

Combined fit to BaBar and Belle Data on $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $\phi f_0(980)$

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Abstract

A combined fit is performed to the BaBar and Belle measurements of the $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $\phi f_0(980)$ cross sections for center-of-mass energy between threshold and 3.0 GeV. The resonance parameters of the $\phi(1680)$ and $Y(2175)$ are determined. The mass is (1681_{-12}^{+10}) MeV/ c^2 and the width is (221_{-24}^{+34}) MeV/ c^2 for the $\phi(1680)$, and the mass is (2117_{-49}^{+59}) MeV/ c^2 and the width is (164_{-80}^{+69}) MeV/ c^2 for the $Y(2175)$. These information will shed light on the understanding of the nature of the excited ϕ and Y states observed in e^+e^- annihilation.

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I. INTRODUCTION

Although vector mesons are produced copiously in e^+e^- annihilation, the resonance parameters of them are not well measured [1]. In the $s\bar{s}$ sector, in a study of initial-state radiation (ISR) events of the type, $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi\pi^+\pi^-$, the BaBar and Belle Collaborations observed two clear structures near $\sqrt{s} = 1.68$ GeV and 2.175 GeV, the latter was produced dominantly via a $\phi f_0(980)$ intermediate state, and was dubbed the $Y(2175)$ [2, 3]. The $\phi(1680)$ was measured by the Belle Collaboration with its mass and width to be $1689 \pm 7 \pm 10$ MeV/ c^2 and $211 \pm 14 \pm 19$ MeV/ c^2 , respectively [3]. For the $Y(2175)$, its mass and width measured by the BaBar Collaboration are $2175 \pm 10 \pm 15$ MeV/ c^2 and $58 \pm 16 \pm 20$ MeV/ c^2 , respectively, by fitting the $\phi f_0(980)$ cross section; while with more luminosity, the Belle experiment found that its mass and width are 2163 ± 32 MeV/ c^2 and 125 ± 40 MeV/ c^2 , respectively. Examining closely the fitting to the $\phi f_0(980)$ cross section, we found that the fitted $Y(2175)$ resonance parameters are sensitive to the background shape. So it is necessary to give a more sophisticated estimation of the resonance parameters of the $Y(2175)$ for a better understanding of its nature. This goal can be reached by performing a combined fit to the $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $e^+e^- \rightarrow \phi f_0(980)$ cross sections measured by the BaBar and Belle experiments since all the data are available via Durham database [2, 3].

Since the $Y(2175)$ resonance is produced via ISR in e^+e^- collisions, its $J^{PC} = 1^{--}$. The $Y(2175)$ was firstly suspected to be an s -quark partner of the $Y(4260)$ [4, 5] since both are produced in e^+e^- annihilation and exhibit similar decay patterns. On the other hand, a number of different interpretations have been proposed, which include: an $s\bar{s}g$ hybrid [6]; a 2^3D_1 $s\bar{s}$ state [7] with a width predicted to be in the range 120-210 MeV/ c^2 ; a tetraquark state [8, 9, 10]; a $\Lambda\bar{\Lambda}$ bound state [11]; a conventional excited ϕ meson [12]; or a structure produced by final state interactions [13, 14]. The possibility that the $Y(2175)$ is a 3^3S_1 $s\bar{s}$ state is disfavored by the rather large predicted width ($\Gamma \sim 380$ MeV/ c^2) [15]. A review [16] discusses the basic problem of the large expected decay widths into two mesons, which are in contrast to experimental observations.

In this paper, we try to perform all the possible fits to the $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $e^+e^- \rightarrow \phi f_0(980)$ cross sections measured by the BaBar and Belle experiments to obtain a better knowledge of the resonance parameters of the $\phi(1680)$ and $Y(2175)$. We also study the possible production of the structure at around 2.4 GeV/ c^2 in both $\phi\pi^+\pi^-$ and $\phi f_0(980)$ modes.

II. THE DATA

Both the BaBar and the Belle experiments reported cross sections of $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $e^+e^- \rightarrow \phi f_0(980)$ for center-of-mass energy ranges from threshold to about 3.0 GeV [2, 3]. The integrated luminosity of the BaBar data sample is 232 fb $^{-1}$ while that of the Belle data sample is 673 fb $^{-1}$, with $\sim 90\%$ of the data collected at the $\Upsilon(4S)$ resonance ($\sqrt{s} = 10.58$ GeV), while the rest were taken off the $\Upsilon(4S)$ peak.

Figure 1 shows the data, good agreement between BaBar and Belle results is observed within errors, and the two structures (the $\phi(1680)$ and $Y(2175)$) are evident in $e^+e^- \rightarrow \phi\pi^+\pi^-$ mode, and one structure (the $Y(2175)$) is evident in $e^+e^- \rightarrow \phi f_0(980)$ mode. Also we notice that there are clusters of events near 2.4 GeV/ c^2 in both modes.

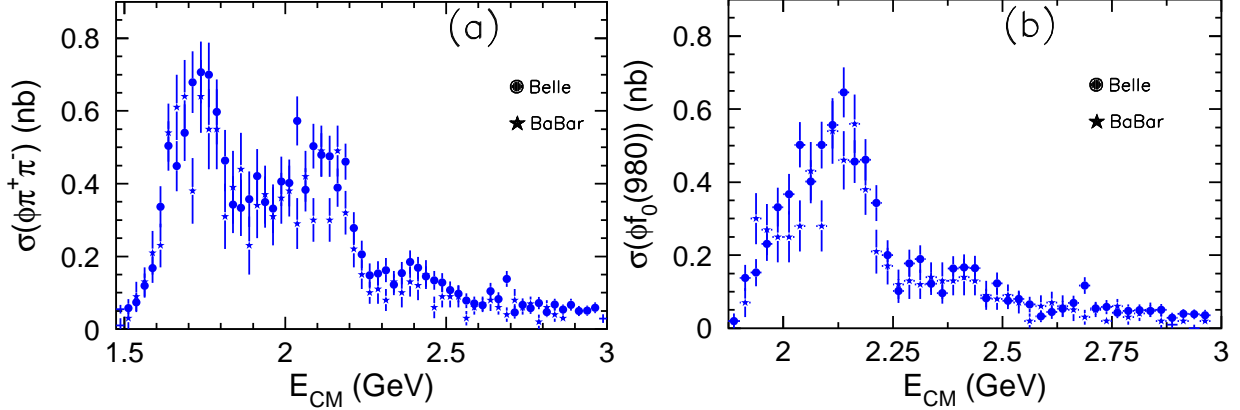


FIG. 1: The cross sections of $e^+e^- \rightarrow \phi\pi^+\pi^-$ (a) and $\phi f_0(980)$ (b) measured at BaBar (stars with error bars) and Belle (dots with error bars).

III. THE FIT TO $e^+e^- \rightarrow \phi\pi^+\pi^-$ CROSS SECTION

In order to obtain the resonance parameters of the $\phi(1680)$ and $Y(2175)$, we fit the $\phi\pi^+\pi^-$ cross section distribution using a least square method with MINUIT in the CERN Program Library [17]. The Breit-Wigner (BW) form of a signal resonance used in $e^+e^- \rightarrow \phi\pi^+\pi^-$ analysis is

$$BW(\sqrt{s}) = \sqrt{\frac{M^2}{s}} \frac{\sqrt{12\pi\Gamma_{e^+e^-}\mathcal{B}(R \rightarrow f)\Gamma_{\text{tot}}}}{s - M^2 + iM\Gamma_{\text{tot}}} \sqrt{\frac{PS(\sqrt{s})}{PS(M)}}$$

where M is the mass of the resonance, Γ_{tot} and $\Gamma_{e^+e^-}$ are the total width and partial width to e^+e^- respectively, $\mathcal{B}(R \rightarrow f)$ is the branching fraction of R decays into final state f , and $PS(\sqrt{s})$ is the three-body decay phase space factor. We fit the Belle data on $\phi\pi^+\pi^-$ between threshold and 3.0 GeV (61 data points) and the BaBar data in the similar energy range (56 data points) simultaneously.

Since the $\phi(1680)$ decays into $\phi\pi^+\pi^-$ while the $Y(2175)$ decays dominantly into $\phi f_0(980)$, we use two incoherent BW functions in the fit first, one for the $\phi(1680)$ and the other for the $Y(2175)$. The fit result is shown in Fig. 2(a), with a goodness-of-the-fit of $\chi^2/ndf = 170/111$, corresponding to a confidence level (C.L.) of 0.03%. The statistical significance of each resonance is greater than 10σ . From the fit we obtain the following resonance parameters of the $\phi(1680)$: $M = (1685 \pm 5) \text{ MeV}/c^2$, $\Gamma_{\text{tot}} = (208 \pm 11) \text{ MeV}/c^2$ and $\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-} = (24.6 \pm 1.2) \text{ eV}/c^2$, while those of the $Y(2175)$ are $M = (2080 \pm 12) \text{ MeV}/c^2$, $\Gamma_{\text{tot}} = (182 \pm 20) \text{ MeV}/c^2$ and $\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-} = (18.1 \pm 1.8) \text{ eV}/c^2$, where the errors are statistical only. A fit with an additional nonresonant component does not improve the fit quality, and the contribution of the nonresonant term is negligibly small and can be neglected.

Since there are some events accumulating at about 2.4 GeV, we also perform a fit with an additional incoherent BW function for this structure. The fitted parameters of this structure are $M = (2419 \pm 25) \text{ MeV}/c^2$, $\Gamma_{\text{tot}} = (49 \pm 42) \text{ MeV}/c^2$ and $\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-} = (1.06 \pm 0.58) \text{ eV}/c^2$ with a goodness-of-the-fit of $\chi^2/ndf = 163/108$, corresponding to a C.L. of 0.05%. The statistical significance of the structure is 1.8σ as determined from the change in the χ^2 value compared to the two-incoherent-resonance fit and the difference in the number of degrees of freedom. The fitted values of the $\phi(1680)$ and $Y(2175)$ resonance parameters

only change a little compared to the two-incoherent-resonance fit, and the difference is quoted as one source of systematic errors. The fit result is shown in Fig. 2(b).

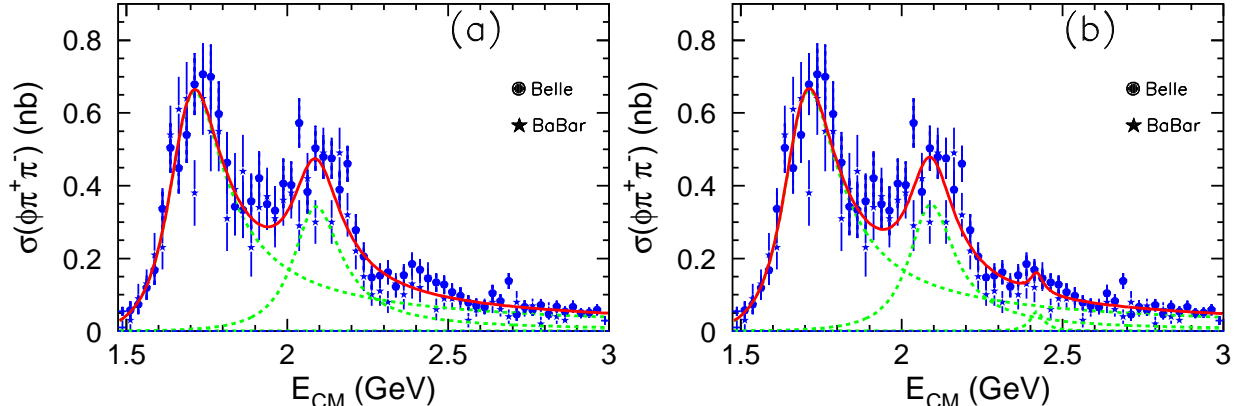


FIG. 2: Fit to $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections with two incoherent BW functions (a), and with three incoherent BW functions (b), respectively. The curves show the projections from the best fit and the contribution from each component.

We also fit with two coherent BW functions. Figure 3 shows the fit results. There are two solutions with equally good fit quality, with a goodness-of-the-fit of $\chi^2/ndf = 158/110$, corresponding to a C.L. of 0.19%. The masses and the widths of the two resonances are identical but the partial widths to e^+e^- and relative phases are different in these two solutions, as shown in Table I. The resonance parameters of the $\phi(1680)$ are $M = (1677 \pm 6)$ MeV/ c^2 and $\Gamma_{\text{tot}} = (233 \pm 16)$ MeV/ c^2 , and those of the $Y(2175)$ are $M = (2112 \pm 16)$ MeV/ c^2 and $\Gamma_{\text{tot}} = (196 \pm 27)$ MeV/ c^2 .

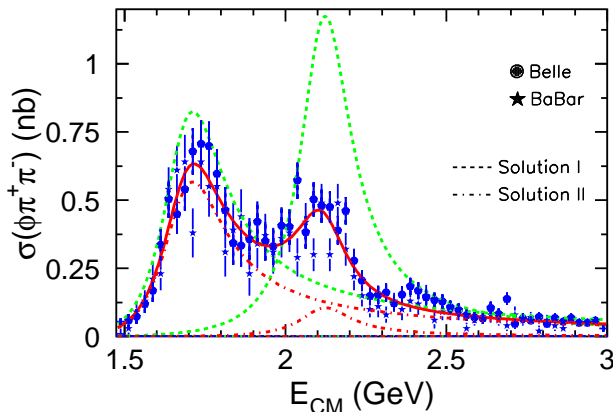


FIG. 3: The results of the fit to $e^+e^- \rightarrow \phi\pi^+\pi^-$ data from Belle and BaBar. The curve show the the projection from the best fit with two coherent BW s. The interference between the two amplitudes is not shown. The two dashed curves at each peak show the two solutions (see text), one is for the destructive solution (Solution I), the other is for the constructive solution (Solution II).

Similarly we also try to perform a fit with an additional coherent BW function at around 2.4 GeV/ c^2 . The result is shown in Fig. 4. There are four solutions. The values of the fitted results for these four solutions are shown in Table II, with a goodness-of-the-fit of

TABLE I: Fit results to the combined BaBar and Belle data on $e^+e^- \rightarrow \phi\pi^+\pi^-$ with two coherent BW functions. The errors are statistical only. M , Γ_{tot} , and $\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}$ are the mass (in MeV/c^2), total width (in MeV/c^2), and product of the branching fraction to $\phi\pi^+\pi^-$ and the e^+e^- partial width (in eV/c^2), respectively. ϕ is the relative phase (in degree).

Parameters	Solution I	Solution II
$M(\phi(1680))$	1677 ± 6	
$\Gamma_{\text{tot}}(\phi(1680))$	233 ± 16	
$\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(\phi(1680))$	33.4 ± 1.3	23.0 ± 1.5
$M(Y(2175))$	2112 ± 16	
$\Gamma_{\text{tot}}(Y(2175))$	196 ± 27	
$\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(Y(2175))$	68.9 ± 7.0	6.2 ± 1.1
ϕ	-122 ± 2	90 ± 8

$\chi^2/ndf = 143/106$. The fitted values of the masses and widths of the $\phi(1680)$ and $Y(2175)$ are consistent with the above results within 2σ , and the statistical significance of the structure at $2.4 \text{ GeV}/c^2$ is estimated to be 2.8σ as determined from the change in the χ^2 value compared to the two-coherent-resonance fit and the difference in the number of degrees of freedom.

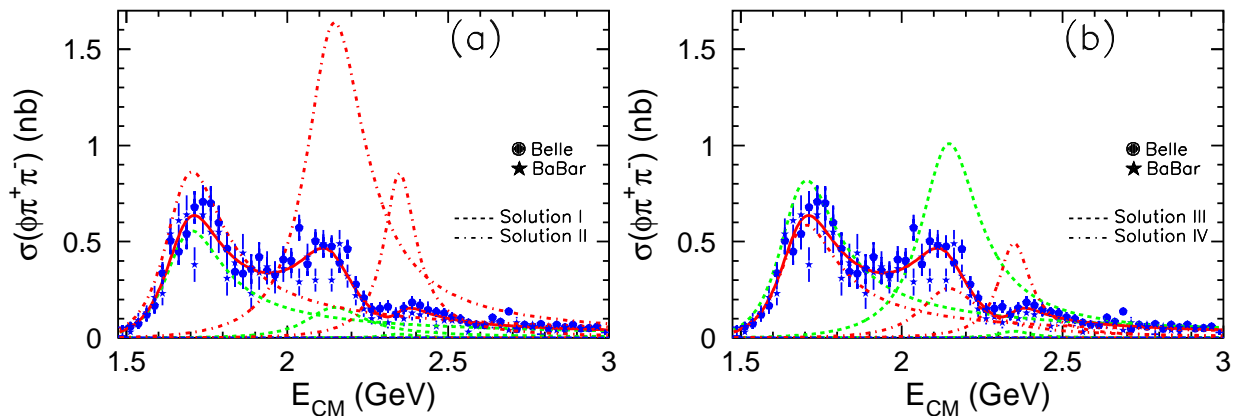


FIG. 4: The results of the fit to $e^+e^- \rightarrow \phi\pi^+\pi^-$ data from Belle and BaBar with three coherent BW functions. The curves show the projections from the best fit with three coherent BW s. There are four solutions.

IV. THE FIT TO $e^+e^- \rightarrow \phi f_0(980)$ CROSS SECTION

We fit the Belle data on $\phi f_0(980)$ between threshold and 3.0 GeV (44 data points) and the BaBar data in the same energy range (44 data points) simultaneously. For the nonresonant component amplitude, we use the same formula used by the BaBar Collaboration [2] as the following:

$$\sigma_{nr} = P(\mu) \times |A_{nr}(\mu)|^2,$$

TABLE II: Fit results to the combined BaBar and Belle data on $e^+e^- \rightarrow \phi\pi^+\pi^-$ with three BW functions that interfere with each other. The errors are statistical only. M , Γ_{tot} , and $\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}$ are the mass (in MeV/c^2), total width (in MeV/c^2), and product of the branching fraction to $\phi\pi^+\pi^-$ and the e^+e^- partial width (in eV/c^2), respectively. ϕ is the relative phase (in degree).

Parameters	Solution I	Solution II	Solution III	Solution IV
$M(\phi(1680))$		1673 ± 6		
$\Gamma_{\text{tot}}(\phi(1680))$		218 ± 13		
$\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(\phi(1680))$	21.2 ± 1.1	32.9 ± 1.5	31.2 ± 1.5	22.3 ± 1.1
$M(Y(2175))$		2133 ± 24		
$\Gamma_{\text{tot}}(Y(2175))$		221 ± 38		
$\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(Y(2175))$	10.7 ± 1.8	110.4 ± 16.0	68.0 ± 5.5	17.3 ± 4.6
ϕ_1	110 ± 5	-111 ± 5	-126 ± 3	125 ± 9
$M(X(2400))$		2346 ± 26		
$\Gamma_{\text{tot}}(X(2400))$		121 ± 35		
$\mathcal{B}(\phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(X(2400))$	0.90 ± 0.37	38.6 ± 15.7	1.6 ± 0.8	22.3 ± 5.9
ϕ_2	28 ± 23	159 ± 5	-127 ± 28	-46 ± 6

$$A_{nr}(\mu) = \sqrt{\sigma_0} \times (1 - e^{-(\mu/a_1)^4}) \times (1 + a_2\mu + a_3\mu^2),$$

$$P(\mu) = \sqrt{1 - m_0/(m_0 + \mu)^2},$$

$$\mu = \sqrt{s} - m_0,$$

where the a_i are free parameters, $P(\mu)$ is an approximation of the two-body phase space for particles with similar masses. Both the $\phi(1020)$ and $f_0(980)$ have small but finite widths, and the selection criterion of $m_{\pi^+\pi^-} > 0.85 \text{ GeV}/c^2$ defines an effective minimum mass, $m_0 = 1.8 \text{ GeV}/c^2$. For the signal resonance, we take the same form as described in $\phi\pi^+\pi^-$ mode, except that the three-body decay phase space factor is replaced by the two-body decay phase space factor.

We fit the $\phi f_0(980)$ cross section distribution with a single BW function that interferes with a nonresonant component. Figure 5 shows the result. There are two solutions. The interference is constructive for one solution (dashed curves) and destructive for the other (dot-dashed curves). The values of the fitted results for these two solutions are shown in Table III, with a goodness-of-the-fit of $\chi^2/ndf = 102/80$.

We also fit the $\phi f_0(980)$ cross section distribution with two coherent BW functions (one for the $Y(2175)$ and the other for the structure at about 2.4 GeV), which interfere with a nonresonant component. There are four solutions. Figures 6(a) and (b) show the result. The values of the fitted results for these four solutions are shown in Table IV, with a goodness-of-the-fit of $\chi^2/ndf = 88/76$. The fitted values of the mass and width of the $Y(2175)$ are consistent with the results obtained with a single BW function interfering with a nonresonant component within 1σ , and the statistical significance of the structure at $2.4 \text{ GeV}/c^2$ is estimated to be 2.7σ .

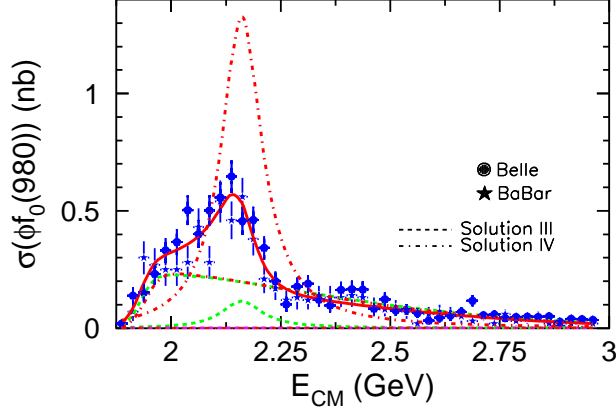


FIG. 5: Fit to $e^+e^- \rightarrow \phi f_0(980)$ cross section with a single BW function that interferes with a nonresonant component. The curves show the projections from the best fit and the contribution from each component. The dashed curves are for constructive solution, and the dot-dashed curves for destructive solution.

TABLE III: Fit results to the combined BaBar and Belle data on $e^+e^- \rightarrow \phi f_0(980)$ with a single BW function that interferes with a nonresonant component. The errors are statistical only. M , Γ_{tot} , and $\mathcal{B}(\phi f_0(980)) \times \Gamma_{e^+e^-}$ are the mass (in MeV/c^2), total width (in MeV/c^2), and product of the branching fraction to $\phi f_0(980)$ and the e^+e^- partial width (in eV/c^2), respectively. ϕ is the relative phase (in degree).

Parameters	Solution I	Solution II
$M(Y(2175))$	2159 ± 11	
$\Gamma_{\text{tot}}(Y(2175))$	113 ± 19	
$\mathcal{B}(\phi f_0(980)) \times \Gamma_{e^+e^-}(Y(2175))$	4.1 ± 1.1	47.6 ± 1.5
ϕ	130 ± 12	-106 ± 2

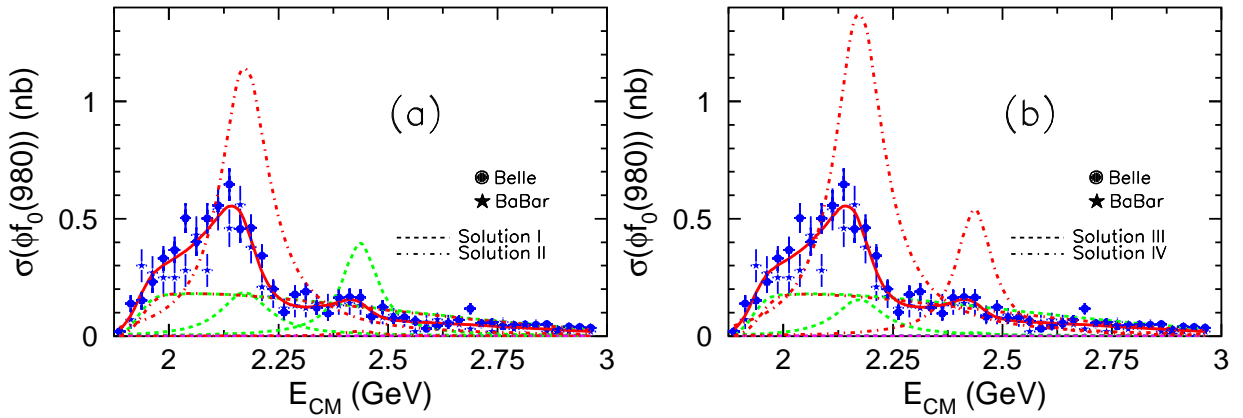


FIG. 6: Fit to $e^+e^- \rightarrow \phi f_0(980)$ cross sections with two coherent BW functions that interfere with a nonresonant component. The curves show the projections from the best fit and the contribution from each component. There are four solutions.

TABLE IV: Fit results to the combined BaBar and Belle data on $e^+e^- \rightarrow \phi f_0(980)$ with two coherent BW functions that interfere with a nonresonant component. The errors are statistical only. M , Γ_{tot} , and $\mathcal{B}(\phi f_0(980)) \times \Gamma_{e^+e^-}$ are the mass (in MeV/c^2), total width (in MeV/c^2), and product of the branching fraction to $\phi f_0(980)$ and the e^+e^- partial width (in eV/c^2), respectively. ϕ is the relative phase (in degree).

Parameters	Solution I	Solution II	Solution III	Solution IV
$M(Y(2175))$			2171 ± 16	
$\Gamma_{\text{tot}}(Y(2175))$			134 ± 22	
$\mathcal{B}(\phi f_0(980)) \times \Gamma_{e^+e^-}(Y(2175))$	8.0 ± 3.3	49.6 ± 2.0	6.7 ± 1.3	59.6 ± 1.9
ϕ_1	172 ± 38	-116 ± 3	153 ± 7	-97 ± 2
$M(X(2400))$			2436 ± 34	
$\Gamma_{\text{tot}}(X(2400))$			99 ± 105	
$\mathcal{B}(\phi f_0(980)) \times \Gamma_{e^+e^-}(X(2400))$	15.8 ± 12.8	0.8 ± 0.4	0.6 ± 0.3	21.5 ± 1.3
ϕ_2	-92 ± 22	91 ± 21	151 ± 17	-150 ± 3

V. SYSTEMATIC ERRORS

The sources of the systematic errors on the resonance parameters measurements are not very different from those listed by the Belle experiment [3]. For the masses and widths of the $\phi(1680)$ and $Y(2175)$ resonances, we have considered the uncertainties in the absolute mass scale, the mass resolution, the background shape, fit range and possible existence of additional resonances as systematic errors. For the products of the branching fractions to $\phi\pi^+\pi^-$ or $\phi f_0(980)$ and the e^+e^- partial widths of the $\phi(1680)$ and $Y(2175)$ resonances, we have considered the uncertainties in the background shape, fit range and cross sections measurements as systematic errors. The sources of the uncertainties in the cross sections measurements from Belle and BaBar are independent, and the total systematic errors on the cross sections measurements are 5.4% and 4.9% for $\phi\pi^+\pi^-$ and $\phi f_0(980)$ modes, respectively.

Finally, the results of the resonance parameters of the $\phi(1680)$ and $Y(2175)$ are listed in Table V, where the first errors are statistical and the second systematic. The fitted results with an additional BW for the structure at around 2.4 GeV/c^2 are not included since the statistical significance is smaller than 3.0σ .

VI. RESULTS AND DISCUSSION

In summary, we present a fit to the cross sections for $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $e^+e^- \rightarrow \phi f_0(980)$ from threshold to $\sqrt{s} = 3.0$ GeV measured by BaBar and Belle experiments. The masses, widths and the products of the branching fraction and the e^+e^- partial width of the $\phi(1680)$ and $Y(2175)$ are determined, as listed in Table V. From the table, we could see that the differences in the $\phi(1680)$ resonance parameters between two-incoherent- BW fit and two-coherent- BW fit to $\phi\pi^+\pi^-$ are not large. We take a simple average as the central value and enlarge the errors to cover all the possibilities. However, the differences in the $Y(2175)$ resonance parameters from different fit are very large due to the assump-

TABLE V: The results of the resonance parameters of the $\phi(1680)$ and $Y(2175)$. The first errors are statistical and the second systematic. M , Γ_{tot} , and $\mathcal{B} \times \Gamma_{e^+e^-}$ are the mass (in MeV/c^2), total width (in MeV/c^2), and product of the corresponding branching fraction and the e^+e^- partial width (in eV/c^2). There are two solutions in fitting with coherent sum of the amplitudes.

State	Mode	Fit Method	Mass	Γ_{tot}	$\mathcal{B} \times \Gamma_{e^+e^-}$
$\phi(1680)$	$\phi\pi^+\pi^-$	incoherent	$1685 \pm 5 \pm 3$	$208 \pm 11 \pm 3$	$24.6 \pm 1.2 \pm 1.3$
	$\phi\pi^+\pi^-$	coherent	$1677 \pm 6 \pm 5$	$233 \pm 16 \pm 15$	$33.4 \pm 1.3 \pm 1.8$ $23.0 \pm 1.5 \pm 1.3$
$Y(2175)$	$\phi\pi^+\pi^-$	incoherent	$2080 \pm 12 \pm 3$	$182 \pm 20 \pm 10$	$18.1 \pm 1.8 \pm 0.9$
	$\phi\pi^+\pi^-$	coherent	$2112 \pm 16 \pm 22$	$196 \pm 27 \pm 25$	$68.9 \pm 7.0 \pm 3.4$ $6.2 \pm 1.1 \pm 0.3$
	$\phi f_0(980)$	coherent	$2159 \pm 11 \pm 13$	$113 \pm 19 \pm 22$	$47.6 \pm 1.5 \pm 2.3$ $4.1 \pm 1.1 \pm 0.4$

tion on the background shape and multi-solution problem, especially for the value of the product of the branching fraction and the e^+e^- partial width, we only give $Y(2175)$ mass and width here. Finally, we obtain $M(\phi(1680)) = (1681_{-12}^{+10}) \text{ MeV}/c^2$, $\Gamma_{\text{tot}}(\phi(1680)) = (221_{-24}^{+34}) \text{ MeV}/c^2$ and $\mathcal{B}(\phi(1680) \rightarrow \phi\pi^+\pi^-) \times \Gamma_{e^+e^-}(\phi(1680)) = 27.0_{-5.9}^{+8.6} \text{ eV}/c^2$, and $M(Y(2175)) = (2117_{-49}^{+59}) \text{ MeV}/c^2$, $\Gamma_{\text{tot}}(Y(2175)) = (164_{-80}^{+69}) \text{ MeV}/c^2$. Here the uncertainties include both statistical and systematic errors.

We find that the central value of the $Y(2175)$ width is larger than the BaBar measurement. However it is consistent with the predicted values with the assumptions that the $Y(2175)$ is an $s\bar{s}g$ hybrid [6] or a $2^3D_1 s\bar{s}$ state [7]. The widths of the $\phi(1680)$ and $Y(2175)$ are quite similar and both are at the $200 \text{ MeV}/c^2$ level. This may suggest that the $Y(2175)$ be an excited ϕ state. Although there is faint evidence of the structure at around $2.4 \text{ GeV}/c^2$ in $e^+e^- \rightarrow \phi\pi^+\pi^-$ and $\phi f_0(980)$ in both BaBar and Belle data, the statistical significance is smaller than 3.0σ . Larger data sample is necessary to confirm it.

Acknowledgments

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