

# Pair Production Constraints on Superluminal Neutrinos Revisited

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## Abstract

We revisit the pair creation constraint on superluminal neutrinos considered by Cohen and Glashow in order to clarify which types of superluminal models are constrained. We show that a model in which the superluminal neutrino is effectively light-like can evade the Cohen-Glashow constraint.

Cohen and Glashow (CG) have shown that the vacuum lepton pair production process ( $\nu \rightarrow \nu e^+ e^-$ ) acts as an efficient energy loss mechanism for propagating superluminal neutrinos [1]; the observation of neutrinos in the OPERA experiment thus appears to exclude the superluminal interpretation of their velocities [2]. However, as we shall show, the requirement of energy-momentum conservation does not limit the possibility that the superluminal neutrino is effectively light-like.

The group velocity  $v_G$  of a particle with energy  $E$  and momentum  $\mathbf{p}$  is given by  $\mathbf{v}_G = dE/d\mathbf{p}$ . Assuming a dispersion relation of form  $E = \sqrt{|\mathbf{p}|^2 c'^2 + m'^2 c'^4}$  [3, 4], where  $m' c'^2 = m c^2$  and  $m$  is the particle mass, yields  $\mathbf{v}_G/c = (|\mathbf{p}|c/E)(c'/c)^2$ . Thus a particle can become superluminal, namely, it can attain  $v_G > c$ , where  $c$  is the measured speed of light in a terrestrial vacuum [5], in two different ways:

1. if  $c'/c > 1$  with  $|\mathbf{p}|c/E \lesssim 1$  or
2. if  $|\mathbf{p}|c > E$  and  $c' = c$ .

Forming  $p = (E, c\mathbf{p})$ , so that  $p^2 = E^2 - c^2|\mathbf{p}|^2$ , then if  $v_G > c$  the superluminal particle is timelike in the former case and spacelike in the latter case. Searches for particles obeying the latter possibility — “tachyons” [6] — have been conducted previously without success [7].

The observation of neutrino oscillations sharply constrains superluminal neutrinos [1], and severe limits exist on the velocity difference of neutrinos of different species [1, 3, 4, 8]. Thus superluminality, if it exists, must pertain to all three neutrino mass eigenstates. CG assume a superluminal neutrino by choosing  $c' > c$ ; thus all neutrino mass eigenstates are placed on the same footing.

If one assumes that the neutrino mass matrix is Hermitian [3, 4], then the possibility of a spacelike superluminal neutrino [6] is excluded from the onset. If one relaxes the Hermiticity assumption, one runs afoul of the observation of neutrinos propagating from distant sources. Moreover, a spacelike superluminal neutrino necessarily becomes *more* superluminal as its energy decreases, although OPERA observes no significant energy dependence in comparing the speed of neutrinos with an average energy of 13.9 and 42.9 GeV [2]. Thus one can safely eliminate spacelike superluminal neutrinos as a possibility.

The mass scale of neutrinos is sufficiently well-known [9] to establish that any effect from nonzero neutrino masses at OPERA energies is negligible [2]; in fact, since  $E \gg m' c'^2$  with  $c' > c$ , then  $E \simeq c'|\mathbf{p}|$ . Since CG define  $p = (E, c\mathbf{p})$ , then  $p^2 = (c'^2 - c^2)|\mathbf{p}|^2 > 0$  for finite  $|\mathbf{p}|$ , and their superluminal neutrinos are timelike. Alternatively, one can define  $p = (E, c'\mathbf{p})$ , so that the superluminal neutrino is effectively lightlike at high energies,  $p^2 = m'^2 c'^4 \simeq 0$  [10]. Note that  $p^2$  in this latter case remains a Lorentz scalar. Indeed  $c$  does not appear explicitly in  $p$ , and Lorentz symmetry is not manifestly broken —  $c'$  can be taken to be the causal velocity. Nevertheless, in this scenario, the neutrino can *appear* superluminal compared to the propagation of photons; this is because light can suffer interactions in matter which the neutrino does not — this is already true in the Standard Model (SM).

The vacuum Cherenkov-like process which CG find of greatest importance is  $\nu(p_1) \rightarrow \nu(p_2) + e^+(l'_e) + e^-(l_e)$ . Energy-momentum conservation implies  $p_1^2 = p_2^2 + 2p_2 \cdot (l_e + l'_e) + (l_e + l'_e)^2$ , where  $(l_e + l'_e)^2 > 0$  and  $p_2 \cdot (l_e + l'_e) \gtrsim 0$ . Thus this process cannot occur for effectively lightlike models in which the four momentum squared does not change  $p_1^2 = p_2^2$ , such as in Ref. [10]. In contrast, a superluminal neutrino as considered by CG has a value of  $p^2$  which depends on energy, so that  $p_1^2 - p_2^2 > 0$  can occur, and the process is indeed allowed. For such neutrinos vacuum Cherenkov processes act as a new source of neutrino decay.

Pair production will of course occur in matter even if vacuum pair production is forbidden — the transfer of 3-momentum to a target or medium is needed to realize energy-momentum conservation, as has been previously studied in the context of pair production in the Coulomb field of a nucleus of charge  $Ze$ , e.g.,  $\mu^+ + Z \rightarrow \mu^+ + e^- + e^+ + Z$  in quantum electrodynamics [11, 12]. In the model of Ref. [10] pair production associated with a propagating superluminal neutrino mimics that of SM neutrinos in ordinary matter — it shares all the same diagrams, except that  $c$  is replaced by  $c'$  throughout. Under this global replacement,  $\pi$  decay and cosmic ray constraints also do not operate [13, 14]: Lorentz symmetry is not broken at the level of particle interactions. Pair bremsstrahlung is much less effective in matter; Standard Model neutrinos do shed such radiation, but Earth-crossing neutrinos are both expected and observed [15].

In the model of Ref. [10]  $c' > c$  is realized through photon interactions with a dark sector which make  $c$  smaller than the causal velocity  $c'$ , so that additional diagrams do appear in that case vis-a-vis final-state interactions of the charged leptons with the dark sector; these are, however, small modifications as such couplings must be much weaker than the usual electromagnetic coupling  $e$ . These calculations do not reference the value of  $c$  per se. The propagation speed of light — or of electrons or of other particles with suitable generalization — in a medium, is fixed by the real part of the forward Compton amplitude in that medium: a different calculation altogether. Thus the possibility of lightlike superluminal neutrinos is not excluded by the observation of neutrinos at OPERA energies and beyond.

An additional constraint on neutrino superluminality comes from the observation of neutrinos from SN 1987A: the close detection in time of the neutrino and optical bursts yields a neutrino-light velocity comparison test, namely, of  $v_\nu - v_\gamma < 2 \times 10^{-9}$  [16]. This constraint does assume that the neutrinos and photons were emitted at the same time. Since supernova neutrinos are of  $\mathcal{O}(10 \text{ MeV})$  in energy, reconciling this result with OPERA evidently requires an energy-dependent neutrino dispersion relation [8, 17–20]. Indeed, the generality of the CG constraint in eliminating superluminal neutrino models rests on the inclusion of the SN 1987A constraint as well [21, 22]. Note, however, that the assumption of contemporary emission of the neutrino and optical bursts is not firmly established; it relies on an understanding of the SN mechanism — which we do not possess. Realistic studies of neutrino transport reveal that considerations such as convection, rotation, and magnetic fields first investigated in the 1990's are in some measure necessary to realize the explosion of a star [23], and the implementation of this program is ongoing [24]. Moreover, observations of SN 1987A show it to be an outlier in many respects [25]. The neutrinos need not come from the initial core collapse; they can be emitted at a later stage from the edge of an accretion disk around a forming black-hole [26–28]. If the neutrinos were not emitted contemporaneously with the light, there is a coincidence in their close temporal detection — thus in order to explain OPERA, one requires fine-tuning at order  $\mathcal{O}(8 \times 10^{-5})$ , which is certainly very improbable but not impossible.

In summary, any model for which the CG pair production process operates is excluded because such timelike neutrinos would not be detected by OPERA or other experiments. However, a superluminal neutrino which is effectively lightlike with fixed  $p^2$  [10] can evade the Cohen-Glashow constraint because of energy-momentum conservation. The coincidence involved in explaining the SN1987A constraint certainly makes such a picture improbable — but it is still intrinsically *possible*. The lightlike model is appealing in that it does not violate Lorentz symmetry in particle interactions, although one would expect Hughes-Drever tests to turn up a violation eventually [10]. Other evasions of the CG constraints are also

possible; perhaps, e.g., the neutrino takes a “short cut” through extra dimensions [29] or suffers anomalous acceleration in matter [30].

Irrespective of the OPERA result, Lorentz-violating interactions [31] remain possible, and ongoing experimental investigation of such possibilities should continue.

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