 0.7$ and $E_\gamma > 5$ GeV. We observe a very nice agreement between simulation and data.

## 2.2 Example – SHERPA's dipole shower

While the CSSHOWER, incorporating $1 \to 2$ splittings, is said to be dipole-like owing to the spectator involvement in constructing the splitting kinematics, the currently unreleased shower model presented in [22] is based on exploiting the QCD property of antenna factorization in soft gluon emissions. This enables a complete $2 \to 3$ treatment of the splitting process employing $2 \to 3$ splitting functions and $2 \to 3$ kinematics. The original idea goes back to the pioneering work of Gustafson [38, 39] resulting in the release of the successful ARIADNE program [40].

Although, as described in Ref. [22], the goal of unifying initial- and final-state radiation into a single perturbative framework was greatly achieved, here we only want to recall a nice result obtained during verification of the (ARIADNE-like) final-state showering of SHERPA's colour dipole model. In Fig. 3 we display various predictions for the all-particle $1 - T$ distribution in $e^+e^-$ annihilation, where $T$ denotes the event-shape variable thrust. By comparing directly to theoretical results from analytic resummations at next-to-LL (NLL) level and beyond, we obtain a stringent and unambiguous test of the resummation as encoded in the dipole shower without the need for hadronization corrections.[3] The pure shower results turn out to be significantly different from the NLL predictions (green curves), they actually are closer to the $N^3$LL resummed results [41], which were calculated using soft-collinear effective theory and matched to NNLO predictions [42, 43]. This is a rather remarkable result for the dipole shower.

# 3 Higher precision for parton shower predictions

Traditionally it was PSs (parton showers) that were used to describe any additional jet activity, including the production of further hard jets. The shower "seeds" are given by QCD LO processes for a fixed final-state multiplicity. For these reasons, parton shower algorithms are said to describe multi-jet production at the LO+LL level. But there are a number of limitations to this description. Shower algorithms only represent the semi-classical picture of the entire branching process: quantum interferences and multi-parton correlations are hardly taken into account, and the whole evolution is only formulated in the limit of a large number of colours, $N_{\mathrm{C}}$. The application of the shower approximations outside the singular regions of QCD leads to uncontrolled behaviour and highly inaccurate predictions for rather energetic and/or large-angle radiation; shower uncertainties can therefore get large, and in general they are not easy to assess, which has the potential danger of partly compensating for missing perturbative effects via the tuning of non-perturbative parameters.

It was clear, to systematically correct for these deficiencies, the shower generators had to be improved by using more precise MEs (matrix elements). Motivated by the ground-breaking advances in efficiently calculating multi-leg MEs at tree and, more recently, even loop level, the theoretical effort in enhancing the accuracy of PSs has resulted in two new developments with significant impact on doing collider phenomenology (cf. e.g. [44]): tree-level matrix elements merged with parton showers (ME+PS), and NLO calculations interfaced (or matched) with parton showers (NLO+PS). The former primarily originated from the Catani–Krauss–Kuhn–Webber (CKKW) paper [45], with the innovative idea to correct the first few hardest shower emissions by using exact tree-level matrix-element expressions. A vast body of literature has appeared subsequently, advocating several variants, implementations and refinements to the original method (see Refs. [46, 47, 1, 48] for a review). Well-known variants include CKKW [45, 49, 50], Lönnblad-CKKW [51, 52], Mangano's MLM method [53, 54] and the versions of matrix-element and truncated-shower merging (ME&TS) [55, 56], all producing so-called *improved* LO+LL descriptions of multi-jet observables.

[3] Hadronization corrections are on the order of $1/Q$ (see Fig. 3), broaden all jets and shift the results towards smaller $T$ as seen by comparing the red and blue histograms.

The NLO+PS development was initiated by the MC@NLO papers [57, 58] and followed later by the POWHEG proposal [59, 60]. Both approaches aim at improving the event generation of a basic process in such a way that NLO accuracy is reached for inclusive observables, while maintaining the LL accuracy of the shower approach. Essentially, this is achieved by raising the order of precision of the underlying core process. In the context of multi-jet production, we then arrive at a description accurate at NLO+LL level (see Ref. [61, 48] for a very recent review). In both cases, ME+PS and NLO+PS, we have to solve two major problems simply because MEs and PSs can describe the same final state: the emission phase space has to be covered in a way that double counting of contributions is removed and dead regions are avoided at the same time.

The theoretical effort behind these two developments has led to enormous progress in the last decade regarding the systematic embedding of higher-order QCD corrections in multi-purpose Monte Carlo event generators. PYTHIA, HERWIG and SHERPA provide solutions (partly relying on interfaces to specialized tools) and implementations to make these developments available in experimental analyses and collider studies. Using the new tools, we have found better agreement to a broad range of QCD jet data taken at lepton and hadron colliders. We have gained better control over the systematic uncertainties of the generator predictions, and generally have been able to reduce these uncertainties. In the remainder of this contribution, we will quickly summarize the status of the ME+PS and NLO+PS techniques in SHERPA.

### 3.1 ME&TS in SHERPA

The ME&TS implementation in SHERPA is state-of-the-art. Predictions are obtained from merging tree-level matrix elements for $X$ plus $0, \ldots, n$-parton final states with the CSSHOWER, while preserving the LL accuracy to which soft and collinear multiple emissions are described by the CSSHOWER. The new ME&TS merging scheme was introduced in Ref. [55] and optimized as documented in Refs. [35, 36] to improve over the original SHERPA implementation based on the CKKW approach [45, 49, 62, 63]. ME&TS guarantees great compatibility between the $(Q)$ scales used to resolve the matrix-element final states and those $(t)$ scales ordering and driving the parton showering. In particular, truncated showering has been enabled to insert important soft emissions between resolved parton jet seeds. These shower emissions themselves do not give rise to jets but are necessary to retain the accuracy of the shower evolution, for example restore soft colour coherence. The very basic steps of the ME&TS algorithm are:

Separate phase space into a "hard" ME ($Q > Q_{\rm cut}$) and "soft" PS ($Q < Q_{\rm cut}$) domain according to a suitably chosen infrared-safe jet criterion. This factorizes the shower kernels similarly and regularizes the matrix elements. Via "inverted" showering one then finds the likely PS histories for the generated $n$-parton MEs. Based on the selected history one further evolves (using the $t$ scales) the ME final state beyond $n$ partons unless one encounters a shower emission above $Q_{\rm cut}$ resulting in the rejection of the event. This way one replaces the shower kernels in the ME domain by exact ME expressions for the hardest $n$ jets, and ensures that rejected events are to be described by $(n+1)$-parton MEs.

We exemplify the performance of SHERPA's ME&TS merging by showing differential jet rates

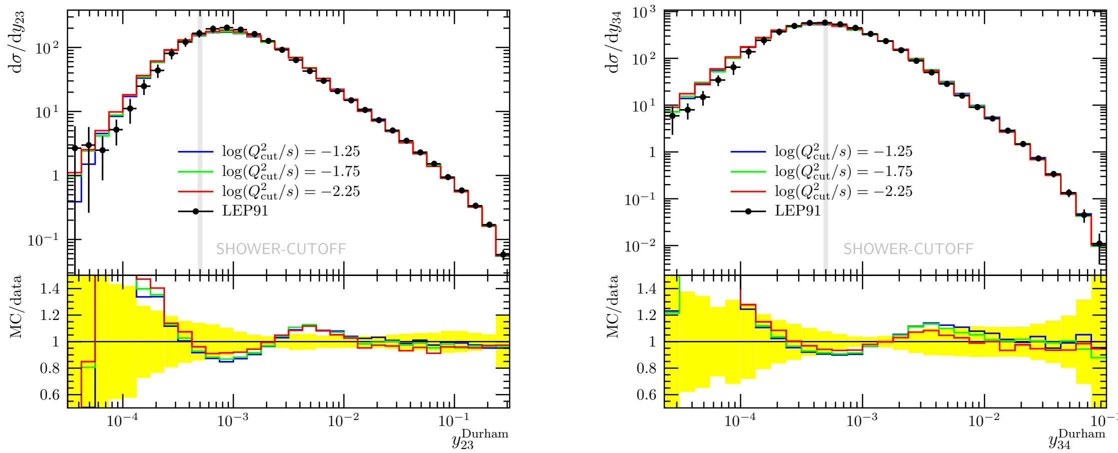


Figure 4: Differential Durham jet rates obtained from a Sherpa ME&TS sample where up to 6 jets are described by MEs. Results are shown for 3 different choices of the merging-cut parameter and compared to LEP1 data [64].

for $e^+e^-$ annihilation into hadrons. The ME&TS sample was generated by including matrix elements with up to four extra partons ($u, d, s, c, b$ quarks and gluons). The $y_{n\,n+1}$ distributions show at which rate, according to the Durham $k_T$ algorithm, $n+1$ jets are clustered into $n$ jets as a function of (the resolution parameter) $y_{n\,n+1} \approx Q^2_{n\,n+1}/s$. The observable is very sensitive to the jet emission pattern, therefore, lends itself eminently to assess the $Q_{\rm cut} = \sqrt{s\,y_{\rm cut}}$ parameter dependence of the ME&TS merging. Fig. 4 shows predictions for various $Q_{\rm cut}$, found to be in good agreement with data from LEP1 ($\sqrt{s} = 91.25$ GeV) [64]. Owing to the ME inclusion the high scales are well described, while it is very reassuring to see that the good shower behaviour is maintained at medium scales. The low scales below the (marked) shower cut-off are affected by hadronization effects and related parameter tuning (not optimized here). We therefore conclude that the merging systematics is well below the 10% level, which is a remarkable improvement over earlier merging variants.

### 3.2 Powheg and MEnloPS in Sherpa

The first results of a NLO+PS effort in Sherpa were published in Ref. [65]. The implementation has been based on the Powheg formalism, which can be understood as an advancement of earlier methods developed to correct the leading shower emission by the corresponding real-emission ME [66, 67]. This was done by invoking the Sudakov veto algorithm with an additional weight to be respected, schematically written as $w(\Phi_R) = R(\Phi_R)/R^{(\rm PS)}(\Phi_R)$ where $\Phi_R$ denotes the full real-emission phase space and $R$ ($R^{(\rm PS)}$) stands for the real-emission ME (shower) expression. The Powheg method reweights similarly, but at the same time accounts for a local $K$-factor implemented through a NLO event weight $\bar{B} = B + V + I + \int d\Phi_{R|B}(R - S)$ where $\Phi_{R|B}$ is the one-particle emission phase space. This way one generates not only observable shapes showing the Sudakov suppression and ME improvement at low and high scales, respectively, but also

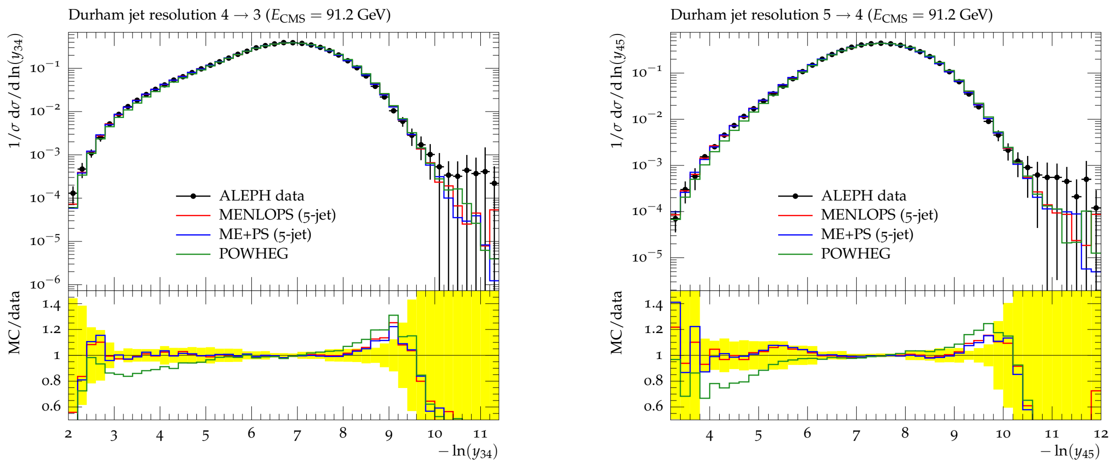


Figure 5: Differential Durham jet rates as predicted by SHERPA using three different matching schemes, POWHEG, ME&TS (up to 5 jets) and MENLOPS (up to 5 jets). The predictions are compared with LEP1 data measured by ALEPH in $e^+e^- \to$ hadrons [77].

NLO accuracy for the event sample, hence featuring a reduced scale dependence. The matching is smooth in a sense that no phase-space cut is needed as in (conventional) ME+PS methods.

SHERPA possesses almost all ingredients that make a POWHEG automation possible: automated tree-level ME generators provide the Born ($B$) and real-emission ($R$) terms [68, 69], the integrated and explicit subtraction terms ($I$ and $S$) are given by the automated implementation of the CS dipole subtraction formalism [70] and the virtual contributions ($V$) are obtained via interfacing to one-loop ME libraries as facilitated e.g. by BLACKHAT [71], GOSAM [72] or MCFM [73] using the Binoth Les Houches Accord [74]. Last but not least the CSSHOWER is well suited for combination with the ME computations; its $R^{(\mathrm{PS})}(\Phi_R)$ often closely approximate the $R(\Phi_R)$ resulting in a very reasonable distribution of the $w(\Phi_R)$ weights.

With the ME+PS facilities in SHERPA at hand, it suggests itself to aim at fusing the POWHEG and ME+PS approaches. This effort goes under the name MENLOPS, and its key idea is to slice the POWHEG phase space in ME+PS style into two domains, the NLO core process domain and the multi-jet domain. MENLOPS has been developed very recently by two groups as documented in Ref. [75] and Ref. [76]. The method exhibits what is cutting edge in combining higher-order calculations with PSs. To understand the slicing into domains, we schematically write down the expression for an observable $\langle O\rangle$ in the MENLOPS scheme:

$$\begin{aligned}\langle O\rangle \;=\; & \int d\Phi_B\, \bar{B}(\Phi_B)\Bigg[\underbrace{\Delta^{(\mathrm{ME})}(t_0,\mu^2)\,O(\Phi_B)}_{\text{unresolved}} + \sum_{ij,k}\frac{1}{16\pi^2}\int_{t_0}^{\mu^2} dt \int_{z_-}^{z_+} dz \int_0^{2\pi}\frac{d\phi}{2\pi}\,\frac{R_{ij,k}(\Phi_R)}{B(\Phi_B)}\,O(\Phi_R) \\ & \times\Big(\underbrace{\Delta^{(\mathrm{ME})}(t,\mu^2)\,\Theta(Q_{\mathrm{cut}}-Q_{ij,k})}_{\text{resolved, PS domain}} + \underbrace{\Delta^{(\mathrm{PS})}(t,\mu^2)\,\Theta(Q_{ij,k}-Q_{\mathrm{cut}})}_{\text{resolved, ME domain}}\Big)\Bigg] \qquad (2)\end{aligned}$$

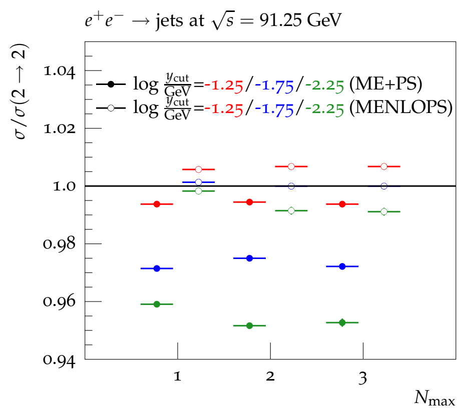


Figure 6: Parameter dependence of the total inclusive cross section as predicted by ME&TS and MEnloPS in Sherpa for $e^+e^- \to$ jets at LEP1. The cross sections are shown as a function of $N_{\max}$ for 3 different values of the merging cut. Note that the ME&TS and MEnloPS results are normalized to the LO and NLO cross sections, respectively. $N_{\max}$ denotes up to which multiplicity $(n+2)$-parton MEs are taken into account.

where $\Phi_B$, $\mu^2$ and $t_0$ denote the Born phase space, factorization/high scale and infrared cut-off, respectively. The one-particle emission phase space $\Phi_{R|B}$ is written explicitly, the sum is over the relevant CS dipoles, and the Powheg and CSshower Sudakov form factors (no-branching probabilities), $\Delta^{(\mathrm{ME})}$ and $\Delta^{(\mathrm{PS})}$, differ from each other by using the $R/B$ and CSshower kernels, respectively. The domains can be read off Eq. (2) pretty conveniently. The PS (or Powheg) domain is restricted to no resolved and soft emissions ($Q < Q_{\mathrm{cut}}$) preserving the NLO accuracy for inclusive observables. The hard (higher-order) emissions ($Q > Q_{\mathrm{cut}}$) are described by the ME (or ME+PS) domain guaranteeing the LO+LL accuracy of each resolved jet emission. Note that before fusing the contributions, the ME+PS part has to be multiplied by the $K$-factor $\bar{B}(\Phi_B)/B(\Phi_B)$, as shown in Eq. (2). In Sherpa this $K$-factor is applied locally, i.e. on an event-by-event basis.

MEnloPS hence inherits the good features of NLO+PS and ME+PS, which we demonstrate in Figs. 5 and 6.[4] The differential jet rates for $e^+e^- \to$ hadrons in Fig. 5 prove that the shapes of MEnloPS and ME&TS essentially are identical, and in very good agreement with the data [77] over the entire perturbative regime (which is to the left in these plots). In contrast the Powheg predictions fall short in describing the region of hard multiple jets. We display in Fig. 6 the parameter dependence of the MEnloPS and ME&TS total inclusive cross sections to show that NLO accuracy for the core process leads to a NLO-like correction and stabilization of the MEnloPS cross sections. In the Powheg case ($Q_{\mathrm{cut}} \to \infty$) we were to find that the term in the square bracket of Eq. (2) would integrate to one, much like as in the pure parton shower case. The phase-space slicing in ME+PS and in MEnloPS necessarily generates a mismatch in the non-logarithmic structure as given by the bracket term resulting in deviations from

[4] The NLO predictions shown in these plots were obtained by using virtual MEs provided by BlackHat [71].

the LO (ME+PS) and NLO (MEnloPS) cross sections.[5] As shown in Fig. 6, the parameter dependence of the MEnloPS cross section is smaller maintaining the NLO accuracy almost completely. This is where MEnloPS improves over ME+PS.

# 4 Conclusions and outlook

Parton showers have been continuously improved and modernized over the last years. The demand for improvement has come from measurements reaching (for) higher precision at current hadron and future linear colliders. The feasibility to aim at improvement came with the fantastic advances in efficiently computing multi-leg tree-level and one-loop amplitudes, including their integration over phase space. Two directions have been established for systematically enhancing the capabilities of parton showers:

1. Parton showers are improved by merging them with real-emission matrix elements for hard radiation (ME+PS). This is the new standard in the LHC era. Alpgen [78], MadGraph/Event [79, 80] and Sherpa are widely used. The new ME+PS scheme in Sherpa, ME&TS (available since versions 1.2), greatly helped reduce the systematic uncertainties of older Sherpa predictions. When compared to data, ME+PS predictions describe plenty of the measured shapes enabling the application of global $K$-factors that can be determined by higher-order calculations of the total inclusive cross section or the measurements themselves.

2. Parton showers are improved by matching them with NLO calculations (NLO+PS). Powheg [81–83] and MC@nlo [84–86] have a number of processes available.[6] For the latter, aMC@nlo [93, 94], the new, automated MC@nlo framework developed by Frixione et al. in principle allows for tackling arbitrary processes provided the necessary amount of computer resources is available. Sherpa's NLO+PS effort has been re-directed towards a MC@nlo-like strategy for many reasons; after gaining experience using a Powheg-like method [65, 76], it became clear that among other things a MC@nlo-like technique allows for much better control of the exponentiated terms, cf. [95, 96].

Both directions are very active fields of research, and MEnloPS actually emerged as a first successful attempt in fusing NLO calculations with tree-level higher-order matrix elements. While MC@nlo and Powheg give shower predictions of improved accuracy in the basic process, MEnloPS and ME+PS give improved multi-jet predictions. MEnloPS capabilities are enhanced over ME+PS regarding stability and accuracy of the total inclusive cross section.

The frontier has been pushed as documented by many recent publications [97–103, 96, 104]; for example, NLO+PS techniques were applied to calculate $W + 2$ and $W + 3$ jets. The former result was computed by aMC@nlo [103], while the latter result is a documentation of the

[5] The "unitarity violations" indicate the potential size of beyond NLO corrections; note that the pure Powheg phase-space slicing effect is shown for $N_{\max} = 1$.

[6] Similar/Alternative approaches have been presented by other groups, e.g. [87, 19, 88, 89, 25, 90–92].

remarkable capabilities of the MC@NLO implementation that has become available in SHERPA lately [96], provided efficient "one-loop engines" like BLACKHAT are interfaced as done in this $W + 3$-jet SHERPA computation.

The above examples clearly demonstrate that NLO+PS for multi-jet final states is no magic anymore, it is doable owing to the advances in NLO calculations in the multi-jet realm. This actually brings ME+PS@NLO within reach. First proposals have already appeared in the literature [105]. The naive combination in form of NLO Exclusive Sums discussed for $W + 0, \ldots, 4$ jets in [48] has been shown to work surprisingly well. To go towards ME+PS@NLO, it will be necessary to replace each naive Exclusive-Sums jet veto at the respective NLO accuracy by a jet veto at least accurate at $\mathcal{O}(\alpha_s^{m+1})$ where $m$ is the highest LO jet multiplicity (i.e. $m = 4$ in the above example).

No matter which of these methods is finally used for phenomenological studies, in all cases it is absolutely crucial to be able to provide reliable estimates of the theoretical uncertainties of the calculations. Comparisons between N(N)LO, NLO+PS, ME+PS, MENLOPS are mandatory to broaden our understanding regarding these issues [48].

# Acknowledgments

JW thanks the organizers for having created an excellent workshop experience. SH's work is supported by the US Department of Energy under contract DE-AC02-76SF00515, and in part by the US National Science Foundation, Grant NSF-PHY-0705682, (The LHC Theory Initiative). MS's work is supported by the Research Executive Agency (REA) of the European Union under the Grant Agreement number PITN-GA-2010-264564 (LHCPhenoNet). FS's is supported by the German Research Foundation (DFG) via Grant DI 784/2-1.

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