# Study of the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay at BABAR 

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#### Abstract

The Dalitz-plot of the decay $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ measured by the BABAR collaboration shows the structure of a final state having quantum numbers $I^{G} J^{P C}=0^{-} 0^{--}$. An isospin analysis of this Dalitz-plot finds that the fraction of the $I=0$ contribution is about $96 \%$. This high $I=0$ contribution is unexpected because the weak interaction violates the isospin.


## 1 Introduction

This communication reports the results of the analysis of the Cabibbo suppressed decay ${ }^{11}$

$$
\begin{equation*}
D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0} \tag{1}
\end{equation*}
$$

made by the BABAR collaboration.
The BABAR detector [1] measured the $e^{+} e^{-}$annihilations at the PEP-II collider. Most of the data were taken at the $\Upsilon(4 S)$ resonance to study the $B$ mesons. The detector measured also the decays of the charm mesons and baryons generated by the decay of the $B$ mesons or by the continuum, i.e. by the $e^{+} e^{-}$annihilations into $q \bar{q}$.

The charged tracks were measured by a silicon vertex tracker and by a drift chamber, and identified by a ring-imaging Cherenkov detector. The photons were measured by an elctromagnetic calorimeter made of $\mathrm{CSI}(\mathrm{Tl})$ crystals. The magnetic field of 1.5 T was generated by a superconducting solenoid. The iron of the flux return was instrumented by RPCs and LSTs for measuring the muons.

[^0]

Figure 1: The Dalitz-plot of the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay. $s_{+}$and $s_{-}$are respectively $m^{2}\left(\pi^{+} \pi^{0}\right)$ and $m^{2}\left(\pi^{-} \pi^{0}\right)$. The fine diagonal line at low $\pi^{+} \pi^{-}$mass corresponds to the events removed by the cut $489<M\left(\pi^{+} \pi^{-}\right)<508 \mathrm{MeV} / c^{2}$. The Dalitz-plot shows three diagonals with low density, indicating the dominance of the isospin zero.

## 2 Data collection

The data used in this analysis include $288 \mathrm{fb}^{-1}$ taken at the $\Upsilon(4 S)$ resonance and $27 \mathrm{fb}^{-1}$ collected below the resonance.

The cuts applied for selecting the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ candidates are [2]: (i) charged tracks with $p_{T}>100 \mathrm{MeV} / c$; (ii) particle identification compatibile with a charged pion; (iii) $E_{\gamma}>100 \mathrm{MeV}$; (iv) $115<M(\gamma \gamma)<150 \mathrm{MeV} / c^{2}$; (v) $E(\gamma \gamma)>350 \mathrm{MeV}$; (vi) $D^{0}$ vertex fit with $P\left(\chi^{2}\right)>0.5 \%$; (vii) $1848<M\left(D^{0}\right)<$ $1880 \mathrm{MeV} / c^{2}$; (viii) $D^{0}$ generated by the $D^{*+}$ decay and selected with the cut $\left|M\left(D^{*+}\right)-M\left(D^{0}\right)-145.4\right|<0.6 \mathrm{MeV} / c^{2}$; (ix) $D^{0}$ momentum in the $e^{+} e^{-}$ c.m.s. $p^{*}>2.77 \mathrm{MeV} / c ;(\mathrm{x})$ exclusion of the events with $489<M\left(\pi^{+} \pi^{-}\right)<508$ $\mathrm{MeV} / c^{2}$.

The Dalitz-plot of the events selected with these cuts is shown in Fig. 1. It was already published in Refs. 3, 4. It contains $44780 \pm 250 D^{0}$ decays and an estimated contamination of $830 \pm 250$ events. The contamination was evaluated
using the sideband $1930<M\left(\pi^{+} \pi^{-} \pi^{0}\right)<1990 \mathrm{MeV} / c^{2}$.
This Dalitz-plot shows three $\rho$ bands and has low density at the centre and on the three diagonals. This structure is typical of a $\pi^{+} \pi^{-} \pi^{0}$ final state with $I^{G} J^{P}=0^{-} 0^{-}[5$. The same properties were also visible in the CLEO analysis of the same decay [6].

## 3 Dalitz-plot analysis

The Dalitz plot density was fitted with the ansatz

$$
\begin{equation*}
D\left(s_{+}, s_{-}\right)=\mathcal{N}\left|a_{\mathrm{NR}} e^{i \phi_{\mathrm{NR}}}+\sum_{n} a_{n} e^{i \phi_{n}} A_{n}\left(s_{+} s_{-}\right)\right|^{2} \tag{2}
\end{equation*}
$$

where $s_{+}=m^{2}\left(\pi^{+} \pi^{0}\right), s_{-}=m^{2}\left(\pi^{-} \pi^{0}\right), \mathcal{N}$ is the normalization factor such that $\int D\left(s_{+}, s_{-}\right) d s_{+} d s_{-}=1$, and $A_{n}\left(s_{+}, s_{-}\right)$is the amplitude for the $n$.th channel.

The amplitude for the decay $D^{0} \rightarrow R_{n} \pi_{3}, R_{n} \rightarrow \pi_{1} \pi_{2}$ was calculated as

$$
\begin{equation*}
A_{n}\left(s_{+} s_{-}\right)=\frac{h_{n} S_{J}}{m_{n}^{2}-m_{12}^{2}-i m_{n} \Gamma_{n}\left(m_{12}\right)} \tag{3}
\end{equation*}
$$

where $h_{n}$ is a normalization factor evaluated such that $\int\left|A_{n}\left(s_{+} s_{-}\right)\right|^{2} d s_{+} d s_{-}=$ $1, m_{n}$ is the mass of the resonance $R_{n}, S_{J}$ is the spin factor for a resonance with spin $J$, and $\Gamma_{n}\left(m_{12}\right)$ is the variable width of the resonance.

The functions $S_{J}$ and $\Gamma_{n}\left(m_{12}\right)$ were written using the formulae written by the CLEO Collaboration in the analysis of the decay $D^{0} \rightarrow K^{-} \pi^{+} \pi^{0}$ [7]

The result of the fit is reported in Table I. It shows that the $D^{0}$ Dalitz-plot is dominated by the three $\rho(770) \pi$ channels, with a small contribution of the three $\rho(1700) \pi$ channels and a very small contribution of the other channels. Furthermore, the sum of the fractions is $147.4 \%$. This fact indicates that there is a strong negative interference between the 16 amplitudes of the channels used in the analysis.

## 4 Isospin decomposition

The first nine channels reported in Table I have the pions 1 and 2 in the isospin eigenstate $I_{12}=1$, the subsequent six channel $I_{12}=0$, and the last, i.e. the non resonant channel, is not an eigenstate of $I_{12}$. The Feynman diagrams of these channels are shown in Fig. 2. They can be grouped into four channels

| Channel | Amplitude $a_{n}$ | Phase $\phi_{n}\left(^{\circ}\right)$ | Fraction $f_{n}(\%)$ |
| :--- | :---: | :---: | :---: |
| $\rho(770)^{+} \pi^{-}$ | $0.823 \pm 0.000 \pm 0.004$ | 0 | $67.8 \pm 0.0 \pm 0.6$ |
| $\rho(770)^{0} \pi^{0}$ | $0.512 \pm 0.005 \pm 0.011$ | $16.2 \pm 0.6 \pm 0.4$ | $26.2 \pm 0.5 \pm 1.1$ |
| $\rho(770)^{-} \pi^{+}$ | $0.588 \pm 0.007 \pm 0.003$ | $-2.0 \pm 0.6 \pm 0.6$ | $34.6 \pm 0.8 \pm 0.3$ |
| $\rho(1450)^{+} \pi^{-}$ | $0.033 \pm 0.011 \pm 0.018$ | $-146 \pm 18 \pm 14$ | $0.11 \pm 0.07 \pm 0.12$ |
| $\rho(1450)^{0} \pi^{0}$ | $0.055 \pm 0.010 \pm 0.006$ | $10 \pm 8 \pm 13$ | $0.30 \pm 0.11 \pm 0.07$ |
| $\rho(1450)^{-} \pi^{+}$ | $0.134 \pm 0.008 \pm 0.004$ | $16 \pm 3 \pm 3$ | $1.79 \pm 0.22 \pm 0.12$ |
| $\rho(1700)^{+} \pi^{-}$ | $0.202 \pm 0.017 \pm 0.017$ | $-17 \pm 2 \pm 2$ | $4.1 \pm 0.7 \pm 0.7$ |
| $\rho(1700)^{0} \pi^{0}$ | $0.224 \pm 0.013 \pm 0.022$ | $-17 \pm 2 \pm 3$ | $5.0 \pm 0.6 \pm 1.0$ |
| $\rho(1700)^{-} \pi^{+}$ | $0.179 \pm 0.011 \pm 0.017$ | $-50 \pm 3 \pm 3$ | $3.2 \pm 0.4 \pm 0.6$ |
| $f_{0}(400) \pi^{0}$ | $0.091 \pm 0.006 \pm 0.006$ | $8 \pm 4 \pm 8$ | $0.82 \pm 0.10 \pm 0.10$ |
| $f_{0}(980) \pi^{0}$ | $0.050 \pm 0.004 \pm 0.004$ | $-59 \pm 5 \pm 4$ | $0.25 \pm 0.04 \pm 0.04$ |
| $f_{0}(1370) \pi^{0}$ | $0.061 \pm 0.009 \pm 0.007$ | $156 \pm 9 \pm 6$ | $0.37 \pm 0.11 \pm 0.09$ |
| $f_{0}(1500) \pi^{0}$ | $0.062 \pm 0.006 \pm 0.006$ | $12 \pm 9 \pm 4$ | $0.39 \pm 0.08 \pm 0.07$ |
| $f_{0}(1710) \pi^{0}$ | $0.056 \pm 0.006 \pm 0.007$ | $51 \pm 8 \pm 7$ | $0.71 \pm 0.07 \pm 0.08$ |
| $f_{2}(1270) \pi^{0}$ | $0.115 \pm 0.003 \pm 0.004$ | $-171 \pm 3 \pm 4$ | $1.32 \pm 0.08 \pm 0.10$ |
| nonresonant | $0.092 \pm 0.011 \pm 0.007$ | $-11 \pm 4 \pm 2$ | $0.84 \pm 0.21 \pm 0.12$ |

Table 1: Result of the fit of the $D^{0}$ Dalitz-plot showing the amplitude $a_{n}$, the phase $\phi_{n}$ and the fraction $f_{n}=a_{n}^{2}$. The mass and width of the $f_{0}(400)$ are 400 and $600 \mathrm{MeV} / c^{2}$. The masses and widths of the other mesons are taken by the 2006 issue of the Review of Particle Physics [8]. The phase of the $\rho(770)^{+} \pi^{-}$is fixed at $0^{\circ}$.

$$
\begin{align*}
\left|\{\rho\}^{+} \pi^{-}\right\rangle= & \left|\rho(770)^{+} \pi^{-}\right\rangle+\left|\rho(1450)^{+} \pi^{-}\right\rangle+\left|\rho(1700)^{+} \pi^{-}\right\rangle= \\
& \frac{1}{\sqrt{2}}(|+0-\rangle-|0+-\rangle),  \tag{4}\\
\left|\{\rho\}^{0} \pi^{0}\right\rangle= & \left|\rho(770)^{0} \pi^{0}\right\rangle+\left|\rho(1450)^{0} \pi^{0}\right\rangle+\left|\rho(1700)^{0} \pi^{0}\right\rangle= \\
& \frac{1}{\sqrt{2}}(|-+0\rangle-|+-0\rangle), \\
\left|\{\rho\}^{-} \pi^{+}\right\rangle= & \left|\rho(770)^{-} \pi^{+}\right\rangle+\left|\rho(1450)^{-} \pi^{+}\right\rangle+\left|\rho(1700)^{-} \pi^{+}\right\rangle= \\
& \frac{1}{\sqrt{2}}(|-0+\rangle-|0-+\rangle), \\
\left|\{f\} \pi^{0}\right\rangle= & \left|f_{0}(400) \pi^{0}\right\rangle+\left|f_{0}(980) \pi^{0}\right\rangle+\left|f_{0}(1370) \pi^{0}\right\rangle+ \\
& \left|f_{0}(1500) \pi^{0}\right\rangle+\left|f_{0}(1710) \pi^{0}\right\rangle+\left|f_{2}(1710) \pi^{0}\right\rangle= \\
& \frac{1}{\sqrt{2}}(|+-0\rangle+|-+0\rangle),
\end{align*}
$$

where $\left|q_{1} q_{2} q_{3}\right\rangle$ indicates the state $\left|\pi^{q_{1}}\right\rangle\left|\pi^{q_{2}}\right\rangle\left|\pi^{q_{3}}\right\rangle, q_{i}=+,-, 0$ being the charge of the $i$.th pion. The amplitudes of these four states are obtained by summing the amplitudes of the channels in the right side of (4).

We use the symbols $\left|I\left(I_{12}\right) ; I_{z}\right\rangle$ for the isospin eigenstates of the $3 \pi$ final


Figure 2: The Feynman diagrams for the first fitheen channels listed in Table I. (a) $\left|\{\rho\}^{+} \pi^{-}\right\rangle$. (b) $\left|\left\{\rho^{-}\right\} \pi^{+}\right\rangle$. (c) and (d) $\left|\left\{\rho^{0}\right\} \pi^{0}\right\rangle$ and $\left|\{f\} \pi^{0}\right\rangle$. $\{\rho\}$ indicates one of the three mesons $\rho(770), \rho(1450)$, and $\rho(1700)$. $\{f\}$ indicates one of the six mesons $f_{0}(400), f_{0}(980), f_{0}(1370), f_{0}(1500), f_{0}(1710)$, and $f_{2}(1270)$.
states. Here $I$ is the total isospin, $I_{12}$ is the isospin of interacting pion pair 12, and $I_{z}$ is the third component of the isospin. The formulae of these eigenstates with $I_{z}=0$ can be found in Eq. (3) of Ref. 4].

Using the relations (4), the three eigenstates with $I_{12}=1$ can be written such a way

$$
\begin{align*}
|2(1) ; 0\rangle & =\frac{1}{\sqrt{6}}\left(\left|\{\rho\}^{+} \pi^{-}\right\rangle-2\left|\{\rho\}^{0} \pi^{0}\right\rangle+\left|\{\rho\}^{-} \pi^{+}\right\rangle\right)  \tag{5}\\
|1(1) ; 0\rangle & =\frac{1}{\sqrt{2}}\left(\left|\{\rho\}^{+} \pi^{-}\right\rangle-\left|\{\rho\}^{-} \pi^{+}\right\rangle\right) \\
|0(1) ; 0\rangle & =\frac{1}{\sqrt{3}}\left(\left|\{\rho\}^{+} \pi^{-}\right\rangle+\left|\{\rho\}^{0} \pi^{0}\right\rangle+\left|\{\rho\}^{-} \pi^{+}\right\rangle\right)
\end{align*}
$$

The non resonant channel $|\mathrm{NR}\rangle$ can have two interpretations: (i) it corrects the $\left|\{f\} \pi^{0}\right\rangle$ amplitude that was not well parametrized; (ii) it describes a pointlike interaction that generates a uniform $\pi^{+} \pi^{-} \pi^{0}$ final state. In the case (i), the channel $|\mathrm{NR}\rangle$ has $I_{12}=0$. Therefore, the contribution of the isospin state $|1(0) ; 0\rangle$ to the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay is

$$
P_{\{+-0\}}|1(0) ; 0\rangle=\left|\{f\} \pi^{0}\right\rangle+|\mathrm{NR}\rangle
$$

$P_{\{+-0\}}$ being the projection operator of a isospin eigenfunction into the final state $\pi^{+} \pi^{-} \pi^{0}$.

| Isospin wave function | Amplitude | Phase $\left(^{\circ}\right.$ ) | Fraction (\%) |
| :--- | :---: | :---: | :---: |
| $\|2(1) ; 0\rangle$ | $0.1368 \pm 0.0016$ | $-42.5 \pm 0.7$ | $1.87 \pm 0.04$ |
| $\|1(2) ; 0\rangle$ | $0.0617 \pm 0.0022$ | $-8.9 \pm 2.6$ | $0.38 \pm 0.03$ |
| $\|1(1) ; 0\rangle$ | $0.0799 \pm 0.0023$ | $18.0 \pm 2.0$ | $0.64 \pm 0.04$ |
| $\|1(0) ; 0\rangle$ | $0.0936 \pm 0.0051$ | $14.5 \pm 2.4$ | $0.87 \pm 0.10$ |
| $\|0(1) ; 0\rangle$ | $0.9810 \pm 0.0006$ | 0 | $96.23 \pm 0.12$ |

Table 2: Amplitudes, phases and fractions of the five isospin channels contributing to the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay. The errors are only statistical. The phase of the state $|0(1) ; 0\rangle$ is fixed at $0^{\circ}$.

In the case (ii), the isospin wave functions of $|N R\rangle$ is symmetrical. There are two $3 \pi$ symmetrical isospin eigenstates. One is $|3(2) ; 0\rangle$, the other is the $I=1$ symmetric state

$$
\begin{aligned}
|1(S) ; 0\rangle \equiv & \frac{2}{3}|1(2) ; 0\rangle+\frac{\sqrt{5}}{3}|1(0) ; 0\rangle=\frac{1}{\sqrt{15}}(|+0-\rangle+|+-0\rangle+ \\
& |0+-\rangle+|0-+\rangle+|-+0\rangle+|-0+\rangle-3|000\rangle)
\end{aligned}
$$

The analysis carried out in Ref. 44 was based on the assumption that the isospin wave-function of the non resonant channels was $|1(S) ; 0\rangle$, because a state generated by a point-like four quark interaction cannot have $I=3$. This interpretation predicts

$$
\begin{aligned}
P_{\{+-0\}}|1(0) ; 0\rangle & =\left|\{f\} \pi^{0}\right\rangle+\frac{\sqrt{5}}{3}|\mathrm{NR}\rangle, \\
P_{\{+-0\}}|1(2) ; 0\rangle & =\frac{2}{3}|N R\rangle .
\end{aligned}
$$

The results are reported in Table II.
These results allow to estimate the branching ratio $\mathcal{B}\left(D^{0} \rightarrow 3 \pi^{0}\right)$. The branching ratio of the decay (11) is $\mathcal{B}\left(D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=(1.44 \pm 0.06) \%$ [9 and Eq. (3) of Ref. 4 tell us that the isospin eigenstates $|1(2) ; 0\rangle$ and $|1(0) ; 0\rangle$ decays into $3 \pi^{0}$ and $\pi^{+} \pi^{-} \pi^{0}$ respectively with the ratios $4: 11$ and 1:2. Therefore, from the fractions $f$ reported in the last coluumn of Table II, we obtain

$$
\begin{aligned}
\mathcal{B}\left(D^{0} \rightarrow 3 \pi^{0}\right)= & \mathcal{B}\left(D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)\left[\frac{4}{11} f_{|1(2) ; 0\rangle}+\frac{1}{2} f_{|1(0) ; 0\rangle}\right]= \\
& (8.3 \pm 0.8) \times 10^{-5}
\end{aligned}
$$

This estimate is in agreement with the measure of CLEO $\mathcal{B}\left(D^{0} \rightarrow 3 \pi^{0}\right)<$ $3.5 \times 10^{-4}$ [10.

## 5 Interpretation and predictions

A $\pi^{+} \pi^{-} \pi^{0}$ final state generated by a pseudoscalar meson decay has $J^{P}=0^{-}$, $G$-parity -1 , and charge conjugation $C=G(-1)^{I}$. The isospin states $|0(1) ; 0\rangle$
and $2(1) ; 0\rangle$ have $C P=+1$ and the other three states with $I=1$ have $C P=-1$. Therefore, the results shown in Table II tell us that the decay $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ proceeds for $(98.11 \pm 0.11) \%$ via the $C P=+1$ eigenstate

$$
D_{1}=\frac{1}{\sqrt{2}}\left(\left|D^{0}\right\rangle+\left|\bar{D}^{0}\right\rangle\right)
$$

The tree graphs shown in Fig. 2 do not predict the dominance of a pure isospin state. Then, if the $I=0$ dominance is not coincidental, there should be a physical explanation. A possible interpretation is that the $I=0$ dominance is due to a final state interaction with an $I^{G} J^{P C}=0^{-} 0^{--}$meson that resonates with the four quark generated by the $D^{0}$ decay. Such a meson must be exotic because a $q \bar{q}$ pair cannot have these quantum number. It could be a state $2 q 2 \bar{q}$.

If this interpretation is right, this meson should also have other decays. The pricipal candidates are the final states $2 \pi^{+} 2 \pi^{-} \pi^{0}$ and $\pi^{+} \pi^{-} 3 \pi^{0}$ because the analysis of the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ found a non negligible contributions of the channels $\rho(1450) \pi$ and $\rho(1700) \pi$, and the mesons $\rho(1450)$ and $\rho(1700)$ decay also in $4 \pi$. Furthermore, it is possible that this meson could also decay into $K \bar{K} \pi$.

## 6 Conclusions

We see a strange behaviour in the $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ Dalitz-plot: this decay is dominated by the $I=0$ final state [3, 4]. We take this as a hint of someting interesting that deserves further studies. We need to analyze the decays $D^{0} \rightarrow$ $K \bar{K} \pi$ and $D^{0} \rightarrow 5 \pi$ to understand if the $I=0$ dominance in $D^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ is coincidental or it is a general rule not predicted by the standard model.

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[^0]:    ${ }^{1}$ Charge conjugate decay modes are implicitely included.

