SLAC-PUB-14920

Electroweak radiative corrections to the parity-violating asymmetry for SLAC experiment E158

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Abstract

Electroweak radiative corrections to observable quantities of Møller scattering of polarized particles are calculated. We emphasize the contribution induced by infrared divergent parts of cross section. The covariant method is used to remove infrared divergences, so that our results do not involve any unphysical parameters. When applied to the kinematics of SLAC E158 experiment, these corrections reduce the parity violating asymmetry by about -6.5% at $E = 48$ GeV and y=0.5, and kinematically weighted "hard" bremsstrahlung effect for SLAC E158 is $\sim 1\%$.

1 Introduction

The SLAC experiment E158 [1] is aimed to measure the parity violating left-right polarization asymmetry A_{PV} in Møller scattering with a precision not reached before: error of measurements $\delta A_{PV}/A_{PV} \approx \pm 8\%$. E158 will determine $\sin^2(\theta_W)$ at momentum transfer $Q^2 \approx 0.02$ GeV^2 with uncertainty $\Delta \sin^2(\theta_W) = \pm 0.0008$ making it the most accurate determination of $\sin^2(\theta_W)$ at low energies. To this aim the 45-48 GeV polarized electron beam scattering off unpolarized electrons in a hydrogen target is used.

To extract the reliable data with high precision, it is necessary to consider higher order electroweak radiative corrections (EWRC). The EWRC to E158 experiment were estimated by Czarnecki and Marciano [2], Denner, Pozzorini [3] and Petriello [4]. We see at least two reasons for new calculation of the EWRC: 1) the problem of radiative corrections is of crucial importance and it is necessary to have various independent calculations, 2) in papers cited above a scheme of infrared singularity removal is used, so the result contains unphysical parameters.

In this paper a calculation of the lowest order EWRC is carried out using the on-shell renormalization scheme, Feynman gauge and the covariant approach in order to cancel explicitly the infrared divergences. We carry out the calculation at E158 energies (and for the situation when only one electron is detected), emphasizing on a contribution induced by infrared divergent parts of cross section and discuss the numerical estimation of corrections.

2 Born cross section of $e^- e^- \to e^- e^-$

The Born cross section of Møller scattering can be written as:

$$\frac{d\sigma^0}{dy} = \frac{2\pi\alpha^2}{s} \sum_{i,j=\gamma,Z} [\lambda_-^{ij}(u^2 D^{it} D^{jt} + t^2 D^{iu} D^{ju}) + \lambda_+^{ij} s^2 (D^{it} + D^{iu})(D^{jt} + D^{ju})], \tag{1}$$

where the four-momenta of the initial and final electrons $k_1,\ p_1$ and $k_2,\ p_2$ (see Fig.1) can be combined to form the Mandelstam invariants

$$s = (k_1 + p_1)^2,\ t = (k_1 - k_2)^2,\ u = (k_2 - p_1)^2. \tag{2}$$

The kinematic variable y is defined as

$$y = -\frac{t}{s} \approx \frac{1 - \cos\Theta}{2} \frac{E'}{E}, \tag{3}$$

where Θ is the center of mass scattering angle of the detected electron with momentum k_2. $E(E')$ is the energy of the initial (detected) electron, respectively. Whenever possible, we ignore the electron mass m (this cannot be done in the collinear singularity regions discussed below).

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Work supported in part by US Department of Energy contract DE-AC03-76SF00515.

The matrix elements in the Born cross section (1) are expressed through the photon and Z^0 propagators

$$D^{ik} = \frac{1}{k - m_i^2}\ (i = \gamma, Z). \tag{4}$$

When squaring matrix elements, we used the combinations of coupling constants and the polarizations of the beam and target electrons:

$$\lambda_{\pm}^{ij} = \lambda_1{}_B^{ij}\lambda_1{}_T^{ij} \pm \lambda_2{}_B^{ij}\lambda_2{}_T^{ij}, \tag{5}$$

$$\lambda_1{}_{B(T)}^{ij} = \lambda_V^{ij} - p_{B(T)}\lambda_A^{ij}, \lambda_2{}_{B(T)}^{ij} = \lambda_A^{ij} - p_{B(T)}\lambda_V^{ij}, \tag{6}$$

$$\lambda_V^{ij} = v^i v^j + a^i a^j,\ \lambda_A^{ij} = v^i a^j + a^i v^j, \tag{7}$$

where

$$v^\gamma = 1,\ a^\gamma = 0,\ v^Z = (I_e^3 + 2s_W^2)/(2s_W c_W),\ a^Z = I_e^3/(2s_W c_W), \tag{8}$$

$I_e^3 = -1/2$ and s_W (c_W) are sine (cosine) of the Weinberg angle.

3 One-loop electroweak radiative corrections

We apply the on-shell renormalization scheme of electroweak standard model to our calculation of the one-loop electroweak radiative corrections. The building blocks needed for explicit calculations according to this scheme have been worked out in the paper of Böhm et al. [5]. We use the results for gauge boson self-energies and vertex functions taken from [5].

3.1 Virtual corrections (V-contribution)

The virtual contributions to Møller scattering can be classified into three categories (see Fig.2): boson self-energies, vertex functions and boxes. In renormalization "on-mass shell" scheme there is no contribution from the self-energy of electrons. The total virtual cross section is the following sum:

$$\frac{d\sigma^V}{dy} = \frac{d\sigma^S}{dy} + \frac{d\sigma^{Ver}}{dy} + \frac{d\sigma^B}{dy}. \tag{9}$$

The self-energies of γ and Z-boson (including the photon vacuum polarization associated with light quarks) have been studied extensively (see [2, 3, 4] and references therein). Calculating the lepton vertices corrections we used the form factors $\delta F_{V,A}^{je}$ from [5] taken at $k^2 = t, u$. Substituting the coupling constants for the vertex form-factors (*e.g.* $v^\gamma \to \delta F_V^{\gamma e}$) in the expressions for the functions $\lambda_\pm$, we get the vertex part of the cross section

$$\frac{d\sigma^{Ver}}{dy} = \frac{4\pi\alpha^2}{s}\mathrm{Re}\sum_{i,j=\gamma,Z}[(\lambda_-^{F^i jij} + \lambda_-^{ijF^i j})(u^2 D^{it}D^{jt} + t^2 D^{iu}D^{ju}) + \\ +(\lambda_+^{F^i jij} + \lambda_+^{ijF^i j})s^2(D^{it} + D^{iu})(D^{jt} + D^{ju})]. \tag{10}$$

The box diagrams with at least one photon (*e.g.* forth and fifth diagrams in Fig.2) also contain infrared divergences. The diagrams with two Z or two W bosons are infrared-convergent. The IR-finite part of cross section looks like

$$\frac{d\sigma_{fin}^B}{dy} = -\frac{2\alpha^3}{s}\sum_{(ij)=1}^{4}\sum_{k=\gamma,Z} B_{(ij)}^k + \bigl(t \leftrightarrow u\bigr), \tag{11}$$

where double subscript (ij) runs $(ij) = \{1, 2, 3, 4\} = \{\gamma\gamma, \gamma Z, ZZ, WW\}$.

The terms B have the form

$$B_{(\gamma\gamma)}^k = D^{kt}\lambda_-^{\gamma k}\delta_{(\gamma\gamma)}^1 + (D^{kt} + D^{ku})\lambda_+^{\gamma k}\delta_{(\gamma\gamma)}^2,\quad B_{(\gamma Z)}^k = D^{kt}\lambda_-^{Zk}\delta_{(\gamma Z)}^1 + (D^{kt} + D^{ku})\lambda_+^{Zk}\delta_{(\gamma Z)}^2,$$

$$B_{(ZZ)}^k = D^{kt}\lambda_-^{Bk}\delta_{(ZZ)}^1 + (D^{kt} + D^{ku})\lambda_+^{Bk}\delta_{(ZZ)}^2,$$

$$B_{(WW)}^k = D^{kt}\lambda_-^{Ck}\delta_{(WW)}^1 + (D^{kt} + D^{ku})\lambda_+^{Ck}\delta_{(WW)}^2. \tag{12}$$

The coupling constants for two heavy bosons look like

$$v^B = (v^Z)^2 + (a^Z)^2, \; a^B = 2v^Z a^Z, \; v^C = a^C = 1/(4s_W^2). \tag{13}$$

The expressions $\delta^{1,2}_{(ij)}$ have the form (here we used the low energy approximation: $s, |t|, |u| \ll m_Z^2$):

$$\begin{aligned}
\delta^1_{(\gamma\gamma)} &= L_s^2(s^2+u^2)/(2t) - L_s u - (L_x^2+\pi^2)u^2/t, \\
\delta^2_{(\gamma\gamma)} &= L_s^2 s^2/t + L_x s - (L_x^2+\pi^2)(s^2+u^2)/(2t), \\
\delta^1_{(\gamma Z)} &= 8u^2(4I_{\gamma Z} - \hat{I}_{\gamma Z}), \; \delta^2_{(\gamma Z)} = 8s^2(I_{\gamma Z} - 4\hat{I}_{\gamma Z}), \\
\delta^1_{(ZZ)} &= 3u^2/(2m_Z^2), \; \delta^2_{(ZZ)} = -3s^2/(2m_Z^2), \\
\delta^1_{(WW)} &= 2u^2/m_W^2, \; \delta^2_{(WW)} = s^2/(2m_W^2);
\end{aligned} \tag{14}$$

logarithms from pure electromagnetic boxes are $L_s = \ln(s/|t|), \; L_x = \ln(u/t)$. The scalar integrals in γZ-part are

$$\begin{aligned}
I_{\gamma Z} &= \frac{1}{2\sqrt{-u}} \int_0^1 z dz \int_0^1 dx \frac{1}{\sqrt{\beta}} \ln \Big| \frac{xz\sqrt{-u} - \sqrt{\beta}}{xz\sqrt{-u} + \sqrt{\beta}} \Big|, \quad \hat{I}_{\gamma Z} = I_{\gamma Z}|_{u \to -s}, \\
&\beta = -ux^2z^2 + 4(1-z)(tz(x-1) + m_Z^2).
\end{aligned}$$

3.2 Extraction of infrared singularity from the one-loop virtual cross section

Thus, let us present the total virtual one-loop cross section as the sum of infrared (IR) divergent and IR-finite parts

$$\frac{d\sigma^V}{dy} = \frac{d\sigma^V_{IR}}{dy} + \frac{d\sigma^V}{dy}(\lambda^2 \to s), \tag{15}$$

where λ is infinitesimal photon mass.

For IR-part we find the expression which is proportional to Born cross section

$$\frac{d\sigma^V_{IR}}{dy} = -\frac{2\alpha}{\pi} \log \frac{s}{\lambda^2} (\log \frac{tu}{m^2 s} - 1) \frac{d\sigma^0}{dy}. \tag{16}$$

3.3 The photon bremsstrahlung $e^-e^- \to e^-e^-\gamma$

To complete the lowest order radiative corrections (and to get an infrared finite result) one needs to include the real bremsstrahlung diagrams (see Fig.3) (R-contribution).

The differential cross section for the process with the emission of one real photon reads

$$\frac{d\sigma^R}{dy} = -\frac{\alpha^3}{4s\pi} \int\limits_0^{v^{max}} dv \int \frac{d^3k}{k_0} \delta[(k_1+p_1-k_2-k)^2 - m^2] \sum_{j,i=1}^{4} M^R_{ij}(-1)^{i+j}. \tag{17}$$

For the calculation of squared matrix elements M^R_{ij}, where $i,j = (1,2,3,4) = (\gamma t, \gamma u, Zt, Zu)$, we used the standard Feynman rules.

As the kinematic variables of the bremsstrahlung process we use in this case

$$\begin{aligned}
z = 2kk_2, \; z_1 = 2kk_1 = z - t_1 + t, \; t_1 = (p_2 - p_1)^2, \\
v_1 = 2kp_1 = s + u + t_1 - 4m^2, \; v = 2kp_2 = s + u + t - 4m^2,
\end{aligned} \tag{18}$$

where k is a 4-momentum of the radiated photon.

The integration region for variable v is given by the Chew–Low diagram [6] (see Fig.4). The upper bound of v at $s \approx 0.05$ GeV^2 is denoted by solid line. We can observe the asymptotic behavior of the function in the regions around $v = s + t$ and $t = 0$. As the upper border $v = v^{max}$ corresponds to point $u = 0$ (collinear singularity), we must cut the region of integration up to the value, which corresponds to experimental set-up of E158 – the energy of detected particle in the lab. system $E^L \geq 13$ GeV. In this case $u^{max} = 2m(m - E^L)$, and $v^{max} = s + t + u^{max} - 4m^2$ (dotted line).

According to the covariant method of Bardin and Shumeiko [7] we can present the cross section of bremsstrahlung by splitting it into a soft infrared-divergent part and IR–finite contribution

$$\frac{d\sigma^R}{dy} = \frac{d\sigma^R_{IR}}{dy} + \frac{d\sigma^R_F}{dy}. \tag{19}$$

The infrared divergent part (the first term) of the expression (19) after integrations over k and v and λ-parametrization reads

$$\frac{d\sigma^R_{IR}}{dy} = \frac{2\alpha}{\pi}(\ln\frac{(v^{max})^2}{4m^2\lambda^2}(\ln\frac{tu}{m^2 s} - 1) + \delta^S + \delta^H_1)\frac{d\sigma^0}{dy}. \tag{20}$$

The corrections δ^S and δ^H_1 can be found in [7]:

$$\delta^S_1 = \ln\frac{s(s+t)}{m^4} - \frac{1}{2}l_m \ln\frac{s^2(s+t)^2}{-tm^6} - \frac{1}{2}l_r^2 - 2l_r l_m + l_m - l_m^2 - \frac{\pi^3}{3} + 1, \tag{21}$$

$$\begin{aligned}
\delta^H_1 \quad = \quad & \int\limits_0^{v^{max}} dv\Big(\frac{2}{v}\Big[\ln(1-\frac{v}{t}) - \ln(1-\frac{v}{s}) + \ln(1-\frac{v}{s+t}) - \frac{1}{2}\ln(1+\frac{v}{m^2})\Big] + \\
+ \quad & \frac{2}{s+t-v}\ln\frac{s+t-v}{m^2} - \frac{1}{s-v}\ln\frac{(s-v)^2}{m^2\tau} - \frac{1}{v-t}\ln\frac{(v-t)^2}{m^2\tau} - \frac{1}{\tau}\Big), \\
& l_m = \ln\frac{-t}{m^2},\ l_r = \ln\frac{s+t}{s}, \tau = v + m^2.
\end{aligned} \tag{22}$$

The second term of (19) is the IR-finite part of bremsstrahlung. To calculate this part it is necessary to integrate analytically over k-4-momenta of photon and then numerically over v. The integral over whole phase space of the radiated photon can be presented in the form

$$I[A] = \frac{1}{\pi}\int\frac{d^3k}{k_0}\delta[(k_1 + p_1 - k_2 - k)^2 - m^2][A] = \frac{1}{\pi}\int\limits_{t_1^{min}}^{t_1^{max}} dt_1 \int\limits_{z^{min}}^{z^{max}}\frac{dz}{\sqrt{R_z}}[A], \tag{23}$$

where R_z is proportional to the Gram determinant. The limits of the double integration $z^{min/max}$ and $t_1^{min/max}$ are the roots of equations $R_z = 0$ and $z^{min} = z^{max}$, respectively.

During the calculation of "hard" part of squared matrix elements M^R_{ij} we calculated more then 100 scalar integrals. Exact results for this part are rather cumbersome and we do not present them here. The total list of expression M^R_{ij} as a set of output REDUCE files and scalar integrals as subroutine-functions can be found in text of FORTRAN code RCORR2A1 ("Radiative CORRections TO asymmetry A1").

3.4 The result of infrared singularity cancellation

Adding the IR-parts of V- and R- contributions (formulas (16) and (20))

$$\frac{d\sigma^C}{dy} = \frac{d\sigma^V_{IR}}{dy} + \frac{d\sigma^R_{IR}}{dy} = \frac{\alpha}{\pi}(4\ln\frac{v^{max}}{m\sqrt{s}}(\ln\frac{tu}{m^2 s} - 1) + \delta^S_1 + \delta^H_1)\frac{d\sigma^0}{dy}, \tag{24}$$

we obtain the final finite expression which is free of infrared divergences and unphysical parameters.

4 Numerical estimates

Numerical calculations of the electroweak radiative corrections to the asymmetry A_{PV} in Møller scattering at the energy of longitudinally polarized electron beam of SLAC experiment E158 were performed using the FORTRAN code RCORR2A1 [8]. The structure of code allows us to estimate the corrections for arbitrary experimental conditions and successfully apply them to the E158 Monte Carlo simulation [9].

The asymmetry corresponds to the usual expression

$$A_{PV} = \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}}, \quad \sigma \equiv d\sigma/dy. \tag{25}$$

So we are interested in the following basic contributions: 1) infrared-finite parts: self-energy of gauge bosons (for this part authors of [2, 3, 4] have found the correction to the asymmetry of $\sim -50\%$), heavy vertices (give rather small contribution to asymmetry), heavy boxes (ZZ and WW) (give $+3\%$ to asymmetry); 2) infrared-divergent parts (IR parts): the rest part of virtual 1-loop contributions, which consist of electron vertices with photon , γ-γ– and γ-Z–boxes, infrared singularity cancellation (this log-term is proportional to Born cross section and does not change the asymmetry), and "hard" bremsstrahlung.

The correction to the asymmetry is defined as follows

$$\delta A_{PV} = \frac{A_{PV}^{RC} - A_{PV}^{0}}{A_{PV}^{0}}, \tag{26}$$

where A_{PV}^0 is the Born asymmetry, and A_{PV}^{RC} is the asymmetry taking into consideration the electroweak radiative corrections.

The influence of the electroweak radiative corrections to the asymmetry is shown in Fig.5. One can observe that for $E = 48$ GeV at $y = 0.5$ the correction δA_{PV} amounts to $\sim -6.5\%$, it is minimal at moderate y, and this minimum shifts to small y with the increasing energy. At last

$$-6.5\% = +1\% \text{ ("hard" brems.)} + (-7.5\%) \text{ (the rest part)}.$$

Kinematically weighted "hard" initial and final state radiation effect for observable A_{PV} under condition of E158 is

$$\mathcal{F}_b = 1.01 \pm 0.01$$

(notation of [9]), it is our contribution to radiative correction procedure for SLAC E158.

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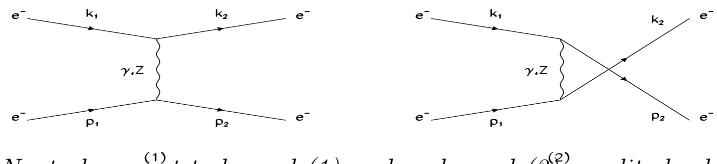

Figure 1: *Neutral current t-channel (1) and u-channel (2) amplitudes leading to the asymmetry* A_{PV} *at tree level.*

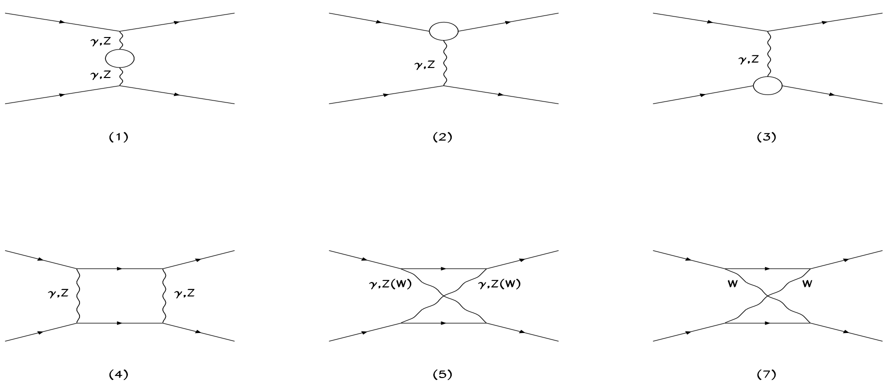

Figure 2: *The virtual t-channel one-loop diagrams for* $e^-e^- \to e^-e^-$ *process. The contributions to the self-energies and vertex corrections are symbolized by the empty loops.*

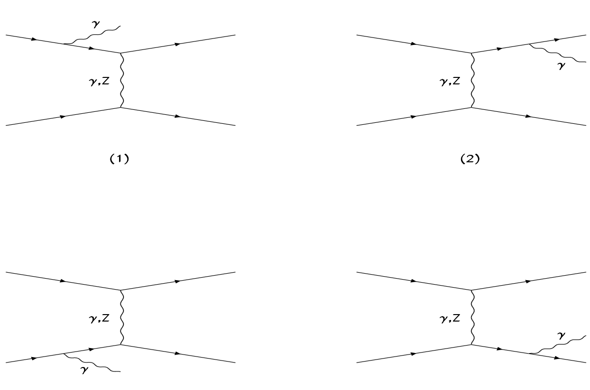

Figure 3: *Bremsstrahlung t-channel diagrams for* $e^-e^- \to e^-e^-\gamma$ *process.*

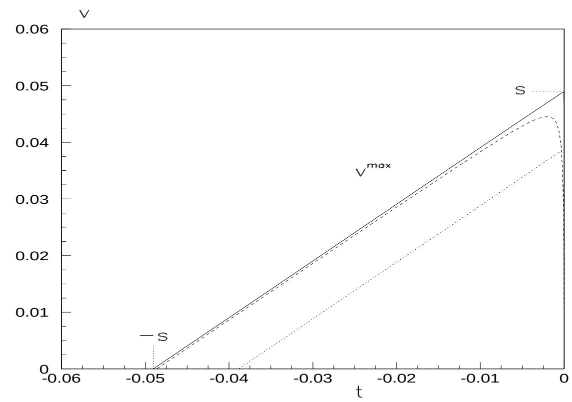

Figure 4: *Chew-Low diagram at $s \approx 0.05\ GeV^2$. Mass of electron is real (solid line) and had been raised in 20 times for illustration (dashed line). Dotted line corresponds to v^{max} for experimental conditions of E158.*

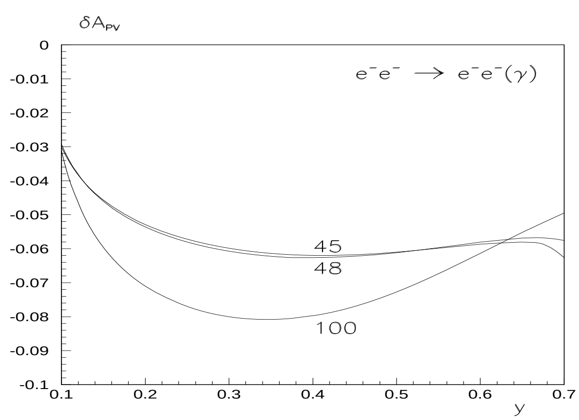

Figure 5: *Corrections to polarization asymmetry A_{PV} as a functions of y at different energies which are denoted by numbers on curves.*